COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Pion-nucleon partial-wave analysis to 2 GeV"

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At incident pion energies below 150 MeV, values for the pion charge exchange cross section, derived from the pion-nucleon partial-wave analysis by Arndt *et al.*, are slightly in error because a factor of $p_{\mathrm{c.m.x}}/p_{\mathrm{c.m.}}$ has been neglected, where $p_{\mathrm{c.m.x}}$ and $p_{\mathrm{c.m.}}$ are the momenta of the scattered neutral and charged pions in the center-of-mass frame, respectively. In addition, an oversight in the center-of-mass to laboratory kinematic transformations has been discovered in the Scattering Analysis Interactive Dial-in (SAID) system, a system through which the aforementioned pion-nucleon partial-wave analysis may be accessed. This oversight, in which the mass differences between charged and neutral pions and between protons and neutrons have been neglected, introduces systematic energy and angular dependent errors to the laboratory differential cross section for pion charge exchange below 150 MeV.

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This Comment points out two small oversights in the pion-nucleon phase shift analysis by Arndt *et al.*, which is presented in Refs. [1] and [2]. First, a factor of $p_{\mathrm{c.m.x}}/p_{\mathrm{c.m.}}$ has been omitted from the pion-nucleon charge exchange cross section, where $p_{\mathrm{c.m.x}}$ and $p_{\mathrm{c.m.}}$ are the momenta of the scattered neutral and charged pions in the center-of-mass frame, respectively. Second, the mass differences between charged and neutral pions and between protons and neutrons have been neglected in the laboratory to center-of-mass kinematic transformations of the scattering snalysis interaction dial-in (SAID) system, which is discussed in Ref. [1]. These oversights introduce significant errors in the pion-nucleon charge exchange, $\pi^- p \rightarrow \pi^0 n$, cross section at incident pion energies below 150 MeV.

Although the authors have included a factor of $(p_{\mathrm{c.m.x}} p_{\mathrm{c.m.}})^{-1/2}$ in the scattering amplitude [3] resulting from the change between energy and momentum states, they have neglected the factor $p_{\mathrm{c.m.x}}/p_{\mathrm{c.m.}}$ resulting from the density of final states [4]. Hence, the differential cross section $d\sigma/d\Omega_{\mathrm{c.m.}}$, which is proportional to $p_{\mathrm{c.m.x}}/p_{\mathrm{c.m.}}$, contains a small error that becomes significant below incident pion energies of 150 MeV. However, this effect may be partially compensated by isospin breaking in the phase shifts themselves. A plot of this correction is shown in Fig. 1. It is expected that isospin-breaking effects play an important role at low energies [5]. In fact, the pion charge exchange databases, used in the partial-wave analyses [1,2], were quite limited below 150 MeV, and a single nuclear amplitude was deemed sufficient for fitting all pion-nucleon channels [1], thus ignoring isospin-breaking effects.

Several groups [6–10] have utilized the scattering analysis interactive dial-in (SAID) system to access values of pion charge exchange cross sections in order to calibrate the π^0 detection efficiency of their instrumentation. SAID computes cross sections and kinematics for the pion-nucleon interaction between 0 and 2.1 GeV and is based on recent phase shift analyses [1,2]. For a given incident pion energy and center-of-mass scattering angle, the program returns the center-of-mass cross section as

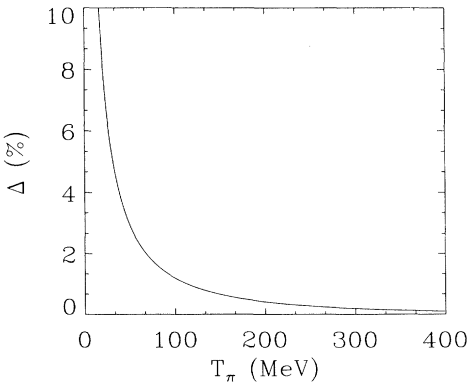

FIG. 1. Correction Δ to the cross section and scattering amplitudes for pion charge exchange as a function of incident pion energy, where $\Delta = (p_{\mathrm{c.m.x}}/p_{\mathrm{c.m.}}) - 1$.

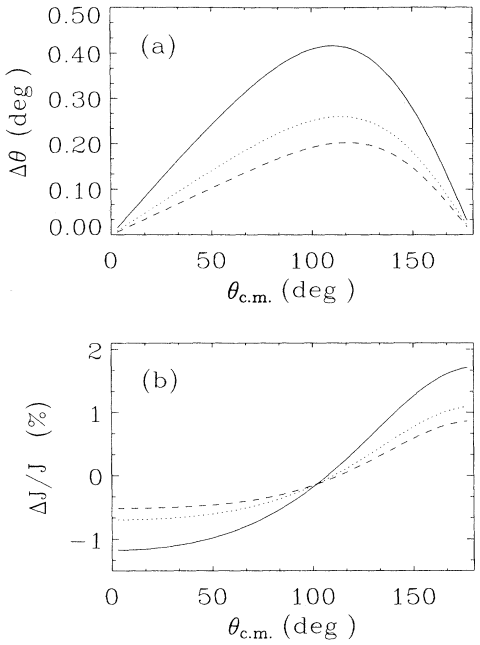

FIG. 2. Corrections (a) $\Delta\theta$ and (b) $\Delta J/J$ as a function of center-of-mass scattering angle $\theta_{\text{c.m.}}$. The solid, dotted, and dashed lines correspond to incident pion energies T_π of 50, 100, and 150 MeV, respectively. The laboratory scattering angle correction is defined as $\Delta\theta = \theta_{LX} - \theta_L$, where θ_{LX} and θ_L are the laboratory scattering angles for the neutral and charged pion, respectively. The correction to the Jacobian is defined as $\Delta J/J = \frac{d\Omega_{\text{c.m.x}}/d\Omega_{LX}}{d\Omega_{\text{c.m.}}/d\Omega_L} - 1$, where $d\Omega_{\text{c.m.x}}/d\Omega_{LX}$ and $d\Omega_{\text{c.m.}}/d\Omega_L$ are the Jacobians for charge exchange and elastic scattering, respectively.

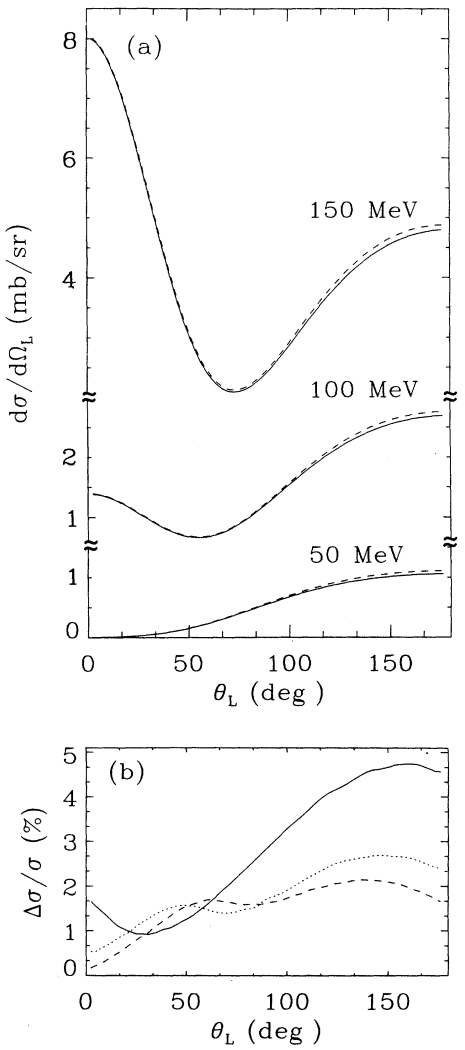

FIG. 3. Laboratory (a) differential cross sections and (b) corrections $\Delta\sigma/\sigma$ for pion charge exchange at T_π = 50, 100, and 150 MeV. The dashed and solid lines in (a) correspond to SAID cross sections with and without kinematic corrections, respectively. The relative magnitudes of these corrections are shown in (b); the solid, dotted, and dashed lines correspond to incident pion energies, T_π, of 50, 100, and 150 MeV, respectively. The individual kinematic contributions are shown in Figs. 1 and 2.

well as the laboratory cross section and scattering angle. However, an oversight in the center of mass to laboratory kinematic transformations for pion charge exchange, $\pi^- p \rightarrow \pi^0 n$, in the SAID system [1] exists in which the authors have neglected the mass difference between the charged and neutral pions and between the protons and neutrons. Therefore, the laboratory differential cross sections $d\sigma/d\Omega_L$ computed by SAID for charge exchange are incorrect. The corrections to θ_L and $d\Omega_{\text{c.m.}}/d\Omega_L$ are shown in Fig. 2. Neglecting these mass differences introduces negligible error in the 200 MeV to 2.1 GeV range and is consistent in an isospin-conserving treatment; however, there is a noticeable effect on the cross section in the neighborhood of 150 MeV and below.

Although the contribution of each of these errors, incorrect θ_L, $d\Omega_{\text{c.m.}}/d\Omega_L$, and omission of $p_{\text{c.m.x}}/p_{\text{c.m.}}$, is small, their combined effect can become significant at incident pion energies near 100 MeV as shown in Fig. 3. Measurements of the $^1\text{H}(\pi^-, \pi^0)n$ reaction at and below 150 MeV, normalized to SAID charge exchange cross cross sections, are slightly in error. Measurements of the $^1\text{H}(\pi^-, \pi^0)n$ reaction have been used previously to calibrate the π^0 detection efficiencies, of the Clinton P. Anderson Meson Physics Facility (LAMPF) π^0 spectrometer [6–9] and Neutral Meson Spectrometer (NMS) [10]. The physics community should be aware of these errata particularly as improved low energy pion-nucleon charge exchange measurements become available [11,12].

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