$U(3)_L \times U(3)_R$ chiral theory of mesons

Bing An Li
Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506
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The chiral theory of mesons of two flavors has been extended to mesons containing strangeness. The theory has been studied at the tree level. Two new mass relations between vector and axial-vector mesons have been obtained. In the chiral limit, the physical processes of normal parity and abnormal parity have been studied. Because of the universality of the coupling of this theory many interesting results have been obtained. In the chiral limit, theoretical results are in reasonable agreement with data.

Chiral symmetry is one of the most important features revealed from quantum chromodynamics (QCD). Based on chiral symmetry and the principle of minimum coupling, a meson theory of two flavors has been proposed [1] and the theoretical results are in good agreement with the phenomenology of pseudoscalar, vector, and axial-vector mesons made of u and d quarks. In this paper we generalize the study of Ref. [1] to K, η, η', $K^*(892)$, ϕ, $K_1(1400)$, and $f_1(1510)$ mesons containing the third flavor-strange quark. The paper is organized in the following way: (1) the formalism of the theory; (2) new mass relations between vector and axial-vector mesons; (3) vector meson dominance (VMD) and kaon-form factors; (4) decays of τ lepton; (5) decays of ϕ, K^*, $K_1(1400)$, $f_1(1510)$, and η' mesons; (6) decays of $K^*(892) \to K\pi\pi$; (7) electromagnetic decays of mesons; (8) summary of the results; (9) conclusions.

I. THE FORMALISM OF $U(3)_L \times U(3)_R$ CHIRAL THEORY OF MESONS

Using $U(3)_L \times U(3)_R$ chiral symmetry and the minimum coupling principle, the Lagrangian of quarks of three flavors and other fields has been constructed as

$$\mathcal{L} = \bar{\psi}(x)[i\gamma\cdot\partial + \gamma\cdot v + e_0 Q\gamma\cdot A + \gamma\cdot a\gamma_5 - mu(x)]\psi(x) + \frac{1}{2}m_1^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu)$$
$$+\frac{1}{2}m_2^2(K_\mu^{*a}\bar{K}^{*a\mu} + K_1^\mu K_{1\mu}) + \frac{1}{2}m_3^2(\phi_\mu\phi^\mu + f_s^\mu f_{s\mu}) + \bar{\psi}(x)_L g_w\gamma\cdot W\psi(x)_L + \mathcal{L}_{\rm EM} + \mathcal{L}_W + \mathcal{L}_{\rm lepton}\,, \quad (1)$$

where $a_\mu = \tau_i a_\mu^i + \lambda_a K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}$ ($i = 1,2,3$ and $a = 4,5,6,7$), $v_\mu = \tau_i\rho_\mu^i + \lambda_a K_\mu^{*a} + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$, A_μ is the photon field, Q is the electric charge operator of u, d, and s quarks, W_μ^i is the W boson, $u = \exp\{\gamma_5 i(\tau_i\pi_i + \lambda_a K^a + \eta + \eta')\}$, and m is a parameter. In Eq. (1) u can be written as

$$u = \frac{1}{2}(1+\gamma_5)U + \frac{1}{2}(1+\gamma_5)U^\dagger, \quad (2)$$

where $U = \exp\{i(\tau_i\pi_i + \lambda_a K^a + \eta + \eta')\}$. In Eq. (1) the ψ are u, d, and s quark fields which carry colors and other quantum numbers of quark. All other fields are colorless. The physical fields related to a_μ and v_μ will be defined below. As mentioned in Ref. [1], in QCD the boson fields v_μ, a_μ, and pseudoscalars are not fundamental fields and they should be bound state solutions of QCD. Therefore, in Eq. (1) there are no kinetic terms for those fields and the kinetic terms will be generated from quark loops (see below). As a matter of fact, the relationship between boson fields and quark fields can be found from the Lagrangian (1). Taking the ρ_μ^i and a_μ^i fields as examples, and using the least action principle

$$\frac{\delta\mathcal{L}}{\delta\rho_\mu^i} = 0, \quad \frac{\delta\mathcal{L}}{\delta a_\mu^i} = 0,$$

we obtain the relationships

$$\rho_\mu^i = -\frac{1}{m_1^2}\bar{\psi}\tau_i\gamma_\mu\psi, \quad a_\mu^i = -\frac{1}{m_1^2}\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi.$$

Substituting these relations into Eq. (1), except for the term $-m\bar{\psi}u\psi$, the hadronic part of the Lagrangian (1) becomes the Nambu–Jona-Lasinio model [2]. The introduction of the pseudoscalar fields into the Lagrangian (1) is based on the nonlinear σ model, where u of Eq. (1) is a series of pseudoscalar fields. In principle, the relationships between pseudoscalar fields and quark fields should be found by the least action principle, but they are not as simple as the relations shown above. This is the difference between the present theory and the Nambu–Jona-Lasinio model.

As in Ref. [1] the use of the method of path integral in integrating out the quark fields leads to the effective Lagrangian of mesons. In terms of dimensional regularization, to the fourth order in covariant derivatives in Minkowski space, the real part of the effective Lagrangian describing the physical processes of normal parity takes the form

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$$\mathcal{L}_{\rm RE} = \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4}\Gamma\left(2-\frac{D}{2}\right)\mathrm{Tr}D_\mu U D^\mu U^\dagger - \frac{1}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma\left(2-\frac{D}{2}\right)\mathrm{Tr}\{v_{\mu\nu}v^{\mu\nu}+a_{\mu\nu}a^{\mu\nu}\}$$
$$+\frac{i}{2}\frac{N_c}{(4\pi)^2}\mathrm{Tr}\{D_\mu U D_\nu U^\dagger + D_\mu U^\dagger D_\nu U\}v^{\nu\mu} + \frac{i}{2}\frac{N_c}{(4\pi)^2}\mathrm{Tr}\{D_\mu U^\dagger D_\nu U - D_\mu U D_\nu U^\dagger\}a^{\nu\mu}$$
$$+\frac{N_c}{6(4\pi)^2}\mathrm{Tr}D_\mu D_\nu U D^\mu D^\nu U^\dagger - \frac{N_c}{12(4\pi)^2}\mathrm{Tr}\{D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger$$
$$+D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U - D_\mu U D_\nu U^\dagger D^\mu U D^\nu U^\dagger\}$$
$$+\frac{1}{2}m_1^2(\rho_i^\mu\rho_{\mu i}+\omega^\mu\omega_\mu+a_i^\mu a_{\mu i}+f^\mu f_\mu)+\frac{1}{2}m_2^2(K_\mu^{*a}\bar{K}^{*a\mu}+K_1^\mu K_{1\mu})+\frac{1}{2}m_3^2(\phi_\mu\phi^\mu+f_s^\mu f_{s\mu}), \quad (3)$$

where

$$D_\mu U = \partial_\mu U - i[v_\mu, U] + i\{a_\mu, U\},$$
$$D_\mu U^\dagger = \partial_\mu U^\dagger - i[v_\mu, U^\dagger] - i\{a_\mu, U^\dagger\},$$
$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu],$$
$$a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu],$$
$$D_\nu D_\mu U = \partial_\nu(D_\mu U) - i[v_\nu, D_\mu U] + i\{a_\nu, D_\mu U\},$$
$$D_\nu D_\mu U^\dagger = \partial_\nu(D_\mu U^\dagger) - i[v_\nu, D_\mu U^\dagger] - i\{a_\nu, D_\mu U^\dagger\}.$$

Following Ref. [1], the effective Lagrangian describing the physical processes with abnormal parity will be evaluated in terms of the quark propagators.

In this paper except for the kaon form factor $f_-(q^2)$, all studies have been done in the chiral limit. In the chiral limit, the following definitions [1] are used in this paper:

$$\frac{F^2}{16} = \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4}\Gamma\left(2-\frac{D}{2}\right), \quad (4)$$
$$g^2 = \frac{8}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma\left(2-\frac{D}{2}\right) = \frac{1}{6}\frac{F^2}{m^2}. \quad (5)$$

According to the arguments of Ref. [1], the physical meson fields have been defined as

$$\rho \to \frac{1}{g}\rho, \quad K^* \to \frac{1}{g}K^*, \quad \omega \to \frac{1}{g}\omega, \quad \phi \to \frac{\sqrt{2}}{g}\phi,$$
$$a_\mu^i \to \frac{1}{g(1-\frac{1}{2\pi^2 g^2})^{\frac{1}{2}}}a_\mu^i - \frac{c}{g}\partial_\mu\pi^i, \quad f_\mu \to \frac{1}{g(1-\frac{1}{2\pi^2 g^2})^{\frac{1}{2}}}f_\mu - \frac{c}{g}\partial_\mu\eta_0,$$
$$K_{1\mu} \to \frac{1}{g(1-\frac{1}{2\pi^2 g^2})^{\frac{1}{2}}}K_{1\mu} - \frac{c}{g}\partial_\mu K, \quad f_{s\mu} \to \frac{\sqrt{2}}{g(1-\frac{1}{2\pi^2 g^2})^{\frac{1}{2}}}f_{s\mu} - \frac{c}{g}\partial_\mu\eta_s,$$
$$\pi \to \frac{2}{f_\pi}\pi, \quad K \to \frac{2}{f_K}K, \quad \eta \to \frac{2}{f_\eta}\eta, \quad \eta' \to \frac{2}{f_{\eta'}}\eta', \quad (6)$$

where $\eta_0 = (\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)\eta + (\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)\eta'$ and $\eta_s = (-\frac{2}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)\eta + (-\frac{2}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)\eta'$; θ is the mixing angle of η and η'. In the chiral limit, we take $f_\pi = f_K = f_\eta = f_{\eta'}$. In the chiral limit, the following two equations of Ref. [1] are held in the case of three flavors:

$$c = \frac{f_\pi^2}{2gm_\rho^2}, \quad (7)$$
$$\frac{F^2}{f_\pi^2}\left(1-\frac{2c}{g}\right) = 1. \quad (8)$$

Following Ref. [1] we have

$$g = 0.35. \quad (9)$$

Use of the substitutions (6) leads to the physical masses of vector mesons as

$$m_\rho^2 = m_\omega^2 = \frac{1}{g^2}m_1^2, \quad m_{K^*}^2 = \frac{1}{g^2}m_2^2, \quad m_\phi^2 = \frac{2}{g^2}m_3^2. \quad (10)$$

II. NEW MASS FORMULAS OF VECTOR MESONS AND ITS CHIRAL PARTNERS

In Ref. [1] two mass relations between a_1, ρ and $f_1(1285)$, ω have been obtained

$$\left(1-\frac{1}{2\pi^2 g^2}\right)m_a^2 = \frac{F^2}{g^2} + m_\rho^2,$$
$$\left(1-\frac{1}{2\pi^2 g^2}\right)m_f^2 = \frac{F^2}{g^2} + m_\omega^2. \quad (11)$$

Using the same method for obtaining Eqs. (11), we obtain two other mass formulas

$$\left(1-\frac{1}{2\pi^2 g^2}\right)m_{K_1}^2 = \frac{F^2}{g^2} + m_{K^*}^2,$$
$$\left(1-\frac{1}{2\pi^2 g^2}\right)m_{f_1(1510)}^2 = \frac{F^2}{g^2} + m_\phi^2. \quad (12)$$

If we input m_a, m_ρ, and f_π, we obtain

$$m_{f_1(1285)} = 1.27\ \text{GeV},\quad m_{K_1(1400)} = 1.38\ \text{GeV},$$
$$m_{f_1(1510)} = 1.51\ \text{GeV}. \tag{13}$$

The deviations from data are about 1%. In Table I, the masses of these mesons are obtained by taking $g = 0.35$.

Weinberg's first sum rule [3] is

$$\frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = \frac{1}{4}f_\pi^2, \tag{14}$$

where g_a and g_ρ are defined in Ref. [1]. In order to compare with this sum rule, the four mass formulas (11),(12) can be rewritten as

$$\frac{m_a^2}{g_a^2} - \frac{m_\rho^2}{g_\rho^2} = \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)},$$
$$\frac{m_{f_1(1285)}^2}{g_a^2} - \frac{m_\omega^2}{g_\rho^2} = \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)},$$
$$\frac{m_{K_1(1400)}^2}{g_a^2} - \frac{m_{K^*}^2}{g_\rho^2} = \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)},$$
$$\frac{m_{f_1(1510)}^2}{g_a^2} - \frac{m_\phi^2}{g_\rho^2} = \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)}. \tag{15}$$

TABLE I. Summary of the results.

	Experimental	Theoretical
f_π	186 MeV	Input
m_π	138 MeV	Input
m_{K^+}	494 MeV	Input
m_{K^0}	498 MeV	Input
m_η	548 MeV	Input
$m_{\eta'}$	958 MeV	Input
m_ρ	770 MeV	Input
m_{K^*}	892 MeV	Input
m_ϕ	1020 MeV	Input
g		0.35 input
m_{K_1}	1402±7 MeV	1510 MeV
$m_{f_1(1510)}$	1512±4 MeV	1640 MeV
$g_{\phi\gamma}$	$0.081(1 \pm 0.05)\ \text{GeV}^2$	$0.086\ \text{GeV}^2$
$\langle r^2\rangle_{K^\pm}$	$0.34 \pm 0.05\ \text{fm}^2$	$0.33\ \text{fm}^2$
$\langle r^2\rangle_{K^0}$	$0.054 \pm 0.026\ \text{fm}^2$[11]	$0.0582\ \text{fm}^2$
$\lambda_+(K_{l3}^+)$	0.0286 ± 0.0022	0.0239
$\xi(K_{l3}^+)$	-0.35 ± 0.15	-0.284
$\lambda_+(K_{l3}^0)$	0.03 ± 0.0016	0.0245
$\xi(K_{l3}^0)$	-0.11 ± 0.09	-0.287
$\Gamma(K_{e3}^+)$	$0.256(1 \pm 0.015) \times 10^{-17}$ GeV	0.233×10^{-17} GeV
$\Gamma(K_{e3}^0)$	$0.493(1 \pm 0.016) \times 10^{-17}$ GeV	0.483×10^{-17} GeV
$B(\tau \to K^*(892)\nu)$	$(1.45 \pm 0.18)\%$	1.46%
$\Gamma(\tau \to K_1(1400)\nu)$		0.373%
$\Gamma(\phi \to K^0\bar{K}^0)$	$1.52(1 \pm 0.03)$ MeV	1.11 MeV
$\Gamma(\phi \to K^+K^-)$	$2.18(1 \pm 0.03)$ MeV	1.7 MeV
$\Gamma(K^*(892) \to K\pi)$	49.8 ± 0.8 MeV	39.4 MeV
$\Gamma(K^{+*} \to K^+\gamma)$	$50.3(1 \pm 0.11)$ keV	43.5 keV
$\Gamma(K^{0*} \to K^0\gamma)$	$116.2(1 \pm 0.10)$ keV	175.4 keV
$B(K^*(892) \to K\pi\pi)$	0.53×10^{-4}	$< 7 \times 10^{-4}$
$f_1(1510) \to K^*(892)\bar{K}$	35 ± 15 MeV	22 MeV
$\Gamma(K_1(1400) \to K^*(892)\pi)$	$163.6(1 \pm 0.14)$ MeV	126 MeV
$B(K_1(1400) \to K\rho)$	$(3.0 \pm 3.0)\%$	11.1%
$B(K_1(1400) \to K\omega)$	$(2.0 \pm 2.0)\%$	2.4%
$\Gamma(K_1 \to K\gamma)$		440 keV
$\Gamma(\eta' \to \eta\pi^+\pi^-)$	$87.8(\pm 0.12)$ keV	85.7 keV
$\Gamma(\eta' \to \eta\pi^0\pi^0)$	$41.8(\pm 0.11)$ keV	48.6 keV
$\Gamma(\eta \to \gamma\gamma)$	$0.466(1 \pm 0.11)$ keV	0.619 keV
$\Gamma(\phi \to \eta\gamma)$	$56.7(1 \pm 0.06)$ keV	91.4 keV
$\Gamma(\rho \to \eta\gamma)$	$57.5(1 \pm 0.19)$ keV	61.4 keV
$\Gamma(\omega \to \eta\gamma)$	$7.0(1 \pm 0.26)$ keV	7.84 keV
$\Gamma(\eta' \to \gamma\gamma)$	$4.26(1 \pm 0.14)$ keV	4.88 keV
$\Gamma(\eta' \to \rho\gamma)$	$60.7(1 \pm 0.12)$ keV	63.0 keV
$\Gamma(\eta' \to \omega\gamma)$	$6.07(1 \pm 0.18)$ keV	5.86 keV

III. VECTOR MESON DOMINANCE (VMD) AND KAON FORM FACTORS

Vector meson dominance (VMD) has been obtained from this theory [1]. Before this paper the Nambu–Jona-Lasinio Lagrangian [2] has been employed to simulate many properties of the VMD [4] and in Ref. [5] a linear σ model has been used to unify many properties of the VMD by computing various constituent quark loops and working in the chiral limit. In these studies [4,5] the vector mesons and pions are coupled to the quarks and these couplings play the key role in obtaining VMD. The Lagrangian [Eq. (1)] of this paper is different from Refs. [4,5]. In this theory the mesons are coupled to the quarks too and the VMD is expected in this theory by the same reason as in Refs. [4,5]. Comparing with Refs. [4,5], the couplings between the mesons and quarks are determined in a different way.

In addition to the ρ and ω dominance

$$\frac{e}{f_\rho}\left\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\rho^0_\nu-\partial_\nu\rho^0_\mu)+A^\mu j^0_\mu\right\},$$

$$\frac{e}{f_\omega}\left\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\omega_\nu-\partial_\nu\omega_\mu)+A^\mu j^\omega_\mu\right\},$$

$$\frac{1}{f_\rho}=\frac{1}{2}g,\quad \frac{1}{f_\omega}=\frac{1}{6}g, \tag{16}$$

the ϕ dominance is derived from Eq. (1):

$$\frac{e}{f_\phi}\left\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\phi_\nu-\partial_\nu\phi_\mu)+A^\mu j^\phi_\mu\right\},\quad f_\phi=-\frac{1}{3\sqrt{2}}g, \tag{17}$$

where the ϕ meson couples to the current j^ϕ_μ.

The electric kaon form factor can be studied by VMD. The effective Lagrangian of $KK\gamma$ consists of two parts: kaons couple to the photon directly and kaons couple first to the vector mesons then the vector mesons couple to the photon. In the chiral limit, the couplings between kaons and ρ, ω, and ϕ mesons can be found from Eq. (3):

$$\begin{aligned}\mathcal{L}_{K\bar{K}v}=&-\frac{2\sqrt{2}i}{g}\phi^\mu(K^+\partial_\mu K^-+K^0\partial_\mu\bar{K}^0),\\&+\frac{2i}{g}\omega^\mu(K^+\partial_\mu K^-+K^0\partial_\mu\bar{K}^0),\\&+\frac{2i}{g}\rho^\mu(K^+\partial_\mu K^--K^0\partial_\mu\bar{K}^0).\end{aligned} \tag{18}$$

In Eq. (18), Eq. (8) has been used. Using the substitutions

$$\rho_\mu\to\frac{e}{f_\rho}A_\mu,\quad \omega_\mu\to\frac{e}{f_\omega}A_\mu,\quad \phi_\mu\to\frac{e}{f_\phi}A_\mu, \tag{19}$$

in Eq. (18), the direct couplings of $KK\gamma$ is obtained. From the couplings $-\frac{1}{2}\frac{e}{f_v}F_{\mu\nu}(\partial_\mu v_\nu-\partial_\nu v_\mu)$ $(v=\rho,\omega,\phi)$ [Eqs. (16),(17),(18)], the indirect couplings of $KK\gamma$ can be obtained. Adding these two couplings together, the electric form factor of the charged kaon is found:

$$F_{K^+}(q^2)=\frac{1}{3}\frac{m^2_\phi}{m^2_\phi-q^2}+\frac{1}{6}\frac{m^2_\omega}{m^2_\omega-q^2}+\frac{1}{2}\frac{m^2_\rho}{m^2_\rho-q^2}. \tag{20}$$

Because of Eq. (8) this form factor is normalized to be one at $q^2=0$ and the radius is determined to be

$$\langle r^2\rangle=\frac{2}{m^2_\phi}+\frac{1}{m^2_\omega}+\frac{3}{m^2_\rho}=0.33\ \text{fm}^2. \tag{21}$$

The theoretical result agrees with the data [6] (see Table I). In the same way, the electric form factor of the neutral kaon is then derived:

$$\begin{aligned}F_{K^0}(q^2)&=\frac{1}{3}\frac{m^2_\phi}{m^2_\phi-q^2}+\frac{1}{6}\frac{m^2_\omega}{m^2_\omega-q^2}-\frac{1}{2}\frac{m^2_\rho}{m^2_\rho-q^2},\\ \left.\frac{\partial F_{K^0}(q^2)}{\partial q^2}\right|_{q^2=0}&=\frac{1}{3m^2_\phi}+\frac{1}{6m^2_\omega}-\frac{1}{2m^2_\rho}=-0.25\ \text{GeV}^{-2},\\ \langle r^2\rangle_{K^0}&=-6\left.\frac{\partial F_{K^0}}{\partial q^2}\right|_{q^2=0}=0.0582\ \text{fm}^2.\end{aligned} \tag{22}$$

The comparison of $\langle r^2\rangle_{K^0}$ with data can be found in Table I.

The VMD can be applied to study the form factors of $K\to\pi l\nu$. Let us study $K^+\to\pi^0 l\nu$ first. The vertex of $K^*(892)K\pi^0$ has normal parity and can be derived from Eq. (3):

$$\begin{aligned}\mathcal{L}_{K^*K\pi^0}=&\frac{i}{g}\{(\pi^0\partial_\mu K^--\partial_\mu\pi^0K^-)K^{+\mu}+(\pi^0\partial_\mu K^+-\partial_\mu\pi^0K^+)K^{-\mu}\}\\&+\frac{4i}{gf^2_\pi}\left\{c^2-\frac{2N_c}{(4\pi)^2}\left(1-\frac{2c}{g}\right)^2\right\}\partial^\nu\pi^0\{(\partial_\mu K^-_\nu-\partial_\nu K^-_\mu)\partial^\mu K^-\}\\&-\frac{i}{8\pi^2g}\left(1-\frac{2c}{g}\right)^2\{(\partial^\nu\pi^0\partial_{\mu\nu}K^--\partial^\nu K^-\partial_{\mu\nu}\pi^0)K^{+\mu}+(\partial_\nu\pi^0\partial^{\mu\nu}K^+-\partial^{\mu\nu}\pi^0\partial_\nu K^+)K^-_\mu\}\ .\end{aligned} \tag{23}$$

In the decays of $K\to\pi l\nu$, there are direct couplings $K\pi W$ and indirect couplings $K\pi K^*$ and K^*W. Using the substitution obtained from Eq. (1),

$$K_\mu\to\frac{g_w}{4}gW_\mu\sin\theta_C\ . \tag{24}$$

In Eq. (23), the direct coupling can be obtained. In terms of VMD, the coupling between K^* and W boson is derived from Eq. (3):

$$\mathcal{L}_i = -\frac{1}{4} g g_w \{(\partial_\mu K_\nu^+ - \partial_\nu K_\mu^+)\partial^\mu W^{-\nu} + (\partial_\mu K_\nu^- - \partial_\nu K_\mu^-)\partial^\mu W^{+\nu}\} \ . \tag{25}$$

From Eqs. (23),(24),(25) the indirect coupling is obtained. Adding the direct and indirect couplings together, the two form factors of $K^+ \to \pi^0 l\nu$ are obtained:

$$f_+(q^2) = \frac{1}{\sqrt{2}} \frac{m_{K^*}^2}{m_{K^*}^2 - q^2}, \quad f_-(q^2) = -\frac{1}{\sqrt{2}} \frac{1}{m_{K^*}^2 - q^2}(m_{K^+}^2 - m_{\pi^0}^2),$$

$$\lambda_+ = -\lambda_- = \frac{m_{\pi^0}^2}{m_{K^*}^2} = 0.0239,$$

$$\xi = \frac{f_-}{f_+} = -\frac{m_{K^+}^2 - m_{\pi^0}^2}{m_{K^{+*}}^2} = -0.284. \tag{26}$$

For K_{l3}^0 we find the following quantities in the same way:

$$f_+(q^2) = \frac{1}{\sqrt{2}} \frac{m_{K^*}^2}{m_{K^*}^2 - q^2}, \quad f_-(q^2) = -\frac{1}{\sqrt{2}} \frac{1}{m_{K^*}^2 - q^2}(m_{K^0}^2 - m_{\pi^+}^2),$$

$$\lambda_+ = -\lambda_- = \frac{m_{\pi^+}^2}{m_{K^*}^2} = 0.0245,$$

$$\xi = \frac{f_-}{f_+} = -\frac{m_{K^0}^2 - m_{\pi^+}^2}{m_{K^{+*}}^2} = -0.287. \tag{27}$$

In Eqs. (26),(27), the leading terms of chiral perturbation have been kept. The decay widths are computed to be

$$\Gamma(K_{e3}^+) = 0.233 \times 10^{-17} \text{ GeV}, \quad \Gamma(K_{e3}^0) = 0.483 \times 10^{-17} \text{ GeV}. \tag{28}$$

The comparisons with the experimental data are shown in Table I.

IV. DECAYS OF $\tau \to K^*(892)\nu$ AND $\tau \to K_1(1400)\nu$

In a calculation similar to $\tau \to \rho\nu$ and $\tau \to a_1\nu$ [1], we obtain

$$\Gamma(\tau \to K^*(892)\nu) = \frac{G^2}{32\pi}\sin^2\theta_C g^2 m_{K^*}^2 m_\tau^3 \left(1 - \frac{m_{K^*}^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_{K^*}^2}{m_\tau^2}\right) = 0.326 \times 10^{-13} \text{ GeV},$$

$$B(\tau \to K_1^*(892)\nu) = 1.46\%,$$

$$\Gamma(\tau \to K_1\nu) = \frac{G^2}{32\pi}\sin^2\theta_C g^2 \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1} \frac{m_{K^*}^4}{m_{K_1}^2} m_\tau^3 \left(1 - \frac{m_{K_1}^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_{K_1}^2}{m_\tau^2}\right) = 0.831 \times 10^{-14} \text{ GeV},$$

$$B(\tau \to K_1(1400)\nu) = 0.373\%. \tag{29}$$

V. DECAYS OF ϕ, $K^*(892)$, $K_1(1400)$, AND $f_1(1510)$ MESONS

In this theory, the vertices of $\phi K\bar{K}$, $K^*K\pi$, $K_1K^*\pi$, $K_1K\rho$, $K_1K\omega$, and $f_1(1510)K^*\bar{K}$ contain even numbers of γ_5; therefore, they are the processes with normal parity and the vertices of these processes can be derived from Eq. (3). In this section the calculation of the decay widths of these processes is provided.

VI. DECAYS OF $\phi \to K\bar{K}$

In the chiral limit, the vertex of this process is derived from Eq. (3):

$$\mathcal{L}_{\phi K\bar{K}} = \frac{i2\sqrt{2}}{g}\phi_\mu(K^+\partial^\mu K^- + K^0\partial^\mu \bar{K}^0). \tag{30}$$

In deriving Eq. (30), Eq. (8) has been used. The numerical results of the decays are

$$\Gamma(\phi \to K^0\bar{K}^0) = 1.11 \text{ MeV}, \quad \Gamma(\phi \to K^+K^-) = 1.7\text{MeV}, \quad \frac{\Gamma(\phi \to K^0\bar{K}^0)}{\Gamma(\phi \to K^+K^-)} = 0.66. \tag{31}$$

VII. DECAYS OF $K^*(892) \to K\pi$

In the chiral limit and using Eq. (3), the vertex of this process is derived:

$$\mathcal{L}_{K^*K\pi} = \frac{2i}{g}\{\sqrt{2}\pi^+(K^-_\mu\partial^\mu K^0 - K^0_\mu\partial^\mu K^-) + \sqrt{2}\pi^-(\bar{K}^0_\mu\partial^\mu K^+ - K^+_\mu\partial^\mu\bar{K}^0) + \pi^0(K^-_\mu\partial^\mu K^+ - K^+_\mu\partial^\mu K^- - \bar{K}^0_\mu\partial^\mu K^0 + K^0_\mu\partial^\mu\bar{\mu}K^0)\}\,. \tag{32}$$

The numerical results of the decay widths are

$$\Gamma(K^* \to K^0\pi^+) = 25.4 \text{ MeV}, \quad \Gamma(K^* \to K^+\pi^0) = 14.0 \text{ MeV}, \quad \Gamma_{\text{tot}} = 39.4 \text{ MeV}. \tag{33}$$

VIII. DECAYS OF $K_1(1400)$

In the chiral limit, the vertex of $K_1 \to K^*\pi$ is found from Eq. (3):

$$\mathcal{L}_{K_1K^*\pi} = f_{abi}\pi^i\{AK^a_{1\mu}K^{b\mu} + Bp^\mu_\pi p^\nu_\pi K^a_{1\mu}K^b_\nu\},$$
$$A = \frac{2}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}(m^2_{K_1} - m^2_{K^*})\left(1-\frac{2c}{g}\right)\left(1-\frac{3}{4\pi^2g^2}\right),$$
$$B = -\frac{2}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}\frac{1}{2\pi^2g^2}\left(1-\frac{2c}{g}\right), \tag{34}$$

where $m^2_{K_1}$ is determined by Eq. (12). In deriving the expression for A [Eq. (34)], Eq. (12) has been applied. The numerical results are

$$\Gamma(K_1 \to K^{*+}\pi^0) = 42 \text{ MeV}, \quad \Gamma(K_1 \to K^{*0}\pi^+) = 2\Gamma(K_1 \to K^{*+}\pi^0), \quad \Gamma_{\text{tot}} = 126 \text{ MeV}. \tag{35}$$

The vertex of $K_1K\rho$ can be found from Eq. (3) and it is just the formula obtained by rearranging Eq. (34):

$$A = \frac{2}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}\left\{m^2_{K_1} - m^2_{K^*} - (m^2_{K_1} - m^2_\rho) \times\left[\frac{2c}{g} + \frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right]\right\}. \tag{36}$$

The decay width of $K_1 \to K\rho$ is calculated to be

$$\Gamma(K_1 \to K\rho) = 19.3 \text{ MeV},$$
$$B(K_1 \to K\rho) = 11.1(1 \pm 0.075)\%. \tag{37}$$

In the same way, if we ignore the mass difference of ρ and ω mesons we obtain

$$\Gamma(K_1 \to K\omega) = \frac{1}{3}\Gamma(K_1 \to K\rho). \tag{38}$$

The numerical results are

$$\Gamma(K_1 \to K\omega) = 4.12 \text{ MeV}, \quad B(K_1 \to K\omega) = 2.4\%. \tag{39}$$

Comparing $\Gamma(K_1 \to K\rho)$ and $\Gamma(K_1 \to K\omega)$ with $\Gamma(K_1 \to K^*\pi)$, the former are much less than the latter. Except for phase space, the differences of the formulas for these three processes are caused by the masses of ρ, ω, and K^* in the amplitude A. The cancellations in A [Eq. (36)] cause the smallness of $\Gamma(K_1 \to K\rho)$ and $\Gamma(K_1 \to K\omega)$.

IX. DECAY OF $K_1 \to K\gamma$

Using VMD [Eq. (16)], following vertex is derived from the vertex of K_1Kv:

$$\mathcal{L}_{K_1K\gamma} = -\frac{i}{2}e\left(\frac{1}{f_\rho}+\frac{1}{f_\omega}-\frac{1}{f_\phi}\right)\frac{2}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}\left\{m^2_{K_1} - m^2_{K^*} - m^2_{K_1}\left[\frac{2c}{g} + \frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right]\right\}. \tag{40}$$

The numerical result is

$$\Gamma(K_1 \to K\gamma) = 440 \text{ keV}. \tag{41}$$

X. DECAYS OF $f_1(1510) \to K^*(892)\bar{K}$

From Eq. (3), the decay amplitude of this decay is found:

$$\langle K^+(p_1)K^{*-}(p_2)|S|f_1(p)\rangle = -(2\pi)^4\delta^4(p-p_1-p_2)\frac{1}{\sqrt{8m_f E_K E_{K^*}}}e^{\lambda}_{\mu}(p)e^{\lambda'*}_{\nu}\{Ag^{\mu\nu}+Bp_1^{\mu}p_2^{\nu}\},$$
$$A = \frac{1}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}(m_f^2-m_{K^*}^2)\left(1-\frac{2c}{g}\right)\left(1-\frac{3}{4\pi^2g^2}\right),$$
$$B = -\frac{1}{f_\pi}\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}\frac{1}{2\pi^2g^2}\left(1-\frac{2c}{g}\right). \tag{42}$$

There are four channels in this decay and the numerical results are

$$\Gamma(f_1\to K^+K^{*-}) = 5.48\ \text{MeV}, \quad \Gamma_{\text{tot}} = 21.9\ \text{MeV}. \tag{43}$$

XI. DECAYS OF $\eta' \to \eta\pi\pi$

In this theory, the vertex of this process contains four factors of γ_5. Therefore, this is a process of normal parity and the vertex should be derived from Eq. (3). It is well known that the masses of the pion and η are proportional to light quark masses [7], therefore, in the chiral limit, $m_\pi, m_\eta \to 0$. However, because of the U(1) problem [8] $m^2_{\eta'}$ does not approach zero in the limit of chiral symmetry. Therefore, in the chiral limit only the mass of the η' meson can be kept in the amplitude of $\eta'\to\eta\pi\pi$. The calculation shows that in Eq. (3) only the terms at the fourth order in derivatives contribute to $\eta'\to\eta\pi\pi$. Consequently, in the amplitude of this decay there is a factor of $\frac{1}{(4\pi)^2}$. Therefore, this theory predicts that the width of this decay is very narrow. The amplitude is found to be

$$\langle\pi^0(k_1)\pi^0(k_2)\eta(p)|S|\eta'(p')\rangle = i(2\pi)^4\delta^4(p'-p-k_1-k_2)\frac{1}{\sqrt{16m_{\eta'}E_\eta\omega_1\omega_2}}$$
$$\times\frac{8}{f_\pi^4}\frac{2}{(4\pi)^2}\left\{\frac{1}{2}\left(1-\frac{2c}{g}\right)^4(q_1^4+q_2^4+q_3^4)+\left(1-\frac{2c}{g}\right)\left[\frac{2c^2}{g^2}-\left(1-\frac{2c}{g}\right)^2\right]m^4_{\eta'}\right.$$
$$\left.+\left(1-\frac{2c}{g}\right)\left[\frac{1}{2}\left(1-\frac{2c}{g}\right)+\frac{4c^3}{g^3}\right]q_3^2m^2_{\eta'}+\frac{1}{2}\left(1-\frac{2c}{g}\right)^2\left(1-\frac{4c^2}{g^2}\right)(q_1^2+q_2^2)m^2_{\eta'}\right\}, \tag{44}$$

where $q_1^2=(p'-k_1)^2$, $q_2^2=(p'-k_2)^2$, and $q_3^2=(p'-p)^2$. The contribution of the quark masses to the mass of η' is about 0.376 GeV; therefore, in the chiral limit $m_{\eta'} = 0.582$ GeV. Using this value we obtain

$$\Gamma(\eta'\to\eta\pi^+\pi^-) = 85.7\ \text{keV},$$
$$\Gamma(\eta'\to\eta\pi^0\pi^0) = 48.6\ \text{keV}. \tag{45}$$

In the range of $(0.958\ \text{GeV})^2 \geq m^2_{\eta'} \geq 0$, we obtain

$$22.1\ \text{keV} \leq \Gamma(\eta'\to\eta\pi^+\pi^-) \leq 145.2\ \text{keV},$$
$$12.5\ \text{keV} \leq \Gamma(\eta'\to\eta\pi^0\pi^0) \leq 82.4\ \text{keV}. \tag{46}$$

Equation (46) shows that, indeed, the decay widths are always small and the data (see Table I) prefers a nonzero $m_{\eta'}$ in the chiral limit. This is consistent with the study of the U(1) problem in $m_{\eta'}$ [8]. Therefore, phenomenologically, in the Lagrangian (1) a mass term of η' should be added. The study of the U(1) problem could bring something new to the present theory. However, this is not the task of this paper.

XII. DECAYS OF $K^*(892) \to K\gamma$ AND $K\pi\pi$

The decays of $K^*\to K\gamma$ and $K\pi\pi$ have been studied in Ref. [9] by using the gauging Wess-Zumino Lagrangian. As mentioned in Ref. [1], the formalism obtained from this theory is the same as the one in Ref. [9]. However, in this theory the couplings are universal and VMD is a result of the present theory and not an input. According to VMD, the decays of $K^*\to K\gamma$ are associated with $K^*\to Kv$. Therefore, the processes of $K^*\to K\gamma$ have abnormal parity. The vertices of K^*Kv can be found from the calculation of $\frac{1}{g}K^*_{a\mu}\langle\bar\psi\lambda_a\gamma^\mu\psi\rangle$, which is similar to $\frac{1}{g}\omega_\mu\langle\bar\psi\gamma^\mu\psi\rangle$ in Ref. [1]:

$$\mathcal{L}_{K^*Kv} = -\frac{N_c}{2g^2\pi^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}d_{abc}K_{a\mu}\partial_\nu v^c_\alpha\partial_\beta P^b, \tag{47}$$

where P^b is a pseudoscalar meson and v^i_α is a vector meson. From Eq. (47) the following vertices are derived:

$$\mathcal{L}_i = -\frac{N_c}{2\pi^2 g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^+_\mu\partial_\beta K^+\left\{\frac{1}{2}\partial_\nu\rho^0_\alpha + \frac{1}{2}\partial_\nu\omega_\alpha + \frac{\sqrt{2}}{2}\partial_\nu\phi_\alpha\right\}. \quad (48)$$

Using VMD [Eqs. (16),(17)], we find

$$\mathcal{L}_{K^{+*}K^+\gamma} = -\frac{e}{4\pi^2 g}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^+_\mu\partial_\beta K^+\partial_\nu A_\alpha. \quad (49)$$

The decay width is computed to be

$$\Gamma(K^{+*}\to K^+\gamma) = 43.5 \text{ keV}. \quad (50)$$

In the same way, we obtain

$$\mathcal{L}_{K^{0*}K^0\gamma} = \frac{e}{2\pi^2 g}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^0_\mu\partial_\beta \bar{K}^0\partial_\nu A_\alpha, \quad (51)$$

and the decay width is

$$\Gamma(K^{0*}\to K^0\gamma) = 175.4 \text{ keV}. \quad (52)$$

The experimental value of the branching ratio of $K^*\to K\pi\pi$ is less than 7×10^{-4} [6]. To understand so small a branching ratio is a crucial test for the present theory. There are three channels:

$$K^{-*}\to K^-\pi^0\pi^0, K^-\pi^+\pi^-, \bar{K}^0\pi^-\pi^0.$$

The decay $K^{-*}\to K^-\pi^0\pi^0$ consists of $K^{-*}\to K^{-*}\pi^0$ and $K^{-*}\to K^-\pi^0$. The vertices are found from Eqs. (47),(32):

$$\mathcal{L}_{K^{-*}K^{+*}\pi^0} = -\frac{N_c}{\sqrt{2}\pi^2 g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^-_\mu\partial_\nu K^+_\alpha\partial_\beta\pi^0,$$

$$\mathcal{L}_{K^{-*}K^+\pi^0} = \frac{2i}{g}K^-_\mu\partial^\mu K^+\pi^0. \quad (53)$$

These two vertices lead to the following amplitude for $K^{-*}\to K^-\pi^0\pi^0$:

$$\mathcal{M} = \frac{\sqrt{2}N_c}{\pi^2 g^3}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon^\lambda_\mu(p')p'_\nu k_{1\alpha}k_{2\beta}\left\{\frac{1}{(p'-k_2)^2-m^2_{K^*}} - \frac{1}{(p'-k_1)^2-m^2_{K^*}}\right\}, \quad (54)$$

where p', p, k_1, and k_2 are the momenta of K^*, K, π^0, and π^0, respectively. It can be seen that there is cancellation in Eq. (54). This cancellation has been obtained in Ref. [9]. The calculated width is

$$\Gamma(K^{-*}\to K^-\pi^0\pi^0) = 0.214 \text{ keV}. \quad (55)$$

The second channel $K^{-*}\to K^-\pi^+\pi^-$ consists of three processes: direct coupling $K^{-*}K^+\pi^+\pi^-$ and indirect couplings: $K^{-*}\to\bar{K}^{0*}\pi^-$ and $\bar{K}^{0*}\to K^-\pi^+$, $K^{-*}\to K^-\rho^0$ and $\rho^0\to\pi^+\pi^-$. The direct coupling is derived from $\frac{1}{g}K^{a\mu}\langle\bar{\psi}\lambda_a\gamma_\mu\psi\rangle$ with a calculation similar to the one from which the direct coupling $\omega\pi\pi\pi$ has been found in Ref. [1]:

$$\mathcal{L}_{K^*K\pi\pi} = \frac{1}{4\pi^2 g}\left(\frac{2}{f_\pi}\right)^3\left(1-\frac{6c}{g}+\frac{6c^2}{g^2}\right)\varepsilon^{\mu\nu\alpha\beta}K^a_\mu\partial_\nu P^b\partial_\alpha P^c\partial_\beta P^d d_{abe}f_{cde}, \quad (56)$$

where P stands for pseudoscalar field. Equation (56) leads to

$$\mathcal{L}_{K^{-*}K^+\pi^+\pi^-} = -\frac{i}{2\sqrt{2}\pi^2 g}\left(\frac{2}{f_\pi}\right)^3\left(1-\frac{6c}{g}+\frac{6c^2}{g^2}\right)\varepsilon^{\mu\nu\alpha\beta}K^-_\mu\partial_\nu K^+\partial_\alpha\pi^-\partial_\beta\pi^+. \quad (57)$$

From Eqs. (32),(47) the following vertices are obtained:

$$\mathcal{L}_{K^{-*}K^{0*}\pi^+} = -\frac{N_c}{2\sqrt{2}\pi^2 g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^-_\mu\partial_\nu K^0_\alpha\partial_\beta\pi^+,$$

$$\mathcal{L}_{\bar{K}^{0*}K^+\pi^-} = \frac{2\sqrt{2}i}{g}\pi^-\bar{K}^0_\mu\partial^\mu K^+,$$

$$\mathcal{L}_{K^{-*}K^+\rho^0} = -\frac{N_c}{4\pi^2 g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K^-_\mu\partial_\beta K^+\partial_\nu\rho^0_\alpha,$$

$$\mathcal{L}_{\rho^0\pi\pi} = \frac{2}{g}\epsilon_{3jk}\rho^0_\mu\pi_j\partial^\mu\pi_k, \quad (58)$$

where $\mathcal{L}_{\rho^0\pi\pi}$ is from Ref. [1]. These vertices lead to the amplitude

$$\mathcal{M}_{K^{-*}\to K^-\pi^+\pi^-} = \frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu(p')p'_\nu k_{+\alpha}k_{-\beta}\left\{\frac{1}{2\sqrt{2}\pi^2 g}\left(\frac{2}{f_\pi}\right)^2\left(1-\frac{6c}{g}+\frac{6c^2}{g^2}\right) + \frac{N_c}{\pi^2 g^3}\left[\frac{1}{(p'-k_-)^2-m^2_{K^*}} - \frac{1}{(p'-p)^2-m^2_\rho}\right]\right\}, \quad (59)$$

where p', p, and k_- are the momenta of K^{-*},K^-, and π^-, respectively. The width is computed to be

$$\Gamma(K^{-*}\to K^-\pi^+\pi^-)=1.21\ \text{keV}. \tag{60}$$

In the same way, the amplitude of $K^{-*}\to \bar{K}^{0*}\pi^-\pi^0$ is obtained:

$$\mathcal{M}_{K^{-*}\to\bar{K}^0\pi^-\pi^0}=\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu(p')p'_\nu k_{0\alpha}k_{-\beta}\left\{\frac{1}{2\pi^2 g}\left(\frac{2}{f_\pi}\right)^2\left(1-\frac{6c}{g}+\frac{6c^2}{g^2}\right)\right.$$
$$\left.-\frac{N_c}{\sqrt{2}\pi^2 g^3}\left[\frac{1}{(p'-k_0)^2-m^2_{K^*}}+\frac{1}{(p'-k_-)^2-m^2_{K^*}}\right]+\frac{\sqrt{2}N_c}{\pi^2 g^3}\frac{1}{(p'-p)^2-m^2_\rho}\right\}, \tag{61}$$

where p', p, k_-, and k_0 are the momenta of K^{-*}, $\bar{K}^0$, π^-, and π^0, respectively. The width is calculated

$$\Gamma(K^{-*}\to\bar{K}^0\pi^0\pi^-)=1.23\ \text{keV}. \tag{62}$$

The total width is 2.65 keV which is below the experimental limit. From Eqs. (59),(61) it can be seen that there are cancellations also in these two amplitudes. In these processes there are subprocesses of normal parity and abnormal parity and the relative signs between these subprocesses have been determined without any ambiguity. Because all the vertices are derived from the Lagrangian (1), we have a universality of the couplings in this theory. Both the smallness of the phase space and the cancellations cause the smallness of the branching ratio of $K^*\to K\pi\pi$.

XIII. ELECTROMAGNETIC DECAYS OF MESONS

In this section the processes $\phi\to\eta\gamma$, $\eta\to\gamma\gamma$, $\eta'\to\rho\gamma,\omega\gamma$, and $\eta'\to\gamma\gamma$ are studied. They have been studied in Ref. [9]. The formulas obtained in this theory are the same as the ones derived from the gauging Wess-Zumino Lagrangian in Ref. [9]. However, as mentioned above, in this theory there is universality of couplings and VMD is not an input. In the vertices of these processes the number of γ_5 is odd and they are processes of abnormal parity. In Ref. [1], $\langle\bar\psi\gamma_5\psi\rangle$ has been evaluated [see Eq. (177) of Ref. [1]]. In the same way $\langle\bar\psi\lambda_8\gamma_5\psi\rangle\rangle$ can be computed. From $\langle\bar\psi\gamma_5\psi\rangle$ and $\langle\bar\psi\lambda_8\gamma_5\psi\rangle$ the vertices of ηvv and $\eta' vv$ are found to be

$$\mathcal{L}_{\eta vv}=\frac{N_c}{(4\pi)^2}\frac{4}{g^2}\varepsilon^{\mu\nu\alpha\beta}\eta\left\{\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)(\partial_\mu\omega_\nu\partial_\alpha\omega_\beta+\partial_\mu\rho^i_\nu\partial_\alpha\rho^i_\beta)-\left(\sqrt{\frac{2}{3}}\sin\theta+\frac{2}{\sqrt{3}}\cos\theta\right)\partial_\mu\phi_\nu\partial_\alpha\phi_\beta\right\},$$
$$\mathcal{L}_{\eta' vv}=\frac{N_c}{(4\pi)^2}\frac{4}{g^2}\varepsilon^{\mu\nu\alpha\beta}\eta'\left\{\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)(\partial_\mu\omega_\nu\partial_\alpha\omega_\beta+\partial_\mu\rho^i_\nu\partial_\alpha\rho^i_\beta)+\left(\sqrt{\frac{2}{3}}\cos\theta-\frac{2}{\sqrt{3}}\sin\theta\right)\partial_\mu\phi_\nu\partial_\alpha\phi_\beta\right\}, \tag{63}$$

where θ is the mixing angle between η and η'. Combining VMD [Eqs. (16),(17)] and Eqs. (63), the decay widths of the physical processes are found:

$$\Gamma(\eta\to\gamma\gamma)=\frac{\alpha^2}{16\pi^3}\frac{m^3_\eta}{f^2_\eta}\left(2\sqrt{\frac{2}{3}}\sin\theta-\frac{1}{\sqrt{3}}\cos\theta\right)^2,$$
$$\Gamma(\eta'\to\gamma\gamma)=\frac{\alpha^2}{16\pi^3}\frac{m^3_{\eta'}}{f^2_{\eta'}}\left(2\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)^2,$$
$$\Gamma(\phi\to\eta\gamma)=\frac{\alpha}{48\pi^4 g^2}\frac{m^3_\phi}{f^2_\eta}\left(1-\frac{m^2_\eta}{m^2_\phi}\right)^3\times\left(\sqrt{\frac{2}{3}}\sin\theta+\frac{2}{\sqrt{3}}\cos\theta\right)^2,$$
$$\Gamma(\omega\to\eta\gamma)=\frac{\alpha}{96\pi^4 g^2}\frac{m^3_\omega}{f^2_\eta}\left(1-\frac{m^2_\eta}{m^2_\omega}\right)^3\times\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)^2,$$
$$\Gamma(\rho\to\eta\gamma)=\frac{3\alpha}{32\pi^4 g^2}\frac{m^3_\rho}{f^2_\eta}\left(1-\frac{m^2_\eta}{m^2_\rho}\right)^3\times\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)^2,$$
$$\Gamma(\eta'\to\rho\gamma)=\frac{9\alpha}{32\pi^4 g^2}\frac{m^3_{\eta'}}{f^2_{\eta'}}\left(1-\frac{m^2_\rho}{m^2_{\eta'}}\right)^3\times\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)^2,$$
$$\Gamma(\eta'\to\omega\gamma)=\frac{\alpha}{32\pi^4 g^2}\frac{m^3_{\eta'}}{f^2_{\eta'}}\left(1-\frac{m^2_\omega}{m^2_{\eta'}}\right)^3\times\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)^2. \tag{64}$$

There are two values for the mixing angle θ [6]. $\theta=-10^\circ$ from the quadratic mass formula and $\theta=-23^\circ$ from the linear mass formula. According to Ref. [10], the two photon decays of η and η' favor $\theta = -20^0$. In this theory, $\theta = -20^0$ gives a better fit too. In the chiral limit, we take $f_\eta = f_{\eta'} = f_\pi$. The numerical results are shown in Table I.

XIV. CONCLUSIONS

In this paper two new mass formulas have been obtained. The theoretical values of the hadronic decay rates are lower than the data. The worst one is $\phi \to K\bar{K}$ which is less than the data by 30%. The corrections from the strange quark mass should alleviate these deviations. In Ref. [7] the corrections of the strange quark mass to f_K and f_η have been studied. All other results agree with the data well. In particular, this theory provides a better understanding of the smallness of $\Gamma(K_1 \to K\rho, K\omega)$, $\Gamma(K^* \to K\pi\pi)$ and the decay of $\eta' \to \eta\pi\pi$. The values for f_π, m_π, m_η, m_ρ, and g are not only inputs here; they are also inputs of Ref. [1]. It should be pointed out that the introduction of vector and axial-vector fields to the theory is not based on gauge invariance, but on the minimum coupling principle. This opens a door to introduce other mesons to the theory. In the chiral limit, the cutoff determined in Ref. [1] is 1.6 GeV. The mass of $f_1(1510)$ is closer to this value. However, we still obtain a reasonable result for the decay $f_1(1510) \to K^*K$.

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