Reexamination of the perturbative pion form factor with Sudakov suppression

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The perturbative pion form factor with Sudakov suppression is reexamined. To guarantee the reliability of the perturbative calculations we suggest that the running coupling constant should be frozen at $\alpha_s(t = \langle k_T\rangle)$ and $\langle k_T\rangle$ is the average transverse momentum which can be determined by the pionic wave function. In addition we correct the previous calculations about the Sudakov suppression factor which plays an important role in the perturbative calculations for the pion form factor.

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It is believed that perturbative QCD (pQCD) can successfully describe exclusive processes at asymptotically large momentum transfers [1]. However, the applicability of pQCD to the electromagnetic pion form factor at the present energy is a matter of controversy [2]. Recent studies [3–5] on the pion electromagnetic form factor have shown that the pQCD contributions become self-consistent for momentum transfers in the range of a few GeV. Li and Sterman [4] give a modified expression for the pion form factor by taking into account the customarily neglected partonic transverse momenta as well as the Sudakov corrections. Jakob and Kroll [5] point out that the dependence of the hadronic wave function on the intrinsic transverse momentum should be considered in the perturbative calculation. Sudakov corrections come from an infinite summation of higher-order effects associated with the elastic scattering of the valence partons. However, because the running coupling constant α_s becomes rather large with b (the distance between a quark-antiquark pair) increasing in the end-point regions, a cutoff on α_s has to be made to evaluate perturbative contributions and to justify the self-consistency of perturbative calculations. In this paper, we will reexamine the perturbative pion form factor with the Sudakov suppression. It is pointed out that $\alpha_s(t)$ should be frozen as t is smaller than a certain value because of the multigluon exchange at low Q^2. We suggest that the frozen point is related to the root-mean-square (rms) transverse momentum, which is determined by the pionic wave function. In addition, we correct the previous calculations about the Sudakov suppression factor, which plays an important role in the perturbative calculations for the pion form factor.

Let us begin with a brief review on the derivation of the expression for the pion form factor in Ref. [4]. Taking into account the transverse momenta $\mathbf{k}_T$ that flow from the wave functions through the hard scattering leads to a factorization form with two wave functions $\psi(x_i, \mathbf{k}_{T_i})$ corresponding to the external pions, combined with a new hard-scattering function $T_H(x_1, x_2, Q, \mathbf{k}_{T_1}, \mathbf{k}_{T_2})$, which depends in general on transverse as well as longitudinal momenta:

$$F_\pi(Q^2) = \int_0^1 dx_1 dx_2 \int d^2\mathbf{k}_{T_1} d^2\mathbf{k}_{T_2} \psi(x_1, \mathbf{k}_{T_1}, P_1) \times T_H(x_1, x_2, Q, \mathbf{k}_{T_1}, \mathbf{k}_{T_2}, \mu)\psi(x_2, \mathbf{k}_{T_2}, P_2), \quad (1)$$

where $Q^2 = 2P_1 \cdot P_2$, and μ is the renormalization and factorization scale.

The hard-scattering amplitude T_H is to be calculated from the one-gluon-exchange diagrams to the lowest order in the perturbation theory. Neglecting the transverse momentum dependence in the fermion propagator, T_H is given by

$$T_H(x_1, x_2, Q, \mathbf{k}_{T_1}, \mathbf{k}_{T_2}) = \frac{16\pi C_F \alpha_s(\mu)}{x_1 x_2 Q^2 + (\mathbf{k}_{T_1} + \mathbf{k}_{T_2})^2}, \quad (2)$$

where C_F is the color factor. The k_T dependence on the fermion propagator contributes to T_H a factor $\frac{x_1 Q^2}{x_1 Q^2 + \mathbf{k}_{T_1}^2}$, which involves only a single transverse momentum corresponding to the one in the external pion, and this factor causes the Fourier transformation for T_H to involve multiple-b integrals. This factor leads to a reduction of the prediction for F_π by about 10% [6]. We may neglect this factor from the mechanisms that we are discussing in this paper.

Through Fourier transformation Eq. (1) can be expressed as

$$F_\pi(Q^2) = \int_0^1 dx_1 dx_2 \frac{d\mathbf{b}_1}{(2\pi)^2} \frac{d\mathbf{b}_2}{(2\pi)^2} \varphi(x_1, \mathbf{b}_1, P_1, \mu) \times T_H(x_1, x_2, Q, \mathbf{b}_1, \mathbf{b}_2, \mu)\varphi(x_2, \mathbf{b}_2, P_2, \mu). \quad (3)$$

In this expression, wave function $\varphi(x_i, \mathbf{b}_i, P_i, \mu)$ takes into account an infinite summation of higher-order effects associated with the elastic scattering of the valence partons, which gives out the Sudakov suppression to the large-b and small-x regions [7]:

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$$\varphi(x,b,P,\mu)=\exp\left[-s(x,b,Q)-s(1-x,b,Q)-2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(g(\bar{\mu}))\right]\phi\left(x,\frac{1}{b}\right), \tag{4}$$

where $\gamma_q=-\alpha_s/\pi$ is the quark anomalous dimension in the axial gauge. $s(\xi,b,Q)$ is the Sudakov exponent factor:

$$\begin{aligned} s(\xi,b,Q)=&\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{-\hat{b}}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{-\hat{b}}-1\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}+\hat{b})-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(-2\hat{b})+1}{-\hat{b}}-\frac{\ln(-2\hat{q})+1}{-\hat{q}}\right]\\ &-\left(\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln(\frac{1}{2}e^{2\gamma-1})\right)\ln\left(\frac{\hat{q}}{-\hat{b}}\right)+\frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q})-\ln^2(-2\hat{b})\right], \end{aligned} \tag{5}$$

where

$$\begin{gathered} \hat{q}=\ln[\xi Q/(\sqrt{2}\Lambda)],\quad \hat{b}=\ln(b\lambda),\\ \beta_1=\frac{33-2n_f}{12},\quad \beta_2=\frac{153-19n_f}{24},\\ A^{(1)}=\frac{4}{3},\quad A^{(2)}=\frac{67}{9}-\frac{1}{3}\pi^2-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln(\frac{1}{2}e^{\gamma}). \end{gathered} \tag{6}$$

n_f is the number of quark flavors, and γ is the Euler constant.

It should be noted that there are some mistakes in the coefficients of the fourth and the sixth terms in $s(\xi,b,Q)$ given by Refs. [4,6]. We find that the correct coefficients should be $-\frac{A^{(1)}\beta_2}{4\beta_1^3}$ and $+\frac{A^{(1)}\beta_2}{8\beta_1^3}$, in place of $-\frac{A^{(1)}\beta_2}{16\beta_1^3}$ and $-\frac{A^{(1)}\beta_2}{32\beta_1^3}$ in Refs. [4,6]. It is $s(\xi,b,Q)$ that plays an important role in the perturbative calculations for the pion form factor. In this paper, we are going to examine the effects brought about by these corrections.

Applying the renormalization-group equation to T_H and substituting the explicit expression for T_H, we have the following expression for the pion form factor:

$$F_\pi(Q^2)=16\pi C_F\int_0^1 dx_1dx_2\int_0^\infty b\,db\alpha_s(t)K_0(\sqrt{x_1x_2}Qb)\phi(x_1,1/b)\phi(x_2,1/b)\exp[-S(x_1,x_2,Q,b,t)], \tag{7}$$

where

$$S(x_1,x_2,Q,b,t)=\left[\sum_{i=1}^{2}[s(x_i,b,Q)+s(1-x_i,b,Q)]-\frac{2}{\beta_1}\ln\frac{\hat{t}}{-\hat{b}}\right]. \tag{8}$$

K_0 is the modified Bessel function of order zero.

Radiative corrections in higher orders will bring logarithms of the form $\ln(t/\mu)$ into T_H, where t is the largest mass scale appearing in T_H. Reference [4] points out that a natural choice for μ in T_H is $\mu=t$ and

$$t=\max(\sqrt{x_1x_2}Q,1/b)\,. \tag{9}$$

If b is small, radiative corrections will be small regardless of the values of x because of the small α_s. When b is large and $x_1x_2Q^2$ is small, radiative corrections are still large in T_H, while φ will suppress these regions. But with b increasing, α_s becomes rather large (for example, $\alpha_s\geq 1$ as $b>5$ GeV^{-1} for $x_1=0.01$, $x_2=0.01$ and $Q=2$ GeV; see Fig. 1) and accordingly the perturbative calculation loses its self-consistency. Therefore, a cutoff on α_s is made to evaluate perturbative contributions and to justify the self-consistency of perturbative calculation. That is to say, if 50% of the result come from the regions where α_s is not very large (say, <0.7), the perturbative calculation can be trusted.

Strictly speaking, the perturbative predictions to the regions where α_s is larger than unity are unreliable, although these regions are suppressed. In fact, in the regions of small $x_1x_2Q^2$ and large b, the multigluon exchange is important and the transverse momentum intrinsic to the bound-state wave functions flows through all the propagators [3,11]. To respect this point, instead of Eq. (9) we suggest that

$$t=\max(\sqrt{x_1x_2}Q,1/b_F)\,, \tag{10}$$

where

$$b_F=\begin{cases} b & \text{if } 1/b\geq\ \langle k_T\rangle\\ 1/\langle k_T\rangle & \text{if } 1/b<\ \langle k_T\rangle, \end{cases} \tag{11}$$

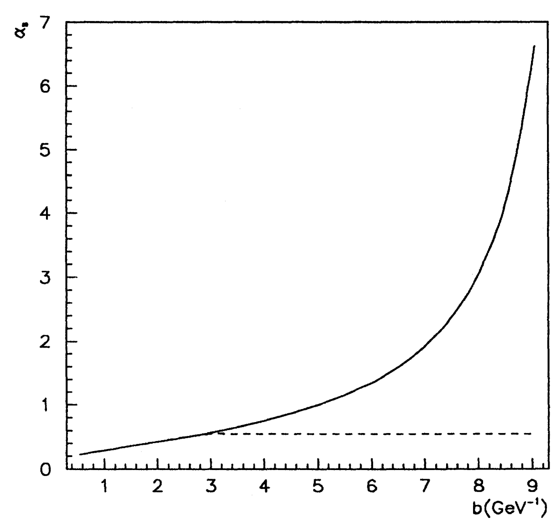

FIG. 1. The evolution of α_s with b for $x_1=0.01, x_2=0.01$, $Q=2$ GeV and $\Lambda_{\rm QCD}=100$ MeV. The solid line is evaluated with Eq. (9). The dashed line is evaluated with Eq. (10) for $\psi^{(a)}$.

and $\langle k_T \rangle$ is the rms transverse momentum of the pion. With such a choice, the running coupling constant will be frozen at $\alpha_s(t = \langle k_T \rangle)$ when b is large and $x_1 x_2 Q^2$ is small. In this way, the perturbative contributions to the pion form factor can be calculated from the present energy with a reasonable α_s. Experimentally the rms quark transverse momentum of the pion is of the order 300 MeV approximately [8]. In the perturbative calculations, $\langle k_T \rangle$ always associates with the pion wave function, which may be a little different for different models of wave functions. Because the perturbative predictions for the pion form factor are insensitive to the values of $\langle k_T \rangle$, the model dependence of our prescription for freezing the running coupling constant is very weak. An effective gluon mass [9] and an infrared cutoff [10] have been taken as the scale to freeze α_s, in which the result are sensitive to the variation of the freezing scale (the gluon mass or the infrared cutoff). According to the present approach the perturbative predictions for the form factor are insensitive to the variation of $\langle k_T \rangle$ with the help of the Sudakov suppression. We would like to point out also that although $\langle k_T \rangle$ appears in the Sudakov suppression [Eq. (8)], as well as in α_s, through the mass scale t [Eqs. (10) and (11)], it affects the Sudakov suppression very weakly while it affects α_s dramatically (see Fig. 1).

The pion wave function. According to the Brodsky-Huang-Lepage prescription [11], one can connect the equal-time wave function in the rest frame and the light-cone wave function by equating the off-shell propagator in the two frames. They got the wave function [11, 3] at the infinite momentum frame from the harmonic oscillator model at the rest frame [12]:

$$\psi^{(a)}(x, \mathbf{k}_T) = A \exp\left[-\frac{\mathbf{k}_T^2 + m^2}{8\beta^2 x(1-x)}\right], \tag{12}$$

where $\beta = 0.385$ GeV and $A = 32$ GeV^{-1} are parameters adjusted [13] by using the constraints derived [11] from $\pi \to \mu\nu$ and $\pi^0 \to \gamma\gamma$ decay amplitudes:

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_T}{16\pi^3} \psi(x, \mathbf{k}_T) = \frac{f_\pi}{2\sqrt{6}}, \tag{13}$$

$$\int_0^1 dx \psi(x, \mathbf{k}_T = 0) = \frac{\sqrt{6}}{f_\pi}. \tag{14}$$

$f_\pi = 0.133$ GeV is the pion decay constant. The quark mass is chosen as $m = 289$ MeV.

The mean-square transverse momentum is defined as

$$\langle \mathbf{k}_T^2 \rangle = \int \frac{d^2\mathbf{k}_T}{16\pi^3} dx |\mathbf{k}_T|^2 |\psi(x, \mathbf{k}_T)|^2 / P_{q\bar{q}}, \tag{15}$$

where

$$P_{q\bar{q}} = \int \frac{d^2\mathbf{k}_T}{16\pi^3} dx |\psi(x, \mathbf{k}_T)|^2 \tag{16}$$

is the probability of finding the $q\bar{q}$ Fock state in the pion. For $\psi^{(a)}(x, \mathbf{k}_T)$, $\langle \mathbf{k}_T^2 \rangle$=(0.356 GeV)^2 and $P_{q\bar{q}} = 0.296$. Expressing $\psi^{(a)}(x, \mathbf{k}_T)$ in the b space, we obtain

$$\phi^{(a)}(x, 1/b) = \frac{2A\beta^2}{(2\pi)^2} x(1-x) \times \exp\left(-\frac{m^2}{8\beta^2 x(1-x)}\right) \times \exp\left[-2\beta^2 x(1-x) b^2\right]. \tag{17}$$

Another model of the wave function adopted in our calculations is the Chernyak-Zhitnitsky (CZ) like [18] wave function [14–16]:

$$\psi^{(b)}(x, \mathbf{k}_T) = A\ (1-2x)^2\ \exp\left[-\frac{\mathbf{k}_T^2 + m^2}{8\beta^2 x(1-x)}\right] \tag{18}$$

and

$$\phi^{(b)}(x, 1/b) = \frac{2A\beta^2}{(2\pi)^2} x(1-x)(1-2x)^2 \times \exp\left(-\frac{m^2}{8\beta^2 x(1-x)}\right) \times \exp[-2\beta^2 x(1-x) b^2], \tag{19}$$

where β=0.455 MeV and $A = 136$ GeV^{-1} with the use of $m = 342$ MeV; $\langle \mathbf{k}_T^2 \rangle$ =(0.343 GeV)^2 and $P_{q\bar{q}} = 0.364$.

Numerical calculations. Numerical evaluations for the pion form factor with $\phi^{(a)}$ and $\phi^{(b)}$ are plotted in Fig. 2. We can find that the perturbative predictions are still smaller than the experimental data. It is expected to take into account the contributions from higher orders and higher Fock states to reach the data at the intermediate energy.

To evaluate the effects due to the errors in the $s(\xi, b, Q)$ expression, we adopt the formalism of Ref. [4] in our numerical calculations. That is, we choose t as defined in Eq. (9) and neglect the evolution of $\phi(x, 1/b)$ with b. In addition, the same two models of the distribution amplitudes in Ref. [4] are used: (a) the asymptotic wave function [17]

$$\phi^{\text{as}}(x) = \frac{3f_\pi}{\sqrt{2N_c}} x(1-x) \tag{20}$$

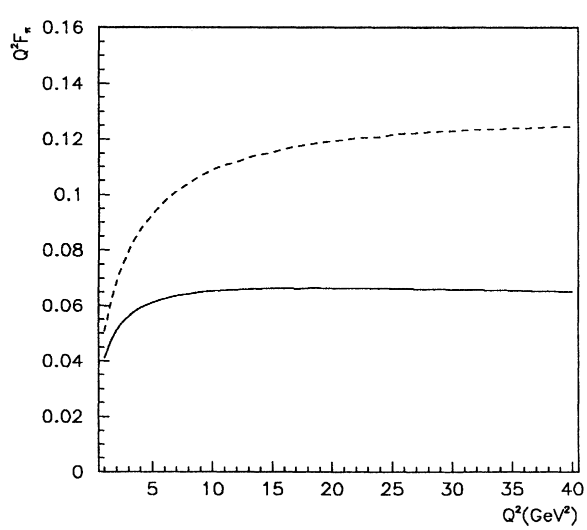

FIG. 2. The pion form factor with $\psi^{(a)}$ (solid line) and $\psi^{(b)}$ (dashed line).

and (b) the Chernyak-Zhitnitsky wave function [18]

$$\phi^{CZ}(x) = \frac{15 f_\pi}{\sqrt{2N_c}} x(1-x)(1-2x)^2. \tag{21}$$

We find that the corrected expression for the Sudakov suppression function $s(\xi, b, Q)$ [Eq. (5)] increases the predictions of the pion form factor by about 0.8% for the ϕ^{as} and about 1.0% for the ϕ^{CZ} at $Q = 20\Lambda_{QCD}$. The effect increases with Q decreasing (reaching about 2.0% for ϕ^{as} and 3.0% for ϕ^{CZ} at $Q = 10\Lambda_{QCD}$). The corrections are sizable individually for the fourth and sixth terms in the $s(\xi, b, Q)$ expression, but fortunately they cancel each other in the final expression. As a result, the whole effect on the pion form factor is mild.

Summary. In this paper, we reexamine the perturbative pion form factor with the Sudakov suppression. It is found that in the previous perturbative calculations there are regions where the running coupling constant $\alpha_s \geq 1$ and the perturbative predictions are unreliable, although these regions are suppressed. Thus a cutoff on α_s has to be made to guarantee the applicability of the pQCD. Observing that in the above regions the multigluon exchange is important, we suggest that the running coupling constant should be frozen at $\alpha_s(t = \langle k_T \rangle)$ when b is large and $x_1 x_2 Q^2$ is small by taking into account the average transverse momentum. In this way, the perturbative contributions to the pion form factor can be calculated from the present energy with a reasonable α_s. Although $\langle k_T \rangle$ depends on the wave function, the perturbative predictions for the pion form factor are not sensitive to the value of $\langle k_T \rangle$. Hence our prescription of freezing the running coupling constant depends on the model of wave function very weakly. In addition, we correct the previous calculations about the Sudakov suppression factor, which plays an important role in the perturbative calculations for the pion form factor.

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