PHYSICAL REVIEW D VOLUME 52, NUMBER 9 1 NOVEMBER 1995

Gluon plasma with a medium-dependent dispersion relation

Mark I. Gorenstein* and Shin Nan Yang
Department of Physics, National Taiwan University, Taipei, Taiwan 10764, Republic of China

(Received 16 January 1995; revised manuscript received 17 May 1995)

The statistical mechanics for systems with a medium-dependent dispersion relation are formulated and applied to construct a model for the gluon plasma equation of state with a temperature-dependent particle mass. This simple model gives us an example of a system with temperature-dependent effective Hamiltonian. We find that to satisfy the thermodynamical relations in these systems, standard statistical mechanics formulas have to be supplemented by special requirements. The self-consistent statistical model formulation is used to describe Monte Carlo lattice data for the thermodynamical functions of SU(2) and SU(3) gluon plasma.

PACS number(s): 12.38.Mh, 11.55.Fv, 12.40.Ee

I. INTRODUCTION

Recently a phenomenological model of the gluon plasma (GP) was considered [1,2] to describe the Monte Carlo (MC) lattice data in SU(2) and SU(3) gluodynamics for the thermodynamical functions of the deconfined phase, i.e., at temperatures T above the critical value T_c. It was assumed that the system of interacting massless gluons (the same arguments can be applied when quarks are included [2]) can be effectively represented at $T > T_c$ as an ideal gas of "massive" noninteracting "gluons" (and "quarks"). The gluon dispersion relation for a particle of energy ω^* and momenta k in such a system is assumed to be

$$\omega^*(k,T) = \left[k^2 + m^2(T)\right]^{1/2} , \tag{1}$$

where a temperature-dependent "gluon mass" $m = m(T)$ appears due to the medium effects. The ideal gas picture of gluons and quarks with "thermal masses" was further used in Refs. [3,4] for some physical applications. The "massive gluons" were always treated as noninteracting "quasiparticles" with quantum numbers of real gluons and have, therefore, only two states of polarization (transverse polarized modes).

In the present paper we investigate some peculiar features of physical systems with a medium-dependent dispersion relation. To illustrate the problem we begin with the ideal Bose gas formulas for a particle of mass m and zero chemical potential. The pressure and energy density are given by [5]

$$p_{\rm id}(T,m) = -T\,\frac{d}{2\pi^2}\int_0^\infty k^2 dk\,\ln\left[1 - \exp(-\omega/T)\right] , \tag{2}$$

$$\epsilon_{\rm id}(T,m) = \frac{d}{2\pi^2}\int_0^\infty k^2 dk\,\frac{\omega}{\exp(\omega/T) - 1} , \tag{3}$$

where $\omega = (k^2 + m^2)^{1/2}$ and d is the degeneracy factor [$d = 6$ for SU(2) and $d = 16$ for SU(3) gluons]. Both Eq. (2) and Eq. (3) with $\omega = \omega^*(k,T)$ (1) together were assumed in Refs. [3,4] as the starting point for the GP equation of state. However, when the particle mass in Eqs. (2) and (3) is T dependent the fundamental thermodynamical relation between the pressure $p(T)$ and energy density $\epsilon(T)$ [5], namely,

$$\epsilon(T) = T\,\frac{dp(T)}{dT} - p(T), \tag{4}$$

is not satisfied. One can easily check that the only solution of Eq. (4) for $m(T)$, with $p(T)$ and $\epsilon(T)$ given by Eqs. (2) and (3), is $m = m_0 = \text{const}$. This implies that the ideal gas model (2) and (3) is thermodynamically consistent for particles of constant mass only—any temperature dependence $m = m(T)$ violates the thermodynamic identity (4).

In Refs. [1,2] only the ideal gas form for the pressure function (2) with a temperature-dependent mass was assumed. The energy density is then calculated from the thermodynamic relation (4). In this case the energy density function of T and m does not have the ideal gas form (3). One can equally well start with the ideal gas form (3) with $\omega = \omega^*(k,T)$ (1) for the energy density and solve differential equation (4) to find a $p(T)$ function. The obtained $p(T)$ is again different from the ideal gas formula (2). One can, therefore, construct two different "ideal gas" formulations with different $m(T)$ functions from the same MC lattice data fitting either $p_{\rm MC}(T)$ by Eqs. (1) and (2) or $\epsilon_{\rm MC}(T)$ by Eqs. (1) and (3). Both these two procedures have, however, a disadvantage. They do not correspond to a system of quasiparticles with the dispersion relation (1) and, as a consequence, the gluon momentum distribution function has no physical meaning.

The aim of this paper is to propose another scheme of statistical mechanics formulation for systems with a medium-dependent dispersion relation. Our procedure respects thermodynamic self-consistency and we find that, for systems with $\omega = \omega^*(k,T)$, neither $p(T)$ nor $\epsilon(T)$ has the simple form (2) and (3). They should contain additional medium contributions. It turns out that

---

*Permanent address: Bogolyuobov Institute for Theoretical Physics, 252143, Kiev-143, Ukraine.

only the entropy density preserves the ideal gas form.

The formulation is then used to construct a statistical mechanics model with $m = m(T)$ to describe the existing MC lattice data in SU(2) and SU(3) gluodynamics.

The outline of the paper is as follows. In Sec. II we formulate the requirements of thermodynamical self-consistency for statistical mechanics for systems with a medium-dependent dispersion relation. The formulation is then used to construct a model for the GP with a temperature-dependent mass in Sec. III. We use this model to fit the MC lattice data of SU(2) and SU(3) gluodynamics in Sec. IV. Section V concludes our work with a summary of the results and an outlook.

II. STATISTICAL MECHANICS OF SYSTEMS WITH A MEDIUM-DEPENDENT DISPERSION RELATION

To construct a thermodynamically consistent statistical mechanics model with a medium-dependent dispersion relation, we briefly recall how thermodynamic identities are obtained in statistical mechanics. For a system with Hamiltonian operator H and conserved charge number operator N, the statistical mechanics definitions of pressure, energy density, and conserved charge number density, are [5]

$$p(T,\mu) = \frac{T}{V}\ \ln\ \mathrm{Tr}\left(e^{-(H-\mu N)/T}\right) \equiv \frac{T}{V}\ \ln\ Z(T,\mu,V)\ , \tag{5}$$

$$\epsilon(T,\mu) = \frac{1}{V}\ \frac{1}{Z(T,\mu,V)}\ \mathrm{Tr}\left(H\ e^{-(H-\mu N)/T}\right)\ , \tag{6}$$

$$n(T,\mu) = \frac{1}{V}\ \frac{1}{Z(T,\mu,V)}\ \mathrm{Tr}\left(N\ e^{-(H-\mu N)/T}\right)\ , \tag{7}$$

where μ is the chemical potential and the "thermodynamic limit" when V goes to infinity is assumed. The thermodynamic identities are

$$\epsilon\ =\ Ts\ +\ \mu n\ -\ p\ , \tag{8}$$

$$s\ =\ \left(\frac{\partial p}{\partial T}\right)_{\mu}\ ,\quad n\ =\ \left(\frac{\partial p}{\partial \mu}\right)_{T}\ , \tag{9}$$

where s is the entropy density. They reduce to Eq. (4) with $s = dp/dT$ when $\mu = 0$ and mean nothing more than the identities

$$\left(\frac{\partial\ \mathrm{Tr}\left(e^{-(H-\mu N)/T}\right)}{\partial T}\right)_{\mu} \equiv \frac{1}{T^2}\ \mathrm{Tr}\left[(H-\mu N)\ e^{-(H-\mu N)/T}\right]\ , \tag{10}$$

$$\left(\frac{\partial\ \mathrm{Tr}\left(e^{-(H-\mu N)/T}\right)}{\partial \mu}\right)_{T} \equiv \frac{1}{T}\ \mathrm{Tr}\left[N\ e^{-(H-\mu N)/T}\right]\ , \tag{11}$$

which are valid, of course, for any T- and μ-independent operators H and N.

For an ideal gas Hamiltonian [5]

$$H_{\mathrm{id}}\ =\ \sum_{i=1}^{d}\sum_{\mathbf{k}} \omega(k)\ a^{\dagger}_{\mathbf{k},i}a_{\mathbf{k},i}\ , \tag{12}$$

where the index i corresponds to the particle internal degrees of freedom, e.g., different spin and color states for gluons, evaluations of Eqs. (5) and (6) are straightforward. For large V, i.e., in the thermodynamic limit, the summation over $\mathbf{k}$ in Eqs. (5) and (6) can be substituted by the integration over the momentum phase space

$$\sum_{i=1}^{d}\sum_{\mathbf{k}}\ \cdots\ =\ d\ \frac{V}{(2\pi)^3}\int d\mathbf{k}\ \cdots$$

and Eqs. (2) and (3) are easily obtained. They correspond to the case $\mu = 0$ in Eqs. (5) and (6) and $a^{\dagger}_{\mathbf{k},i}$, and $a_{\mathbf{k},i}$ in Eq. (12) are the Bose creation and annihilation operators, respectively. The calculations of Eqs. (5)–(7) with H given by Eq. (12) for fermions and nonzero μ values are also obvious. They lead to the well known ideal gas expressions where the function $\omega(k)$ for particle ("quasiparticle") excitation energy can take any arbitrary form.

When $\omega(k)$ in Eq. (12) is replaced by $\omega^*(k,T,\mu)$, we still obtain ideal gas expressions from Eqs. (5)–(7). For example, both $p(T)$ and $\epsilon(T)$ as obtained from Eqs. (5) and (6) with $\omega = \omega^*(k,T)$ (1) and $\mu = 0$ would still have the ideal gas form (2) and (3). However, a problem immediately arises. The Hamiltonian of the system becomes temperature dependent and the statistical mechanics definition of the energy density (6) contradicts the thermodynamic relation (4). This occurs because the identity (10) is no longer valid. The recipe for solving this problem is evident: upon introducing a temperature- and/or chemical-potential-dependent effective Hamiltonian H_{eff} one should additionally be required to satisfy identities (10) and (11) to guarantee the thermodynamic consistency of the model.

Similar requirements take place in the theory of nuclear matter [6] where the quasinucleon dispersion relation

$$\omega^*(k,T,\mu)\ =\ \left(k^2\ + M^2\right)^{1/2}\ +\ U\ , \tag{13}$$

with an effective nucleon mass $M(T,\mu)$ and "potential energy" $U(T,\mu)$, appears due to the presence of the interaction between nucleons and the scalar and vector fields in the mean-field approximation. These "fields" contribute to the effective Hamiltonian of the system and, therefore, produce additional "field" terms in the p and ϵ functions to restore the thermodynamical consistency of the model (see Ref. [7]).

To generalize this procedure, we consider an effective Hamiltonian $H_{\mathrm{eff}}(c_1, c_2, ...)$ depending on phenomenological parameters $c_1, c_2, ...$ which are assumed to be functions of temperature T and chemical potential μ (or chemical potentials μ_i if the system has several conserved charges). To satisfy the thermodynamic identities (8) and (9) for $H = H_{\rm eff}$ with T- and/or μ-dependent parameters $c_1, c_2, \ldots$, one should require additionally

$$\left(\frac{\partial p}{\partial c_1}\right)_{T,\mu,c_2,\ldots} = 0\,, \quad \left(\frac{\partial p}{\partial c_2}\right)_{T,\mu,c_1,\ldots} = 0\,, \ldots \tag{14}$$

for the pressure function $p = p(T,\mu,c_1,c_2,\ldots)$ defined in Eq. (5) with $H = H_{\rm eff}$. Requirements (14) are equivalent to Eqs. (10) and (11).

To apply this formulation to our statistical system of quasiparticle excitations we observe that in general the Hamiltonian (12) should be rewritten as

$$H_{\rm eff} = \sum_{i=1}^{d}\sum_{\mathbf{k}} \omega^*(k)\; a^\dagger_{\mathbf{k},i} a_{\mathbf{k},i} + E_0^*\,, \tag{15}$$

where

$$\omega^*(k) \equiv \omega(k, c_1, c_2, \ldots)\,, \qquad E_0^* \equiv E_0(c_1, c_2, \ldots)\,. \tag{16}$$

E_0^* in Eq. (16) is the system energy in the absence of quasiparticle excitations. This zero point energy is of constant value E_0 in the standard case of a medium-independent $\omega(k)$. It is usually subtracted from the system energy spectrum. It cannot be done, however, for a T- and/or μ-dependent dispersion relation $\omega^*(k)$ as the system's lowest state energy E_0^* becomes also a function of T and/or μ. This function is defined by the requirement of thermodynamic self-consistency (14).

Calculating now the thermodynamic functions $p(T,\mu)$ and $\epsilon(T,\mu)$ from Eqs. (5) and (6) with the effective Hamiltonian (15) and quasiparticle number operator $N = \sum_{i=1}^{d}\sum_{\mathbf{k}} a^\dagger_{\mathbf{k},i} a_{\mathbf{k},i}$ (for simplicity, we do not write down explicitly possible antiquasiparticle terms where $\mu \to -\mu$) we find

$$p(T,\mu,c_1,c_2,\ldots) = \mp\, T\,\frac{d}{2\pi^2}\int_0^\infty k^2 dk \times \ln\left[1 \mp \exp\left(-\frac{(\omega^* - \mu)}{T}\right)\right] - B^*\,, \tag{17}$$

$$\epsilon(T,\mu,c_1,c_2,\ldots) = \frac{d}{2\pi^2}\int_0^\infty \frac{k^2 dk\;\omega^*}{\exp[(\omega^* - \mu)/T] \mp 1} + B^*\,, \tag{18}$$

where the upper sign is for bosons and the lower for fermions. The first terms in Eqs. (17) and (18) are the standard ideal gas expressions $p_{\rm id}(T,\mu,\omega^*)$, and $\epsilon_{\rm id}(T,\mu,\omega^*)$ and the second ones are additional medium contributions with

$$B^* = B^*(c_1, c_2, \ldots) \equiv \lim_{V\to\infty} \frac{E_0^*}{V}\,.$$

It is a T- and/or μ-dependent system energy density without quasiparticle excitations. However, the statistical mechanics expressions for quasiparticle number density $n(T,\mu)$ (7) and entropy density $s(T,\mu)$ (9) preserve their ideal gas form:

$$n(T,\mu,c_1,c_2,\ldots) = \frac{d}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\exp[(\omega^* - \mu)/T] \mp 1} \equiv n_{\rm id}(T,\mu,\omega^*)\,, \tag{19}$$

$$s(T,\mu,c_1,c_2,\ldots) = \frac{\epsilon + p - \mu n}{T} = \frac{\epsilon_{\rm id} + p_{\rm id} - \mu n_{\rm id}}{T} \equiv s_{\rm id}(T,\mu,\omega^*). \tag{20}$$

As we shall see this last fact is an important consequence of the above formulation for phenomenological applications.

III. GLUON PLASMA WITH $m = m(T)$

Turning to our problem of the GP with $m = m(T)$, we have $p(T,m) = p_{\rm id}(T,m) - B^*(m)$, and $\epsilon(T,m) = \epsilon_{\rm id}(T,m) + B^*(m)$ from Eqs. (17) and (18). $p_{\rm id}(T,m)$ and $\epsilon_{\rm id}(T,m)$ are given by Eqs. (2) and (3) with $\omega = \omega^*(k,T)$ defined by Eq. (1). For $m(T) = m_0 =$ const, these expressions satisfy the thermodynamic identity (4) with $B^* = B =$ const. If $m_0 = 0$ Eqs. (19) and (20) coincide with the well known bag model equation of state for the GP, e.g., Ref. [8].

If the particle mass is temperature dependent $m = m(T)$, the additional requirement

$$\left(\frac{\partial p(T,m)}{\partial m}\right)_T = 0 \tag{21}$$

should be satisfied. It is a special case of our general requirements (14). The condition (21) was introduced in Ref. [7] as a self-consistency equation for phenomenological extension of the mean-field theory approach of Ref. [6], where B^* has the meaning of the scalar field energy density. Explicit evaluation of $[\partial p_{\rm id}(T,m)/\partial m]_T$ in Eq. (21) leads to

$$\frac{dB^*}{dm} = -m\frac{d}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T)}\,\frac{1}{\exp[\omega^*(k,T)/T] - 1}\,. \tag{22}$$

If $B^*(m)$ is known, Eq. (22) gives us an equation for the function $m(T)$ and the model is completely defined. If, on the other hand, the function $m(T)$ is known we can calculate $B^*(T)$ from Eq. (22) as

$$B^*(T) = B_0 - \frac{d}{2\pi^2}\int_{T_0}^{T} dT'\; m\,\frac{dm}{dT'} \times \int_0^\infty \frac{k^2 dk}{\omega^*(k,T')}\,\frac{1}{\exp[\omega^*(k,T')/T'] - 1}\,, \tag{23}$$

up to an arbitrary integration constant $B_0 = B^*(T_0)$. Our equations will then take the form

$$p(T,m) = p_{\rm id}(T,m) - B^*(T) , \quad (24)$$

$$\epsilon(T,m) = \epsilon_{\rm id}(T,m) + B^*(T) . \quad (25)$$

For a constant zero value of the gluon mass Eqs. (24) and (25) are, as we already mentioned, reduced to the standard bag model equation of state [8] $p(T,m = 0) = p_{\rm SB} - B, \epsilon(T,m = 0) = \epsilon_{\rm SB} + B$, where a positive constant value of B ("vacuum pressure") is assumed and $p_{\rm SB}$ and $\epsilon_{\rm SB}$ stand for the corresponding Stefan-Boltzmann expressions: $p_{\rm SB} \equiv d\pi^2T^4/90$ and $\epsilon_{\rm SB} \equiv d\pi^2T^4/30$. This equation of state cannot reproduce MC lattice data $p_{\rm MC}(T)$ and $\epsilon_{\rm MC}(T)$ for SU(2) [9] and SU(3) [10] gluodynamics, since $\epsilon = \epsilon_{\rm SB} + B > \epsilon_{\rm SB}$, while $\epsilon_{\rm MC} < \epsilon_{\rm SB}$.

The form of Eqs. (24) and (25) was first used in Ref. [11] to fit MC lattice data for the SU(3) GP. It was considered as an extension of the bag model with two independent functions $m(T)$ and $B^*(T)$. The MC lattice data used in Ref. [11] were, however, rather poor and their fitting led to the constant values $m = m_o \cong 2.5T_c$ and $B^{*\ 1/4} = B_0^{1/4} \cong 1.25T_c$. Therefore the problem of a temperature-dependent gluon mass and the self-consistency condition (22) [or (23)] which relates these two functions was not discussed.

The physical meaning of our $B^*(T)$ terms in Eqs. (24) and (25) can be rather different from that in the bag model, and we have no *a priori* estimate of its numerical value and even of its sign.

IV. MODEL RESULTS FOR SU(2) AND SU(3) GLUODYNAMICS

In this section, we use the GP model constructed in the preceding section to describe MC lattice data for the pressure and energy density of SU(2) [9] and SU(3) [10] GP's. A straightforward way to find the function $m = m(T)$ is to compare the model (23)–(25) with MC lattice data for the entropy density because it does not depend on $B^*(T)$:

$$s(T,m) \equiv \frac{p(T,m) + \epsilon(T,m)}{T} = \frac{p_{\rm id}(T,m) + \epsilon_{\rm id}(T,m)}{T} \equiv s_{\rm id}(T,m) . \quad (26)$$

Namely, as we have already shown in the more general case of Eq. (20), just the entropy density function preserves its simple ideal gas form without any additional medium contributions.

In Fig. 1 the MC lattice data of Ref. [9] are shown for the energy density, pressure, and entropy density of an SU(2) GP (at $T > T_c$) normalized to their Stefan-Boltzmann limits $\epsilon_{\rm SB}(T)$, $p_{\rm SB}(T)$, and $s_{\rm SB}(T) \equiv s_{\rm id}(T,m = 0) = 4\pi^2T^3/15$. The corrections of all thermodynamic function data for the finite size lattice effects are done according to the procedure of Ref. [12]. Requiring our entropy density function (26) to be equal to the MC lattice data $s_{\rm MC}(T)$ of Fig. 1, we find the $m(T)$ values at the corresponding values of temperature. They are shown in Fig. 2, where we also show the $m(T)$ values obtained from the prescription $p_{\rm id}(T,m) = p_{\rm MC}(T)$, as it has been done in Ref. [1], and from the prescription $\epsilon_{\rm id}(T,m) = \epsilon_{\rm MC}(T)$. As seen in Fig. 2 and Fig. 5 below for the SU(3) case our $m(T)$ functions (full triangles) are qualitatively similar to those obtained in Refs. [1,2] (circles), but they are, of course, rather different quantitatively, as our statistical mechanics model is quite different

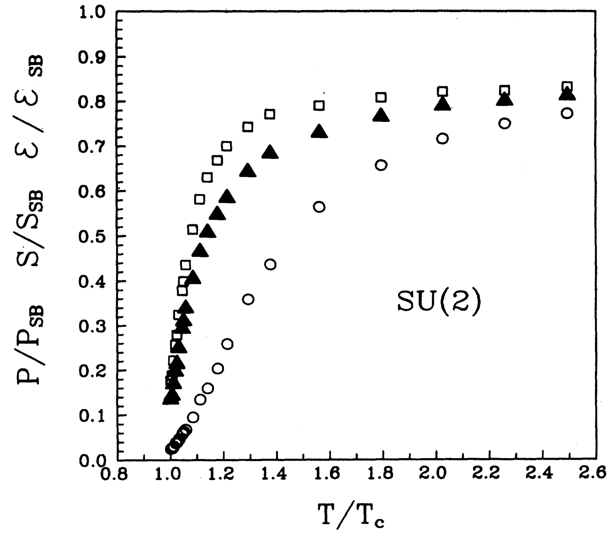

FIG. 1. MC lattice data of SU(2) gluodynamics [9] for the energy density (open boxes), pressure (open circles), and entropy density (full triangles) at $T > T_c$ normalized to their Stefan-Boltzmann limits.

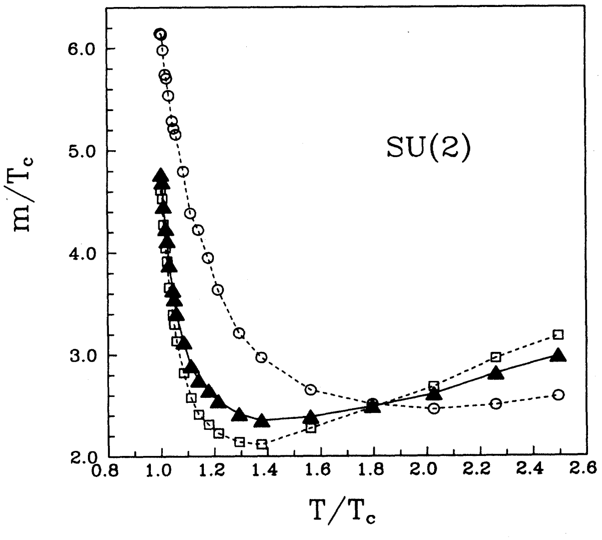

FIG. 2. Gluon "thermal mass" $m = m(T)$ obtained from the fit to MC SU(2) lattice data [9] for the entropy density (full triangles), energy density (open boxes), and pressures (open circles) in Fig. 1 by the corresponding expressions for the ideal gas entropy density (26), ideal gas energy (3), and ideal gas pressure (2). The "pressure fit" (open circles) was done in Ref. [1].

from the formulation of Refs. [1,2].

Then, with condition (23) and the MC lattice value for $p_{\rm MC}(T_0)$ to define the integration constant B_0, we reconstruct corresponding $B^*(T)$ values as shown in Fig. 3. Once $m(T)$ and $B^*(T)$ are found we can proceed to calculate the functions $p(T,m)$ and $\epsilon(T,m)$ according to Eqs. (24) and (25). The obtained pressure and the energy density are compared with MC lattice data [9] in Fig. 4. Ideal gas formulas (2) and (3) for $p(T)$ and $\epsilon(T)$ calculated with the same $m = m(T)$ of Fig. 2 determined from $s_{\rm MC}(T)$ are also shown in Fig. 4. It is seen that the lack of thermodynamic self-consistency in this approach leads to rather strong deviations of $p_{\rm id}(T,m)$ and $\epsilon_{\rm id}(T,m)$ from $p_{\rm MC}(T)$ and $\epsilon_{\rm MC}(T)$ data.

The complete agreement of our model results with MC lattice data is not surprising. If the MC lattice data for $p(T)$, $\epsilon(T)$, and $s(T)$ are thermodynamically consistent [i.e., they satisfy identity (4), or, equivalently, $s(T) = dp/dT = (p+\epsilon)/T$], as they are, then only one of these functions, say $p(T)$, is really independent: $\epsilon(T)$ and $s(T)$ can be calculated from $p(T)$ using thermodynamic identities. But in our model we also have one "independent" function $m(T)$. Therefore, if we fit completely the MC lattice data for $s(T)$ and satisfy the thermodynamical identity (4) due to our Eq. (23), the model pressure and energy density (24) and (25) have to coincide with the MC lattice data too.

For the MC lattice data of an SU(3) GP [10] our fitting procedure is essentially the same. We show only our results for the functions $m(T)$ and $B^*(T)$ in Figs. (5) and (6). These two functions are connected by Eq. (23) and our model equations (24) and (25) are again in complete agreement with the MC lattice data $p_{\rm MC}(T)$ and $\epsilon_{\rm MC}(T)$ [10].

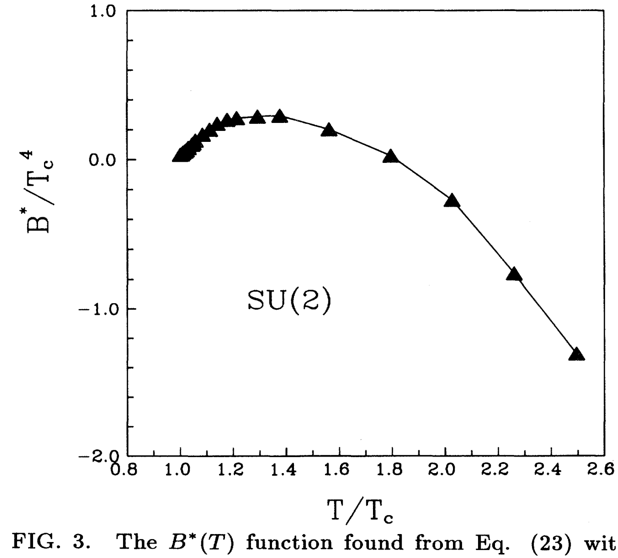

FIG. 3. The $B^*(T)$ function found from Eq. (23) with $m(T)$ in Fig. 2 obtained from the MC SU(2) lattice data for the entropy density.

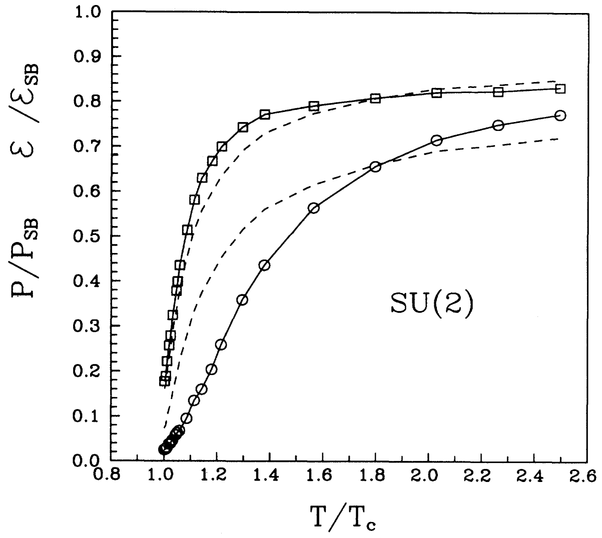

FIG. 4. MC lattice data of SU(2) gluodynamics [9] for the pressure (open circles) and energy density (open boxes) at $T > T_c$ normalized to their Stefan-Boltzmann limits. Full lines are obtained from Eqs. (24) and (25) with $m(T)$ in Fig. 2 found from the entropy density (full triangles) and the function $B^*(T)$ given in Fig. (3). The dashed lines are obtained from Eqs. (2) and (3) with the same $m(T)$ function but without $B^*(T)$ contributions.

V. DISCUSSIONS AND CONCLUSIONS

We construct a thermodynamically self-consistent statistical mechanics model with $m = m(T)$ to describe MC lattice data for SU(2) and SU(3) GP's. Complete agreement with MC lattice data for $p(T)$ and $\epsilon(T)$ can be achieved as we have an arbitrary function $m = m(T)$

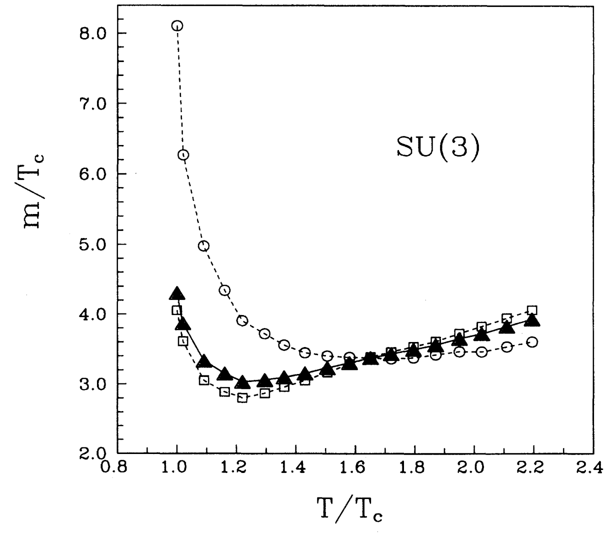

FIG. 5. Gluon "thermal mass" $m = m(T)$ obtained from the fit to MC lattice SU(3) data [10] for the entropy density (full triangles), energy density (open boxes), and pressures (open circles) by the corresponding expressions for the ideal gas entropy density (26), ideal gas energy (3) and ideal gas pressure (2). The "pressure fit" (circles) for these SU(3) data was done in Ref. [2].

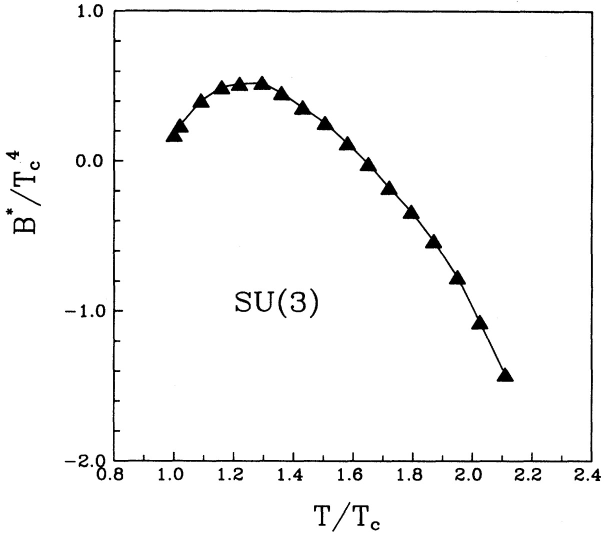

FIG. 6. The $B^*(T)$ function found from Eq. (23) with $m(T)$ in Fig. 5 obtained from the MC SU(3) lattice data [10] for the entropy density.

which is found from $s_{\rm MC}(T)$ data. We cannot, however, make definite conclusions about the physical validity of this model. To clarify its efficiency some other properties of the GP should be calculated and compared with the MC lattice data. Note that $\omega^*(k,T)$ is more a system "microscopic" characteristic than the thermodynamic functions themselves and all other properties of the GP (e.g., the "heavy-quark potential" in the GP) can be found in terms of the gluon distribution function with a medium-modified dispersion relation.

One can be surprised that $m(T)$ and the absolute value of $B^*(T)$ found from MC lattice data increase at large T when the GP is expected to approach the Stefan-Boltzmann limiting behavior of massless noninteracting gluons. We note that to have the essential deviations of the thermodynamic functions (24) and (25) from their Stefan-Boltzmann limits one needs large values of m/T and B^*/T^4. These quantities in fact are large at T near T_c, and we expect they will gradually go to zero at $T \to \infty$. A possible physical origin of the $m = m(T)$ dependence was discussed in Ref. [1].

The analysis of systems with $\omega(k) = \omega^*(k,T,\mu)$ presented here can be useful for other models of equations of state of strongly interacting matter. We mention only two examples. The first one is the "cutoff" model of the GP [13] with a temperature-dependent "cutoff" parameter $K(T)$ used in Ref. [9]. It corresponds to the following dispersion relation of the "ideal gas" excitations in the GP:

$$\omega^*(k,T) = k\ \theta(k - K(T)) + (k^2 + M^2)^{1/2}\ \theta(K(T) - k)\ . \qquad (27)$$

The first term in Eq. (27) represents massless high-momenta gluons and the second one is low-momenta glueballs with mass M. A statistical mechanics treatment of the system (27) leads to equations similar to Eqs. (17) and (18) with $\mu = 0$ and the function $K(T)$ should be extracted from the equation $s_{\rm id}(T,K) = s_{\rm MC}(T)$. The second example concerns a temperature-dependent pion dispersion relation. It was suggested by Shuryak [14] in the form

$$\omega^*(k,T) = \left[u^2(T)k^2 + m_\pi^2\right]^{1/2}\ , \qquad (28)$$

where $u(T)$ is the temperature-dependent refraction index. It was extracted in Ref. [14] from some "data" for the pion energy density $\epsilon_\pi(T)$, i.e., using the equation $\epsilon_{\rm id}(T,\omega^*) = \epsilon_\pi(T)$, while our statistical mechanics formulation requires again $s_{\rm id}(T,\omega^*) = s_\pi(T)$. We consider both these problems in separate publications [15]. Note that one cannot reconstruct a function of two variable $\omega^*(k,T)$ from a function $s(T)$ of one variable and some model assumptions which reduce ω^* to one unknown function of one variable must be made [e.g., $m(T)$ in Eq. (1), $K(T)$ in Eq. (27), or $u(T)$ in Eq. (28)].

Statistical mechanics with any T- and μ-independent Hamiltonian operator satisfies automatically the thermodynamic identities. A statistical mechanics model with a temperature-dependent particle mass gives us the simplest example of systems with $H = H_{\rm eff}(T,\mu)$. For such models the standard statistical mechanics formulas have to be supplemented by special requirements to guarantee that fundamental thermodynamic identities will be satisfied. We have shown that these additional requirements can be presented in the simple mathematical form of Eq. (14). Particular examples of this kind have been known in the mean-field theory of nuclear matter [6] and its phenomenological extension [7].

We have suggested a procedure to construct statistical mechanics models with a T- and/or μ-dependent dispersion relation $\omega^*(k,T,\mu)$ which should be constructed according to Eqs. (15) and (16). The lowest state energy E_0^* without quasiparticle excitations becomes a function of T and/or μ in this case and has to be included in the statistical model formulation for thermodynamic consistency. The connection between ω^* and E_0^* is given by Eq. (14).

In conclusion we repeat two main points of our study for systems with $\omega(k) = \omega^*(k,T,\mu)$.

(1) The quasiparticle dispersion relation, e.g., Eqs. (1), (13), (27), and (28), should be used for or extracted from the entropy density function. It leads to an unambiguous definition for the quasiparticle distribution function.

(2) The pressure and and energy density for such systems contain an additional medium contribution which is to be found from the requirements of thermodynamic self-consistency (14).

ACKNOWLEDGMENTS

M.I.G. is indebted to V. L. Sobolev for many fruitful discussions and to O. Borisenko, K. A. Bugaev, and O. P. Pavlenko for valuable comments. He also expresses his gratitude for the warm hospitality at the Physics Department of National Taiwan University in Taipei. The work has been supported in part by the National Science Council, ROC under Contract No. NSC84-2112-M002-022 and by the International Science Foundation under Grant No. U4D000.

[1] V. Goloviznin and H. Satz, Z. Phys. C **57**, 671 (1993).

[2] A. Peshier, B. Kämpfer, O.P. Pavlenko, and G. Soff, Phys. Lett. B **337**, 235 (1994).

[3] J. Cleymans, S.V. Ilyin, S.A. Smolyanski, and G.M. Zinovjev, Z. Phys. C **62**, 62 (1994).

[4] J. Letessier, J. Rafelski, and A. Tounsi, Phys. Lett. B **323**, 393 (1994).

[5] L.D. Landau and E.M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1975).

[6] J.D. Walecka, Phys. Lett. **59B**, 109 (1975).

[7] K.A. Bugaev and M.I. Gorenstein, Z. Phys. C **43**, 261 (1989).

[8] E.V. Shuryak, Phys. Rep. **61**, 71 (1980); J. Cleymans, R.V. Gavai, and E. Suhonen, *ibid.* **130**, 217 (1986).

[9] J. Engels, J. Fingberg, K. Redlich, H. Satz, and M. Weber, Z. Phys. C **42**, 341 (1989).

[10] J. Engels, J. Fingberg, F. Karsch, D. Miller, and M. Weber, Phys. Lett. B **252**, 625 (1990).

[11] T.S. Biro, P. Levai, and B. Müller, Phys. Rev. D **42**, 3078 (1990).

[12] J. Engels, F. Karsch, and H. Satz, Nucl. Phys. **B205**, 239 (1982).

[13] F. Karsch, Z. Phys. C **38**, 147 (1988).

[14] E.V. Shuryak, Phys. Rev. D **42**, 1764 (1990).

[15] M.I. Gorenstein and S.N. Yang, J. Phys. G **21**, 1053 (1995); M.I. Gorenstein, H.G. Miller, R.A. Ritchie, and S.N. Yang (unpublished).