Constraints on light quark masses from the heavy meson spectrum

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We use the observed SU(3) breaking in the mass spectrum of mesons containing a single heavy quark to place restrictions on the light quark current masses. A crucial ingredient in this analysis is our recent first-principles calculation of the electromagnetic contribution to the isospin-violating mass splittings. We also pay special attention to the role of higher-order corrections in chiral perturbation theory. We find that large corrections are necessary for the heavy meson data to be consistent with $m_u = 0$.

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I. INTRODUCTION

The light quark current masses m_u, m_d, and m_s are among the fundamental parameters in the standard model of particle interactions, but an accurate and reliable determination of these parameters remains elusive. The reason is that these masses are small compared to the mass scale associated with confinement, $\Lambda \sim 1$ GeV, so that the light quarks are tightly bound inside hadrons and their mass cannot be measured directly. However, we can expand around the chiral limit $m_{u,d,s} \to 0$ to obtain information about current mass *ratios* using chiral perturbation theory [1–3]. The idea is that in the chiral limit, there is a $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry that is spontaneously broken to $\mathrm{SU}(3)_{L+R}$ at the scale Λ. In this limit, the theory contains 8 Nambu-Goldstone bosons, which are identified with the light pseudoscalar meson octet (π, K, η). The low-energy interactions of these states can be parametrized by an effective Lagrangian with a few unknown parameters [4]. In addition, the chiral symmetry is explicitly broken by the light quark masses and by electromagnetism. This breaking can be treated perturbatively in the quark masses and the electromagnetic coupling, and selection rules associated with the chiral symmetry again tightly constrain the form of these perturbations.

The main difficultly in this approach is that $m_s/\Lambda \sim 20 - 30\,\%$, so that higher order effects can significantly change low-order results. This is the source of the interesting (and controversial) issue of whether higher order corrections can be large enough to allow $m_u = 0$ [5–7], thus solving the strong CP problem. This will be a large part of the focus of this paper.

In this paper , we apply the methods of heavy quark symmetry [8–10] and heavy hadron chiral perturbation theory [11] to the determination of the light quark masses from the mass spectrum of mesons containing a single heavy quark. These states have quantum numbers $P_q \sim Q\bar{q}$, where $Q = b, c$ and $q = u, d, s$. If we could ignore electromagnetic effects and higher order corrections in the quark masses, we would obtain

$$R \equiv \frac{m_s - \hat{m}}{m_d - m_u} \stackrel{?}{=} \frac{P_s - \hat{P}}{P_d - P_u} \simeq \begin{cases} 20 & \text{for} \quad P = D, \\ 280 & \text{for} \quad P = B, \end{cases} \tag{1.1}$$

where $\hat{m} \equiv \frac{1}{2}(m_u + m_d)$, etc. (We use the names of the heavy-meson states to denote their masses.) The enormous discrepancies between these numbers clearly cannot be accounted for by the higher order quark mass corrections. The reason for these discrepancies is simply that the electromagnetic contribution to $P_d - P_u$ is numerically comparable to the contribution from the light quark masses. This underlines the importance of determining the electromagnetic contribution to the mass differences.

We will carry out an improved analysis of R making use of the heavy-meson electromagnetic mass differences computed in Ref. [12]. (For earlier related work, see Ref. [13].) Our analysis will also include important nonanalytic corrections in the quark masses [14,15] and will pay careful attention to the role of higher order corrections in chiral perturbation theory.

II. COMPUTATION OF ELECTROMAGNETIC MASS DIFFERENCES

In Ref. [12], we computed the electromagnetic mass differences of heavy mesons in terms of measurable data using techniques similar to those used in the classic calculation of the π^+-π^0 mass difference [16]. The basic idea in both calculations is to use dispersion-theoretic arguments together with the ultraviolet properties of QCD to relate the electromagnetic mass difference to measured strong-interaction matrix elements.

The electromagnetic mass differences computed in Ref. [12] depend on matrix elements β (and β') which measure $P^*P\gamma(P^*P^*\gamma)$ couplings. Heavy-quark symmetry gives $\beta' = \beta + O(1/m_Q)$, and our results are not sensitive to the $O(1/m_Q)$ corrections for reasonable values. We can determine β from our computation by using the fact that to order m_q/m_Q in chiral perturbation theory, one finds [15]

$$(D^{*+} - D^{*0}) - (D^+ - D^0) = [(D^{*+} - D^{*0}) - (D^+ - D^0)]^{(\mathrm{EM})} + \frac{1}{R}[(D_s^* - \hat{D}^*) - (D_s - \hat{D})]. \quad (2.1)$$

The last term on the right-hand side is experimentally negligible, and using the results in Ref. [12] we obtain

$$\beta \simeq -1.6 \text{ GeV}^{-1} + O(1/m_c). \quad (2.2)$$

This is consistent with the bounds obtained in Refs. [17,18].[1]
For comparison the constituent quark model prediction is $\beta \simeq -3$ GeV^{-1}. Using Eq. (2.2), the results of Ref. [12] give

$$\tfrac{1}{4}[(D^+ - D^0) + 3(D^{*+} - D^{*0})]^{(\mathrm{EM})} \simeq 2.5 \text{ MeV}, \quad (2.3)$$

$$(B^+ - B^0)^{(EM)} \simeq 1.8 \text{ MeV}. \quad (2.4)$$

The electromagnetic mass differences are not very sensitive to β, so despite the uncertainties discussed above, we expect these values to be correct to about 30%, which is the accuracy of the comparable calculation of the π^+-π^0 mass difference [12].

III. CONSTRAINTS ON R

We now turn to the analysis of R defined in Eq. (1.1). We pay special attention to the role of higher order corrections in chiral perturbation theory, since it is known that these can have an important effect on the determination of the light quark mass ratios [5,7]. Keeping terms of order m_q, $m_q^{3/2}$, m_q/m_Q, m_q^2, and the corresponding logs,[2] the mass differences of the heavy mesons can be written in the form

$$P_d - P_u = (P_d - P_u)^{(EM)} + A_0(m_d - m_u) + \Delta_{d-u}^{(3/2)} + [A_1(\mu) + \Delta_1(\mu)]\frac{m_d - m_u}{m_Q} + [A_2(\mu) + \Delta_2(\mu)]m_s(m_d - m_u) + O(m_{u,d}^2 \ln m_s), \quad (3.1)$$

$$P_s - \hat{P} = A_0 m_s + \Delta_s^{(3/2)} + [A_1(\mu) + \Delta_1(\mu)]\frac{m_s}{m_Q} + [A_2(\mu) + \Delta_2(\mu)]m_s^2 + [A_3(\mu) + \Delta_3(\mu)]m_s^2 + O(m_s^{5/2}). \quad (3.2)$$

[For the D system, $P = \frac{1}{4}(D + 3D^*)$.] The meaning of these terms is as follows. $(P_d - P_u)^{(EM)}$ is the electromagnetic contribution discussed in the previous sections. $A_0 m_q$ is the term linear in quark masses that gives rise to Eq. (1.1). $\Delta^{(3/2)}$ are the $O(m_q^{3/2})$ nonanalytic corrections, which can be expressed in terms of the light meson masses and couplings [14,15]:[3]

$$\Delta_{d\text{-}u}^{(3/2)} = -\frac{g^2}{16\pi f^2}\left(M_{K^0}^3 - M_{K^+}^3 + \frac{1}{2R}M_\eta^3\right), \quad (3.3)$$

$$\Delta_s^{(3/2)} = -\frac{g^2}{16\pi f^2}\left(M_K^3 + \frac{1}{2}M_\eta^3\right). \quad (3.4)$$

Here, $f \simeq 110$ MeV is the K decay constant and g measures the strength of the $PP\pi$ coupling [11]. The experimental limit on the D^* width gives the bound $g^2 \leq 0.5$ [11]. (To the order we are working, the value of g is the same in the D and B system by heavy quark flavor symmetry.) However, the analysis of Refs. [17,18] reveals that the magnitudes of g and β are correlated, so that $g^2 \simeq 0.15$ for the value of β in Eq. (2.2). Their analysis also shows that $g^2 \geq 0.1$. We will conservatively assume $0.1 \leq g^2 \leq 0.3$, which corresponds to $-\beta \leq 3$ GeV^{-1}.

The terms proportional to A_1 and $A_{2,3}$ in Eqs. (3.1), (3.2) arise from analytic counterterms in the effective Lagrangian of order m_q/m_Q and m_q^2, respectively. The terms proportional to $\Delta_{1,2,3} \sim \ln m_s$ are the chiral logs that renormalize them. The counterterms and the chiral logs each depend on a renormalization scale μ in such a way that the terms proportional to $A_j + \Delta_j$ are independent of μ. The fact that the chiral logs are proportional to the same function of quark masses as the corresponding counterterms is a consequence of the simple structure of the SU(3) group theory for this system.

Perhaps surprisingly, the chiral logs in $\Delta_{2,3}$ cannot be computed in terms of known quantities. The reason is the presence of higher order terms in the effective heavy-meson Lagrangian such as (in the notation of Ref. [11])

$$\delta\mathcal{L} = \frac{c}{\Lambda}\mathrm{tr}[H^\dagger H(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)], \quad (3.5)$$

that are not constrained by present data. These terms

[1]Our convention for β differs by a sign from that of Ref. [17].
[2]This corresponds to assuming that $m_s/\Lambda \sim \Lambda/m_Q$. This probably overestimates the $O(m_q/m_Q)$ corrections for the B system, but this is harmless because our results are identical if these corrections are omitted.
[3]Reference [14] contains an error that was corrected in Ref. [15].

give rise to nonanalytic contributions such as

$$\Delta_3 m_s^2 \sim \frac{c}{\Lambda}\frac{M_K^4}{16\pi^2 f^2}\ln\frac{M_K^2}{\mu^2} = O(m_s^2 \ln m_s). \qquad (3.6)$$

(A similar fact was noted for baryons in Ref. [19].) In principle, the log corrections are enhanced over the counterterm contributions by $\sim \ln M_K^2/\mu^2$ for $\mu \sim 1$ GeV, but in practice the logs are not significantly larger than unity. We will therefore treat the terms proportional to $A_j + \Delta_j$ as unknown corrections.

We can solve for R from Eqs. (3.1), (3.2) to obtain

$$R = \frac{(P_s - \hat{P}) - \Delta_s^{(3/2)}(K) - \bar{A}_3}{(P_d - P_u) - (P_d - P_u)^{(EM)} - \Delta_{d\text{-}u}^{(3/2)}(K)} \qquad (3.7)$$

where $\Delta^{(3/2)}(K)$ are the terms in the $O(m_q^{3/2})$ corrections which depend on the K masses [see Eqs. (3.3), (3.4)]; $\bar{A}_3 \equiv [A_3(\mu) + \Delta_3(\mu)]m_s^2$ parametrizes the unknown $O(m_q^2)$ and $O(m_q^2 \ln m_q)$ deviations from the $O(m_q^{3/2})$ mass relations for both the D and B systems. Since such relations are expected to work to 20–30 %, it is reasonable to assume that $\bar{A}_3$ should not be much larger than 30%. Reference [5] advocates a different measure of the chiral corrections. They demand that the second-order corrections to individual masses be less than 30%, but allow this to be the result of cancellations between larger corrections. Following this criterion, we would allow $\bar{A}_3$ to be 60%, since this can cancel against -30% corrections arising from the term proportional to $A_2 + \Delta_2$ to give 30% corrections to P mass differences.

We show R as a function of $(P_d - P_u)^{(EM)}$ in Figs. 1 and 2 for values of $\bar{A}_3$ corresponding to 0, 30%, and 60% of $P_s - \hat{P}$. We see that if the chiral corrections parametrized by $\bar{A}_3$ are $\sim$ 30%, and the computed electromagnetic mass differences have errors $\lesssim$ 30%, then the results for both the D and B systems can comfortably accommodate $R = 44$. This is the value of R predicted by lowest-order chiral perturbation theory for the light meson masses; it is also the value obtained by the $O(m_q^2)$ analysis of Ref. [3].

We can use our results to address the question of whether $m_u = 0$ by using a relation between R and m_u/m_d obtained from the light pseudoscalar masses that is valid to $O(m_q^2)$ [5,6] and that predicts $R = 24 \pm 2$ for $m_u = 0$. From Figs. 1 and 2, we see that if we assume that $\bar{A}_3$ is a 30% correction, we require very large corrections in both the D and B electromagnetic mass differences in order to be consistent with $m_u = 0$. If $\bar{A}_3$ is a 60% correction, the data can accommodate $m_u = 0$.

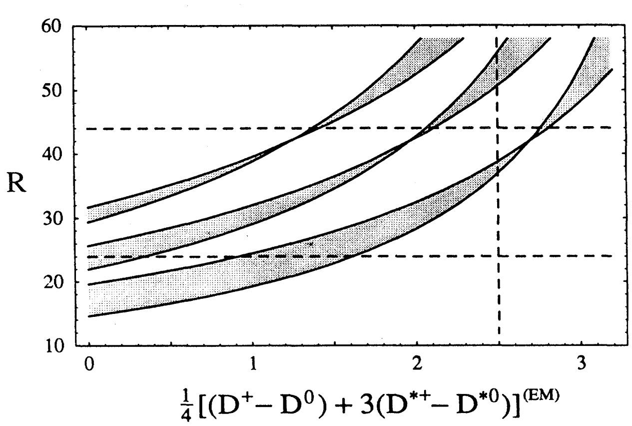

FIG. 1. The quark mass ratio R as a function of the spin-independent electromagnetic mass difference in the D system. Our prediction for the electromagnetic mass difference is shown by the vertical dashed line. The bands correspond to the range $0.1 \le g^2 \le 0.3$. The chiral corrections parametrized by $\bar{A}_3$ are assumed to be 0% in the upper band, 30% in the middle band, and 60% in the lower band. The value $R = 44$ is the value obtained from a leading order analysis of the light meson masses, while the value $R = 24$ is the value required by $m_u = 0$.

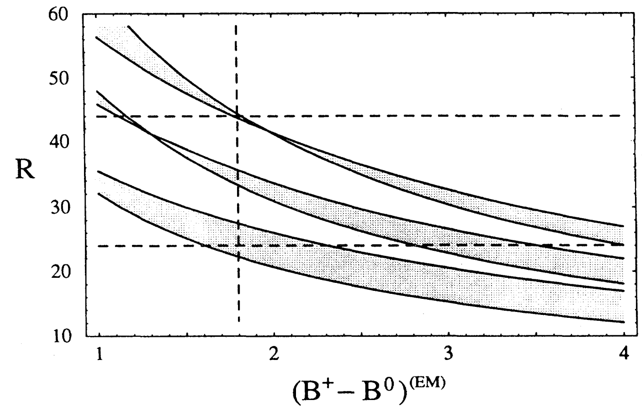

FIG. 2. The quark mass ratio R as a function of the B electromagnetic mass difference. Our prediction for the electromagnetic mass difference is shown by the vertical dashed line. The bands correspond to the range $0.1 \le g^2 \le 0.3$. The chiral corrections parametrized by $\bar{A}_3$ are assumed to be 0% in the upper band, 30% in the middle band, and 60% in the lower band.

IV. CONCLUSIONS

We have derived constraints on the light quark mass ratio R from the spectrum of mesons containing a single heavy quark, using the QCD-based computation of the heavy-meson electromagnetic mass differences of Ref. [12]. Our results, summarized in Figs. 1 and 2, indicate that even when 30% uncertainties are assigned to both the electromagnetic mass differences and the unknown $O(m_q^2)$ chiral corrections, the value of R is bounded away from the value required by $m_u = 0$. While we do not regard this as definitive proof that $m_u \neq 0$, it is striking that the central values of higher order analyses of both the light pseudoscalar mesons and the heavy mesons prefer $m_u \neq 0$, and large chiral corrections must be invoked in both cases to be consistent with $m_u = 0$.

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