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Quantization dependence in a constituent quark model

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We study the implications of Lorentz symmetry for hadronic structure by formulating a variable quantization front constituent quark model. We conclude that there is little sensitivity of the calculated observables to the choice of the formalism, provided relativity is properly implemented.

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I. INTRODUCTION

The explicit quark picture of hadrons has been investigated by an increasing number of models using the constituent quarks as the effective degrees of freedom [1–3]. Despite the fact that a formal link is missing between such QCD-inspired models and fundamental QCD there are many indications that the constituent quark representation does emerge as a result of strong vacuum correlations which generate the quark condensate and lead to dynamical chiral symmetry breaking [4,5]. Central to these approaches is a globally gauge-invariant potential confining the quarks. Justification for an effective potential is provided by the flux tube configurations generated by the gluon field. Nevertheless, the validity of the constituent quark model remains an open issue. For example, it is not clear how to reconcile the simple, valence constituent quark model spin-spin interaction as the source of the π-ρ mass splitting [1] with the explanation put forth from chiral symmetry-breaking arguments [4]. The above issues become even more complex when attempts are made to develop a consistent relativistic quark approach. Relativity is indispensable [6,7] due to the large velocities associated with quarks having masses of the order of a few hundred MeV but bound on a typical hadronic scale of the order of 1 GeV. The implementation of Lorentz symmetry is the thrust of this paper.

The problem of formulating relativistic dynamics for a fixed number of particles originated with the pioneering work of Dirac [8] and has been extensively studied over the years on both classical and quantum levels. Classically this is a Cauchy problem to determine the world lines in Minkowski space, while in the quantum case one seeks the probability amplitude distributions, as an initial value problem on a three-dimensional spacelike or lightlike surface. We refer the reader to the extensive literature on the subject [9–13]. The possible choices of the initial surface are commonly classified according to the dimension of the corresponding stability group consisting of purely kinematical transformations within the subspace defined by the initial conditions. Conveniently, the group representation does not require a full solution of the evolution equations. With this classification scheme the surface of the light cone, also referred to as the light front, is very appealing because it generates the largest stability group [12]. The light cone stability group acts transitively on the mass shell, and so the light cone wave function of arbitrary momentum can be determined from the wave function in the rest frame. At the field theoretic level there are also practical reasons for quantizing on the light front [14]. At short distances color interactions can be treated perturbatively and such separations, which are lightlike in a Minkowski metric, usually dominate high energy processes [15]. Further, the QCD vacuum is rigorously decoupled from excited Fock states when formulated on the light cone and this permits a more sensible quark Fock state expansion for hadron states [16]. However, until a formal connection is established between field theory and QCD-inspired models, such arguments should cautiously be regarded.

Irrespective of quantization surface orientation formalisms using impulse approximated currents lack complete Lorentz covariance. This is because such currents do not include interactions and will therefore not properly commute with the interaction-dependent generators of the Poincaré group. Consequently, matrix elements of the current operator will also not be properly constrained and will acquire an incorrect four-vector structure which is represented by a dependence upon n^μ.

Of course full covariance is not always necessary, provided strong arguments exist for disposing of unphysical degrees of freedom or spurious form factors arising in a noncovariant formulation [16,17]. However, in a particular framework with a fixed number of constituents such as the valence constituent quark model any extensions such as to nonvalence degrees of freedom should be taken with caution until full implementation of symmetries including Lorentz covariance is properly done. The main objective of this paper is to provide insight and criteria for assessing when Lorentz covariance violations are quantitatively important and should be addressed. To this end a comparative study of alternative relativistic calculations of observables for pseudoscalar mesons is performed using various quantization schemes to document framework sensitivity.

In the following two sections the basic ingredients entering the quark model calculation of physical observables are reviewed and the method for calculation of matrix elements for arbitrary quantization scheme is outlined. Numerical results are presented in Sec. IV with major findings summarized in Sec. V.

II. MESONIC PROPERTIES IN A CONSTITUENT QUARK MODEL

All meson observables investigated in this work are specified in terms of hadronic matrix elements of local operators, having one of two forms:

$$\mathcal{A}_1 \equiv \langle 0|J(0)|P\rangle, \\ \mathcal{A}_2 \equiv \langle P'|J(0)|P\rangle. \tag{2.1}$$

Here $|P\rangle$ denotes a pseudoscalar meson state, with four-momentum P^μ, $J = \overline{\psi}(0)\Gamma\psi(0)$ is a current operator with the fermion field operators $\overline{\psi}, \psi$ describing the QCD quark fields, and $|0\rangle$ denotes the vacuum. In the constituent quark picture a meson state is entirely represented by a valence constituent $q\overline{q}$ pair and the exact QCD field operators are replaced by the effective constituent fields [18]. In the following we shall concentrate on this two-body Fock sector.

A flat instant *time* ($= n\cdot x$), spacelike or lightlike quantization surface Σ is defined by a timelike or null four-vector n^μ, $n^2 \geq 0$:

$$\Sigma = \Sigma_1 \cup \cdots \cup \Sigma_A \\ \equiv \{\{x_1^\mu \cdots x_A^\mu\} : (n\cdot x_1) = \cdots (n\cdot x_A) = 0\}, \tag{2.2}$$

with A being the number of constituents ($A = 2$ for the $q\overline{q}$ mesons). Since *time* parameters do not appear in the matrix elements in Eq. (2.1), only the knowledge of the single-time wave functions defined on Σ are needed. The hadronic matrix elements, defined in Eq. (2.1), in the light cone quantization, $n^2 = 0$, will be obtained in the limit as $n^2 \to 0$. For each constituent i described by configuration or momentum space four-vector A^μ we define the associated transverse four-vector A_T in the ith subspace of Σ and the longitudinal component A_L by projections:

$$A_L \equiv \frac{A\cdot n}{\sqrt{n^2}}, \quad A_T^\mu \equiv A^\mu - A_L \frac{n^\mu}{\sqrt{n^2}}\,. \tag{2.3}$$

Since $A_T \cdot n = 0$, A_T has only three independent components denoted in three-vector form by $A_T = (0, \mathbf{A})$. The three components of a transverse vector $\mathbf{A}$ do not necessary correspond to the Minkowski coordinates since in general $n^\mu \neq (1,0,0,0)$. The scalar product becomes

$$A\cdot B = A_L B_L + A_T \cdot B_T = A_L B_L - \mathbf{A}\cdot\mathbf{B}, \tag{2.4}$$

where

$$\mathbf{A}\cdot\mathbf{B} \equiv \sum_{i,j=1}^{3} A^i g_{ij} B^j, \tag{2.5}$$

A_i (B_i) are the components of the three-vector $\mathbf{A}$ ($\mathbf{B}$) and g_{ij} is the 3×3 matrix of nonvanishing components of $\mathbf{g}$, the projection of $-g$ onto Σ:

$$\begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{g} \end{pmatrix} = g_{T\mu\nu} = -g_{\mu\nu} + \frac{n_\mu n_\nu}{n^2}. \tag{2.6}$$

In a similar way the transverse antisymmetric tensor $\epsilon_{T,ijk}$ is defined by the nonvanishing components of $\epsilon_{\mu\nu\rho\sigma}$ projected onto Σ, i.e., $\frac{n^\mu}{|n|}\epsilon_{\mu ijk}$. Thus, if x and p are canonically conjugate, $[x^\mu, p^\nu] = ig^{\mu\nu}$, then the three-dimensional transverse components $\mathbf{x}$ and $\mathbf{p}$ are also canonically conjugate:

$$[\mathbf{x}{,}\mathbf{p}] = ig_T. \tag{2.7}$$

The longitudinal p_L, component of a four-momentum defines particle's *energy* and because it is conjugate to *time* x_L it describes dynamical evolution. Returning to the fermion field operators ψ and $\overline{\psi}$, these can be canonically quantized on the surface Σ and represented by a Hilbert space expansion in terms of operators $b = b(\mathbf{p},\lambda,\tau), d = d(\mathbf{p},\lambda,\tau)$:

$$\{b(\mathbf{p}',\lambda',\tau'), b^\dagger(\mathbf{p},\lambda,\tau)\} = (2\pi)^3 \frac{\mathbf{P}}{m}\delta^3(\mathbf{p}'-\mathbf{p})\delta_{\lambda'\lambda}\delta_{\tau'\tau}, \tag{2.8}$$

$$\psi(0,\mathbf{x}) = 2m\int \frac{d^4p}{(2\pi)^4} 2\pi\delta(p^2-m^2)\theta(p_L), \sum_{\tau,\lambda=\pm}\left[bu(\mathbf{p},\lambda)e^{-ipx} + d^\dagger v(\mathbf{p},\lambda)e^{ipx}\right]_{|\Sigma}, \\ \overline{\psi}(0,\mathbf{x}) = 2m\int \frac{d^4p}{(2\pi)^4} 2\pi\delta(p^2-m^2)\theta(p_L), \sum_{\tau,\lambda=\pm}\left[b^\dagger \overline{u}(\mathbf{p},\lambda)e^{ipx} + d\overline{v}(\mathbf{p},\lambda)e^{-ipx}\right]_{|\Sigma}, \tag{2.9}$$

where λ is a spin projection on a specified direction in Σ, τ represents other quark quantum numbers, i.e., flavor and color, m is the constituent quark mass, and u, v are the Dirac spinors normalized according to $\overline{u}u = 1$. The quark and antiquark ($q\overline{q}$) momentum space wave function $\Psi_{NJJ_3\alpha}$ is defined as the probability amplitude for finding the $q\overline{q}$ pair with given individual $(\mathbf{p}_i, \lambda_i, \tau_i)$ quantum numbers in a meson state $|\mathbf{P}, N, J, J_3, \alpha\rangle$ which belongs to an irreducible representation of the Poincaré group characterized by the meson mass M_N, total angular momentum J, J_3, and other quantum numbers (flavor, orbital angular momentum, and spin of the $q\overline{q}$) denoted collectively by α ($J, J_3 = 0$ and $\alpha = 1,\ldots,8$ specify the low-lying octet state):

$$|\mathbf{P}, N, J, J_3\alpha\rangle = \sum_{\lambda,\tau}\int [d\mathbf{p}_i]_{\mathbf{P}}\, \Psi_{NJJ_3\alpha}(\mathbf{p}_i,\lambda_i,\tau_i)|\mathbf{p}_1,\lambda_1,\tau_1;\mathbf{p}_2,\lambda_2,\tau_2\rangle, \tag{2.10}$$

where

$$[d\mathbf{p}_i]_{\mathbf{P}} \equiv \frac{d^3\mathbf{p}_1}{(2\pi)^3\sqrt{p_{1L}}}\frac{d^3\mathbf{p}_2}{(2\pi)^3\sqrt{p_{2L}}}\sqrt{P_L\mathcal{E}}(2\pi)^3\delta^3\left(\mathbf{P}-\sum_i^2\mathbf{p}_i\right), \tag{2.11}$$

$p_{iL} = p_{iL}(|\mathbf{p}|_i) = \sqrt{m_i^2+\mathbf{p}_i^2}$, $\mathcal{E} = p_{1L}+p_{2L}$, and $P_L = \sqrt{M_N^2+\mathbf{P}^2}$ is the meson *energy*. The particular form of the measure, $[d\mathbf{p}_i]$, will be discussed in the next section. Combining Eqs. (2.1), (2.9), and (2.10) yields

$$\begin{aligned}\mathcal{A}_1 &= \langle 0|\overline{\psi}(0)\Gamma\psi(0)|\mathbf{P},N,J,J_3,\alpha\rangle \\ &= \sum_{\lambda,\tau}\int[d\mathbf{p}_i]_{\mathbf{P}}\Psi_{NJJ_3\alpha}(\mathbf{p}_i,\lambda_i,\tau_i)\left[\sqrt{\frac{m_1}{p_{1L}}}\overline{v}(\mathbf{p}_2,\lambda_2)\Gamma_{\tau_1,\tau_2}\sqrt{\frac{m_2}{p_{2L}}}u(\mathbf{p}_1,\lambda_1)\right],\end{aligned} \tag{2.12}$$

$$\begin{aligned}\mathcal{A}_2 &= \langle \mathbf{P}',N',J',J_3'\alpha'|\overline{\psi}(0)\Gamma\psi(0)|\mathbf{P},N,J,J_3\alpha\rangle \\ &= \sum_{\lambda\lambda'\tau\tau'}\int[d\mathbf{p}_i']_{\mathbf{P}'}[d\mathbf{p}_i]_{\mathbf{P}}\Psi^\dagger_{N'J'J_3'\alpha'}(\mathbf{p}_i',\lambda_i',\tau_i')\Psi_{NJJ_3\alpha}(\mathbf{p}_i,\lambda_i,\tau_i) \\ &\times\left[\left(\sqrt{\frac{m_1}{p_{1L}'}}\overline{u}(\mathbf{p}_1',\lambda_1')\Gamma_{\tau_1',\tau_1}\sqrt{\frac{m_1}{p_{1L}}}u(\mathbf{p}_1,\lambda_1)\right)\delta_{\tau_2'\tau_2}\delta_{\lambda_2'\lambda_2}(2\pi)^3\delta^3(\mathbf{p}_2'-\mathbf{p}_2)\right. \\ &-\left.\left[\sqrt{\frac{m_1}{p_{2L}'}}\overline{v}(\mathbf{p}_2',\lambda_2')\Gamma_{\tau_2',\tau_2}\sqrt{\frac{m_2}{p_{2L}}}v(\mathbf{p}_2,\lambda_2)\right]\delta_{\tau_1'\tau_1}\delta_{\lambda_1'\lambda_1}(2\pi)^3\delta^3(\mathbf{p}_1'-\mathbf{p}_1)\right],\end{aligned} \tag{2.13}$$

where pointlike constituent quarks have been assumed [2,3].

As mentioned in the Introduction the current matrix elements can be written as explicit four-dimensional integrals depending on the physical momenta and n^μ. This is achieved by writing

$$\begin{aligned}\delta^3(\mathbf{p}) &= \int d\xi\delta^4(p-\xi n), \\ d^3\mathbf{p}_i &= d^4p_i\delta(p_{iL})\end{aligned} \tag{2.14}$$

in Eqs. (2.12) and (2.13).

III. COVARIANCE

Under a Poincaré transformation, hadronic states should transform as an element of a unitary representation while a matrix element is expected to transform covariantly (i.e., consistent with the tensorial rank of the matrix element operator).

A. Hadronic states

In the constituent quark model the meson and baryon mass spectrum corresponds to eigenvalues of a phenomenological Hamiltonian. The phenomenological description of hadrons as the few-body bound states can be extended to satisfy the requirements of Poincaré symmetry by an explicit construction of interaction-dependent generators of the symmetry group. The method, for a fixed number of constituents, has been introduced by Bakamjian and Thomas [9], and extensively discussed by Osborn [10] and Foldy and Krajcik [11]. First, an interaction-independent set of internal and center of mass (c.m.) variables is introduced and the product of noninteracting generators for individual constituents is cast into a free, single-particle form that specifies symmetry properties for the system as a whole. Since the free mass of the constituents in the single-particle representation enters only as a function of internal variables, the generators for an interacting system are modeled by replacing the free mass by an interaction dependent operator. In particular for two spin-1/2 particles with individual momenta $\mathbf{p}_i$ and spins $\mathbf{s}_i$, the relative momentum $\mathbf{k}$ and c.m. momentum $\mathbf{P}$ are defined by

$$\mathbf{p}_{1,2} \equiv \pm\mathbf{k} \pm \frac{(\mathbf{k}\cdot\mathbf{P})}{\mathcal{M}(\mathcal{E}+\mathcal{M})}\mathbf{P} + \frac{\omega_{1,2}}{\mathcal{M}}\mathbf{P}, \tag{3.1}$$

with $\omega_{1,2} = \omega_{1,2}(|\mathbf{k}|) = \sqrt{m_{1,2}^2+\mathbf{k}^2}$, $\mathcal{M} = \mathcal{M}(|\mathbf{k}|) = \omega_1+\omega_2$, and $\mathcal{E} = \mathcal{E}(|\mathbf{k}|,|\mathbf{P}|) = \sqrt{\mathcal{M}^2+\mathbf{P}^2}$. With the components $[\mathbf{s}']_{\sigma\sigma'}$ of the c.m spin operators $\mathbf{s}_i'$ obtained through an interaction-independent Lorentz boost to the c.m. frame given by

$$[\mathbf{s}_i']_{\sigma\sigma'} \equiv \sum_{\lambda\lambda'}D(\mathbf{p}_i,\tilde{\mathbf{p}}_i)_{\sigma\lambda}[\mathbf{s}_i]_{\lambda\lambda'}D^\dagger(\mathbf{p}_i,\tilde{\mathbf{p}}_i)_{\lambda'\sigma'}, \tag{3.2}$$

where the Wigner rotations are given by ($\tilde{\mathbf{p}}_i \equiv \pm\mathbf{k}$)

$$D(\mathbf{p}_i,\tilde{\mathbf{p}}_i)_{\sigma\lambda} = \frac{[(p_{iL}+m_i)(\omega_i+m_i)+\mathbf{p}_i\cdot\tilde{\mathbf{p}}_i]\delta_{\sigma\lambda}+i[\sigma]_{\sigma\lambda}\cdot[\mathbf{p}_i,\tilde{\mathbf{p}}_i]}{\sqrt{2(p_{iL}+m_i)(\omega_i+m_i)(p_{iL}\omega_i+\mathbf{p}_i\cdot\tilde{\mathbf{p}}_i+m_i^2)}}, \tag{3.3}$$

the ten generators of the Poincaré group, $M_{\mu\nu}$, P^μ, $K_T = (0,\mathbf{K}) \equiv (0, M_{L,i})$, $J_T = (0,\mathbf{J}) = (0, \frac{1}{2}\epsilon_{T,ijk}(g_T^t \cdot M \cdot g_T)_{jk})$ [g_T and ϵ_T being defined in Eq. (2.7)], for an interacting system are constructed as

$$\begin{aligned}
\mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2,\\
P_L &= \sqrt{\tilde{\mathcal{M}}^2 + \mathbf{P}^2},\\
\mathbf{J} &= \left[i\frac{\partial}{\partial \mathbf{P}}, \mathbf{P}\right] + \mathbf{S}' + \mathbf{L}',\ \mathbf{S}' = \mathbf{s}_1' + \mathbf{s}_2',\ \mathbf{L}' = \left[i\frac{\partial}{\partial \mathbf{k}}, \mathbf{k}\right],\\
\mathbf{K} &= -\frac{1}{2}\left\{i\frac{\partial}{\partial \mathbf{P}}, P_L\right\} - \frac{1}{\tilde{\mathcal{M}} + P_L}[\mathbf{P}, \mathbf{S}'],
\end{aligned} \tag{3.4}$$

with

$$\tilde{\mathcal{M}} = \mathcal{M} + V. \tag{3.5}$$

In general, as long as $n^\mu \neq (1,0,0,0)$, the generators are not the usual Cartesian generators. The states are eigenstates of the Casimir operators M^2 and W^2, the latter being the square of the Pauli-Lubanski vector

$$W_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}, \tag{3.6}$$

whose components in the rest frame of a particle (now for an arbitrary n^μ defined as $\mathbf{P} = \mathbf{0}$) are given by

$$\begin{aligned}
W_\mu &= (0, \mathbf{W}_T) = (0, W_i),\\
W_i &= \frac{1}{2}\epsilon_{T,ijk}M_{ik} = MJ_i,
\end{aligned}$$

and are also labeled by the eigenvalues of $\mathbf{P}$ and a eigenvalue of a component of $\mathbf{J}$. States characterized by the same eigenvalue of M and W in two different quantization schemes, characterized by vectors n' and n, respectively, are related to each other by means of the Wigner rotation given in Eq. (3.3). In particular for $n'^\mu = n^\mu_{IN} \equiv (1,0,0,0)$ (instant) and $n^\mu = n^\mu_{LC} \equiv (\cos(\alpha), 0, 0, -\sin(\alpha))/\sqrt{\cos(2\alpha)}$ in the limit $\epsilon \to 0$ when $\alpha \to 45° - \epsilon$ (light cone) the Wigner rotation in Eq. (3.3) reduces to the Melosh rotation. From the first two equations of Eqs. (3.4) this limit also yields

$$\frac{P^0 + P^3}{2\sqrt{\epsilon}} + (P^0 - P^3)\frac{\sqrt{\epsilon}}{2} = \frac{1}{2\sqrt{\epsilon}}\sum_{i=1}^{2}(p_i^0 + p_i^3) + \frac{M^2 + (\sum_{i=1}^2 p_{\perp i})^2}{\sum_{i=1}^2 (p_i^0 + p_i^3)}\frac{\sqrt{\epsilon}}{2} + O(\epsilon), \tag{3.7}$$

thus, taking $\epsilon \to 0$

$$\begin{aligned}
P_\perp &= \sum_{i=1}^{2} p_{\perp i},\\
P^+ \equiv P^0 + P^3 &= \sum_{i=1}^{2}(p_i^0 + p_i^3),\\
P^- \equiv P^0 - P^3 &= \frac{M^2 + P_\perp^2}{P^+},
\end{aligned}$$

and thus the generator P^+ becomes interaction independent while the interactions are contained in P^- in agreement with the standard light cone quantization rules. Similarly it can be shown that the generators $\mathbf{J}$ and $\mathbf{K}$ are mapped into the standard light cone generators [12,14].

The restrictions on the potential V which can be either local or nonlocal are that it commutes with $\mathbf{J}$, $\mathbf{P}$, and $i\nabla_{\mathbf{P}}$. The basis of an unitary representation of the Poincaré algebra given in Eq. (3.4) is obtained from the set of eigenstates of $\tilde{\mathcal{M}}$, and the basis of the algebra for the noninteracting system. The reducible (with respect to $\mathbf{J}$) representation for the noninteracting system can be expressed in either individual, $|\mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2\rangle$, or relative and c.m. variables, $|\mathbf{P}, \mathbf{k}, \sigma_1, \sigma_2\rangle$, related by

$$\begin{aligned}
\langle \mathbf{P}, \mathbf{k}, \sigma_1, \sigma_2 | \mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2\rangle &= D^\dagger(\mathbf{p}_1, \tilde{\mathbf{p}}_1)_{\lambda_1,\sigma_1} D^\dagger(\mathbf{p}_2, \tilde{\mathbf{p}}_2)_{\lambda_2,\sigma_2}\\
&\quad\times 2\frac{p_{L1}}{\sqrt{m_1}}(2\pi)^3\delta^3(\mathbf{p}_1 - \mathbf{p}_1(\mathbf{P},\mathbf{k}))\frac{p_{L2}}{\sqrt{m_2}}(2\pi)^3\delta^3(\mathbf{p}_2 - \mathbf{p}_2(\mathbf{P},\mathbf{k})),
\end{aligned} \tag{3.8}$$

with $\mathbf{p}_i(\mathbf{P},\mathbf{k})$ given by Eq. (3.1). The states are normalized according to

$$\begin{aligned}
\langle \mathbf{P}', \mathbf{k}', \sigma_1', \sigma_2' | \mathbf{P}, \mathbf{k}, \sigma_1, \sigma_2\rangle &= 2(2\pi)^3\mathcal{E}\delta^3(\mathbf{P}' - \mathbf{P})\rho^{-1}(|\mathbf{k}|)\delta^3(\mathbf{k}' - \mathbf{k})\delta_{\sigma_1'\sigma_1}\delta_{\sigma_2'\sigma_2},\\
\langle \mathbf{p}_1', \lambda_1'; \mathbf{p}_2', \lambda_2' | \mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2\rangle &= \frac{p_{L1}}{m_1}(2\pi)^3\delta^3(\mathbf{p}_1' - \mathbf{p}_1)\frac{p_{L2}}{m_2}(2\pi)^3\delta^3(\mathbf{p}_2' - \mathbf{p}_2)\delta_{\lambda_1'\lambda_1}\delta_{\lambda_2'\lambda_2},
\end{aligned} \tag{3.9}$$

where the factor

$$\rho(|\mathbf{k}|) = \frac{\mathcal{M}}{2(2\pi)^3\omega_1\omega_2} \tag{3.10}$$

assures the unitarity of noninteracting generators in the relative and c.m. variables representation [19,20]. For the time being we have neglected the flavor and color quantum numbers. The irreducible representation classified by the eigenstates of $\mathcal{M}$, $\mathbf{P}$, $\mathbf{J}^2$, $\mathbf{J}_3$, $\mathbf{L}'^2$, and $\mathbf{S}'^2$ is obtained by projecting $|\mathbf{P},\mathbf{k},\sigma_1,\sigma_2\rangle$ onto eigenstates of spin and angular momentum

$$|\mathbf{P},|\mathbf{k}|,J,J_3,L',S'\rangle = \sum_{\sigma_1\sigma_2 S_3' L_3'} \int \frac{d\Omega_{\mathbf{k}}}{4\pi} |\mathbf{P},\mathbf{k},\sigma_1,\sigma_2\rangle Y_{L'L_3'}\left(\frac{\mathbf{k}}{|\mathbf{k}|}\right) C^{S'S_3'}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2} C^{JJ_3}_{S'S_3'L'L_3'}. \tag{3.11}$$

The eigenvalue equation for the interacting mass operator can be written in the form

$$M_N\psi_N(\mathbf{k},\sigma_1,\sigma_2) = \sum_{\sigma_1'\sigma_2'} \int d^3\mathbf{k}'\rho(|\mathbf{k}'|)\tilde{M}_{\sigma_1\sigma_1',\sigma_2\sigma_2'}(\mathbf{k}',\mathbf{k})\psi_N(\mathbf{k}',\sigma_1',\sigma_2'), \tag{3.12}$$

with N denoting the total (radial and orbital) quantum number,

$$2(2\pi)^3\mathcal{E}\delta^3(\mathbf{P}'-\mathbf{P})\psi_N(\mathbf{k},\sigma_1\sigma_2) = \langle \mathbf{P},\mathbf{k},\sigma_1,\sigma_2|\mathbf{P}',N\rangle, \tag{3.13}$$

and $\tilde{M}_{\sigma_1\sigma_1',\sigma_2\sigma_2'}$ defined by

$$2(2\pi)^3\mathcal{E}\delta^3(\mathbf{P}'-\mathbf{P})\tilde{M}_{\sigma_1\sigma_1',\sigma_2\sigma_2'}(\mathbf{k}',\mathbf{k}) = \langle \mathbf{P}',\mathbf{k}',\sigma_1',\sigma_2'|\tilde{\mathcal{M}}|\mathbf{P},\mathbf{k},\sigma_1,\sigma_2\rangle. \tag{3.14}$$

For example, the harmonic oscillator basis $|\mathbf{P},N=n_1+n_2+n_3=2n+l\rangle$ results from the mass operator

$$\tilde{M}_{\sigma_1\sigma_1',\sigma_2\sigma_2'}(\mathbf{k}',\mathbf{k}) = \rho^{-1/2}(|\mathbf{k}'|)\left[\left(\frac{\mathbf{k}^2}{2\mu}+\frac{\mu\omega^2}{2}(i\nabla_{\mathbf{k}})^2\right)\delta^3(\mathbf{k}'-\mathbf{k})\right]\delta_{\sigma_1'\sigma_1}\delta_{\sigma_2'\sigma_2}\rho^{-1/2}(|\mathbf{k}|), \tag{3.15}$$

with the reduced mass, $\mu = m_1m_2/(m_1+m_2)$, and leads to

$$\psi_N(\mathbf{k},\sigma_1,\sigma_2) = \rho^{-1/2}(|\mathbf{k}|)\psi_N^{\rm nr}(\mathbf{k},\sigma_1\sigma_2), \tag{3.16}$$

where $\psi^{\rm nr}$ is the solution of the nonrelativistic harmonic oscillator. The two factors of $\rho^{-1/2}$ in Eq. (3.15) assure Hermiticity of the mass operator in the relativistic norm in Eq. (3.9). Since $[M,\mathbf{J}]=0$, the eigenstates of the mass operator can be classified by J and J_3 and in the $L'S'$ coupling scheme the irreducible representation of the interacting algebra is spanned by

$$|\mathbf{P},NJ,J_3,L',S'\rangle = \int d^3\mathbf{k}\rho(|\mathbf{k}|)\frac{d^3\mathbf{P}}{2(2\pi^3)\mathcal{E}}2(2\pi^3)\mathcal{E}\delta^3(\mathbf{P}-\mathbf{P}')|\mathbf{P},|\mathbf{k}|,J,J^3,L',S'\rangle\sqrt{\frac{P_L}{\mathcal{E}}}\psi_{NJJ^3L'S'}(|\mathbf{k}|), \tag{3.17}$$

with the wave function $\psi_{NJJ^3L'S'}(|\mathbf{k}|)$ related to $\psi_N(\mathbf{k},\sigma_1,\sigma_2)$ by

$$\psi_{NJJ^3L'S'}(|\mathbf{k}|) = \sum_{\sigma_1\sigma_2S_3'L_3'}\int\frac{d\Omega_{\mathbf{k}}}{4\pi}\psi_N(\mathbf{k},\sigma_1,\sigma_2)Y_{L'L_3'}\left(\frac{\mathbf{k}}{|\mathbf{k}|}\right)C^{S'S_3'}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{JJ_3}_{S'S_3'L'L_3'}. \tag{3.18}$$

The square root factor in front of ψ in Eq. (3.17) guarantees unitarity of generators of the interacting algebra in the covariant norm

$$\langle \mathbf{P}',N'J',J_3',\alpha'|\mathbf{P},NJ,J_3,\alpha\rangle = 2(2\pi)^3P_L\delta^3(\mathbf{P}'-\mathbf{P})\delta_{N'N}\delta_{J'J}\delta_{J_3'J_3}\delta_{\alpha'\alpha}. \tag{3.19}$$

Using Eqs. (3.11), (3.9), and (3.8) the mass operator eigenstate in Eq. (3.17) may be expanded in terms of single-particle states. After including flavor and color quantum numbers it has the form [$\alpha \equiv (a,L'S')$, $a=1,\ldots,8$ for the SU(3) flavor octet and $L'=S'=0$ for the lowest-lying pseudoscalars]

$$\begin{aligned}|\mathbf{P},NJ,J_3,\alpha\rangle = \sum_{\lambda_i\tau_i}\int &\frac{d^3\mathbf{p}_1}{(2\pi)^3\sqrt{p_{1L}}}\frac{d^3\mathbf{p}_2}{(2\pi)^3\sqrt{p_{2L}}}(2\pi)^3\sqrt{P_L\mathcal{E}}\delta^3(\mathbf{P}-\mathbf{p}_1-\mathbf{p}_2)\\ &\times\Bigg[\left(\frac{\lambda^a}{\sqrt{2}}\otimes\frac{I}{\sqrt{3}}\right)_{\tau_1\tau_2}\sum_{\sigma_i,S_3',L_3'}D^\dagger(\mathbf{p}_1,\tilde{\mathbf{p}}_1)_{\lambda_1,\sigma_1}D^\dagger(\mathbf{p}_2,\tilde{\mathbf{p}}_2)_{\lambda_2,\sigma_2}Y_{L'L'^3}\left(\frac{\mathbf{k}}{|\mathbf{k}|}\right)\\ &\times C^{S'S_3'}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}C^{JJ_3}_{S'S_3'L'L_3'}\sqrt{\frac{m_1m_2}{p_{1L}p_{2L}}}\psi_{NJJ_3\alpha}(|\mathbf{k}|)\Bigg]|\mathbf{p}_1,\lambda_1,\tau_1;\mathbf{p}_2,\lambda_2,\tau_2\rangle,\end{aligned} \tag{3.20}$$

with $\mathbf{k}$ being a function of $\mathbf{p}_i$ determined from Eq. (3.1), λ^a denoting the Gell-Mann matrices in the SU(3) flavor space, and I being an identity in the color space. Comparing Eq. (3.20) with Eq. (2.10) the $q\bar{q}$ wave function $\Phi_{NJJ_3\alpha}$ is easily identified with the term in the square bracket in Eq. (3.20) . The covariant normalization in Eq. (3.19) follows from the normalization of the mass eigenstates:

$$\langle N|M\rangle = \delta_{NM}, \tag{3.21}$$

which in turn is equivalent to the normalization of eigenstates of a "nonrelativistic Hamiltonian" defined as $\rho^{1/2}(|\mathbf{k}'|)\tilde{\mathcal{M}}(\mathbf{k}',\mathbf{k})\rho^{1/2}(|\mathbf{k}|)$.

B. Matrix elements

Covariance under a Lorentz transformation Λ for a matrix element, say, $\mathcal{A}_2$, evaluated for a current J with k-Lorentz indices means

$$\begin{aligned}\mathcal{A}_2^{\mu_1\cdots\mu_k}(P',P) &= \langle P'|J^{\mu_1\cdots\mu_k}(0)|P\rangle \\ &= \Lambda^{\mu_1}_{\nu_1}\cdots\Lambda^{\mu_k}_{\nu_k}\mathcal{A}_2^{\nu_1\cdots\nu_k}(\Lambda^{-1}P',\Lambda^{-1}P).\end{aligned} \tag{3.22}$$

If $U(\Lambda)$ are the unitary operators corresponding to the transformation Λ in the space of physical states $|P\rangle$, Eq. (3.22) demands

$$U^{-1}(\Lambda)J^{\mu_1\cdots\mu_k}(0)U(\Lambda) = \Lambda^{\mu_1}_{\nu_1}\cdots\Lambda^{\mu_k}_{\nu_k}J^{\mu_1\cdots\mu_k}(0) \tag{3.23}$$

or, in infinitesimal form,

$$\begin{aligned}[J^{\mu_1\cdots\mu_k}(0),M_{\alpha,\beta}] = i\sum_i^k &\delta^{\mu_i}_{\alpha}J^{\mu_1\cdots\beta\cdots\mu_k}(0) \\ &-\delta^{\mu_i}_{\beta}J^{\mu_1\cdots\alpha\cdots\mu_k}(0).\end{aligned} \tag{3.24}$$

These conditions are not satisfied by the free field, one-body current operators used in Sec. II because, as described in a previous subsection, the generators $M_{\mu\nu}$ contain interactions. Thus two-particle and perhaps even more complex currents must be included to restore covariance. Consequently, hadronic matrix elements are expressed in terms of four-vectors representing the particles momenta and polarizations will violate covariance and as shown in Sec. II will contain an overall dependence on the four-vector n^μ. The nonrelativistic expansion of the electromagnetic currents in the presence of internal interactions has been extensively studied [11,21]; however, a solution to all orders in V/m required to maintain covariance is still lacking. Although the spurious n dependence of matrix elements results from the Lorentz symmetry breaking, it permits us to assess quantitatively the extent covariance is violated in a model calculation and, as detailed in the next section, to establish a potentially useful model criterion.

IV. NUMERICAL RESULTS

As mentioned in the Introduction, the valence, constituent quark model does not reflect the Goldstone nature of the pion. This paper does not address the relation between the quark model pion mass and the physical Goldstone boson mass. Rather, the main purpose is to examine the validity of the impulse approximation with pointlike constituents in the quark model analysis of hadronic matrix elements.

All key observables for the low-lying pseudoscalar octet mesons, $J^P = 0^-$, can be obtained from the matrix elements $\mathcal{A}_i$. Of special interest are the meson decay constants and form factors. For the decay constant $\mathcal{A}_1$ is used with the SU(3) octet axial vector current, $A^{a,\mu}(0) = \overline{\psi}(0)\lambda^a/2\gamma^\mu\gamma^5\psi(0)$ while for the electromagnetic form factors the current in $\mathcal{A}_2$ is given by $J^\mu_{em}(0) = \overline{\psi}(0)[\lambda^8/\sqrt{3}+\lambda^3]\gamma^\mu\psi(0)$. Because of the presence of n^μ, the matrix elements written in four-vector notation will in general acquire additional form factors in their Lorentz decomposition. Accordingly, the two matrix elements $\mathcal{A}_i$ can be written as $|\mathbf{P},a\rangle = |\mathbf{P},M_0,0,0,(a,0,0)\rangle$ in the notation of Eqs. (2.10) and

$$\begin{aligned}\langle 0|A^{a\mu}(0)|\mathbf{P},b\rangle &= \delta_{ab}\left[P^\mu f_p + \frac{n^\mu}{|n|}f_n\right], \\ \langle \mathbf{P}',a|J^\mu_{\rm em}(0)|\mathbf{P},b\rangle &= \delta_{ab}\left[\Sigma^\mu F_p + \frac{n^\mu}{|n|}F_n\right].\end{aligned} \tag{4.1}$$

Covariance requires $f_n = F_n = 0$, $f_p = \text{const}$, and $F_p = F_p(Q^2)$, $Q^2 \equiv -(P'-P)^2$, $\Sigma^\mu \equiv (P'+P)^\mu$. Here, these conditions are not satisfied. We also define the *charge* form factors f_L and F_L as matrix elements of $n\cdot A/(n\cdot P)$ and $n\cdot J_{\rm em}/(n\cdot\Sigma)$, respectively. Thus

$$\begin{aligned}f_L &= f_p + \frac{1}{P_L}f_n, \\ F_L &= F_p + \frac{1}{\Sigma_L}F_n.\end{aligned} \tag{4.2}$$

For the ground state mesons having $N = L' = S' = J = 0$, after summing over the spin components σ_i, the $q\bar{q}$ wave function $\Psi_a = \Psi_{000a}$ can be written in a compact form

$$\Psi_a(\mathbf{p}_i,\lambda_i,\tau_i) = \left(\frac{\lambda^a}{\sqrt{2}}\otimes\frac{I}{\sqrt{3}}\right)_{\tau_1\tau_2}\frac{\sqrt{2}m_1m_2\overline{u}(\mathbf{p}_1,\lambda_1)\gamma_5C\overline{u}^T(\mathbf{p}_2,\lambda_2)}{\sqrt{p_{1L}p_{2L}[\mathcal{M}^2-(m_1-m_2)^2]}}\psi_0(|\mathbf{k}|), \tag{4.3}$$

with $p_{iL} = \sqrt{m_i^2+\mathbf{p}_i}$, $\mathcal{M}^2 = (p_{1L}+p_{2L})^2 - (\mathbf{p}_1+\mathbf{p}_2)^2$ and

$$|\mathbf{k}| = \left[\frac{(\mathcal{M}^2+(m_1-m_2)^2)[\mathcal{M}^2+(m_1+m_2)^2]}{4\mathcal{M}^2}\right]^{1/2}. \tag{4.4}$$

The radial wave function ψ_0 is taken to be the normalized ground state solution of the mass operator given in Eq. (3.15):

$$\psi_0(|\mathbf{k}|) = \frac{\rho(|\mathbf{k}|)^{-1/2}}{(\beta^2\pi)^{3/4}} \exp(-\mathbf{k}^2/2\beta^2), \tag{4.5}$$

where $\beta^2 = \mu\omega$ [μ is the reduced mass defined below Eq. (3.15)] has a weak quark mass dependence and typically 200 MeV $\lesssim \beta, m \lesssim$ 300 MeV.

In the light cone quantization it can be easily shown that in terms of the Minkowski coordinates the radial wave function depends on $x_i = p_i^+/P^+$ and $k_\perp = (k^x, k^y) = p_{1\perp} - x_1 P_\perp$ through $\mathcal{M}$ given by

$$\mathcal{M} = \sum_i \frac{m_i^2 + k_\perp^2}{x_i}. \tag{4.6}$$

The form factors defined in Eq. (4.1) can now be calculated using Eqs. (2.12) and (2.13):

$$f_p = f_p(\mathbf{P}^2) = f_p((P\cdot n)^2/n^2 - P^2) = \frac{\sqrt{3}}{\mathbf{P}^2}\int [d\mathbf{p}_i]_{\mathbf{P}} \frac{\psi(|\mathbf{k}|)}{\sqrt{\rho(|\mathbf{k}|)p_{1L}p_{2L}}} \frac{m_1\mathbf{p}_2\cdot\mathbf{P} + m_2\mathbf{p}_1\cdot\mathbf{P}}{\sqrt{[\mathcal{M}^2 - (m_1-m_2)^2]}}, \tag{4.7}$$

$$f_L = f_L(\mathbf{P}^2) = f_L((P\cdot n)^2/n^2 - P^2) = \frac{\sqrt{3}}{P_L}\int [d\mathbf{p}_i]_{\mathbf{P}} \frac{\psi(|\mathbf{k}|)}{\sqrt{\rho(|\mathbf{k}|)p_{1L}p_{2L}}} \frac{m_1 p_{2L} + m_2 p_{1L}}{\sqrt{[\mathcal{M}^2 - (m_1-m_2)^2]}}, \tag{4.8}$$

$$\begin{aligned} F_p &= F_p(\mathbf{Q}^2, \mathbf{\Sigma}^2) = F_p(Q^2, -(P'+P)^2 + [(P'+P)\cdot n]^2/n^2) \\ &= \frac{e_1}{2\mathbf{\Sigma}^2}\int [d\mathbf{p}_i]_{\mathbf{P}'}[d\mathbf{p}_i]_{\mathbf{P}'}\delta^3(\mathbf{p}_2' - \mathbf{p}_2)\frac{\psi_0(|\mathbf{k}'|)\psi_0(|\mathbf{k}|)}{\sqrt{\rho(|\mathbf{k}'|)\rho(|\mathbf{k}|)p_{1L}'p_{1L}}} \\ &\times \frac{\mathcal{M}^2\mathbf{p}_1'\cdot\mathbf{\Sigma} + \mathcal{M}'^2\mathbf{p}_1\cdot\mathbf{\Sigma} + ((p_{1L}'-p_{1L})^2 - \mathbf{Q}^2)\mathbf{p}_2\cdot\mathbf{\Sigma}}{\sqrt{[\mathcal{M}'^2 - (m_1-m_2)^2][\mathcal{M}^2-(m_1-m_2)^2]}} + (1\to 2), \end{aligned} \tag{4.9}$$

$$\begin{aligned} F_L &= F_L(\mathbf{Q}^2, \mathbf{\Sigma}^2) = F_L(Q^2, (P'+P)^2 - [(P'+P)\cdot n]^2/n^2) \\ &= \frac{e_1}{2\Sigma_L}\int [d\mathbf{p}_i]_{\mathbf{P}'}[d\mathbf{p}_i]_{\mathbf{P}'}\delta^3(\mathbf{p}_2' - \mathbf{p}_2)\frac{\psi_0(|\mathbf{k}'|)\psi_0(|\mathbf{k}|)}{\sqrt{\rho(|\mathbf{k}'|)\rho(|\mathbf{k}|)p_{1L}'p_{1L}}} \\ &\times \frac{\mathcal{M}^2 p_{1L}' + \mathcal{M}'^2 p_{1L} + ((p_{1L}'-p_{1L})^2 - \mathbf{Q}^2)p_{2L}}{\sqrt{[\mathcal{M}'^2 - (m_1-m_2)^2][\mathcal{M}^2-(m_1-m_2)^2]}} + (1\to 2), \end{aligned} \tag{4.10}$$

where e_i is the quark charge, $\mathbf{P}' = \mathbf{P} + \mathbf{Q}$, $p_{iL}' = p_{iL}(\mathbf{p}_i')$, $\mathcal{M}' = \mathcal{M}(\mathbf{p}_i')$ with p_{iL} and $\mathcal{M}$ defined below Eq. (4.3) and $\mathbf{k}'$, $\mathbf{k}$ are related to $\mathbf{p}_i'$ and $\mathbf{p}_i$ through Eq. (4.4). The only constraint imposed on the variables Σ, Q, and n is that the momentum transfer $P' - P$ be spacelike, $Q^2 > 0$, and $n\cdot Q = 0$, i.e., $Q^2 = \mathbf{Q}^2 = -Q_T^2$. From Eqs. (4.7)–(4.10) it follows that the form factors do not depend explicitly on n^2 but only on the magnitude of the transverse projections of external momenta. Violation of covariance can be studied and documented by this dependence. It is worth noting that the $|P_{iT}|$ dependence can be interpreted as either a quantization scheme or reference frame dependence. Sensitivity of the observables to changes in the reference frame corresponds to a dependence on the components of external momenta for fixed values of n^μ while sensitivity to the quantization surface corresponds to n^μ dependence for fixed values of the external momenta similarly to the effect of passive vs active Lorentz transformations. For example, the $-P_{iT}^2 = (n\cdot P_i)^2/n^2 - M_i^2 \to \infty$ limit gives a form factor in the *instant* $n^2 = 1$ quantization in a frame where some components of particle momenta are infinite [14] or a form factor in a *light cone* quantization, $n^2 \to 0$, in a frame with finite components of the momenta [12]. It can be shown that in the $P_{iT}^2 \to \infty$ limit Eqs. (4.7)–(4.10) become identical to those derived using explicit *light cone* quantization. The form factors are finite in this limit and thus the unphysical dependence upon P_{iT}^2 disappears. Also from Eq. (4.2) it follows that in this limit $f(F)_L \to f(F)_P$. However, there is *a priori* no reason to expect that the unphysical form factors $[f(F)_n]$ vanish.

In Fig. 1 the form factors f_L (solid line), f_p (dashed line), and f_n (dotted line) for the pion are plotted against $|P_T|$ for m = 220 MeV and β = 290 MeV. As expected, for large $|P_T|$, $f_L \to f_p \to$ const and $f_n \to$ const; however, the unphysical form factor f_n becomes the largest in the *light cone* limit $f_n(\infty)/f_p(\infty) \sim 70\%$. At low $|P_T|$, $f_n(0)/f_L(0) \sim 35\%$, but the variation of $f_p(|P_T|)$ is now the largest. Similar results are obtained for the electromagnetic form factor, as shown in Fig. 2. The upper solid line represents $F_{p\pi}^\infty(Q^2) \equiv F_{p\pi}(Q^2,\infty)$, while $F_{p\pi}^0(Q^2) \equiv$

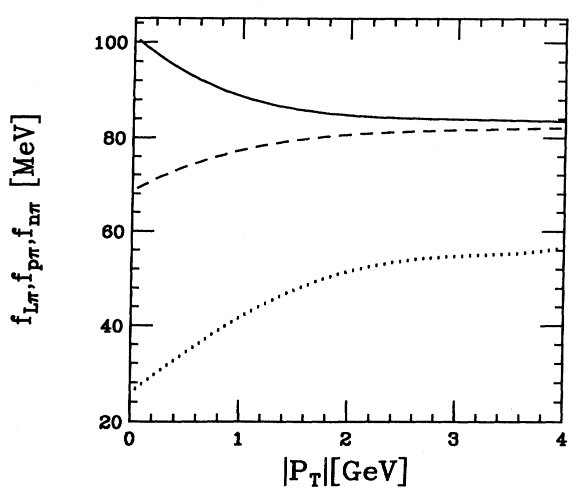

FIG. 1. Quantization surface and/or frame dependence of pion decay constant. $f_{L\pi}$, $f_{p\pi}$, and $f_{n\pi}$ are denoted by solid, dashed, and dotted curves, respectively.

$F_{p\pi}(Q^2, 0)$ is depicted by the lower solid line. The upper dashed line corresponds to $F_{n\pi}^{\infty}(Q^2) \equiv F_{n\pi}(Q^2, \infty)$ while the lower dashed line is $F_{n\pi}^{0}(Q^2) \equiv F_{n\pi}(Q^2, 0)$. There is little of the order of 10–20% sensitivity to $|P_T|$ for both F_P and F_N and similarly for f_p and f_n. Again the nonphysical form factor $F_{n\pi}$ is comparable to $F_{p\pi}$ and largest in the $|P_T| \to \infty$ limit ($F_{n\pi}^{\infty}$). In Figs. 3–5 the charge form factors (F_L) for π, K^+, and K^0 are plotted for $|P_T = 0|$ (dashed lines) and for $|P_T| \to \infty$ (solid lines). The kaon decay constant and charge radii can be well reproduced for the same parametrization $m \sim 220$ MeV and $\beta \sim 290$ MeV within a 10% accuracy. Significantly and related, the overall model sensitivity due to quantization or frame dependence, as detailed from the figures, is also at this level. For additional insight we also have compared our results for the pion form factor with the previous light cone analyses of Dziembowski *et al.* [2] [dashed-dotted line in Fig. 3(b)] and to the nonrelativistic description [dotted line in Fig. 3(b)]. In Ref. [2], an ansatz is taken for the light cone valence wave function. The main difference between the wave function in Eq. (4.3) and the wave function used in Ref. [2] is the use of a constant value instead of the invariant quark mass $\mathcal{M}$ in the spinor wave functions in Ref. [2]. While this difference is not significant for the description of static properties, it is quite substantial for form factors for $Q^2 \gtrsim 1$ GeV^2.

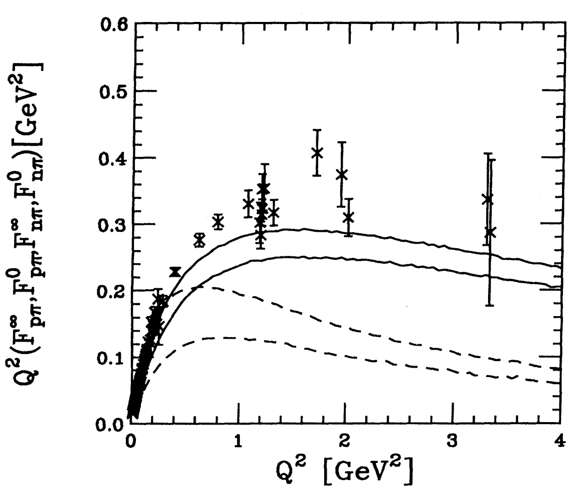

FIG. 2. Quantization surface and/or frame dependence of pion electromagnetic form factor. The assignment of curves is explained in the text.

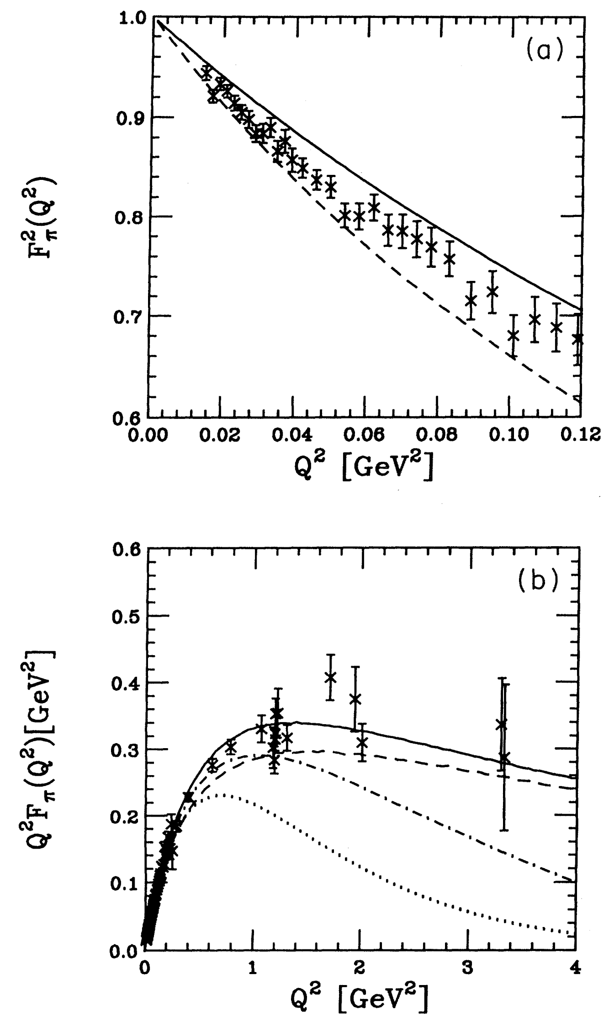

FIG. 3. Pion electromagnetic form factor for spacelike $q^2 = -Q^2$ momentum transfer. Data were taken from Ref. [24].

V. SUMMARY AND CONCLUSIONS

We have developed a variable front approach for investigating hadronic structure involving matrix elements

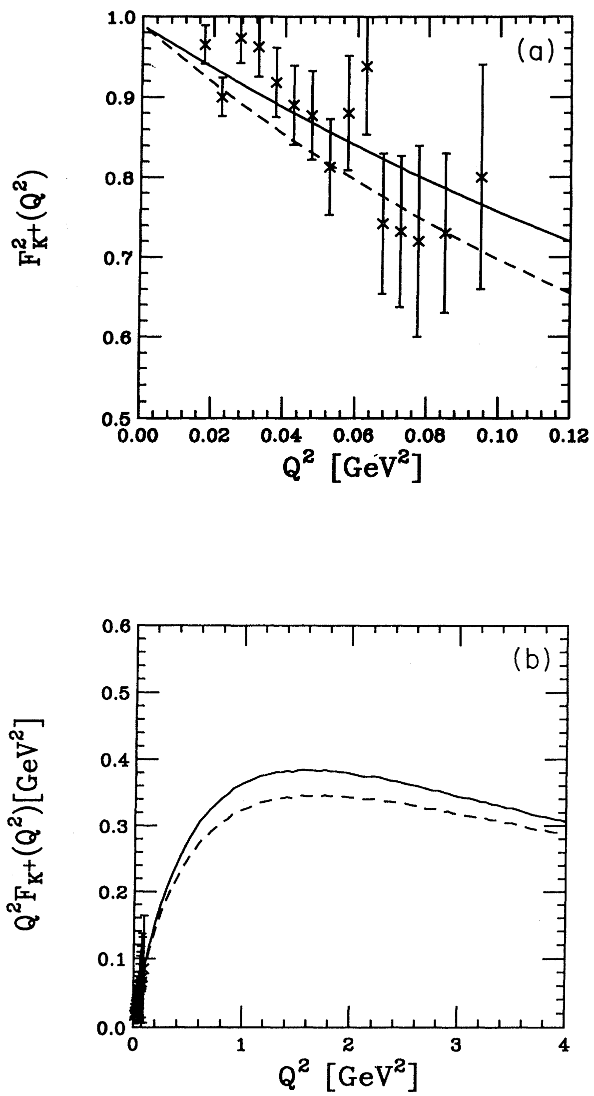

FIG. 4. K^+ electromagnetic form factor ($m_s = 480$ MeV). Data are taken from Ref. [25].

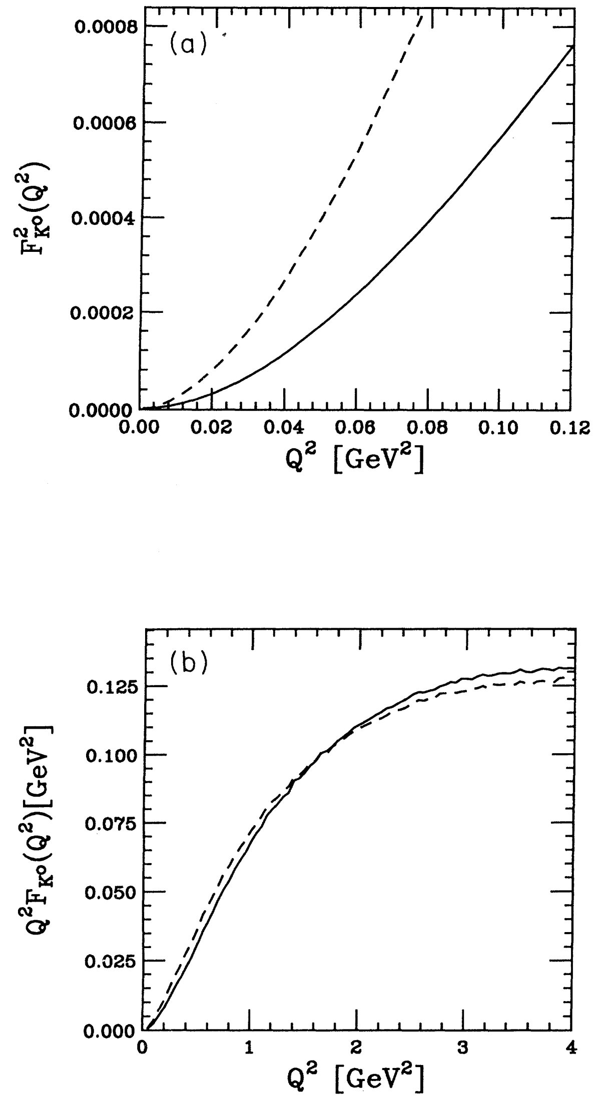

FIG. 5. K^0 electromagnetic form factor.

of constituent quark operators. The source of Lorentz symmetry breaking of the current matrix elements comes from the improper behavior of single-particle currents under Lorentz symmetry. Many successful predictions from the constituent quark model, including early description of baryon magnetic moments, do, however, rely on one-body currents only. Thus there is hope that such currents may give dominant contributions at least for the static properties. Our model incorporates all relevant symmetries of the wave function by construction and permits one to assess the sensitivity of computed observables to the choice in quantization scheme or reference frame. As such our method can be regarded as a covariant criteria since it can be implemented in any quark model to detail sensitivity to front orientation. A large sensitivity indicates the need for restoring full covariance and perhaps a more sophisticated hadronic model. We have calculated various mesonic properties in this model and compared the results in different schemes. The key findings are that relativity is crucial but that alternative relativistic approaches (different n^μ) which properly treat Wigner spin rotations and appropriate *Ansätze* for the relative, spin-independent wave function can achieve equivalent phenomenological descriptions at least for the present quality of data. For π and K^+ we have found that both the *physical* form factors $[f(F)_p]$ and the charge form factors $f(F)_L$ for π are comparable to within 10–20 % which is also the sensitivity level to the front orientation or to the choice in the reference frame. Thus ignoring the spurious form factors, covariance imposes a $\sim$ 20% uncertainty in the applicability of the quark model. In the case of the K^0 form factor at low Q^2, violations of covariance are much larger. In general, we expect the quantities which are intrinsically small to be much more sensitive to model details and uncertainties as similarly concluded in a recent $E2/M1$ N-Δ transition analysis [22]. In the light cone limit as the momentum transfer increases the unphysical form factors become largest. Such behavior is also consistent with recent analysis by Keister [23] of current matrix elements for a spin-1 particle where significant rotational symmetry violations occur for $Q^2/4M^2 \gtrsim 1$.

Although encouraging, small violations of covariance found in the physical form factors are necessary but not sufficient to validate the quark model calculations. Complete confirmation awaits demonstrating that interaction-dependent current operators reduce or eliminate the unphysical pieces of matrix elements without appreciably affecting the physical ones. Ideally, as more precise data become available it may be possible to determine the form of effective current operators and distinguish alternative formulations along with clarifying the role of additional degrees of freedom such as effective gluonic excitations and/or exotic quark configurations, as in principle expected in QCD.

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