Nonresonant Cabibbo suppressed decay $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ and signal for CP violation

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We consider various contributions to the nonresonant decay $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$, both of the long-distance and short-distance types, with the former providing for most of the branching ratio, predicted to be $\mathcal{B}(B^{\pm} \to \pi^+\pi^-\pi^{\pm}) = (1.0\text{–}5.0) \times 10^{-5}$. We also discuss an application to CP violation resulting from the interference of that nonresonant background [with $m(\pi^+\pi^-) \approx 3.4$ GeV] and $B^{\pm} \to \chi_{c0}\pi^{\pm}$ followed by $\chi_{c0} \to \pi^+\pi^-$. The resulting value of the partial rate asymmetry is (0.44–0.49)sinγ, where $\gamma = \arg(V_{ub}^*)$.

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Two-body and quasi-two-body nonleptonic decays of heavy mesons have been extensively studied [1]. Multibody nonleptonic decays are more difficult to estimate, and one usually resorts to statistical or phase space models [2]. In this paper we will not discuss, for reasons that will become clear, heavy meson decays through a chain of real resonances [3]; i.e., we consider only the nonresonant background, and confine ourselves to $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ though similar results are expected for other modes. Our motivation is twofold.

(1) $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ is expected to be larger than $B \to \pi\pi$, which, though not separated yet experimentally from $B \to K\pi$, is estimated to have a branching ratio of the order 10^{-5} [4]. It is therefore challenging to find a viable dynamical description of $B \to \pi\pi\pi$.

(2) Recently [5], it has been suggested that large CP asymmetries should occur in $B^{\pm} \to h\pi^{\pm}$ where the hadronic state $h = \pi^+\pi^-$ has energy corresponding to the resonance $\chi_{c0}(3.4)$.

The absorptive phase necessary to observe CP violation in partial rate asymmetries is provided by the χ_{c0} width (subtracting the small partial width of χ_{c0} to $\pi^+\pi^-$). The CP odd phase γ results from the interference of the two quark processes responsible for the background decay $B \to \pi\pi\pi$ and $B \to \chi_{c0}\pi$, which are $b \to u\bar{u}d$ and $b \to c\bar{c}d$, respectively. The partial rate asymmetry obtained in Ref. [5] suffers from a large uncertainty due mostly to the unknown background and especially its angular dependence. Note that only $h = \pi^+\pi^-$ with spin parity 0^+ leads to interference with the resonant amplitude. Therefore, knowledge of the angular dependence is crucial, and this will come out directly once one has a reliable model for the background process $B \to \pi\pi\pi$. The interference between the resonance and the background amplitudes will then automatically project out the 0^+ component of $h = \pi^+\pi^-$. Thus $\pi^+\pi^-$ arising from resonances such as ρ do not interfere and need not be considered, and other possible resonances in the crossed channel will also not be considered as we aim to estimate here the genuine nonresonant background in the measured asymmetry. Resonances in all channels can be subtracted for a B and a $\bar{B}$ separately, before forming the asymmetry itself.

In this paper we will consider three contributions to nonresonant $B \to \pi\pi\pi$ background and identify the leading one. Estimates for this nonleptonic process will suffer from large uncertainties because of the nature of approximations one has to use. Nevertheless the CP-violating partial rate asymmetry will be affected only mildly by this uncertainty as we will show below.

Let us now consider the three possible contributions to the nonresonant background $B \to \pi\pi\pi$, as depicted in Figs. 1(a)–1(c). We choose our momenta as follows: $B^-(p_B) \to \pi^-(p_1)\pi^+(p_2)\pi^-(p_3)$ and always symmetrize by $p_1 \leftrightarrow p_3$. Furthermore we define $s = (p_B - p_1)^2 = (p_2 + p_3)^2$ and $t = (p_B - p_3)^2 = (p_1 + p_2)^2$.

Figure 1(a) is the short-distance contribution to $B \to$

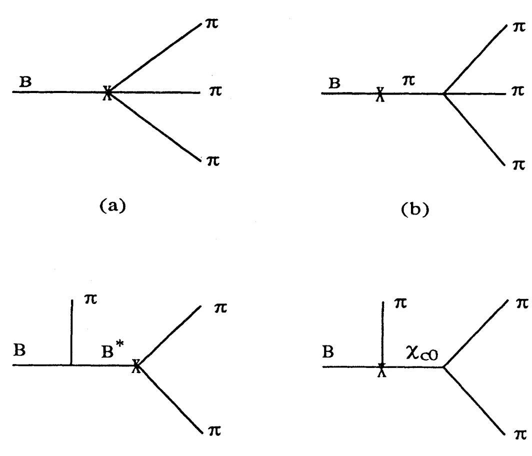

FIG. 1. Diagrams contributing to $B^-(B^+) \to \pi^+\pi^-\pi^-(\pi^+)$. The weak vertices are indicated by $\times$.

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$\pi\pi\pi$, for which the effective weak Hamiltonian is

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* (C_1 O_1 + C_2 O_2) , \quad (1)$$

where $C_1 \approx -0.313$, $C_2 \approx 1.15$, and

$$O_1 = \bar{d}\gamma_\mu (1-\gamma_5) b \bar{u} \gamma^\mu (1-\gamma_5) u ,$$
$$O_2 = \bar{u}\gamma_\mu (1-\gamma_5) b \bar{d} \gamma^\mu (1-\gamma_5) u . \quad (2)$$

Within the factorization approximation, we have the amplitude

$$\begin{aligned} M_a &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \langle \pi^+\pi^-\pi^- | C_1 O_1 + C_2 O_2 | B^- \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left(C_1 + \frac{C_2}{N_c} \right) \langle \pi^-(p_1) | \bar{d}\gamma_\mu b | B^-(p_B) \rangle \langle \pi^+(p_2) \pi^-(p_3) | \bar{u}\gamma_\mu u | 0 \rangle + (p_1 \leftrightarrow p_3) . \end{aligned} \quad (3)$$

The matrix elements in Eq. (3), neglecting m_π^2, are

$$\begin{aligned} \langle \pi^- | \bar{d}\gamma_\mu b | B^- \rangle &= (p_B + p_1)_\mu F_1^{B\pi}(s) + \frac{m_B^2}{s} (p_B - p_1)_\mu [F_0^{B\pi}(s) - F_1^{B\pi}(s)] , \\ \langle \pi^+(p_2) | \bar{u}\,\gamma_\mu\, u | \pi^+(-p_3) \rangle &= (p_2 - p_3)_\mu F_1^{\pi\pi}(s) . \end{aligned} \quad (4)$$

Substituting in Eq. (3) and performing the scalar products lead to

$$M_a = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| e^{-i\gamma} a_2 [F_1^{B\pi}(s) F_1^{\pi\pi}(s)(2t + s - m_B^2) + F_1^{B\pi}(t) F_1^{\pi\pi}(t)(2s + t - m_B^2)] . \quad (5)$$

We have defined $a_2 = C_1 + C_2/N_c$, but will take the phenomenological value $a_2 \approx 0.24$ [6], and $\gamma = \arg(V_{ub}^*)$. For the form factors above we use the pole model forms [7]

$$\begin{aligned} F_{1,0}^{B\pi}(q^2) &= \frac{F_{1,0}^{B\pi}(0)}{1 - q^2/m_{1,0}^2} , \\ F_1^{\pi\pi}(q^2) &= \frac{1}{1 - q^2/m_{\pi\pi}^2 + i\Gamma_\sigma/m_{\pi\pi}} , \end{aligned} \quad (6)$$

where $F_1^{B\pi}(0) = F_0^{B\pi}(0) = 0.333$ [8], or 0.53 ± 0.12 [9] and $m_1 = 5.32$ GeV, $m_0 = 5.78$ GeV, $m_{\pi\pi} \approx m_\sigma = 0.7$ GeV, and $\Gamma_\sigma = 0.2$ GeV.

Substituting the appropriate numerical values, integrating over phase space and using [10] $\tau_B = 1.54 \times 10^{-12}$ s, we find that the contribution of Fig. 1(a) to the branching ratio is

$$B_a = \frac{\Gamma_a}{\Gamma_B} = 0.9 \times 10^{-6} \left(\frac{F_1^{B\pi}(0)}{0.333} \right)^2 \quad (7)$$

which ranges between 0.9×10^{-6} and 2.3×10^{-6}.

Figure 1(b) which is obviously of the long-distance type is harder to calculate than Fig. 1(a). It is nevertheless small as the intermediate pion is highly off shell. The weak transition $B \to \pi$ is easy to evaluate, and leads to

$$\begin{aligned} T(B \to \pi) &= \left\langle \pi^- \left| \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* (C_1 O_1 + C_2 O_2) \right| B^- \right\rangle \\ &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 f_B f_\pi m_B^2 , \end{aligned} \quad (8)$$

where $a_1 = C_1/N_c + C_2 \approx 1.1$, $f_B = 0.2$ GeV, and $f_\pi = 0.13$ GeV. Then, again neglecting m_π, we find

$$M_b = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 f_B f_\pi A(\pi\pi\pi\pi) . \quad (9)$$

$A(\pi\pi\pi\pi)$ is not known for one highly off-shell pion and three on-shell ones. If we assume only S wave, and use the unitarity limit, $A(\pi\pi\pi\pi) \sim 1$, the branching ratio contribution of M_b is

$$B_b = \frac{\Gamma_b}{\Gamma_B} < 10^{-8} . \quad (10)$$

Of course it is unrealistic to assume only S-wave contribution to M_b, and waves with angular momenta up to ka contribute, where k is the momentum in the center of mass and a is a typical size. It is difficult to make our estimates more quantitative since one of the pions is highly off shell. However we cannot envision this contribution to be large, and we shall neglect it.

Turning to Fig. 1(c), which is also of a long-distance type, we will show that it is the dominant diagram and its branching ratio is equal or larger than $B(B \to \pi\pi)$ which should clearly be the case, since even in the charmed meson system [10] $\Gamma(D \to \pi\pi\pi) \geq \Gamma(D \to \pi\pi)$. The calculation of the amplitude M_c involves the application of both heavy quark effective theory (HQET) and chiral perturbation theory (CHPT). For a review of both see Ref. [11]. First we write

$$\begin{aligned} M_c = A^\mu_{BB^*\pi} & \frac{-g_{\mu\nu} + p_{B^*\mu} p_{B^*\nu}/m_{B^*}^2}{p_{B^*}^2 - m_{B^*}^2} A^\nu_{B^*\pi\pi} \\ & + (p_1 \leftrightarrow p_3) . \end{aligned} \quad (11)$$

Note that the B^* is off shell and since we are interested in the nonresonant part of $B \to \pi\pi\pi$, no on-shell intermediate resonances are introduced. Our main aim now is to calculate the strong and weak vertices $A^\mu_{BB^*\pi}$ and $A^\nu_{B^*\pi\pi}$, respectively, using the methods of HQET and for the strong vertex combining them with CHPT [12].

Let us start by calculating $A^\mu_{BB^*\pi}$. The heavy-chiral Lagrangian density [11,12] relevant to us is

$$\mathcal{L}_{\rm int} = ig\sqrt{m_B m_{B^*}}\langle H_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{H}_a \rangle \ , \qquad (12)$$

where $\langle\rangle$ stands for trace. The field H_a describes the heavy-quark–light-quark ($Q\bar{q}_a$) system and

$$H_a = \frac{1+\not{v}}{2}(P^*_{a\mu}\gamma^\mu - P_a\gamma_5) \ , \quad \bar{H}_a = \gamma_0 H^\dagger \gamma_0 \ ,$$
$$A^\mu_{ba} = \frac{1}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ba} \ , \qquad (13)$$

where $P_a = (B^-, \bar{B}^0_d, \bar{B}^0_s)$ and similarly for $P^*_{a\mu}$ in terms of the vector meson states, v is the heavy meson velocity, and $\xi = \exp(iM/f_\pi)$ with M given by

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \ . \qquad (14)$$

We obtain

$$A^\mu_{BB^*\pi}\epsilon_\mu = -\frac{2g}{f_\pi}\sqrt{m_B m_{B^*}} B^- B^*_\nu \partial^\nu \pi^+ \ . \qquad (15)$$

Using the flavor symmetry of HQET the coupling constant g is determined to be 0.6 [11,12]. The main uncertainty in the application of Eq. (15) to our case is that in Fig. 1(c) the B^* is off shell. We therefore define μ as a measure of the off-shellness of the B^* and consider two cases: (1) $\mu = \sqrt{m_B m_{B^*}}$ in Eq. (15). (2) $\mu = \sqrt{m_B \sqrt{p^2_{B^*}}}$, where p_{B^*} is the momentum of the B^*.

To calculate $A_{B^*\pi\pi}$ in Eq. (11), we employ the spin independence of HQET and write

$$\begin{aligned} A^\nu_{B^*\pi\pi}\epsilon_\nu &= \frac{G_F}{\sqrt{2}} V_{ub} V^*_{ud} \langle \pi^+\pi^- | C_1 O_1 + C_2 O_2 | B^* \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{ub} V^*_{ud} a_1 \langle \pi^+ | \bar{u}\gamma_\mu (1-\gamma_5) b | B^* \rangle \langle \pi^- | \bar{d}\gamma^\mu (1-\gamma_5) u | 0 \rangle \ . \end{aligned} \qquad (16)$$

The form factors T_{1-4} are defined as

$$\begin{aligned} \langle \pi^+ | \bar{u}\gamma_\mu b | B^* \rangle &= 2T_1 i \epsilon_{\mu\nu\lambda\sigma} \epsilon^\nu p^\lambda_{B^*} p^\sigma_2 \ , \\ \langle \pi^+ | \bar{u}\gamma_\mu \gamma_5 b | B^* \rangle &= 2T_2 m^2_{B^*} \epsilon_\mu + 2T_3 (\epsilon \cdot q)(p_{B^*} + p_2)_\mu + 2T_4 (\epsilon \cdot q)(p_{B^*} - p_2)_\mu \ , \end{aligned} \qquad (17)$$

where $q = p_{B^*} - p_2$. Relations between $T_i's$ and $f_\pm$ defined through

$$\langle \pi^+ | \bar{u}\gamma_\mu b | \bar{B}^0 \rangle = f_+ (p_B + p_\pi)_\mu + f_- (p_B - p_\pi)_\mu \ , \qquad (18)$$

are [13]

$$\begin{aligned} T_1 &= -i\frac{f_+ - f_-}{2m_B} \ , \quad T_2 = \frac{1}{2m^2_B}\left((f_+ + f_-) m_B + (f_+ - f_-)\frac{p_B \cdot p_\pi}{m_B} \right) \ , \\ T_3 &= \frac{f_+ - f_-}{4m_B} \ , \quad T_4 = T_3 \ . \end{aligned} \qquad (19)$$

Substituting the above relations in Eq. (16), we have

$$A^\nu_{B^*\pi\pi}\epsilon_\nu = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{ud} a_1 f_\pi (\epsilon \cdot p_3) \left(\frac{3f_+}{2} m_B + \frac{f_-}{2} m_B + (f_+ - f_-)\frac{p_2 \cdot p_3}{m_B} \right) \ . \qquad (20)$$

The amplitude for Fig. 1(c), obtained from Eqs. (11), (15), and (20), expressed in terms of $F^{B\pi}_{1,0}$, is

$$\begin{aligned} M_c = &-\frac{G_F}{\sqrt{2}} V_{ub} V^*_{ud} (2ga_1) F^{B\pi}_1 (m^2_\pi) \frac{\mu}{s - m^2_{B^*}} \left[\frac{3}{2} m_B + \frac{s}{2m_B} \right. \\ &\left. - \frac{m_B}{2} \frac{m^2_B - s}{m^2_\pi} \left(1 - \frac{F^{B\pi}_0 (m^2_\pi)}{F^{B\pi}_1 (m^2_\pi)} \right) \right] \left[-\frac{s^2}{4m^2_{B^*}} + \left(\frac{1}{2} + \frac{m^2_B}{4m^2_{B^*}} \right) s + \frac{t - m^2_B}{2} \right] + (s \leftrightarrow t) \ . \end{aligned} \qquad (21)$$

The branching ratio implied by Fig. 1(c) gives

$$B_c = \frac{\Gamma_c}{\Gamma_B} = \begin{cases} 2.0 \times 10^{-5} \left(\frac{F^{B\pi}_1 (0)}{0.333} \right)^2 & \text{case 1} \ , \\ 1.0 \times 10^{-5} \left(\frac{F^{B\pi}_1 (0)}{0.333} \right)^2 & \text{case 2} \ , \end{cases}$$

and yields $B_c = (1.0\text{–}5.0)\times 10^{-5}$. The spread is caused by the two different prescriptions for taking into account the off-shellness of the B^*, and by the fact that $0.333 \leq F^{B\pi}_1(0) \leq 0.53$. Since B_c is the largest branching ratio as compared to B_a and B_b, and is not smaller than the branching ratio for $B \to \pi\pi$, we take B_c as a good estimate for the branching ratio of the nonresonant decay $B \to \pi\pi\pi$, and obviously $M(B \to \pi\pi\pi) = M_c$. It should be noted that we use here CHPT beyond its region of being a reliable model, but we believe that case 2 takes care, at least in part, of that problem by cutting the high energy pions significantly.

It is not surprising that three-body decays are dominated by a long-distance contribution in contrast to the two-body decays which are dominated by factorization and a short-distance amplitude. The mechanism of producing additional pions must necessarily involve the strong interaction.

Turning now to the CP-violating asymmetry, we interfere M_c with the resonance amplitude $M_{\rm res}$ for $B^\pm \to \chi_{c0}\pi^\pm \to \pi^+\pi^-\pi^-$ from Fig. 1(d), where

$$M_{\rm res} = A(B^\pm \to \chi_{c0}\pi^\pm)\frac{1}{s - m_\chi^2 + i\Gamma_\chi m_\chi}A(\chi_{c0} \to \pi^+\pi^-) + (s \leftrightarrow t)\ . \tag{22}$$

Following Ref. [5] we integrate the decay rate in the phase space from $s_{\rm min} = (m_\chi - 2\Gamma_\chi)^2$ to $s_{\rm max} = (m_\chi + 2\Gamma_\chi)^2$ where m_χ and Γ_χ are the mass and width, respectively, of χ_{c0}. We define the partial width $\Gamma_p \sim \int dsdt|M_c+M_{\rm res}|^2$, where $0 \le t \le m_B^2 - s$ and the s integral has the above limits. Therefore the absolute value of the asymmetry

$$|A| = \left|\frac{\Gamma_p - \bar{\Gamma}_p}{\Gamma_p + \bar{\Gamma}_p}\right| = (0.44\text{–}0.49)\sin\gamma\ . \tag{23}$$

Since, unlike the case for Ref. [5], where the amplitude for the nonresonant background is unknown as a function of both s and t (and therefore its angular dependence is unknown), here the model used dictates the angular dependence which gives more confidence in the asymmetry obtained. Although we use CHPT beyond its region of validity, we believe the amplitude M_c used for the interference with $M_{\rm res}$ is more reliable after integration than its values used for calculating B_c, as the dominant contribution only comes from the resonant region and not the end of phase space. It is interesting that the large uncertainty in the background $\mathcal{B}(B \to \pi\pi\pi)$ does not translate into a large spread in the values for $|A|$ since it affects both numerator and denominator in $|A|$. Even if our estimate of the background goes down by a factor of 2, we still would expect coefficient of $\sin\gamma$ in Eq. (23) to go down only by 20%. We did not consider uncertainties stemming from the experimental errors on input parameters such as $|V_{ub}|$, masses, and widths. From the very large direct CP-violation asymmetry obtained for $\sin\gamma = 1$ and using $\mathcal{B}(B^\pm \to \chi_{c0}\pi^-)\mathcal{B}(\chi_{c0} \to \pi^+\pi^-) \approx 5 \times 10^{-7}$, the number of events N required experimentally to detect such an asymmetry at the 3σ level is $9 \times 10^7 \le N \le 13 \times 10^7$. One expects future B factories to be able to reach such a number of events.

We would like to comment that even smaller numbers of events are required to observe CP-violating asymmetry in $B \to h\pi$ where now $h = 2(\pi^+\pi^-)\ , \pi^+\pi^-K^+K^-$ for which $\mathcal{B}(\chi_{c0} \to h)$ is at a level of a few percent. Estimates of the nonresonant background unfortunately become more difficult. The same situation [5] (large asymmetry, but difficult to predict the nonresonant background) is expected in $B \to h\pi$ where $h = \eta'\pi\pi\ , \rho\rho$, etc., and the nonresonant amplitude interferes with $B \to \eta_c\pi$.

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