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$K_L \to \pi^+\pi^- e^+ e^-$

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We calculate all of the form factors for the one-photon, $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$ contribution to the $K_L \to \pi^+\pi^- e^+ e^-$ decay amplitude at leading order in chiral perturbation theory. These form factors depend on one unknown constant that is a linear combination of coefficients of local $O(p^4)$ operators in the chiral Lagrangian for weak radiative kaon decay. We determine the differential rate for $K_L \to \pi^+\pi^- e^+ e^-$ and also the magnitude of two CP-violating observables.

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I. INTRODUCTION

At the present time about 20 $K_L \to \pi^+\pi^- e^+ e^-$ events have been observed and a detailed experimental study of this decay mode will be possible in future experiments [1]. The $K_L \to \pi^+\pi^- e^+ e^-$ weak decay amplitude is dominated by the process $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$, where a single virtual photon creates the e^+e^- pair. This one-photon contribution to the decay amplitude has the form

$$M^{(1\gamma)} = \frac{s_1 G_F \alpha}{4\pi f q^2}[iG\varepsilon^{\mu\lambda\rho\sigma} p_{+\lambda} p_{-\rho} q_\sigma + F_+ p_+^\mu + F_- p_-^\mu] \cdot \overline{u}(k_-)\gamma_\mu v(k_+) , \tag{1.1}$$

where G_F is Fermi's constant, α is the electromagnetic fine-structure constant, $s_1 \simeq 0.22$ is the sine of the Cabibbo angle, and $f \simeq 132$ MeV is the pion decay constant. The π^+ and π^- four-momenta are denoted by p_+ and p_- and the e^+ and e^- four-momenta are denoted by k_+ and k_-. The sum of the electron and positron four-momenta is $q = k_- + k_+$. The Lorentz scalar form factors $G, F_\pm$ depend on the scalar products of the four-momenta q, p_+, and p_-. Neglecting CP nonconservation, under interchange of the pion four-momenta

$$p_+ \to p_- \quad \text{and}\ p_- \to p_+ , \tag{1.2}$$

the form factors become

$$G \to G , \quad F_+ \to F_- , \quad F_- \to F_+ . \tag{1.3}$$

In this paper we compute the CP-conserving contribution to the form factors $G, F_\pm$ using chiral perturbation theory at one-loop order [the $O(p^2)$ amplitude vanishes]. The coefficients of some of the local operators appearing at the same order in the chiral expansion (i.e., order p^4 counterterms, where p is a typical momentum) are determined by the experimental value of the pion charge radius and the measured $K^+ \to \pi^+ e^+ e^-$ and $K_L \to \pi^+\pi^-\gamma$ decay rates and spectra.

We also compute (in chiral perturbation theory) an important tree-level contribution to the form factors $F_\pm$ that arises from the small CP-even component of the K_L state. This contribution to the $F_\pm$ form factors from indirect CP nonconservation has the opposite symmetry property under interchange of pion momenta when compared with the CP-conserving contribution to $F_\pm$ [see Eqs. (1.2) and (1.3)]. If

$$p_+ \to p_- \quad \text{and}\ p_- \to p_+ , \tag{1.4}$$

then the CP-violating one-photon form factors become

$$F_+ \to -F_- , \quad F_- \to -F_+ . \tag{1.5}$$

The decay amplitude that follows from squaring the invariant matrix element in Eq. (1.1) and summing over e^+ and e^- spins is symmetric under interchange of e^+ and e^- momenta, $k_- \leftrightarrow k_+$. Physical variables that are antisymmetric under interchange of the e^+ and e^- momenta arise from the interference of the short-distance contributions (Z-penguin and W-box diagrams) and the two-photon piece with the one-photon amplitude given in Eq. (1.1).

In the minimal standard model the coupling of the quarks to the W bosons has the form

$$\mathcal{L}_{\text{int}} = -\frac{g_2}{\sqrt{2}}\overline{u}_L^j\gamma_\mu V^{jk} d_L^k W^\mu + \text{H.c.} \tag{1.6}$$

Here repeated generation indices j, k are summed over 1,2,3 and g_2 is the weak SU(2) gauge coupling. V is a 3×3 unitary matrix [the Cabibbo-Kobayashi-Maskawa (CKM) matrix] that arises from diagonalization of the quark mass matrices. By redefining phases of the quark fields it is possible to write V in terms of four angles θ_1, θ_2, θ_3, and δ. The θ_j are analogous to the Euler angles and δ is a phase that, in the minimal standard model, is responsible for the observed CP violation. Explicitly

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} , \tag{1.7}$$

where $c_1 \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. It is possible to choose the θ_j to lie in the first quadrant. Then the quadrant of δ has physical significance and cannot be chosen by a phase convention of the quark fields. A value of δ not equal to 0 or π gives rise to CP violation.

The short-distance W-box and Z-penguin Feynman diagrams depend on the V_{ts} element of the Cabibbo-Kobayashi-Maskawa matrix. It is very important to be able to determine this coupling experimentally. In this paper we calculate the contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude arising from the Z-penguin and W-box diagrams which can be determined using chiral perturbation theory since the left-handed current $\bar{s}\gamma_\mu(1-\gamma_5)d$ is related to a generator of chiral symmetry. At the present time all observed CP nonconservation has its origin in K^0-$\overline{K}^0$ mass mixing. A CP-violating variable can be constructed in the decay $K_L \to \pi^+\pi^-e^+e^-$ that gets an important contribution from CP nonconservation in the Z-penguin and W-box diagrams, that is, direct CP violation. The variable (in the K_L rest frame)

$$A_{CP} = \left\langle \frac{(\vec{p}_- \times \vec{p}_+)\cdot(\vec{k}_- - \vec{k}_+)}{|(\vec{p}_- \times \vec{p}_+)\cdot(\vec{k}_- - \vec{k}_+)|} \right\rangle \tag{1.8}$$

is even under charge conjugation and odd under parity. It is also odd under interchange of $\vec{k}_+$ and $\vec{k}_-$. The real and imaginary parts of V_{ts} are comparable, and hence the CP-conserving and CP-violating parts of the Z-penguin and W-box diagrams are of roughly equal importance. A_{CP} gets a significant contribution from this direct source of CP nonconservation. In this paper we calculate A_{CP} in the minimal standard model but unfortunately we find that it is quite small; $|A_{CP}| \approx 10^{-4}$.

The decay $K_L \to \pi^+\pi^-e^+e^-$ has been studied previously by Sehgal and Wanninger [2] and by Heiliger and Sehgal [3]. These authors adopted a phenomenological approach, relating the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude to the measured $K_L \to \pi^+\pi^-\gamma$ decay amplitude. In the systematic expansion of chiral perturbation theory we find that there may be important additional contributions to the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude for $q^2 = (k_- + k_+)^2 \gg 4m_e^2$ that were not included in this previous work. It was pointed out in [2,3] that indirect CP nonconservation from K^0-$\overline{K}^0$ mixing gives an important contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay rate and consequently there is a CP-violating observable B_{CP} that is quite large. We reexamine B_{CP} using the form factors determined in this paper.

II. THE ONE-PHOTON AMPLITUDE

Chiral perturbation theory provides a systematic approach to understanding the one-photon part of the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude. It uses an effective field theory that incorporates the $SU(3)_L \times SU(3)_R$ chiral symmetry of QCD and an expansion in powers of momentum to reduce the number of operators that occur. In the chiral Lagrangian the π's, K's, and η are incorporated into a 3×3 special unitary matrix:

$$\Sigma = \exp\left(\frac{2iM}{f}\right) , \tag{2.1}$$

where

$$M = \begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \overline{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} . \tag{2.2}$$

At leading order in chiral perturbation theory $f \simeq 132$ MeV is the pion decay constant. Under $SU(3)_L \times SU(3)_R$ transformations the Σ field transforms as

$$\Sigma \to L\Sigma R^\dagger , \tag{2.3}$$

where $L \in SU(3)_L$ and $R \in SU(3)_R$.

At leading order in chiral perturbation theory (i.e., order p^2, where p is a typical four-momentum) the strong and electromagnetic interactions of the pseudo Goldstone bosons are described by the chiral Lagrange density

$$\mathcal{L}_S^{(1)} = \frac{f^2}{8}\,\mathrm{Tr}(D_\mu\Sigma D^\mu\Sigma^\dagger) + v\,\mathrm{Tr}(m_q\Sigma + m_q\Sigma^\dagger) , \tag{2.4}$$

where v is a parameter with dimensions of mass to the third power and m_q is the quark mass matrix:

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} . \tag{2.5}$$

In this paper we neglect isospin violation in the quark mass matrix and set $m_u = m_d$. In this approximation the K^0 and K^+ have equal masses which we denote by m_K, and the Gell-Mann–Okubo mass relation

$$3m_\eta^2 - 4m_K^2 + m_\pi^2 = 0 \tag{2.6}$$

holds.

The effective Lagrangian for $\Delta S = 1$ weak nonleptonic decays transforms as $(8_L, 1_R) + (27_L, 1_R)$ under $SU(3)_L \otimes SU(3)_R$. The $(8_L, 1_R)$ amplitudes are much larger than the $(27_L, 1_R)$ amplitudes and so we will neglect the $(27_L, 1_R)$ part of the effective Lagrangian. The effective Lagrangian for weak radiative kaon decay is obtained by gauging the effective Lagrangian for weak nonleptonic decays with respect to the $U(1)_Q$ of electromagnetism. At leading order in chiral perturbation theory the $\Delta S = 1$ transitions are described by

$$\mathcal{L}_W^{(1)} = \frac{g_8 G_F s_1 f^4}{4\sqrt{2}}\,\mathrm{Tr}[D_\mu\Sigma D^\mu\Sigma^\dagger T] + \mathrm{H.c.} \tag{2.7}$$

The matrix T in (2.7) projects out the correct flavor structure of the octet

$$T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} , \tag{2.8}$$

and g_8 is a constant determined by the measured $K_S \to$

$\pi^+\pi^-$ decay rate; $|g_8| \simeq 5.1$. In (2.4) and (2.7) D_μ represents a covariant derivative

$$D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu [Q, \Sigma] , \tag{2.9}$$

where

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \tag{2.10}$$

is the electromagnetic charge matrix for the three lightest quarks, u, d, and s.

The K_L state

$$|K_L\rangle \simeq |K_2\rangle + \epsilon |K_1\rangle \tag{2.11}$$

is mostly the CP-odd state

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) , \tag{2.12}$$

with small admixture of the CP-even state

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) . \tag{2.13}$$

The parameter ϵ characterizes CP nonconservation in K^0-$\overline{K}^0$ mixing. At leading order in chiral perturbation theory the $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$ decay amplitude arises though the CP-even component of K_L. Writing the form factors contributing to $K_L \to \pi^+\pi^-\gamma^*$ as a power series in the chiral expansion,

$$F_\pm = F_\pm^{(1)} + F_\pm^{(2)} + \cdots , \quad G = G^{(1)} + G^{(2)} + \cdots, \tag{2.14}$$

where the superscript denotes the order of chiral perturbation theory, we find that the Feynman diagrams in Fig. 1 give

$$G^{(1)} = 0 ,$$

$$F_+^{(1)} = -\frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{[q^2 + 2 \cdot p_+]} , \tag{2.15}$$

$$F_-^{(1)} = +\frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{[q^2 + 2q \cdot p_-]} .$$

Despite the fact that $\epsilon \simeq 0.0023 e^{i44^\circ}$ [in a phase convention where the $K^0 \to \pi\pi(I=0)$ decay amplitude is real] is small, it is important to keep this part of the decay amplitude. Other contributions not proportional to ϵ do not occur until higher order in chiral perturbation theory. We neglect direct sources of CP nonconservation in the one-photon part of the decay amplitude. Experimental information on ϵ' suggests that they are small.

At the next order in the chiral expansion the form factors $G^{(2)}$, $F_\pm^{(2)}$ arise from $O(p^4)$ local operators and from one-loop Feynman diagrams involving vertices from the leading Lagrange densities in (2.4) and (2.7). However, the form factor $G^{(2)}$ arises solely from local operators as the one-loop Feynman diagrams and tree graphs involving the Wess-Zumino term [4,5] do not contribute. The contribution of the $O(p^4)$ local operators to $G^{(2)}$ is fixed by the measured $K_L \to \pi^+\pi^-\gamma$ decay rate [6,7] to be

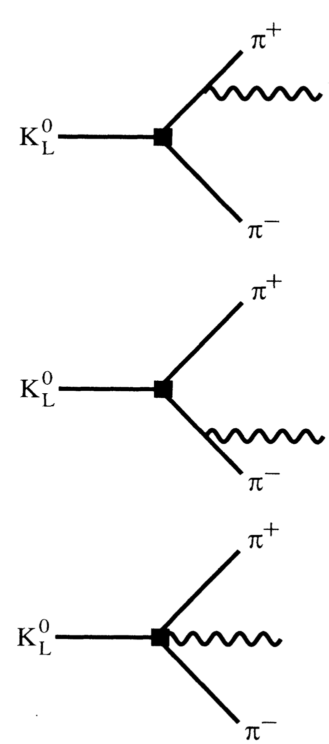

FIG. 1. Feynman diagrams contributing to $F_\pm^{(1)}$.

$$|G^{(2)}| \simeq 40 . \tag{2.16}$$

The experimentally observed $K_L \to \pi^+\pi^-\gamma$ Dalitz plot suggests that the form factor G has significant momentum dependence. This indicates that $G^{(3)}$ is not negligible, and that our extraction of $G^{(2)}$ from the rate is not completely justified [8].

The form factors $F_\pm^{(2)}$ get contributions both from local operators of $O(p^4)$ [9] and from one-loop diagrams involving vertices from the leading Lagrange densities in (2.4) and (2.7). For $K_L \to \pi^+\pi^- e^+ e^-$ the local operators that contribute are

$$\mathcal{L}_S^{(2)} = \frac{-ie\lambda_{\rm cr}(\mu)}{16\pi^2} F^{\mu\nu}\, \mathrm{Tr}[Q(D_\mu \Sigma D_\nu \Sigma^\dagger + D_\mu \Sigma^\dagger D_\nu \Sigma)] , \tag{2.17}$$

and

$$\mathcal{L}_W^{(2)} = i\frac{G_F s_1 e f^2 g_8}{\sqrt{2}16\pi^2}[a_1(\mu)F^{\mu\nu}\,\mathrm{Tr}[QT(\Sigma D_\mu \Sigma^\dagger)(\Sigma D_\nu \Sigma^\dagger)] + a_2(\mu)F^{\mu\nu}\,\mathrm{Tr}[Q(\Sigma D_\mu \Sigma^\dagger)T(\Sigma D_\nu \Sigma^\dagger)]$$
$$+a_3(\mu)F^{\mu\nu}\,\mathrm{Tr}[T[Q,\Sigma]D_\mu \Sigma^\dagger \Sigma D_\nu \Sigma^\dagger - TD_\mu \Sigma D_\nu \Sigma^\dagger \Sigma[\Sigma^\dagger, Q]] + a_4(\mu)F^{\mu\nu}\,\mathrm{Tr}[T\Sigma D_\mu \Sigma^\dagger [Q,\Sigma] D_\nu \Sigma^\dagger]] + \text{H.c.} \tag{2.18}$$

The coefficients $\lambda_{\rm cr}, a_1, a_2, a_3$ and a_4 depend on the renormalization procedure used and we employ dimensional regularization with the modified minimal subtraction scheme ($\overline{\rm MS}$). The dependence of the coefficients $\lambda_{\rm cr}, a_{1,2,3,4}$ on the subtraction point μ cancels that coming from the one-loop diagrams. Note that the basis of operators in Eq. (2.18) is slightly different than that used in [9]. With this basis of operators the combination of counterterms

$$w_L = a_3 - a_4 \tag{2.19}$$

is independent of the subtraction point μ at one loop.

The value of $\lambda_{\rm cr}$ is fixed by the measured π^+ charge radius; $\langle r_\pi^2\rangle = 0.44 \pm 0.02$ fm^2. The one-loop diagrams in Fig. 2 give (using $\overline{\rm MS}$)

$$\lambda_{\rm cr}(\mu) = -\left(\frac{2\pi^2}{3}\right) f^2 \langle r_\pi^2\rangle - \tfrac{1}{24}[2\ \ln(m_\pi^2/\mu^2) + \ln(m_K^2/\mu^2)]\ , \tag{2.20}$$

which implies that (at the subtraction point $\mu = 1$ GeV)

$$\lambda_{\rm cr}(1\ \text{GeV}) = -0.91 \pm 0.06\ . \tag{2.21}$$

A linear combination of a_1 and a_2 is fixed by the measured $K^+ \to \pi^+ e^+ e^-$ decay amplitude. Fortunately it is the same combination of a_1 and a_2 that enters into the $K_L \to \pi^+\pi^- e^+ e^-$ decay amplitude. The one-photon part of the $K^+ \to \pi^+ e^+ e^-$ decay amplitude can be written in terms of a single form factor $f(q^2)$:

$$M^{(1\gamma)}(K^+ \to \pi^+ e^+ e^-) = \frac{s_1 G_F}{\sqrt{2}} \frac{\alpha}{4\pi} f(q^2) p_\pi^\mu \overline{u}(k_-)\gamma_\mu v(k_+)\ . \tag{2.22}$$

The one-loop diagrams in Fig. 3 and the operators in (2.17) and (2.18) give [10]

$$\begin{aligned} f(q^2) &= 2g_8\{\phi_K(q^2) + \phi_\pi(q^2) - \tfrac{1}{6}\ \ln(m_K^2/\mu^2) \\ &\quad - \tfrac{1}{6}\ \ln(m_\pi^2/\mu^2) \\ &\quad + \tfrac{2}{3}[a_1(\mu) + 2a_2(\mu)] - 4\lambda_{\rm cr}(\mu) + \tfrac{1}{3}\} \\ &= 2g_8[\phi_K(q^2) + \phi_\pi(q^2) + w_+]\ , \end{aligned} \tag{2.23}$$

where

$$\phi_i(q^2) = \int_0^1 dx \left(\frac{m_i^2}{q^2} - x(1-x)\right) \times \ln\left(1 - \frac{q^2}{m_i^2}x(1-x)\right)\ . \tag{2.24}$$

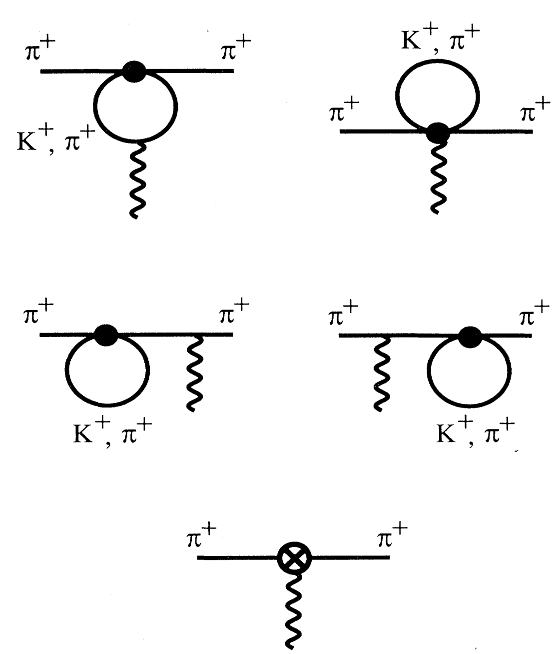

FIG. 2. Feynman diagrams contributing to the $\pi^\pm$ charge radius, $\langle r_\pi^2\rangle$, at leading order in chiral perturbation theory.

This relation defines the μ independent constant w_+ [10] which has been experimentally determined to be [11]

$$w_+ = 0.89^{+0.24}_{-0.14}\ . \tag{2.25}$$

Using the central values of $\lambda_{\rm cr}$ (1 GeV) and w_+ we find that

$$a_1(1\ \text{GeV}) + 2a_2(1\ \text{GeV}) = -6.0\ , \tag{2.26}$$

with an associated error around 10% [which is correlated with the uncertainty in $\lambda_{\rm cr}$ (1 GeV)]. Throughout the remainder of this work we will use the central values of $\lambda_{\rm cr}$ (1 GeV) and a_1(1 GeV)+$2a_2$(1 GeV) and suppress the associated uncertainties. Note that the contributions $\lambda_{\rm cr}$ (1 GeV) and $(a_1 + 2a_2)$ (1 GeV) to $f(q^2)$ are separately quite large but they almost cancel against each other.

At $O(p^4)$ the form factors $F_\pm^2$ for $K_L \to \pi^+\pi^- e^+ e^-$ decay follow from the Feynman diagrams in Fig. 4 and tree-level matrix elements of the operators in (2.17) and (2.18). We find, using $\overline{\rm MS}$ subtraction, that

$$\begin{aligned} F_-^{(2)} = g_8\Bigg\{ &-\frac{2}{3}q^2[a_1(\mu) + 2a_2(\mu) + 6a_3(\mu) - 6a_4(\mu)] - 4q^2\lambda_{\rm cr}(\mu) + \frac{2}{3}q^2 + \phi_{K\eta} + \phi_{K\pi} \\ &-4\int_0^1 dx \left[q^2 x(1-x)\ \ln\left(\frac{m_\pi^2 - q^2 x(1-x)}{\mu^2}\right) - m_\pi^2\ \ln\left(1 - \frac{q^2 x(1-x)}{m_\pi^2}\right)\right] \\ &+2q^2\left(\frac{m_K^2 - m_\pi^2}{q^2 + 2q\cdot(p_+ + p_-)}\right)\left[\phi_K(q^2) - \phi_\pi(q^2) + \tfrac{1}{6}\ \ln\left(\frac{m_\pi^2}{m_K^2}\right)\right]\Bigg\}\ , \end{aligned} \tag{2.27}$$

where

$$\begin{aligned}\phi_{K\eta} = {} & \frac{2}{9}(m_K^2 - m_\pi^2)^2 \left[\int_0^1 dyy \int_0^{1-y} dx \frac{1}{\mu_1^2} + \frac{1}{(q^2+2q\cdot p_-)} \int_0^1 dx \ln\left(1 - \frac{x(1-x)(q^2+2q\cdot p_-)}{m_K^2(1-x)+m_\eta^2 x - m_\pi^2 x(1-x)}\right)\right] \\ & + \frac{1}{3}(m_K^2 - m_\pi^2)\left\{ 2\int_0^1 dy \int_0^{1-y} dx \ln\left(1 - \frac{q^2x(1-x)+2q\cdot p_- xy}{m_K^2(1-y)+m_\eta^2 y - m_\pi^2 y(1-y)}\right) \right. \\ & + 3\int_0^1 dy \int_0^{1-y} dx \ln\left(1 - \frac{q^2x(1-x)+2q\cdot p_+ xy}{m_K^2(1-y)+m_\eta^2 y - m_\pi^2 y(1-y)}\right) \\ & + \int_0^1 dyy \int_0^{1-y} \frac{1}{\mu_1^2}[[q(1-x)+p_- y]\cdot(4q+6p_+ + 4p_-) - 2m_K^2 - 2p_+\cdot(q+p_-)] \\ & \left. + \int_0^1 dx \left(1 + x + \frac{(x-1)(3m_K^2 - 2m_\pi^2)}{q^2+2p_-\cdot q}\right) \ln\left(1 - \frac{x(1-x)(q^2+2q\cdot p_-)}{m_K^2(1-x)+m_\eta^2 x - m_\pi^2 x(1-x)}\right)\right\} \end{aligned} \tag{2.28}$$

with

$$\mu_1^2 = m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y) - q^2x(1-x) - 2q\cdot p_- xy\ . \tag{2.29}$$

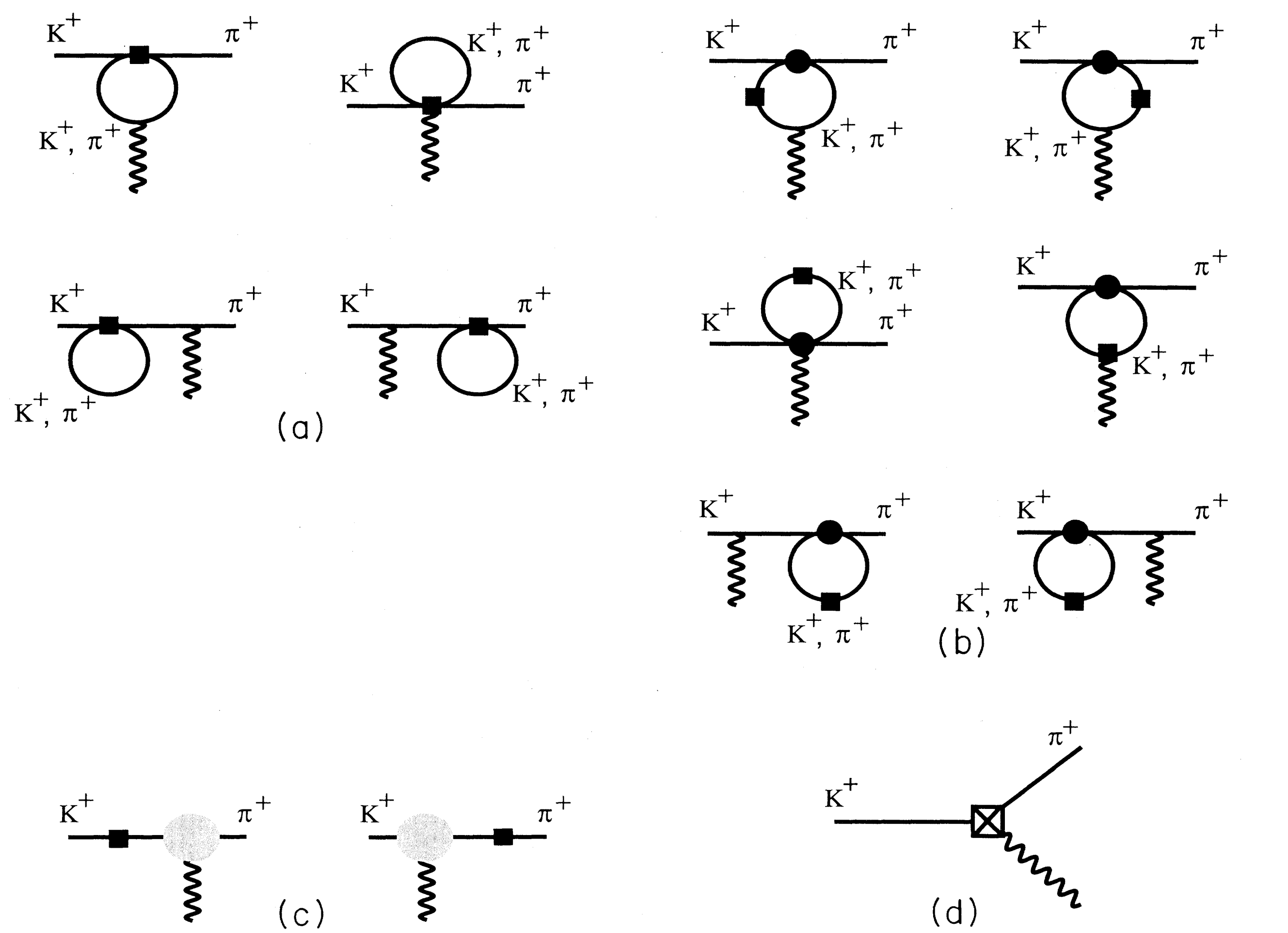

FIG. 3. The Feynman diagrams contributing to the amplitude for $K^+ \to \pi^+\gamma^*$ at leading order in chiral perturbation theory. The solid square denotes a vertex from the gauged weak Lagrangian in (2.7), the solid circle denotes a vertex from the gauged strong Lagrangian in (2.4). (a) involves only weak and electromagnetic vertices while (b) also has a strong vertex. (c) is the contribution from the kaon and pion charge radii (including both loop graphs and the tree-level counterterm). (d) is the contribution of the weak counterterm as given by (2.18). We have not shown the wave-function renormalization of the tree graphs for the process as the sum of these graphs vanish.

The other function is

$$\begin{aligned}\phi_{K\pi} = (m_K^2 - m_\pi^2)\Bigg[&\int_0^1 dy \int_0^{1-y} dx \ln\left(1 - \frac{q^2 x(1-x) + 2q\cdot p_- xy}{m_K^2 y + m_\pi^2(1-y)^2}\right) \\ &+ \int_0^1 dy \int_0^{1-y} dx \ln\left(1 - \frac{q^2 x(1-x) + 2q\cdot p_+ xy}{m_K^2 y + m_\pi^2(1-y)^2}\right) \\ &+2\int_0^1 dyy \int_0^{1-y} dx \frac{(p_+ + p_- + q)\cdot[q(1-x) + yp_- - p_+]}{\mu_2^2} \\ &+ \int_0^1 dx\left(1 + x + \frac{m_K^2(x-1)}{q^2 + 2q\cdot p_-}\right)\ln\left(1 - \frac{x(1-x)(q^2 + 2q\cdot p_-)}{m_\pi^2(1-x)^2 + m_K^2 x}\right)\Bigg] \end{aligned} \tag{2.30}$$

with

$$\mu_2^2 = m_K^2 y + m_\pi^2(1-y)^2 - q^2 x(1-x) - 2q\cdot p_- xy \ . \tag{2.31}$$

The Gell-Mann–Okubo mass formula (2.6) has been used to simplify some of the dependence on the pseudoscalar masses in (2.28) and (2.30). $F_+^{(2)}$ is obtained from (2.27) by taking $p_+ \to p_-$ and $p_- \to p_+$. Notice that the combination of coefficients $(a_2 + 2a_2)$ and $\lambda_{\rm cr}$ that appear in the expression for $F_\pm^{(2)}$ has a relative sign difference compared to the combination that appears in the expression for $f(s)$ given in Eq. (2.23). The uncertainty in $\lambda_{\rm cr}(1\ {\rm GeV})$ and w_+ gives rise to about a 10% uncertainty in the combination of counterterms that appears in (2.27). The one-photon part of the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude is the largest and dominates the rate. In the next section we use the form factors calculated here to obtain $d\Gamma(K_L \to \pi^+\pi^-e^+e^-)/dq^2$. One (scale-independent) linear combination of counterterms $w_L = a_3 - a_4$ is not determined by the present experimental data and consequently we cannot predict the rate for $K_L \to \pi^+\pi^-e^+e^-$. However, this is the only undetermined constant and the entire function $d\Gamma(K_L \to \pi^+\pi^-e^+e^-)/dq^2$ is experimentally accessible.

III. THE DIFFERENTIAL DECAY RATE

The $K_L \to \pi^+\pi^-e^+e^-$ decay rate is obtained by squaring the invariant matrix element (1.1), summing over the e^+ and e^- spins, and integrating over the phase space. Since the e^+ and e^- four-momenta only occur in the lepton trace, ${\rm Tr}[\not k_- \gamma_\nu \not k_+ \gamma_\mu]$, the phase-space integrations over k_- and k_+ produce a factor

$$\int \frac{d^3k_-}{(2\pi)^3 2k_-^0}\int \frac{d^3k_+}{(2\pi)^3 2k_+^0}(2\pi)^4\delta^4(q - k_- - k_+)\,{\rm Tr}[\not k_-\gamma_\nu \not k_+\gamma_\mu] = \frac{1}{6\pi}(q_\mu q_\nu - q^2 g_{\mu\nu}) \ . \tag{3.1}$$

The remaining phase-space integrations can be taken to be over q^2 and the sum and difference of the pion energies in the K_L rest frame, $E_S = p_+^0 + p_-^0, E_D = p_+^0 - p_-^0$. The contribution of the form factors $F_\pm$ and G to $d\Gamma/dq^2$ do not interfere. Therefore, we can write

$$\frac{d\Gamma}{dq^2}(K_L \to \pi^+\pi^-e^+e^-) = \frac{d\Gamma_G}{dq^2} + \frac{d\Gamma_F}{dq^2}\ , \tag{3.2}$$

where

$$\begin{aligned}\frac{d\Gamma_G}{dq^2} = \frac{G_F^2\alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3q^2}&\int dE_S \int dE_D |G|^2 \\ &\times[m_\pi^4 q^2 - m_\pi^2(p_-\cdot q)^2 - m_\pi^2(p_+\cdot q)^2 + 2(p_+\cdot p_-)(q\cdot p_+)(q\cdot p_-) - q^2(p_+\cdot p_-)^2]\ ,\end{aligned}$$

$$\begin{aligned}\frac{d\Gamma_F}{dq^2} = \frac{G_F^2\alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3q^4}&\int dE_S \int dE_D[|F_+ q\cdot p_+ - F_- q\cdot p_-|^2 \\ &-q^2(|F_+|^2 m_\pi^2 + |F_-|^2 m_\pi^2 + 2\,{\rm Re}(F_+F_-^*)p_+\cdot p_-)]\ .\end{aligned} \tag{3.3}$$

In Eqs. (3.2) and (3.3) the difference of pion energies is integrated over the region $-E_D^{(\max)} < E_D < E_D^{(\max)}$ where

$$E_D^{(\max)} = \sqrt{\frac{2m_K E_S + q^2 - m_K^2 - 4m_\pi^2}{2m_K E_S + q^2 - m_K^2}}\sqrt{(m_K - E_S)^2 - q^2}\ , \tag{3.4}$$

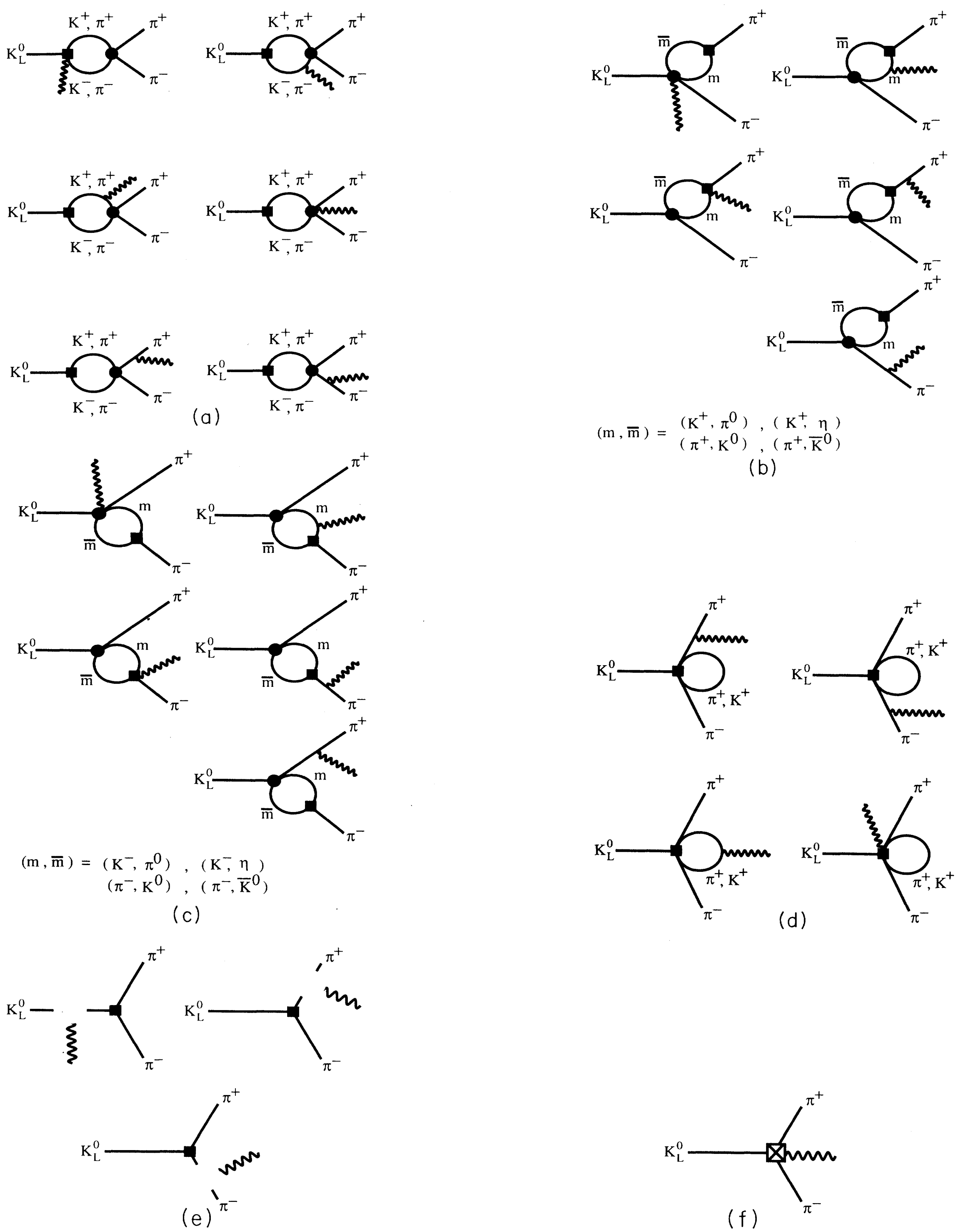

FIG. 4. Feynman diagrams contributing to the CP-conserving amplitude for $K_L \rightarrow \pi^+\pi^-\gamma^*$ at leading order in chiral perturbation theory. The notation is the same as in Fig. 3 and we have not shown the wave-function renormalization of the tree graphs for the process as the sum of these graphs vanish.

and the sum of pion energies is integrated over the region $E_S^{(\min)} < E_S < E_S^{(\max)}$ where the boundaries are

$$E_S^{(\max)} = m_K - \sqrt{q^2}\ ,$$

$$E_S^{(\min)} = \frac{m_K^2 - q^2 + 4m_\pi^2}{2m_K}\ . \tag{3.5}$$

The scalar products appearing in the expression for the rates are easily expressed in terms of E_S, E_D, and q^2:

$$p_+ \cdot p_- = \tfrac{1}{2}(q^2 - m_K^2 - 2m_\pi^2 + 2m_K E_S)\ ,$$

$$q \cdot p_+ = \tfrac{1}{2}(-m_K E_S + m_K E_D - q^2 + m_K^2)\ , \tag{3.6}$$

$$q \cdot p_- = \tfrac{1}{2}(-m_K E_S - m_K E_D - q^2 + m_K^2)\ .$$

The form factors $F_\pm^{(1)}$ and $F_\pm^{(2)}$ have the opposite property under interchange of pion momenta and consequently they do not interfere in $d\Gamma/dq^2$. Neglecting terms in chiral expansion of $O(p^6)$ and higher the differential decay rate given in (3.2) becomes

$$\frac{d\Gamma}{dq^2}(K_L \to \pi^+\pi^-e^+e^-) = \frac{d\Gamma_{G^{(2)}}}{dq^2} + \frac{d\Gamma_{F^{(1)}}}{dq^2} + \frac{d\Gamma_{F^{(2)}}}{dq^2}\ . \tag{3.7}$$

In Fig. 5, we have graphed, for each of the three terms on the right-hand side of (3.7),

$$\frac{1}{\Gamma_{K_L}}\frac{d\Gamma}{dy} = 2y(m_K - 2m_\pi)^2 \frac{1}{\Gamma_{K_L}}\frac{d\Gamma}{dq^2}\ , \tag{3.8}$$

where $y = \sqrt{q^2}/(m_K - 2m_\pi)$, Γ_{K_L} is the total width of the K_L, and we have set $w_L = 0$.

Integrating the three terms on the right-hand side (RHS) of (3.7) over the invariant mass interval $q^2 > (30\ \text{MeV})^2$ (corresponding to $y > 0.14$) we find that for $w_L = 0$

$$10^8 \times \mathcal{B}(K_L \to \pi^+\pi^-e^+e^-; q^2 > (30\ \text{MeV})^2) = 3.8 + 0.78 + 3.4 = 8.0\ . \tag{3.9}$$

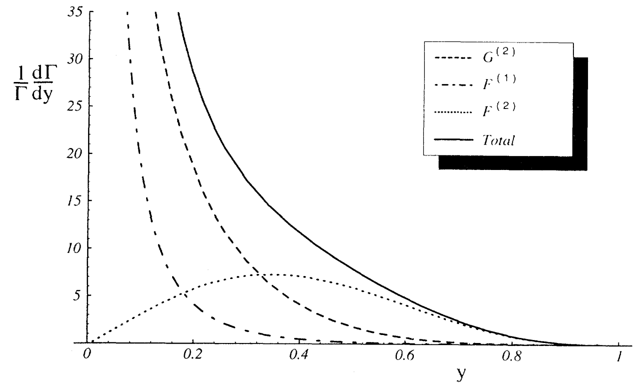

FIG. 5. The differential decay spectrum as a function of y the invariant mass of the lepton pair normalized to $m_K - 2m_\pi$. The dot-dashed curve is the contribution from $F_\pm^{(1)}$, the dotted curve is the contribution from $F_\pm^{(1)}$ with $w_L = 0$ and the dashed curve is the contribution from $G^{(2)}$. The total differential decay rate for $w_L = 0$ is given by the solid curve.

The branching fraction over this range of e^+e^- invariant mass is dominated by the region of low q^2 and for typical values of w_L it receives comparable contributions from the form factors G and $F^{(2)}$. However, in the region of high q^2 the branching fraction is likely to be dominated by the $F_\pm^{(2)}$ form factor. For $q^2 > (80\ \text{MeV})^2$ (corresponding to $y > 0.37$) and $w_L = 0$ the three terms on the RHS of (3.7) contribute

$$10^8\mathcal{B}(K_L \to \pi^+\pi^-e^+e^-; q^2 > (80\ \text{MeV})^2) = 0.61 + 0.07 + 1.8 = 2.6\ . \tag{3.10}$$

A summary of our results for the rate can be found in Table I. We have displayed the contribution to the branching ratio (in units of 10^{-8}) from the three form factors G $F^{(1)}$, and $F^{(2)}$ for different values of the minimum lepton pair invariant mass $q^2_{\min}$. Since the loop contribution to the form factor $F_\pm^{(2)}$ is small, it will be difficult to extract a unique value for w_L for $d\Gamma/dq^2$ data alone; a twofold ambiguity in the value of w_L will persist. The contribution to the rate from G and $F^{(1)}$ are numerically similar to that computed in [2,3], differing only

TABLE I. Contributions to the branching ratio (10^{-8}) for a range of $q^2_{\min}$.

Lower cut $q^2_{\min}$	$\mathcal{B}(10^{-8})_G$	$\mathcal{B}(10^{-8})_{F^{(1)}}$	$\mathcal{B}(10^{-8}_{F^{(2)}})$
$(10\ \text{MeV})^2$	8.8	3.3	$3.6 - 3.4w_L + 0.8w_L^2$
$(20\ \text{MeV})^2$	5.6	1.5	$3.5 - 3.3w_L + 0.8w_L^2$
$(30\ \text{MeV})^2$	3.8	0.8	$3.4 - 3.2w_L + 0.8w_L^2$
$(40\ \text{MeV})^2$	2.7	0.5	$3.1 - 3.0w_L + 0.7w_L^2$
$(60\ \text{MeV})^2$	1.3	0.2	$2.6 - 2.4w_L + 0.6w_L^2$
$(80\ \text{MeV})^2$	0.6	0.07	$1.9 - 1.8w_L + 0.4w_L^2$
$(100\ \text{MeV})^2$	0.3	0.03	$1.3 - 1.2w_L + 0.3w_L^2$
$(120\ \text{MeV})^2$	0.1	0.01	$0.74 - 0.68w_L + 0.16w_L^2$
$(180\ \text{MeV})^2$	0.000 72	0.000 1	$0.027 - 0.025w_L + 0.006w_L^2$

because we have retained the q^2 dependence in $F^{(1)}$. For $w_L = 2$ the contribution of $F_\pm^{(2)}$ to the rate is small and our results are similar to those in [2,3].

IV. THE Z-PENGUIN AND W-BOX AMPLITUDE

The short-distance W-box and Z-penguin diagrams give the effective Lagrange density

$$\mathcal{L}_{\rm SD} = \xi \frac{s_1 G_F \alpha}{\sqrt{2}} \bar{s}\gamma_\mu(1-\gamma_5) d \bar{e}\gamma^\mu \gamma_5 e + \text{H.c.} \quad (4.1)$$

Here we only keep the part that contains the lepton axial vector current (the vector current is neglected). It is only the axial vector current that gives rise to observables that are antisymmetric under interchange of e^+ and e^- four-momenta, $k_+ \leftrightarrow k_-$.

In (4.1) the quantity ξ receives significant contributions from both the top quark and charm quark loops and is given by

$$\xi = -\tilde{\xi}_c + \left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right)\tilde{\xi}_t \ , \quad (4.2)$$

where

$$\tilde{\xi}_q = \tilde{\xi}_q^{(Z)} + \tilde{\xi}_q^{(W)} \quad (4.3)$$

is the sum of the contributions of the Z-penguin and W-box diagrams. It is convenient to express the combination of elements of the Cabibbo-Kobayashi-Maskawa matrix that enters in ξ in terms of $|V_{cb}|$ and the standard coordinates $\rho + i\eta$ of the unitarity triangle

$$V_{ts}^* V_{td}/V_{us}^* V_{ud} = (\rho - 1 + i\eta)|V_{cb}|^2 \ . \quad (4.4)$$

A value of $|V_{cb}| \simeq 0.04$ is obtained from inclusive $B \to X_c e \bar{\nu}_e$ decay and from exclusive $B \to D^* e\bar{\nu}_e$ decay. Although the values of ρ and η are not determined by present data, they are expected to be of order unity.

The quantities $\tilde{\xi}_c$ and $\tilde{\xi}_t$ have been calculated including perturbative QCD corrections at the next-to-leading logarithmic level [12,13]. There is some sensitivity to the values of $\Lambda_{\rm QCD}$, m_c, and m_t but $\tilde{\xi}_c$ is of order 10^{-4} and $\tilde{\xi}_t$ is of order unity.

The quark-level Lagrange density in Eq. (4.3) can be converted into a Lagrange density involving the π, K, and η hadrons using the Noether procedure. Equating the QCD chiral currents with those obtained from chiral variations of the effective Lagrangian in Eq. (2.4) leads to

$$\mathcal{L}_{\rm SD} = -\xi \frac{i G_F \alpha s_1}{2\sqrt{2}} f^2\, \text{Tr}(\partial^\mu \Sigma \Sigma^\dagger T) \bar{e}\gamma_\mu\gamma_5 e + \text{H.c.} \quad (4.5)$$

Expanding out Σ in terms of the meson fields M we find that the Lagrange density (4.5) implies that the short-distance contribution to the $K_L \to \pi^+\pi^- e^+ e^-$ decay amplitude from the W-box and Z-penguin diagrams is

$$M^{(\rm SD)} = \frac{G_F s_1 \alpha}{f}(\xi p_-^\mu + \xi^* p_+^\mu)\bar{u}(k_-)\gamma_\mu\gamma_5 v(k_+) \ . \quad (4.6)$$

V. THE ASYMMETRY A_{CP}

It is the interference of $M^{(\rm SD)}$ in (4.6) with $M^{(1\gamma)}$ in (1.1) that produces the asymmetry A_{CP} defined in (1.8) (essentially K_7 of [3]). For calculation of A_{CP} it is convenient to use the phase-space variables used by Pais and Treiman [14] for K_{l4} decay (rather than those used for the total rate in Sec. III). They are $q^2 = (k_+ + k_-)^2$, $s = (p_+ + p_-)^2$, θ_π, the angle formed by the π^+ three-momentum and the K_L three-momentum in the $\pi^+\pi^-$ rest frame; θ_l, the angle between the e^- three-momentum and the K_L three-momentum in the e^+e^- rest frame; ϕ, the angle between the normals to the planes defined in the K_L rest frame by the $\pi^+\pi^-$ pair and the e^+e^- pair. In terms of these variables

$$\frac{(\vec{p}_- \times \vec{p}_+)\cdot(\vec{k}_- - \vec{k}_+)}{|(\vec{p}_- \times \vec{p}_+)\cdot(\vec{k}_- - \vec{k}_+)|} = \text{sgn}(\sin\phi) \ , \quad (5.1)$$

and the asymmetry is

$$A_{CP} = \frac{1}{2^7(2\pi)^6 m_K^3 \Gamma_{K_L}}\left(\int_0^{2\pi} d\phi\, \text{sgn}(\sin\phi)\right)\int dc_\pi dc_e ds dq^2 \beta X\, \text{Re}(M^{(\rm SD)^*} M^{(1\gamma)}) \ , \quad (5.2)$$

where $c_\pi = \cos\theta_\pi$, $c_e = \cos\theta_e$. The other kinematic functions appearing in this expression are

$$\beta = [1 - 4m_\pi^2/s]^{1/2} \ ,$$

$$X = \left[\left(\frac{m_K^2 - s - q^2}{2}\right)^2 - s q^2\right]^{1/2} \ . \quad (5.3)$$

In order to evaluate the contributing form factors the following scalar products of four vectors are required:

$$
\begin{aligned}
q\cdot p_+ &= \tfrac{1}{4}(m_K^2 - s - q^2) - \tfrac{1}{2}\beta X \cos\theta_\pi \ , \\
q\cdot p_- &= \tfrac{1}{4}(m_K^2 - s - q^2) + \tfrac{1}{2}\beta X \cos\theta_\pi \ , \\
p_+\cdot p_- &= \tfrac{1}{2}(s - 2m_\pi^2) \ , \\
\varepsilon_{\alpha\beta\sigma\rho}p_+^\alpha p_-^\beta k_+^\sigma k_-^\rho &= -\tfrac{1}{4}\beta X\sqrt{sq^2}\,\sin\theta_e\,\sin\theta_\pi\,\sin\phi \ .
\end{aligned}
\tag{5.4}
$$

If the variables s and q^2 are not integrated over the complete phase space then it is understood that the same is to be done for the K_L width Γ_{K_L} in the denominator of Eq. (5.2).

The form factor G does not enter into $\mathrm{Re}(M^{(\mathrm{SD})^*}M^{(1\gamma)})$ (a sum over e^+ and e^- spins is understood). Integrating our $\cos\theta_e$ and ϕ we find that

$$
\begin{aligned}
A_{CP} = {}& \frac{G_F^2 s_1^2\alpha^2}{2^8(2\pi)^6 f^2 m_K^3\Gamma_{K_L}}\int dc_\pi ds dq^2\,\sin\theta_\pi \\
&\times\beta^2X^2\sqrt{\frac{s}{q^2}}[\mathrm{Im}(\xi)[\mathrm{Re}(F_+)+\mathrm{Re}(F_-)]+\mathrm{Re}(\xi)[\mathrm{Im}(F_+)-\mathrm{Im}(F_-)]] \ .
\end{aligned}
\tag{5.5}
$$

The integration over $\cos\theta_\pi$ implies that at leading nontrivial order of chiral perturbation theory $\mathrm{Im}(F_+) - \mathrm{Im}(F_-) \to \mathrm{Im}(F_+^{(1)}) - \mathrm{Im}(F_-^{(1)})$ reflecting indirect CP violation from ϵ and $\mathrm{Re}(F_+) + \mathrm{Re}(F_-) \to \mathrm{Re}(F_+^{(2)}) + \mathrm{Re}(F_-^{(2)})$ in Eq. (5.5).

Using (4.2) and (4.4) we can write the CP-violating asymmetry in terms of the real and imaginary parts of the CKM elements

$$
A_{CP} = A_1((\rho-1)|V_{cb}|^2\tilde{\xi}_t - \tilde{\xi}_c) - A_2\eta|V_{cb}|^2\tilde{\xi}_t \ , \tag{5.6}
$$

where A_1 arises from indirect CP nonconservation (i.e., $\overline{K}^0 - K^0$ mixing) and A_2 arises from direct CP nonconservation. We are only able to predict $|A_{CP}|$ since the sign of g_8 is not known. Our expressions for $F_\pm^{(1)}$ and $F_\pm^{(2)}$ with $w_L = 0$ give (up to an overall sign)

$$
A_1 = 2.7\times10^{-2} \ , \quad A_2 = 3.9\times10^{-2} \ , \tag{5.7}
$$

for $q^2 \geq (30\ \mathrm{MeV})^2$ and

$$
A_1 = 2.4\times10^{-2} \ , \quad A_2 = 8.4\times10^{-2} \ , \tag{5.8}
$$

for $q^2 \geq (80\ \mathrm{MeV})^2$. In Table II we present A_1 and A_2 for a range of values of the minimum lepton pair invariant mass q^2_{min} normalized to the branching ratios given in Table I assuming $w_L = 0$.

We find that direct and indirect sources of CP nonconservation give comparable contributions to A_{CP}. In our computation we have neglected final-state $\pi\pi$ interactions which are formally higher order in chiral perturbation theory. With the values of A_1 and A_2 given in Table II, $|A_{CP}|$ is only of order 10^{-4} and further refinements of our calculation do not seem warranted.

VI. THE ASYMMETRY B_{CP}

Using the kinematic variables introduced in the previous section the CP-violating observable B_{CP} is defined as

$$
B_{CP} = \langle \mathrm{sgn}(\sin\phi\,\cos\phi)\rangle \ . \tag{6.1}
$$

At leading order in chiral perturbation theory it arises from the interference of $F_\pm^{(1)}$ with $G^{(2)}$. The CP-violating form factors $F_\pm^{(1)}$ are not small because they occur at a lower order in chiral perturbation theory than the other form factors, $F_\pm^{(2)}$ and $G^{(2)}$. Consequently, as was noted in [2,3], B_{CP} is quite large. Neglecting $M^{(\mathrm{SD})}$ we find after integrating over ϕ and $\cos\theta_e$ that

TABLE II. The CP-violating quantities A_1, A_2 with $w_L = 0$ for different values of q^2_{min}.

Lower cut q^2_{min}	A_1	A_2
$(10\ \mathrm{MeV})^2$	2.0×10^{-2}	2.0×10^{-2}
$(20\ \mathrm{MeV})^2$	2.5×10^{-2}	3.0×10^{-2}
$(30\ \mathrm{MeV})^2$	2.7×10^{-2}	3.9×10^{-2}
$(40\ \mathrm{MeV})^2$	2.8×10^{-2}	4.8×10^{-2}
$(60\ \mathrm{MeV})^2$	2.7×10^{-2}	6.8×10^{-2}
$(80\ \mathrm{MeV})^2$	2.4×10^{-2}	8.5×10^{-2}
$(100\ \mathrm{MeV})^2$	2.1×10^{-2}	9.8×10^{-2}
$(120\ \mathrm{MeV})^2$	1.8×10^{-2}	0.11
$(180\ \mathrm{MeV})^2$	1.3×10^{-2}	0.13

TABLE III. The CP-violating observable $|B_{CP}\times\mathcal{B}(10^{-8})|$ for a range of values of q^2_{min}.

Lower cut q^2_{min}	$\lvert B_{CP}\times\mathcal{B}(10^{-8})\rvert$ (%)
$(10\ \mathrm{MeV})^2$	134
$(20\ \mathrm{MeV})^2$	78
$(30\ \mathrm{MeV})^2$	50
$(40\ \mathrm{MeV})^2$	33
$(60\ \mathrm{MeV})^2$	14
$(80\ \mathrm{MeV})^2$	6.3
$(100\ \mathrm{MeV})^2$	2.5
$(120\ \mathrm{MeV})^2$	0.92
$(180\ \mathrm{MeV})^2$	0.0086

$$B_{CP} = \frac{G_F^2 s_1^2 \alpha^2}{32^7 (2\pi)^8 f^2 m_K^3 \Gamma_{K_L}} \int dc_\pi ds dq^2 \sin^2\theta_\pi \beta^3 X^2 \frac{s}{q^2} \operatorname{Im}[G(F_+^* - F_-^*)] \ . \tag{6.2}$$

If the variables s and q^2 are not integrated over the entire phase space then it is understood that the same is to be done to the K_L width Γ_{K_L} in the denominator of (6.2). The form factor G is real at leading order in chiral perturbation theory and the imaginary part arises from the phase in $F_+ - F_-$ induced by K^0-$\overline{K}^0$ mixing. The integration over $\cos\theta_\pi$ implies that $F_+ - F_- \to F_+^{(1)} - F_-^{(1)}$ in Eq. (6.2). Using our expressions for $F_\pm^{(1)}$ and the value of $|G^{(2)}|$ we find that with $w_L = 0, |B_{CP}| \simeq 6.3\%$ for $q^2 > (30 \text{ MeV})^2$ and $|B_{CP}| \simeq 2.4\%$ for $q^2 > (80 \text{ MeV})^2$. The asymmetries for a range of values of $q^2_{\min}$ are shown in Table III. Note that in Table III $\mathcal{B}(10^{-8})$ denotes the $K_L \to \pi^+\pi^-e^+e^-$ branching ratio in units of 10^{-8} with the same cut on q^2 imposed. We have neglected final-state $\pi\pi$ interactions because they arise at higher order in chiral perturbation theory. Our prediction for $|B_{CP}|$ has considerable uncertainty because of the neglect of final-state $\pi\pi$ interactions and because neglected $O(p^6)$ contributions to G seem to be important.

VII. CONCLUSION

In this paper we have calculated the one-photon contribution to the $K_L \to \pi^+\pi^-e^+e^-$ decay rate. We used chiral perturbation theory to determine the form factors and for e^+e^- pairs with high invariant mass ($q^2 \gg 4m_e^2$) found that there may be important new contributions that were not included in previous work [2,3]. The amplitude for $K_L \to \pi^+\pi^-e^+e^-$ depends on the undetermined (renormalization scale-independent) combination of counterterms w_L. We found that for $q^2 = (k_+ + k_-)^2 > (30 \text{ MeV})^2$ the branching ratio for $K_L \to \pi^+\pi^-e^+e^-$ is approximately $(8.0 - 3.2w_L + 0.8w_L^2) \times 10^{-8}$ and for $q^2 > (80 \text{ MeV})^2$ the branching ratio is approximately $(2.6 - 1.8w_L + 0.4w_L^2) \times 10^{-8}$.

One interesting aspect of this decay mode is that the CP-even component of the K_L state contributes at a lower order in chiral perturbation theory than the CP-odd component. This enhances CP-violating effects in $K_L \to \pi^+\pi^-e^+e^-$ decay. For example, the CP-violating observable [2,3] $B_{CP} = \langle \text{sgn}(\sin\phi \cos\phi)\rangle$, where ϕ is the angle between the normals to the $\pi^+\pi^-$ and e^+e^- planes, is about 6% for $q^2 > (30 \text{ MeV})^2$ if $w_L = 0$. The CP-violating observable $A_{CP} = \langle \text{sgn}(\sin\phi)\rangle$ arises from the interference of W-box and Z-penguin amplitudes with the one-photon part of the decay amplitude. Unfortunately, we find that A_{CP} is of order 10^{-4} and hence most likely unmeasurable.

Chiral perturbation theory has been extensively applied to nonleptonic, semileptonic, and radiative kaon decays. The study of $K_L \to \pi^+\pi^-e^+e^-$ offers an opportunity to determine the linear combination of coefficients in the $O(p^4)$ chiral Lagrangian that we call w_L and to test the applicability of $O(p^4)$ chiral perturbation theory for kaon decay.

Some improvements in our calculations are possible. While a full computation of the $O(p^6)$ contribution to $F_\pm$ and G arising from two-loop diagrams and new local operators does not seem feasible it should be possible to calculate the leading contribution to the absorptive parts of G and $F_+ - F_-$. Note that the absorptive parts come from both $\pi\pi \to \pi\pi$ rescattering and because of CP nonconservation from $\pi\pi \to \pi\pi\gamma^*$. We hope to present results for this in a future publication.

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[1] Y. W. Wah, in *Proceedings of the XXVI International Conference on High Energy Physics*, Dallas, Texas, 1992, edited by James R. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1992).
[2] L. M. Sehgal and M. Wanninger, Phys. Rev. D **46**, 1035 (1992).
[3] P. Heiliger and L. M. Sehgal, Phys. Rev. D **48**, 4146 (1993).
[4] G. Ecker, H. Neufeld, and A. Pich, Phys. Lett. B **278**, 337 (1992).
[5] J. Bijnens, G. Ecker, and A. Pich, Phys. Lett. B **286**, 341 (1992).
[6] G. Donaldson *et al.*, Phys. Rev. Lett. **33**, 554 (1974); Phys. Rev. D **14**, 2839 (1976).
[7] E. J. Ramberg *et al.*, Phys. Rev. Lett. **70**, 2525 (1993).
[8] G. Ecker, H. Neufeld, and A. Pich, Nucl. Phys. **B413**, 321 (1994).
[9] G. Ecker, J. Kambor, and D. Wyler, Nucl. Phys. **B394**, 101 (1993).
[10] G. Ecker, A. Pich, and E. deRafael, Nucl. Phys. **B291**, 692 (1987).
[11] C. Alliegro *et al.*, Phys. Rev. Lett. **68**, 278 (1992).
[12] G. Buchalla and A. J. Buras, Nucl. Phys. **B398**, 285 (1993).
[13] G. Buchalla and A. J. Buras, Nucl. Phys. **B400**, 225 (1993).
[14] A. Pais and S. Trieman, Phys. Rev. **168**, 1858 (1968).