Method of constraining the CP-violating phase γ from charged B meson decays

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Gronau, Rosner, and London have presented a method of determining the CP-violating phase γ of the CKM matrix from the decays $B^+ \to \pi^+ K^0$ and $B^\pm \to K^\pm \pi^0$. We show that this method could provide a powerful constraint on γ even if only a weak upper limit on the CP violation is obtained. We also point out a consequence of the assumptions they make.

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In a recent paper Gronau, Rosner, and London [1] presented a method for determining the CP-violating Cabibbo-Kobayashi-Maskawa (CKM) phase γ from rate measurements on $B^\pm$ decays. Here we point out the surprising power of this method in certain cases. We also point out that their assumptions yield a relation between strong phases.

The decays of interest are $B^+ \to K^+\pi^0$ and $B^+ \to K^0\pi^+$. These decays are believed to have significant contributions from penguin diagrams [2] as well as doubly Cabibbo-suppressed tree diagrams. The transition amplitude to the final $I = \frac{3}{2}$ state comes only from the tree diagram since the penguin diagram has $\Delta I = 0$; we write this as

$$\sqrt{\tfrac{1}{3}}A(K^0\pi^+) + \sqrt{\tfrac{2}{3}}A(K^+\pi^0) = \sqrt{\tfrac{2}{3}}Ae^{+i\gamma}e^{i\delta_3} . \quad (1)$$

The tree diagram is proportional to $V_{ub}^* V_{us}$ and so has the weak phase factor $e^{i\gamma}$. Gronau, Rosner, and London show that using SU(3) the magnitude of A can be approximated from the rate of $B^\pm \to \pi^\pm\pi^0$.

Because the tree diagram contains no d or $\bar{d}$ Gronau, Rosner, and London argue that the decay $B^+ \to K^0\pi^+$ is a pure penguin diagram which we write as

$$A(K^0\pi^+) = \sqrt{2}Be^{i\delta_P} . \quad (2)$$

Since the penguin diagram is proportional to $V_{tb}^* V_{ts}$ it contains no weak phase factor. Combining Eqs. (1) and (2),

$$A(K^+\pi^0) = Ae^{i\gamma}e^{i\delta_3} + Be^{i\delta_P} . \quad (3)$$

The amplitude for $B^- \to K^-\pi^0$ is obtained by changing γ to $-\gamma$.

We define

$$S = [\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^- \to K^-\pi^0)]/2 ,$$
$$D = [\Gamma(B^+ \to K^+\pi^0) - \Gamma(B^- \to K^-\pi^0)]/2 .$$

Then, from Eq. (3),

$$\cos\delta\,\cos\gamma = (S - A^2 - B^2)/2AB , \quad (4a)$$
$$\sin\delta\,\sin\gamma = D/2AB , \quad (4b)$$

where $\delta = \delta_3 - \delta_p$.

Since A is assumed to be determined via SU(3) from the $\pi^\pm\pi^0$ rate and B from the $K^0\pi^+$ rate (which here is equal to the $\bar{K}^0\pi^-$ rate) the measurement of the $K^+\pi^0$ and $K^-\pi^0$ rates allows the determination of γ and δ from Eqs. (4) up to a twofold ambiguity.

The measurement of a nonzero value for the asymmetry D/S would be a sign of direct CP violation and of great importance. The method discussed above allows the quantitative determination of γ from the asymmetry. Detailed discussions of this asymmetry have been given by Gerard and Hou [3] and Simma *et al.* [4]. Their results suggest that the asymmetry is only a few percent and so will be undetectable for the foreseeable future.

Nevertheless it is possible that a significant constraint on γ may be achieved by this method without detecting any asymmetry. This is best illustrated by a specific example. Let us suppose that γ is close to 90° and δ is close to 0°. In this case the results of the measurements discussed above will be limits on $\cos\delta\,\cos\gamma$ and $\sin\delta\,\sin\gamma$. Assume one finds

$$\sin\delta\,\sin\gamma < L ,$$
$$\cos\delta\,\cos\gamma < L . \quad (5)$$

Then the constraint on γ is

$$\sin^2\gamma < \tfrac{1}{2}(1 - \sqrt{1 - 4L^2})$$

or

$$\sin^2\gamma > \tfrac{1}{2}(1 + \sqrt{1 - 4L^2}) .$$

Assuming the fairly weak limit of $L = 0.4$, this yields

$$|\sin\gamma| < 0.45 \ \text{ or } |\sin\gamma| > 0.9 . \quad (6)$$

If we assume, for example, that the value of V_{ub} gives $(\rho^2 + \eta^2)^{1/2} = 0.4$, Eq. (6) rules out values η between 0.18 and 0.36. Since fitting ϵ requires η greater than about 0.14 and the assumed V_{ub} says $\eta < 0.4$, the very weak results of Eq. (5) provide extremely strong constraints on η.

The example given is a very reasonable case but was selected to demonstrate the power of this method. If γ were close to 50° instead of 90° the method would be much less restrictive with the weak statistical accuracy

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we have assumed above. Also if L is raised from 0.4 to 0.5 the constraint on γ is lost.

To see the implications for the strong phases that follow from the assumptions used in this method we write the *tree* contribution going to the final $I = \frac{1}{2}$ state

$$\left(\frac{2}{3}\right)^{1/2} A_T(K^0\pi^+) - \left(\frac{1}{3}\right)^{1/2} A_T(K^+\pi^0) = \left(\frac{1}{3}\right)^{1/2} Ce^{i\gamma}e^{i\delta_1} . \quad (7)$$

Considering only the tree contribution on the left-hand side (LHS) of Eq. (1) and combining this with Eq. (7),

$$\left(\frac{1}{3}\right)^{1/2} A_T(K^0\pi^+) + \frac{2}{\sqrt{3}}A_T(K^0\pi^+) = \left(\frac{2}{3}\right)^{1/2} e^{i\gamma}(Ae^{i\delta_3} + Ce^{i\delta_1}) . \quad (8)$$

However, the assumption made in Eq. (2) was that $A_T(K^0\pi^+)$ vanishes requiring

$$Ae^{i\delta_3} = -Ce^{i\delta_1} . \quad (9)$$

This can only hold if $\delta_3 = \delta_1$.

In practice we have found no way to calculate these strong phases. In previous papers [3,4] the main strong phase included came from the absorptive part of the penguin diagram corresponding to a virtual $c\bar{c}$ state following Bander, Silverman, and Soni [5]. While this may be reasonable in looking at the inclusive process $b \to s + u + \bar{u}$ it makes little sense for the exclusive [6]. Here a major final-state interaction is the scattering from $K + \pi$ to $K + n\pi$. Indeed for the final state of $K + \pi$ in an s state one expects the major strong interaction to be such inelastic scattering. Unfortunately we do not know how to calculate δ for this nor how to show that δ_3 does not differ from δ_1.

A weaker assumption than Eq. (9) would be to assume $A = -C$ but to allow a difference between δ_3 and δ_1. This is somewhat arbitrary since the final-state interactions also affect the magnitudes. If we do assume $A = -C$ and expand to first order in $(\delta_3 - \delta_1)$ we obtain

$$\cos\delta \, \cos\gamma = (S - S^2 - B^2)/2AB + \frac{1}{3}\sin\frac{\Delta}{2}\cos\gamma \, \sin\delta , \quad (10a)$$

$$\sin\delta \, \sin\gamma = D/2AB + \frac{1}{3}\sin\frac{\Delta}{2}\sin\gamma \, \cos\delta , \quad (10b)$$

where

$$\Delta = \delta_3 - \delta_1$$

and δ is now understood as $\frac{1}{2}(\delta_3 + \delta_1) - \delta_p$. For the example considered above the correction is relatively unimportant. Allowing $\Delta = 30°$ would only lower the limit of 0.9 in Eq. (6) by 3%.

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