

Lattice study of semileptonic B decays: $\bar{B} \rightarrow D l \bar{\nu}$ decays

K. C. Bowler, N. M. Hazel, D. S. Henty, H. Hoerber,*

R. D. Kenway, D. G. Richards, H. P. Shanahan,[†] and J. N. Simone[‡]

Department of Physics & Astronomy, The University of Edinburgh, Edinburgh EH9 3JZ, Scotland

L. Lellouch[§]

Centre de Physique Théorique, CNRS Luminy, Case 907, F-13288 Marseille Cedex 9, France

J. Nieves, C. T. Sachrajda, and H. Wittig

Physics Department, The University, Southampton SO7 1BJ, United Kingdom

(UKQCD Collaboration)

(Received 5 April 1995)

We present a study of semileptonic $\bar{B} \rightarrow D l \bar{\nu}$ decays in quenched lattice QCD through a calculation of the matrix element $\langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle$ on a $24^3 \times 48$ lattice at $\beta = 6.2$, using an $O(a)$ -improved fermion action. We perform the calculation for several values of the initial and final heavy-quark masses around the charm mass, and three values of the light-(anti)quark mass around the strange mass. Because the charm quark has a bare mass which is almost $\frac{1}{3}$ the inverse lattice spacing, we study the ensuing mass-dependent discretization errors, and propose a procedure for subtracting at least some of them nonperturbatively. We extract the form factors h^+ and h^- . After radiation corrections, we find that h^+ displays no dependence on the heavy-quark mass, enabling us to identify it with an Isgur-Wise function ξ . Interpolating the light-quark mass to that of the strange, we obtain an Isgur-Wise function relevant for $\bar{B}_s \rightarrow D_s^* l \bar{\nu}$ decays which has a slope $-\xi'_s = 1.2^{+2}_{-2}(\text{stat})^{+2}_{-1}(\text{syst})$ at zero recoil. An extrapolation to a massless light quark enables us to obtain an Isgur-Wise function relevant for $\bar{B} \rightarrow D^{(*)} l \bar{\nu}$ decays. This function has a slope $-\xi'_{u,d} = 0.9^{+2}_{-3}(\text{stat})^{+4}_{-2}(\text{syst})$ at zero recoil. We observe a slight decrease in the magnitude of the central value of the slope as the mass of the light quark is reduced; given the errors, however, the significance of this observation is limited. We then use these functions, in conjunction with heavy-quark effective theory, to extract V_{cb} with no free parameters from the $\bar{B} \rightarrow D^* l \bar{\nu}$ decay rate measured by the ALEPH, ARGUS, and CLEO Collaborations. Using the CLEO data, for instance, we obtain $|V_{cb}| = 0.037^{+1+2+4}_{-1-2-1}(0.99/1 + \beta^{A_1}(1))1/1 + \delta_{1/m_c^2}$, where δ_{1/m_c^2} is the power corrections inversely proportional to the square of the charm quark mass, and $\beta^{A_1}(1)$ is the relevant radiative correction at zero recoil. Here, the first set of errors is experimental, the second represents the statistical error, and the third represents the systematic error in our evaluation of the Isgur-Wise function. We also use our Isgur-Wise functions and heavy-quark effective theory to calculate branching ratios for $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ and $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays.

PACS number(s): 13.20.He, 12.15.Hh, 12.38.Gc, 12.39.Hg

I. INTRODUCTION

Semileptonic decays of B mesons have been the focus of much activity in the last few years. Experimentally, their rather large branching ratios have allowed thorough studies of their properties. Theoretically, they have been a fertile ground for new ideas. Moreover, the interplay between these experimental studies and new theoretical ideas has led to a greater understanding of the flavor sector of the standard model and, in partic-

ular, to measurements of the less well-known Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{cb} and V_{ub} [1].

The main theoretical development in the study of hadrons containing a heavy quark, such as the b or c quarks, is undoubtedly the discovery of heavy-quark symmetry [2,3] and the development of the heavy-quark effective theory (HQET) [4], which describes the strong interactions of a heavy quark with gluons and light quarks at low energies. If one considers the masses of the b and c quarks to be much larger than the QCD scale, Λ_{QCD} , one finds that the dynamics of the light quarks and gluons coupled to a b or a c quark become independent of this heavy quark's flavor and spin. In this limit, QCD exhibits a new $SU(4)_{\text{spin} \times \text{flavor}}$ symmetry, known as heavy-quark symmetry, which acts on the multiplet ($c \uparrow, c \downarrow, b \uparrow, b \downarrow$). This symmetry simplifies considerably the description of the decays of hadrons containing

*Present address: HLRZ, 52425 Jülich, Germany.

[†]Present address: Dept. of Physics and Astronomy, The University, Glasgow G12 8QQ, Scotland.

[‡]Present address: Fermilab, P.O. Box 500, Batavia, IL 60510.

[§]Electronic address: Lellouch@cpt.univ-mrs.fr

a heavy quark. For instance, the 20 form factors required to describe the semileptonic decays $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ and $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ as well as the elastic form factors of $B_{(s)}^{(*)}$ and $D_{(s)}^{(*)}$ mesons,¹ can all be expressed in terms of two universal form factors, $\xi_{u,d}$ and ξ_s , known as Isgur-Wise functions [3], which parametrize the nonperturbative dynamics of the light degrees of freedom. $\xi_{u,d}$ describes the decays of mesons containing a heavy quark and a \bar{u} or \bar{d} antiquark, and ξ_s describes the decays of mesons containing a heavy quark and an \bar{s} antiquark. Moreover, heavy-quark symmetry requires these Isgur-Wise functions to be 1 when q^2 , the square of the four-momentum transfer, is maximum [3].

In an earlier work [5], we obtained the Isgur-Wise functions $\xi_{u,d}$ from a lattice study of elastic D meson scattering. A similar approach, but with a different lattice action, was taken by Bernard *et al.* [6] and led to very similar results. In the present paper, we extend our earlier work to include decays of the form $P \rightarrow P' l \bar{\nu}$, where $P^{(\prime)}$ is a heavy-light pseudoscalar meson composed of a heavy quark $Q^{(\prime)}$ with a mass around that of the charm quark, and a light antiquark \bar{q} . These processes are described by matrix elements of the vector current $\bar{Q}' \gamma^\mu Q$. These matrix elements can, in turn, be decomposed in terms of two form factors, $h^+(\omega; m_Q, m_{Q'})$ and $h^-(\omega; m_Q, m_{Q'})$, given by

$$\frac{\langle P'(\mathbf{p}') | \bar{Q}' \gamma^\mu Q | P(\mathbf{p}) \rangle}{\sqrt{M_P M_{P'}}} = (v + v')^\mu h^+(\omega; m_Q, m_{Q'}) + (v - v')^\mu h^-(\omega; m_Q, m_{Q'}) , \quad (1)$$

where $v^{(\prime)} = p^{(\prime)}/M_{P^{(\prime)}}$, $\omega = v \cdot v' = (M_P^2 + M_{P'}^2 - q^2)/2M_P M_{P'}$ and $m_{Q^{(\prime)}}$ is the mass of $Q^{(\prime)}$.

In the limit of exact heavy-quark symmetry, the two form factors become independent of the masses of the initial and final heavy quarks and

$$\begin{aligned} h^-(\omega; m_Q, m_{Q'}) &\equiv 0 , \\ h^+(\omega; m_Q, m_{Q'}) &\equiv \xi(\omega) , \end{aligned} \quad (2)$$

where $\xi(\omega)$ is an Isgur-Wise function of the type described above, whose exact functional form only depends on the quantum numbers of the light spectator antiquark. The only change we make to these quantum numbers in the present paper is to vary the light-antiquark mass. For simplicity of notation, this dependence will be left implicit unless stated otherwise.

For heavy quarks of finite mass, there are two sources of corrections to the simple results of Eq. (2). The first is hard-gluon exchange between Q and Q' across the vector current vertex. The second results from the modifications of the vector current and meson states by higher-dimension operators in HQET. These latter corrections

are proportional to inverse powers of the heavy-quark masses. Thus, we have

$$h^i(\omega, m_Q, m_{Q'}) = [\alpha^i + \beta^i(\omega; m_Q, m_{Q'}) + \gamma^i(\omega; m_Q, m_{Q'})] \xi(\omega) , \quad (3)$$

for $i = +, -$, where $\alpha^+ = 1, \alpha^- = 0$, β^i represents the radiative corrections and γ^i , the power corrections. It is important to note that these two corrections incorporate all of the heavy-quark mass dependence of the form factors h^i . As defined in Eq. (3), the Isgur-Wise function, $\xi(\omega)$, is renormalization-group invariant [7] and normalized to one at zero recoil as required by heavy-quark symmetry [3]:

$$\xi(1) = 1 . \quad (4)$$

The radiative corrections can be evaluated analytically in QCD since they are perturbative. To quantify them, we use Neubert's short-distance expansion of heavy-quark currents [7]. He considers semileptonic $\bar{B} \rightarrow D l \bar{\nu}$ and $\bar{B} \rightarrow D^* l \bar{\nu}$ decays and computes radiative corrections to the corresponding heavy-quark matrix elements to order α_s as a function of m_c and m_b . His calculation improves the previous leading logarithmic evaluation of these corrections [8] in two ways: first, he includes next-to-leading logarithms in running the $O((m_c/m_b)^0)$ heavy-quark operators from m_b down to scales at which HQET can be safely used, and second, he obtains, to order α_s , the full dependence of the heavy-quark current on the mass ratio $z = m_c/m_b$. The sum of these new contributions is as large as the leading logarithmic term. Corrections to Neubert's computation² are of order $\alpha_s^2(z \ln z)^n$ with $n = 0, 1, 2$ and should be smaller than 1%. The fact that Neubert's result accounts for the full order α_s dependence of the heavy-quark current on the mass ratio z is important for us, because our range of heavy-quark masses is quite small (see Table I): z ranges from 0.6 to 1.

TABLE I. Physical heavy-quark masses corresponding to different values the heavy-quark hopping parameter κ_Q . They are obtained from the corresponding chirally extrapolated pseudoscalar and vector meson masses, as described in Eq. (26). For completeness, we also tabulate these chirally extrapolated masses in lattice units ($a^{-1} \approx 2.7$ GeV [12]). They were obtained by covariant linear extrapolation of the masses M_P and M_V obtained at three values of the light antiquark hopping parameter: $\kappa_q = 0.141\,44, 0.142\,26, 0.142\,62$. The pseudoscalar meson masses were computed as described in Sec. IID, while the vector meson masses were obtained as in [12], with a fitting range $11 \leq t \leq 23$.

κ_Q	M_P^X	M_V^X	m_Q (GeV)
0.121	0.874_{-3}^{+4}	0.896_{-4}^{+5}	1.90
0.125	0.773_{-3}^{+3}	0.799_{-3}^{+4}	1.64
0.129	0.665_{-3}^{+3}	0.696_{-4}^{+4}	1.36
0.133	0.547_{-3}^{+3}	0.588_{-5}^{+4}	1.06

¹The subscript s is used to distinguish mesons in which the light, spectator antiquark is \bar{s} from those in which it is either \bar{u} or \bar{d} .

²Neubert runs the $O(m_c/m_b)$ contribution at one loop.

The power corrections are proportional to powers of $\epsilon_{Q^{(\prime)}} = \bar{\Lambda}/(2m_{Q^{(\prime)}})$ where $\bar{\Lambda}$ is the energy carried by the light degrees of freedom in the mesons. $\bar{\Lambda}$ will of course depend on what these light degrees of freedom are. In what follows, we will use $\bar{\Lambda} = \bar{\Lambda}_\chi = 500$ MeV [9] when working with light degrees of freedom with spin $\frac{1}{2}$ and isospin $\frac{1}{2}$. Because $\epsilon_{Q^{(\prime)}} \simeq \frac{1}{6}$ for the heavy quarks we are considering, we would naively expect power corrections in $h^+(\omega)$ and $h^-(\omega)$ to be of order 15–30%. These corrections are difficult to quantify because they involve the light degrees of freedom and are therefore nonperturbative. Luke's theorem [10], however, guarantees that there are no $O(\epsilon_Q)$ corrections to $h^+(\omega)$ at zero recoil and one may expect that power corrections to h^+ remain small away from zero recoil. This is not expected to be true for h^- which is not protected by Luke's theorem.

For degenerate transitions where $Q = Q'$, conservation of the vector current $\bar{Q}\gamma^\mu Q$ provides further constraints on the radiative and power corrections:

$$\begin{aligned} \beta^+(1; m_Q, m_Q) &= 0, \\ \gamma^+(1; m_Q, m_Q) &= 0 \\ \beta^-(\omega; m_Q, m_Q) &\equiv 0, \\ \gamma^-(\omega; m_Q, m_Q) &\equiv 0, \end{aligned} \quad (5)$$

where the last two equations hold for all ω .

Our results come from a quenched simulation on a $24^3 \times 48$ lattice at $\beta = 6.2$ on a sample of 60 gauge field configurations [11]. The lattice has an inverse lattice spacing of around 2.7 GeV [12]. We do not suffer much here from errors associated with uncertainties in the determination of the lattice spacing since our main results are dimensionless and depend at most logarithmically on the scale. Our light quarks have masses which bracket the strange quark mass. Because our heavy quarks have masses in the region of the charm-quark mass which are large in lattice units (up to one-half or more), we must contend with discretization errors proportional to powers of am_Q , where m_Q is the mass of the heavy quark. In order to reduce these discretization errors, we use the $O(a)$ -improved fermion action originally proposed by Sheikholeslami and Wohlert [13] with which discretization errors in operator matrix elements and hence in our form factors are reduced from $O(am_Q)$ to $O(\alpha_s am_Q)$ [14].

The remainder of the paper is organized as follows. In Sec. II, we present the details of our simulation, as well as our strategy for obtaining the form factors h^+ and h^- from the calculated three-point functions. In Sec. III, we discuss discretization errors and describe a procedure which enables us to subtract some of these errors nonperturbatively. In Sec. IV we present our results for the form factors h^+ and h^- for three values of the light-antiquark mass and all available initial and final heavy-quark combinations. We also extrapolate h^+ in the light-antiquark mass to the chiral limit, and interpolate it to the strange quark mass. In Sec. V, we study the dependence of h^+ and h^- on heavy-quark mass and attempt to extract the leading power corrections. We find that h^+ displays no measurable dependence on heavy-quark mass which enables us to conclude that this form factor is an Isgur-

Wise function once radiative corrections are subtracted. In Sec. VI, we study the dependence of h^+ on the light-quark mass and extract the Isgur-Wise functions $\xi_{u,d}$ and ξ_s . We find that the slopes of these functions at $\omega = 1$ are

$$\xi'_{u,d}(1) = -[0.9^{+2}_{-3}(\text{stat})^{+4}_{-2}(\text{syst})] \quad (6)$$

and

$$\xi'_s(1) = -[1.2^{+2}_{-2}(\text{stat})^{+2}_{-1}(\text{syst})]. \quad (7)$$

We thus observe a slight decrease in the magnitude of the slope with light-antiquark mass; given the errors, however, the significance of this observation is limited. We compare our results for these Isgur-Wise functions to other theoretical as well as experimental determinations. We find excellent agreement with experiment. In Sec. VII, we use our Isgur-Wise function $\xi_{u,d}$ to extract the CKM matrix element V_{cb} from different experimental measurements of the differential decay rate for $\bar{B} \rightarrow D^* l \bar{\nu}$ decays. Our results for $|V_{cb}|$ are summarized in Table XVII and are compared to other determinations of this matrix element. Our procedure for extracting $|V_{cb}|$ differs from that proposed by Neubert [15] in that we fix the ω dependence of the differential decay rate with our calculation instead of obtaining it from experiment. This enables us not only to extract $|V_{cb}|$ with no free parameters, but also to check the validity of nonperturbative QCD against experiment. We find that the ω dependence predicted by our calculation agrees very well with the results of the ALEPH [16] and CLEO [17] Collaborations. In Sec. VIII, we use $\xi_{u,d}$ and ξ_s to compute the branching ratios for $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ and $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays, and our results are summarized in Table XVIII. We also compute ratios of semileptonic widths and find

$$\frac{\Gamma(\bar{B} \rightarrow D^* l \bar{\nu})}{\Gamma(\bar{B} \rightarrow D l \bar{\nu})} = 3.2^{+3}_{-2}(\text{lat}) \pm 1.0(\text{hqs}) \quad (8)$$

and

$$\frac{\Gamma(\bar{B}_s \rightarrow D_s^* l \bar{\nu})}{\Gamma(\bar{B}_s \rightarrow D_s l \bar{\nu})} = 3.3^{+2}_{-1}(\text{lat}) \pm 1.0(\text{hqs}), \quad (9)$$

where the first set of errors was obtained by adding our lattice statistical and systematic errors in quadrature and the second set of errors, denoted by hqs, quantifies the uncertainty due to neglected power and radiative corrections. We confront our predictions for these branching ratios and ratios of widths with experimental measurements where available and find that they compare quite favorably. Finally, in Sec. IX we present our conclusions.

II. DETAILS OF THE CALCULATION

A. Lattice action and operators

Since we are studying the decays of quarks whose masses are large in lattice units, we must control discretization errors. In order to reduce these errors, we use an $O(a)$ -improved fermion action originally proposed by

Sheikholeslami and Wohlert (SW) [13], given by

$$S_F^{\text{SW}} = S_F^W - i \frac{\kappa}{2} \sum_{\mathbf{x}, \mu, \nu} \bar{q}(x) F_{\mu\nu}(x) \sigma_{\mu\nu} q(x), \quad (10)$$

where S_F^W is the Wilson action:

$$S_F^W = \sum_x \left\{ \bar{q}(x) q(x) - \kappa \sum_{\mu} [\bar{q}(x) (1 - \gamma_{\mu}) U_{\mu}(x) q(x + \hat{\mu}) + \bar{q}(x + \hat{\mu}) (1 + \gamma_{\mu}) U_{\mu}^{\dagger}(x) q(x)] \right\}. \quad (11)$$

The leading discretization errors in matrix elements for heavy-quark decays obtained from numerical simulations with the fermion action Eq. (10) are reduced from $O(am_Q)$ to $O(\alpha_s am_Q)$ and $O(a^2 m_Q^2)$, provided one also uses “improved” operators obtained by “rotating” the field of the heavy quark Q :

$$Q(x) \rightarrow (1 - \frac{1}{2} \gamma \cdot \vec{D}) Q(x). \quad (12)$$

Thus, to obtain an $O(a)$ -improved evaluation of the matrix element of Eq. (1), we use a “rotated” vector current

$$V_I^{\mu} \equiv \bar{Q}'(x) \tilde{\Gamma}^{\mu} Q(x), \quad (13)$$

where

$$\tilde{\Gamma}^{\mu} = (1 + \frac{1}{2} \gamma \cdot \vec{D}) \gamma^{\mu} (1 - \frac{1}{2} \gamma \cdot \vec{D}) \quad (14)$$

and where the subscript I indicates that V_I^{μ} is an improved lattice current.

B. Extended interpolating operators

In order to isolate the ground state in correlation functions effectively, it is useful to use extended (or “smeared”) interpolating operators for the mesons. In this study we use gauge-invariant Jacobi smearing on the heavy-quark field (described in detail in [18]), in which the smeared field, $Q^S(\mathbf{x}, t)$, is defined by

$$Q^S(\mathbf{x}, t) \equiv \sum_{\mathbf{x}'} K(\mathbf{x}, \mathbf{x}') Q(\mathbf{x}', t), \quad (15)$$

where

$$K(\mathbf{x}, \mathbf{x}') = \sum_{n=0}^N \kappa_S^n \Delta^n(\mathbf{x}, \mathbf{x}') \quad (16)$$

and

$$C_3^{\mu}(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'} \underset{t, t_f \rightarrow \infty}{\sim} \frac{Z_P(\mathbf{p}^2) Z_{P'}(\mathbf{p}'^2)}{4E_P E_{P'}} e^{-E_P t - E_{P'}(t_f - t)} \langle P'(\mathbf{p}') | V_I^{\mu}(0) | P(\mathbf{p}) \rangle, \quad (21)$$

where E_P ($E_{P'}$) is the energy of the initial (final) meson and $Z_P(\mathbf{p}^2)$ is the matrix element $\langle 0 | J_P(0) | P(\mathbf{p}) \rangle$. To cancel the above time dependence, we normalize the three-point function by two two-point functions and consider the ratio

$$R^{\mu}(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'} \equiv \frac{C_3^{\mu}(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'}}{C_2(t, \mathbf{p})_Q C_2(t_f - t, \mathbf{p}')_{Q'}}, \quad (22)$$

where

$$\Delta(\mathbf{x}, \mathbf{x}') = \sum_{i=1}^3 \{ \delta_{\mathbf{x}', \mathbf{x} - \hat{i}} U_i^{\dagger}(\mathbf{x} - \hat{i}, t) + \delta_{\mathbf{x}', \mathbf{x} + \hat{i}} U_i(\mathbf{x}, t) \}. \quad (17)$$

Following the discussion in [18], we choose $\kappa_S = 0.25$ and use the parameter N to control the smearing radius, defined by

$$r^2 \equiv \frac{\sum_{\mathbf{x}} |\mathbf{x}|^2 |K(\mathbf{x}, 0)|^2}{\sum_{\mathbf{x}} |K(\mathbf{x}, 0)|^2}. \quad (18)$$

We use $N = 75$, giving $r \approx 5.2$.

In terms of the operator Q^S of Eq. (15), the spatially extended source J_P we use to create pseudoscalar mesons composed of a heavy quark Q and a light antiquark \bar{q} is given by

$$J_P(x) = \bar{q}(x) (1 + \frac{1}{2} \gamma \cdot \vec{D}) \gamma^5 (1 - \frac{1}{2} \gamma \cdot \vec{D}) Q^S(x). \quad (19)$$

C. Three-point functions and lattice form factors

The computation of the matrix element $\langle P'(\mathbf{p}') | \bar{Q}' \gamma^{\mu} Q | P(\mathbf{p}) \rangle$ proceeds along lines similar to earlier calculations of the electromagnetic form factor of the pion and to determinations of the form factors corresponding to semileptonic decays of the D meson into light mesons. (For recent reviews of lattice computations of weak matrix elements and references to the original literature see, for example, the reviews in [19].) Thus, we calculate the three-point correlator

$$C_3^{\mu}(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'} \equiv \sum_{\mathbf{x}, \mathbf{y}} e^{-i\mathbf{q} \cdot \mathbf{x}} e^{-i\mathbf{p}' \cdot \mathbf{y}} \langle J_{P'}(t_f, \mathbf{y}) V_I^{\mu}(t, \mathbf{x}) J_P^{\dagger}(0, \mathbf{0}) \rangle, \quad (20)$$

where J_P is the spatially extended interpolating field for P defined in Eq. (19), V_I^{μ} is the $O(a)$ -improved vector current of Eq. (13), and $\mathbf{p} = \mathbf{q} + \mathbf{p}'$. To evaluate these correlators, we use the standard source method reviewed in [20].

Provided the three points in the correlator of Eq. (20) are sufficiently separated in time, the ground-state contribution dominates and

$$C_2(t, \mathbf{p})_Q \equiv \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle J_P(t, \mathbf{x}) J_P^\dagger(0) \rangle \quad (23)$$

and

$$C_2(t, \mathbf{p})_Q \underset{t \rightarrow \infty}{\sim} \frac{Z_P^2(\mathbf{p}^2)}{E_P} e^{-E_P T/2} \cosh[E_P(T/2 - t)]. \quad (24)$$

Here, T is the temporal extent of the lattice. [For $t \ll T/2$, $\exp(-E_P T/2) \cosh[E_P(T/2 - t)] \rightarrow \frac{1}{2} \exp(-E_P t)$.] Thus, in terms of the form factors defined in Eq. (1),

$$R^\mu(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'} \underset{t, t_f \rightarrow \infty}{\sim} \frac{1}{Z_P(\mathbf{p}^2) Z_{P'}(\mathbf{p}'^2)} \sqrt{M_P M_{P'}} \times ((v + v')^\mu h_{\text{lat}}^+(\omega; m_Q, m_{Q'}) + (v - v')^\mu h_{\text{lat}}^-(\omega; m_Q, m_{Q'})), \quad (25)$$

where h_{lat}^\pm are related to the continuum form factors, h^\pm , by a multiplicative renormalization, up to discretization errors, as discussed in Sec. III.

To obtain the desired form factors, we fit the ratio R^μ of Eq. (22) to the asymptotic form of Eq. (25) by minimizing, with respect to the parameters h_{lat}^+ and h_{lat}^- , a χ^2 function which takes into account correlations between the different times (labeled by t), but not between the different equations (labeled by μ). We neglect correlations between equations, because spatial and temporal components of Eq. (25) may be affected differently by discretization errors, as we discuss at the end of Sec. III C. The χ^2 value that we quote indicates not only whether our ratios R^μ are asymptotic, but also whether the decomposition of R^μ in terms of $h_{\text{lat}}^+(\omega)$ and $h_{\text{lat}}^-(\omega)$ is good. In fitting the ratio R^μ , we fix the wave-function factors $Z_{P^{(\prime)}}$, the energies, $E_{P^{(\prime)}}$, and masses, $M_{P^{(\prime)}}$, of the mesons to the values obtained from a fit of the relevant two-point functions to the asymptotic form of Eq. (24), taking into account correlations in time.

We first obtain $h_{\text{lat}}^+(\omega)$ from the time component of Eq. (25) alone, assuming that the contribution proportional to $h_{\text{lat}}^-(\omega)$ can be neglected. This approximation is exact, up to discretization errors, for degenerate transitions, i.e., transitions in which the initial and final heavy mesons are the same, and true up to radiative and power corrections for nondegenerate transitions, i.e., transitions between mesons which contain the same light antiquark, but different heavy quarks [see Eq. (5)]. For these nondegenerate transitions we can get *a posteriori* some idea of the size of the contribution of $h_{\text{lat}}^-(\omega)$ to the time component of Eq. (25). Holding $h_{\text{lat}}^+(\omega)$ fixed to its time-component value, we use all nonvanishing components of Eq. (25) to obtain $h_{\text{lat}}^-(\omega)$. We find (see Sec. IV A) that $h_{\text{lat}}^-(\omega)$'s contribution to the time component of Eq. (25) is less than about 1%, thereby justifying the approximation we make in obtaining $h^+(\omega)_{\text{lat}}$.

D. Lattice parameters and details of the analysis

We compute the three-point function of Eq. (20) for four values of both the initial and final heavy-quark hopping parameters, κ_Q and $\kappa_{Q'}$ taken from 0.121, 0.125, 0.129, 0.133 (see Table I); three values of the light-

antiquark hopping parameter, κ_q (0.141 44, 0.142 26, 0.142 62); two values of the initial meson momentum [(0,0,0) and (1,0,0) in lattice units]; and ten values of the momentum carried by the vector current [$\mathbf{q}a(12/\pi) = (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,-1,0), (-1,0,1), (0,1,-1)$]. To improve statistics, we average the ratios of Eq. (22) over all equivalent momenta. Moreover, data with initial or final momenta greater than $\pi/12a$ are excluded because they have larger systematic and statistical uncertainties. Finally, we choose t_f , the time at which the final meson is destroyed [see Eq. (20)], to be half-way across the lattice (i.e., $t_f = 24$) and symmetrize the three-point functions about that point using Euclidean time reversal, also to reduce the statistical errors.

We observe a plateau in the ratio $R^\mu(t)$ of Eq. (22) around $t = 12$, typically extending over five time slices. Therefore, we fit the ratio $R^\mu(t)$ over the range $t =$

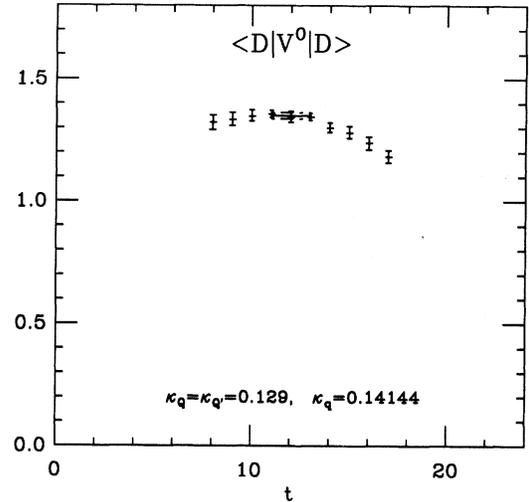


FIG. 1. The ratio $R^4(t)$, up to constant factors, vs t for the case where the initial meson has momentum (0,0,0) and the final meson, momentum $(\pi/12a, 0, 0)$. Here, the initial and final heavy-quark hopping parameters are $\kappa_Q = \kappa_{Q'} = 0.129$ while the light-quark hopping parameter is $\kappa_q = 0.14144$. The solid line is obtained from our fit of $R^4(t)$ to the asymptotic form of Eq. (25). The dashed lines indicate the errors of this fit.

TABLE II. Wave-function Z^2 and energies E_P for our heavy-light, pseudoscalar mesons and for two values of momentum $|\mathbf{p}|$. The energies are quoted in lattice units ($a^{-1} \simeq 2.7$ GeV [12]). The χ^2/N_{DF} for the fits which give these results are all on the order of 1.

κ_Q	$ \mathbf{p} $	0.121		0.125		0.129		0.133	
		Z^2	E_P	Z^2	E_P	Z^2	E_P	Z^2	E_P
0.141 44	0	17.9^{+6}_{-5}	0.924^{+2}_{-2}	16.3^{+5}_{-5}	0.823^{+2}_{-2}	14.5^{+4}_{-4}	0.716^{+2}_{-2}	12.4^{+4}_{-4}	0.600^{+2}_{-2}
	$\pi/12a$	12.3^{+5}_{-5}	0.958^{+3}_{-2}	11.4^{+4}_{-5}	0.816^{+3}_{-2}	10.3^{+4}_{-5}	0.760^{+3}_{-2}	9.0^{+4}_{-4}	0.653^{+3}_{-3}
0.142 26	0	15.5^{+5}_{-5}	0.901^{+3}_{-2}	14.2^{+5}_{-5}	0.800^{+3}_{-2}	12.7^{+4}_{-4}	0.692^{+3}_{-2}	10.8^{+5}_{-3}	0.575^{+3}_{-2}
	$\pi/12a$	10.5^{+5}_{-5}	0.937^{+3}_{-4}	9.7^{+4}_{-4}	0.840^{+3}_{-3}	8.8^{+4}_{-4}	0.739^{+3}_{-3}	7.7^{+4}_{-3}	0.631^{+4}_{-3}
0.142 62	0	14.7^{+8}_{-7}	0.892^{+4}_{-4}	13.5^{+7}_{-6}	0.791^{+3}_{-3}	12.0^{+6}_{-5}	0.683^{+3}_{-3}	10.3^{+4}_{-4}	0.565^{+3}_{-2}
	$\pi/12a$	9.8^{+7}_{-5}	0.928^{+5}_{-4}	9.1^{+6}_{-4}	0.832^{+4}_{-4}	8.3^{+5}_{-4}	0.730^{+5}_{-4}	7.3^{+5}_{-4}	0.623^{+4}_{-3}

11, 12, 13 to the form given in Eq. (25) for all momentum and heavy-quark mass combinations. For the purpose of illustration we plot, in Fig. 1, the ratio $R^4(t)$ vs t for the case where the initial meson has momentum $(\pi/12a, 0, 0)$ and the final meson, momentum $(0, 0, 0)$. We fit the two-point functions to the asymptotic form of Eq. (24) in the range $t = 11 - 22$. The results of these later fits are given in Table II.

Statistical errors are obtained from a bootstrap procedure [21]. This involves the creation of 200 bootstrap samples from the original set of 60 configurations by randomly selecting 60 configurations per sample (with replacement). Statistical errors are then obtained from the central 68% of the corresponding bootstrap distributions as detailed in [11].

Use of the HQET implies a choice of the expansion parameter, m_Q , and this requires some care [22,23]. We define m_Q as follows:

$$m_Q = \frac{a^{-1}}{4} (3M_V^X + M_P^X) - \bar{\Lambda}_\chi, \quad (26)$$

where M_P^X and M_V^X are the relevant, chirally extrapolated pseudoscalar meson and vector meson masses in lattice units (see Table I). Since these masses correspond to heavy-light mesons whose antiquark is massless, the light degrees of freedom carry an energy $\bar{\Lambda}_\chi = 0.50$ GeV as discussed after Eq. (4).

In Tables III and IV, we tabulate the results that we obtain for the radiative corrections, $\beta^+(\omega; m_Q, m_{Q'})$ and $\beta^-(\omega; m_Q, m_{Q'})$, of Eq. (3) for various combinations of

TABLE III. $\beta^+(\omega)$ vs ω for all combinations of initial and final heavy-quark mass.

$\kappa_Q \rightarrow \kappa_{Q'}$	ω				
	1.0	1.1	1.2	1.3	1.4
0.121 \rightarrow 0.121	0	-0.025	-0.047	-0.068	-0.088
0.122 \rightarrow 0.125	0.017	-0.008	-0.030	-0.051	-0.071
0.121 \rightarrow 0.129	0.037	0.013	-0.009	-0.030	-0.050
0.121 \rightarrow 0.133	0.063	0.040	0.018	-0.002	-0.022
0.125 \rightarrow 0.125	0	-0.023	-0.045	-0.065	-0.085
0.125 \rightarrow 0.129	0.024	0.001	-0.021	-0.041	-0.060
0.125 \rightarrow 0.133	0.055	0.033	0.012	-0.008	-0.027
0.129 \rightarrow 0.129	0	-0.022	-0.042	-0.061	-0.079
0.129 \rightarrow 0.133	0.039	0.017	-0.003	-0.022	-0.039
0.133 \rightarrow 0.133	0	-0.019	-0.038	-0.055	-0.071

the heavy-quark masses and for a few values of ω . As mentioned in Sec. I, we determine these corrections with the help of Neubert's work [7]. Since our results for the form factors are obtained in the quenched approximation, we set the number of quark flavors to zero and assume no particle thresholds in Neubert's expressions.³

III. Z_V , DISCRETIZATION ERRORS AND HOW TO SUBTRACT THEM NONPERTURBATIVELY

Throughout this study of semileptonic weak decays of heavy mesons, we use an $O(a)$ -improved fermion action and take for the lattice vector current, the "improved" operator V_I^μ of Eq. (13), as discussed in Sec. II A. In concrete terms, this means that we expect mass-dependent discretization errors to be of $O(\alpha_s am_Q) \sim 5\%$ and $O((am_Q)^2) \sim 10\%$ at the charm mass⁴ instead of $O(am_Q) \sim 40\%$ and $O((am_Q)^2) \sim 10\%$ as they would be without $O(a)$ improvement. Thus, despite the improvement expected, discretization errors in our calculation could be significant.

Discretization errors in the lattice evaluation of the matrix element of Eq. (1) can be parametrized as

TABLE IV. $\beta^-(\omega)$ vs ω for all combinations of initial and final heavy-quark mass.

$\kappa_Q \rightarrow \kappa_{Q'}$	ω				
	1.0	1.1	1.2	1.3	1.4
0.121 \rightarrow 0.121	0	0	0	0	0
0.121 \rightarrow 0.125	0.001	0.000	-0.001	-0.001	-0.002
0.121 \rightarrow 0.129	-0.003	-0.004	-0.005	-0.006	-0.007
0.121 \rightarrow 0.133	-0.014	-0.016	-0.017	-0.019	-0.021
0.125 \rightarrow 0.125	0	0	0	0	0
0.125 \rightarrow 0.129	0.000	-0.001	-0.001	-0.002	-0.003
0.125 \rightarrow 0.133	-0.008	-0.009	-0.011	-0.012	-0.014
0.129 \rightarrow 0.129	0	0	0	0	0
0.129 \rightarrow 0.133	-0.001	-0.002	-0.004	-0.005	-0.006
0.133 \rightarrow 0.133	0	0	0	0	0

³There is, in fact, no rigorous way of running quenched lattice QCD results since the lattice cutoff a^{-1} is adjusted to incorporate in part the effects of quenching.

⁴For this estimate, we use the boosted value of the coupling constant $g_{\text{boost}}^2 = (8\kappa_{\text{crit}})^4 g^2 \simeq 1.66$ and the improved bare mass defined before Eq. (33) with $\kappa_Q = 0.129$.

$$Z_V(\alpha_s(a)) \frac{\langle P'(\mathbf{p}') | \bar{Q}' \tilde{\Gamma}^\mu Q | P(\mathbf{p}) \rangle_{\text{lat}}}{\sqrt{M_P M_{P'}}} = (v + v')^\mu [1 + d^+(\omega)] h^+(\omega) + (v - v')^\mu [1 + d^-(\omega)] h^-(\omega) + O(a^2), \quad (27)$$

where $Z_V(\alpha_s(a))$ is the usual renormalization constant which relates the lattice vector current to the continuum one.⁵ Because this constant describes physics that takes place above and around the lattice cutoff, it is perturbative and independent of the initial and final states.

In Eq. (27), d^+ and d^- are the Euclidean-invariant discretization errors to all orders in a . At $O(a^2)$ the hypercubic group allows for additional errors which depend on the Lorentz index of the vector current. The discretization errors are nonperturbative and depend on the initial and final states, because they correspond to matrix elements of higher-dimension operators which are artifacts of lattice regularization. In addition, they depend on the procedure used to cancel all the factors which relate the three-point function to the matrix element [see Eq. (21)]. We adopt the expedient of assuming that we can absorb the Euclidean-invariant discretization errors into an effective renormalization constant Z_V^{eff} .

In the remainder of this section, we will attempt to quantify the discretization errors in our calculation more precisely and describe a procedure which enables us to subtract them, at least partially.

A. Determination of Z_V^{eff}

To study discretization errors, we define an effective renormalization constant Z_V^{eff} for vector currents composed of degenerate quark fields (i.e., of the form $\bar{q} \gamma^\mu q$) by

$$Z_V^{\text{eff}} = \frac{1}{2} \frac{C_2(t_f; \mathbf{p})}{C_3^4(t; \mathbf{p}, \mathbf{0})} \quad (28)$$

for $t_f = T/2$, where T is the temporal extent of the lattice, C_3^μ and t_f are defined in Eq. (20) and C_2 in Eq. (23). In the absence of discretization errors, Eq. (28) yields a very accurate nonperturbative determination of the renormalization constant Z_V . To see that the ratio of Eq. (28) is in effect Z_V , one must use the fact that the forward matrix element of the temporal component of the vector current is the charge, up to a trivial normalization factor. The factor of $\frac{1}{2}$ comes from our boundary conditions [see Eq. (24)]. Unless stated otherwise, we will take $\mathbf{p} = \mathbf{0}$. In the presence of the discretization errors described in Eq. (27), however, the ratio of Eq. (28) becomes

$$Z_V^{\text{eff}} = Z_V [1 - d^+(1) + O(a^2)]. \quad (29)$$

We start the discussion with a review of the determinations of Z_V^{eff} for currents composed of degenerate

light-quark fields, between pseudoscalar states composed of degenerate, light quarks and antiquarks where we expect Z_V^{eff} to be close to Z_V . Using 10 gluon configurations from our simulation at $\beta = 6.2$, we find [24]

$$\begin{aligned} Z_V^{\text{eff}} &= 0.8314(4) \quad \text{at } \kappa = 0.14144, \\ Z_V^{\text{eff}} &= 0.8245(4) \quad \text{at } \kappa = 0.14226, \\ Z_V^{\text{eff}} &= 0.8214(6) \quad \text{at } \kappa = 0.14262. \end{aligned} \quad (30)$$

These results confirm that discretization errors are small for light quarks (less than about 2%), and we take

$$Z_V = 0.82(1) \quad (31)$$

as our best estimate for Z_V . This value is also consistent with the expectations from one-loop perturbation theory [25]:

$$Z_V = 1 - 0.10g^2 + O(g^4) \simeq 0.83 \quad \text{at } \beta = 6.2 \quad (32)$$

when evaluated using the boosted value of the coupling constant, obtained from the mean field resummation of tadpole diagrams [26].

We now turn to the evaluation of Z_V^{eff} using Eq. (28) for degenerate heavy-quark currents between pseudoscalar mesons consisting of a heavy quark Q and a light antiquark \bar{q} . The results and, in particular, the difference from the value in Eq. (31), give us a measure of the size of the discretization errors. In Table V, we present the results for Z_V^{eff} , obtained from the simulation at $\beta = 6.2$ for four values of the heavy-quark mass, and with the light-quark mass corresponding to $\kappa_q = 0.14144$, and from a simulation at $\beta = 6.0$ for three values of the heavy-quark mass and with the light-quark hopping parameter equal to 0.144.⁶ Also tabulated are estimates of the improved, bare mass of the heavy quark, m_Q^I , defined by $am_Q^I = am_Q [1 - (\frac{1}{2})am_Q]$, where

TABLE V. Values of the effective normalization constant Z_V^{eff} as a function of the improved bare mass of the heavy quark. The value of κ_q is 0.14144 at $\beta = 6.2$ and 0.144 at $\beta = 6.0$.

$\beta = 6.2$			$\beta = 6.0$		
κ_Q	$m_Q^I a$	$Z_V^{\text{eff}}(\kappa_Q)$	κ_Q	$m_Q^I a$	$Z_V^{\text{eff}}(\kappa_Q)$
0.133	0.231	0.8913_{-1}^{+2}	0.129	0.344	0.920_{-1}^{+1}
0.129	0.310	0.9177_{-2}^{+3}	0.125	0.405	0.945_{-1}^{+1}
0.125	0.379	0.9428_{-2}^{+4}	0.120	0.464	0.973_{-2}^{+3}
0.121	0.435	0.9659_{-3}^{+5}			

⁵It is important to note that similar discretization errors are present for all definitions of the current, even the conserved current away from the forward direction.

⁶The simulation at $\beta = 6.0$ was performed with 36 quenched gauge field configurations on a $16^3 \times 48$ lattice using the $O(a)$ -improved SW action of Eq. (10). For details of the simulation, please see [27].

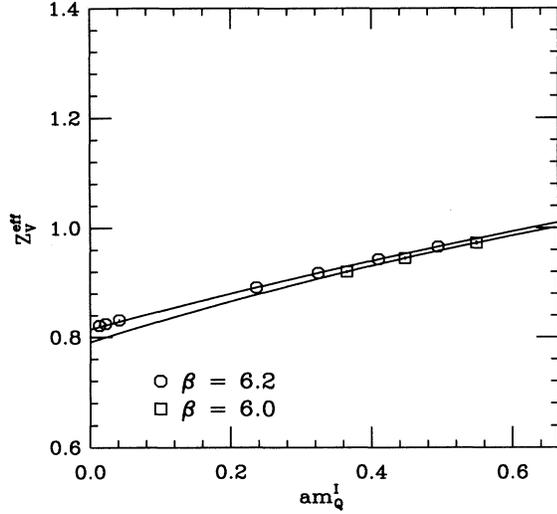


FIG. 2. Values of Z_V^{eff} as functions of $m_Q^I a$. The solid lines represent fits to quadratic functions of $m_Q^I a$ for the data at the two different values of β . We have also plotted the light-quark values of Z_V^{eff} given in Eq. (30) but have not included them in the fit.

$$am_Q = \left(\frac{1}{2}\right)(1/\kappa_Q - 1/\kappa_{\text{crit}}).$$

In Fig. 2, we plot the results for Z_V^{eff} as a function of $m_Q^I a$ for the two values of β . Fitting this behavior to a quadratic function of $m_Q^I a$,

$$Z_V^{\text{eff}}(\kappa_Q) = A + Bm_Q^I a + C(m_Q^I a)^2, \quad (33)$$

we find $A = 0.814(2)$ [$A = 0.791(4)$], $B = 0.342(12)$ [$B = 0.397(18)$], and $C = -0.072(18)$ [$C = -0.120(20)$] at $\beta = 6.2$ ($\beta = 6.0$). These fits are excellent. It is interesting to note that the results extrapolate to approximately 0.81 (0.79) in the chiral limit and are thus in good agreement with the values determined using light quarks as can be seen in Fig. 2 where we have also plotted the light-quark values for Z_V^{eff} given in Eq. (30). This fact together with the observation that the size of the mass-dependent effects for a given am_Q^I is very similar at the two values of β gives us confidence that the mass dependence we observe is indeed due to discretization errors.

Further results from the simulation at $\beta = 6.2$ are presented in Table VI and in Fig. 3. For $\kappa_Q = 0.129$ and 0.121 we have evaluated $Z_V^{\text{eff}}(\kappa_Q)$ at three values of the mass of the light quark. The results can be seen to be practically independent of the mass of the light quark. We have also evaluated Z_V^{eff} using Eq. (28) with $\mathbf{p} = (\pi/12a, 0, 0)$ and $\kappa_q = 0.14144$.⁷ The difference between the results obtained with $\mathbf{p} = (\pi/12a, 0, 0)$ and with $\mathbf{p} = \mathbf{0}$ is less than 1%. Finally, we have determined $Z_V^{\text{eff}}(\kappa_Q)$ using

$$Z_V^{\text{eff}} = \frac{p^1}{2E_P(\mathbf{p}^2)} \frac{C_2(t_f; \mathbf{p})}{C_3^1(t; \mathbf{p}, \mathbf{0})}, \quad (34)$$

for $\mathbf{p} = (\pi/12a, 0, 0)$ and $t_f = T/2$ and where $E_P(\mathbf{p}^2)$ is the energy of the meson with momentum \mathbf{p} . Now it is no longer the charge operator which appears in C_3^1 , and the statistical errors increase significantly (see Fig. 3). The values of Z_V^{eff} given by Eq. (34) are consistent with those obtained with $\mu = 4$ to within 1.5 standard deviations.

TABLE VI. Values of Z_V^{eff} for different choices of the Lorentz index μ , momenta \vec{p} in units of a^{-1} , and light-quark masses (given by κ_q) from the simulation at $\beta = 6.2$.

μ and \vec{p}	κ_Q	Z_V^{eff}		
		$\kappa_q = 0.14144$	$\kappa_q = 0.14226$	$\kappa_q = 0.14262$
$\mu = 4, \vec{p} = \vec{0}$	0.133	0.8913_{-1}^{+2}		
$\mu = 4, \vec{p} = \vec{0}$	0.129	0.9177_{-2}^{+3}	0.9168_{-4}^{+4}	0.9165_{-6}^{+5}
$\mu = 4, \vec{p} = \vec{0}$	0.125	0.9428_{-2}^{+4}		
$\mu = 4, \vec{p} = \vec{0}$	0.121	0.9659_{-3}^{+5}	0.9656_{-6}^{+6}	0.9658_{-11}^{+8}
$\mu = 4, \vec{p} = (\pi/12, 0, 0)$	0.133	0.8976_{-6}^{+10}		
$\mu = 4, \vec{p} = (\pi/12, 0, 0)$	0.129	0.9248_{-7}^{+9}	0.9242_{-12}^{+14}	0.9240_{-24}^{+24}
$\mu = 4, \vec{p} = (\pi/12, 0, 0)$	0.125	0.9498_{-8}^{+7}		
$\mu = 4, \vec{p} = (\pi/12, 0, 0)$	0.121	0.9729_{-9}^{+7}	0.9734_{-16}^{+12}	0.9746_{-25}^{+27}
$\mu = 1, \vec{p} = (\pi/12, 0, 0)$	0.133	0.949_{-56}^{+57}		
$\mu = 1, \vec{p} = (\pi/12, 0, 0)$	0.129	0.994_{-63}^{+57}	0.982_{-86}^{+83}	0.924_{-118}^{+134}
$\mu = 1, \vec{p} = (\pi/12, 0, 0)$	0.125	1.042_{-67}^{+53}		
$\mu = 1, \vec{p} = (\pi/12, 0, 0)$	0.121	1.084_{-75}^{+50}	1.089_{-116}^{+94}	1.059_{-160}^{+165}

⁷The statistical errors in $Z_V^{\text{eff}}(\kappa_Q)$ are tiny, due to a cancellation in the ratio (28) of the fluctuations in the numerator and denominator. In order to get such a dramatic cancellation of the fluctuations it is necessary to have precisely the same momentum in the numerator and denominator. If, for example, we take $\mathbf{p} = (\pi/12a, 0, 0)$ in C_3 but average over all six equivalent momenta in C_2 , $((\pm\pi/12a, 0, 0), (0, \pm\pi/12a, 0), (0, 0, \pm\pi/12a))$, then the statistical error in the ratio increases enormously.

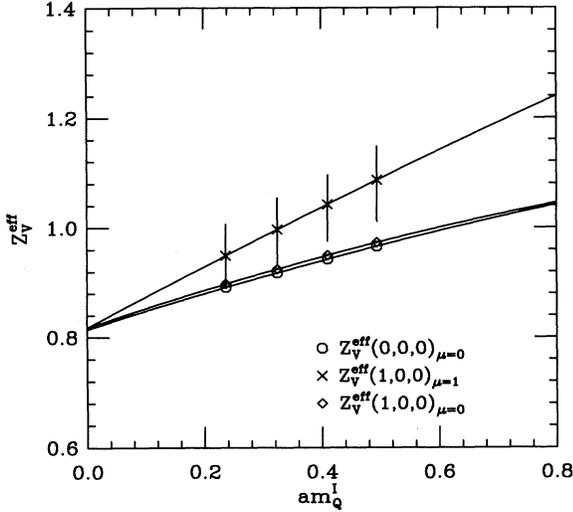


FIG. 3. Values of $Z_V^{\text{eff}}(\kappa_Q)$ obtained from the simulation at $\beta = 6.2$ at different momenta and Lorentz indices. The three curves are quadratic fits to the three sets of data.

B. Implications of the results for Z_V^{eff}

The results for $Z_V^{\text{eff}}(\kappa_Q)$ with $\mathbf{p} = \mathbf{0}$ presented above differ from the value of Z_V given in Eq. (31) by about 10–20% for the range of quark masses used in our simulations (for $\kappa_Q = 0.129$, which corresponds approximately to the charm quark for both values of β , the difference is about 12%). This difference is a good indication of the size of mass-dependent discretization errors in our calculation; it is consistent with our expectation that they should be of $O(\alpha_s am_Q)$ and $O(a^2 m_Q^2)$.

Our results for Z_V^{eff} also enable us to quantify the dependence of discretization errors on momentum as well as on the Lorentz component of the current used to obtain them. As noted in the previous subsection, the difference between the results obtained with $\mathbf{p} = (\pi/12a, 0, 0)$ and with $\mathbf{p} = \mathbf{0}$ is less than 1%. This is a clear indication that as long as we limit ourselves to momenta \mathbf{p} such that $|\mathbf{p}| \leq \pi/12a$, discretization errors proportional to $a\mathbf{p}$ are small. As for the dependence of Z_V^{eff} on the Lorentz index of the current, the situation is less clear. The ratio $Z_V^{\text{eff}}(0.121; \mu = 1)/Z_V^{\text{eff}}(0.121; \mu = 4)$ for $\mathbf{p} = (\pi/12a, 0, 0)$ indicates that this dependence could be as large as 11%. However, given that the statistical errors on $Z_V^{\text{eff}}(0.121; \mu = 1)$ are quite large, much of this dependence could be a statistical fluctuation.

C. Nonperturbative subtraction of am_Q errors

Having isolated and quantified the different sources of discretization errors, we now investigate the possibility of subtracting these errors. It is important to remember that these discretization errors are given by matrix elements of higher-dimension operators: they are nonperturbative and will depend on the initial and final states

between which the current V_I^μ is sandwiched. This means that in any attempt to subtract them, one must evaluate the relevant corrections with states as similar as possible to the ones which appear in the matrix element of interest. With this in mind, we have devised the following subtraction procedure.

First, as mentioned at the beginning of Sec. III, we assume that the mass-dependent discretization errors can be absorbed into an overall effective normalization:

$$\langle P'(\mathbf{p}') | \bar{Q}' \tilde{\Gamma}^\mu Q | P(\mathbf{p}) \rangle_{\text{lat}} = \frac{\langle P'(\mathbf{p}') | \bar{Q}' \gamma^\mu Q | P(\mathbf{p}) \rangle}{Z_V^{\text{eff}}(aM_P, aM_{P'}; \mu)}, \quad (35)$$

where $\tilde{\Gamma}^\mu$ is defined in Eq. (14).

Second, we find a normalization condition, i.e., a kinematical point at which we know the physical value of the matrix element. For the case of degenerate transitions, this normalization condition is simple; electromagnetic charge conservation requires that $h^+(1; m_Q, m_Q) = 1$. For the case of nondegenerate transitions, the normalization condition is slightly more complicated. HQET requires, as we saw earlier, that

$$h^+(1; m_Q, m_{Q'}) = 1 + \beta^+(1; m_Q, m_{Q'}) + \gamma^+(1; m_Q, m_{Q'}).$$

The radiative corrections, $\beta^+(1; m_Q, m_{Q'})$, we know from perturbation theory. The power corrections, $\gamma^+(1; m_Q, m_{Q'})$, are nonperturbative and are yet to be determined in a model-independent and reliable way. We are, however, helped here by Luke's theorem which guarantees that $h^+(1; m_Q, m_{Q'})$ is free of corrections proportional to a single power of the inverse heavy-quark masses. Thus, $\gamma^+(1; m_Q, m_{Q'}) \sim \epsilon_{Q, Q'}^2 + O(\epsilon_{Q, Q'}^3)$ and is small. In fact, as we shall see shortly, the exact size of $\gamma^+(1; m_Q, m_{Q'})$ is not important for determining the Isgur-Wise function. Thus, we will take our normalization condition to be

$$h^+(1; m_Q, m_{Q'}) \equiv 1 + \beta^+(1; m_Q, m_{Q'}) \quad (36)$$

for both degenerate and nondegenerate transitions.

This condition determines Z_V^{eff} . With Z_V^{eff} defined by Eq. (35) we find

$$Z_V^{\text{eff}} = \frac{1 + \beta^+(1; m_Q, m_{Q'})}{h_{\text{lat}}^+(1; m_Q, m_{Q'})} + O(a^2), \quad (37)$$

where $h_{\text{lat}}^+(1; m_Q, m_{Q'})$ is the zero-recoil form factor obtained from our lattice calculation and the $O(a^2)$ stands for discretization errors which are not Euclidean invariant. Because, as we mentioned earlier, discretization errors made in the evaluation of a matrix element depend not only on the initial and final states considered, but also on the procedure used to obtain the matrix element, it is very important to obtain $h_{\text{lat}}^+(1; m_Q, m_{Q'})$ with a procedure as similar as possible to the one used to obtain $h^+(\omega; m_Q, m_{Q'})$ for $\omega \neq 1$. Thus, we get $h_{\text{lat}}^+(1; m_Q, m_{Q'})$ from the time component of the ratio of Eq. (22) with $\mathbf{p}' = \mathbf{q} = \mathbf{0}$. For degenerate transitions, there is another zero-recoil channel, which corresponds to the forward scattering of a meson with one unit of lattice momentum. We do not use the $h_{\text{lat}}^+(1)$ from this channel to determine Z_V^{eff} because it is statistically much noisier

than the one at zero momentum, and because it does not correspond to a zero-recoil transition in the nondegenerate case.

Now, to subtract the discretization errors that Z_V^{eff} incorporates, we simply define the continuum form factors to be

$$h^+(\omega; m_Q, m_{Q'}) \equiv [1 + \beta^+(1; m_Q, m_{Q'})] \frac{h_{\text{lat}}^+(\omega; m_Q, m_{Q'})}{h_{\text{lat}}^+(1; m_Q, m_{Q'})},$$

$$h^-(\omega, m_Q, m_{Q'}) \equiv [1 + \beta^+(1; m_Q, m_{Q'})] \frac{h_{\text{lat}}^-(\omega; m_Q, m_{Q'})}{h_{\text{lat}}^+(1; m_Q, m_{Q'})}. \quad (38)$$

This definition yields

$$h^+(\omega; m_Q, m_{Q'}) \simeq [1 + \beta^+(\omega; m_Q, m_{Q'}) + \gamma^+(\omega; m_Q, m_{Q'}) - \gamma^+(1; m_Q, m_{Q'}) + d^+(\omega; m_Q, m_{Q'}) - d^+(1; m_Q, m_{Q'})] \xi(\omega) \quad (39)$$

up to higher-order discretization errors, radiative and power corrections. It is clear from Eq. (39) that part of the discretization errors have been subtracted. The subtraction is only complete, however, if $d^+(\omega)$ is a constant. For the form factor $h^-(\omega)$ it is less clear that we are subtracting the relevant discretization errors. Indeed, according to the definition of Eq. (38) the discretization errors in h^- are $[d^-(\omega) - d^-(1)] + O(a^2)$. However, the assumption behind this subtraction is the same as the one made by Lepage, Mackenzie, and Kronfeld [28] in their attempt to remove discretization errors by modifying the normalization factors which match fermion fields to their continuum counterparts.

We wish to emphasize here that our subtraction procedure removes nonperturbatively all discretization errors which do not break Euclidean invariance and does so to all orders in a . Thus, amongst others, all discretization errors which are removed in mean-field theory by the procedure of Kronfeld, Lepage, and Mackenzie will be removed nonperturbatively by our procedure.

As Eq. (39) indicates, in subtracting discretization errors in h^+ , we also subtract the zero-recoil power corrections, $\gamma^+(1)$, thereby losing the ability to determine them. This is not a serious concern in practice because these ought to be small—they are proportional to the square of the inverse heavy-quark mass—and therefore difficult to isolate reliably. It does mean, however, that even if we can reduce all of our errors to the percent level, we will be unable to obtain the zero-recoil power corrections to the form factor h_{A_1} relevant for $\bar{B} \rightarrow D^* l \bar{\nu}$ decays if we use an analogous subtraction procedure for these decays. This is unfortunate because these $1/m_c^2$

corrections are one of the dominant theoretical uncertainties in the extraction of the CKM matrix element V_{cb} from experimental studies of these decays (see Sec. VII).

For obtaining the Isgur-Wise function, however, the fact that our normalization procedure subtracts these zero-recoil power corrections, which are nonperturbative and difficult to quantify, is an advantage. Our hope is that, once these corrections are subtracted, the resulting form factor will have smaller power corrections away from zero recoil.

There is one additional issue surrounding normalization that we wish to address. As indicated in the previous subsection, the discretization errors on our three-point functions are typically larger for spatial than for temporal channels (see Table VI). Thus, we ought to normalize spatial and temporal channels differently. For degenerate transitions, this is possible because there is a zero-recoil three-point function which has a nonzero spatial component: $C_3^\mu(t; \mathbf{q}, \mathbf{p} = (\pi/12, 0, 0), \mathbf{0})_{Q \rightarrow Q}$. As mentioned above, however, this three-point function does not correspond to a zero-recoil decay when $Q \neq Q'$. We have no zero-recoil three-point function with a nonvanishing spatial component for nondegenerate transitions (momenta are quantized on the lattice). So, in order to treat degenerate and nondegenerate transitions in the same way, we will normalize h^+ and h^- as described in Eq. (38).

It is important to note that because h^+ is obtained from the temporal component of Eq. (25) alone (see end of Sec. II C) and is correctly normalized, it does not suffer from the possible discrepancy in normalization between temporal and spatial channels. It is h^- , obtained from both temporal and spatial components, which in fact will absorb this discrepancy. For degenerate transitions, where h^- is, in principle, zero, the values of h^- that we obtain are therefore an indication of how large an error this discrepancy can induce in the form factors. For nondegenerate transitions, the values of h^- we obtain, though contaminated to some extent by discretization errors, can be used to put bounds on the physical h^- .

IV. THE FORM FACTORS $h^+(\omega)$ AND $h^-(\omega)$

A. Results at fixed light-quark mass

In Tables VII–IX we present the measurements of $h^+(\omega)$, $h^+(\omega)/[1 + \beta^+(\omega)]$, and $h^-(\omega)$ which we obtain for all available combinations of the initial and final heavy-quark masses for light antiquarks with $\kappa_q = 0.14144$ (Table VII), 0.14226 (Table VIII), and 0.14262 (Table IX). In these tables, the first χ^2/N_{DF} column corresponds to the fit which yields h_{lat}^+ from the temporal component of the ratio R^μ assuming $h_{\text{lat}}^-(\omega) = 0$. The second χ^2/N_{DF} column corresponds to the fit which gives $h_{\text{lat}}^-(\omega)$ from both temporal and spatial components when holding h_{lat}^+ fixed to its temporal-component value. The number of degrees of freedom (N_{DF}) that we quote in this second column depends on the momentum channel because the number of nonvanishing equations for h_{lat}^+ and h_{lat}^- varies

TABLE VII. Results for $h^+(\omega)$, $h^+(\omega)/[1 + \beta^+(\omega)]$ and $h^-(\omega)$ obtained with the fitting procedure described in Sec. II C. The light-quark hopping parameter is fixed to $\kappa_q = 0.14144$ and all heavy-quark mass combinations are presented. Only transitions with initial and final meson momenta less or equal to $\pi/(12a)$ are included.

\mathbf{p}	\mathbf{p}'	ω	$h^+(\omega)$	$h^+(\omega)/[1 + \beta^+(\omega)]$	χ^2/N_{DF}	$h^-(\omega)$	χ^2/N_{DF}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.037_{-1}^{+1}	0.95_{-1}^{+1}	0.96_{-1}^{+1}	3.6/2	0.12_{-3}^{+2}	23.6/5
(1,0,0)	(1,0,0)	0.995_{-3}^{+3}	0.96_{-4}^{+4}	0.96_{-4}^{+4}	0.5/2	0.00_{-0}^{+0}	8.3/6
(1,0,0)	(0,0,0)	1.037_{-1}^{+1}	0.90_{-1}^{+1}	0.90_{-1}^{+1}	1.0/2	-0.05_{-2}^{+3}	1.3/5
(1,0,0)	(0,1,0)	1.075_{-3}^{+3}	0.86_{-2}^{+2}	0.87_{-2}^{+2}	0.4/2	0.03_{-2}^{+2}	16.9/8
(1,0,0)	(-1,0,0)	1.156_{-3}^{+3}	0.78_{-3}^{+3}	0.81_{-3}^{+4}	1.6/2	0.03_{-2}^{+2}	2.9/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.037_{-1}^{+1}	0.98_{-1}^{+1}	0.96_{-1}^{+1}	2.6/2	0.22_{-2}^{+2}	22.6/5
(1,0,0)	(1,0,0)	0.997_{-3}^{+4}	0.99_{-4}^{+4}	0.96_{-4}^{+4}	0.1/2	-0.74_{-35}^{+47}	1.7/5
(1,0,0)	(0,0,0)	1.062_{-2}^{+3}	0.89_{-1}^{+1}	0.87_{-1}^{+1}	0.7/2	0.01_{-2}^{+3}	0.9/5
(1,0,0)	(0,1,0)	1.101_{-3}^{+3}	0.84_{-2}^{+2}	0.83_{-2}^{+2}	0.8/2	0.07_{-2}^{+2}	15.4/8
(1,0,0)	(-1,0,0)	1.205_{-3}^{+3}	0.75_{-3}^{+3}	0.75_{-3}^{+3}	5.0/2	0.08_{-2}^{+2}	6.8/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.047_{-1}^{+2}	0.95_{-1}^{+1}	0.95_{-1}^{+1}	3.9/2	0.08_{-2}^{+2}	23.3/5
(1,0,0)	(1,0,0)	0.995_{-3}^{+3}	0.98_{-4}^{+4}	0.96_{-4}^{+4}	0.7/2	1.23_{-94}^{+71}	3.0/5
(1,0,0)	(0,0,0)	1.037_{-1}^{+1}	0.91_{-1}^{+1}	0.91_{-1}^{+2}	0.7/2	-0.07_{-3}^{+3}	0.9/5
(1,0,0)	(0,1,0)	1.085_{-3}^{+3}	0.86_{-2}^{+2}	0.86_{-2}^{+2}	0.2/2	0.00_{-2}^{+2}	13.5/8
(1,0,0)	(-1,0,0)	1.175_{-3}^{+3}	0.77_{-2}^{+3}	0.79_{-3}^{+3}	1.1/2	0.01_{-2}^{+2}	2.9/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.047_{-1}^{+2}	0.95_{-1}^{+1}	0.94_{-1}^{+1}	2.8/2	0.17_{-2}^{+2}	23.3/5
(1,0,0)	(1,0,0)	0.995_{-3}^{+4}	0.98_{-5}^{+5}	0.96_{-4}^{+4}	0.3/2	-1.10_{-60}^{+86}	2.4/5
(1,0,0)	(0,0,0)	1.062_{-2}^{+2}	0.88_{-1}^{+1}	0.87_{-1}^{+1}	0.6/2	0.00_{-2}^{+3}	0.7/5
(1,0,0)	(0,1,0)	1.111_{-3}^{+3}	0.82_{-2}^{+2}	0.82_{-2}^{+2}	0.5/2	0.05_{-2}^{+2}	12.6/8
(1,0,0)	(-1,0,0)	1.228_{-3}^{+3}	0.72_{-2}^{+3}	0.74_{-3}^{+3}	4.3/2	0.06_{-2}^{+2}	6.5/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.062_{-2}^{+2}	0.95_{-1}^{+1}	0.93_{-1}^{+1}	4.4/2	0.03_{-3}^{+2}	22.5/5
(1,0,0)	(1,0,0)	0.997_{-3}^{+4}	0.99_{-4}^{+5}	0.96_{-4}^{+4}	0.9/2	0.30_{-41}^{+38}	3.8/5
(1,0,0)	(0,0,0)	1.037_{-1}^{+1}	0.93_{-1}^{+1}	0.91_{-1}^{+1}	0.4/2	-0.09_{-3}^{+3}	0.5/5
(1,0,0)	(0,1,0)	1.101_{-3}^{+3}	0.85_{-2}^{+2}	0.84_{-2}^{+2}	0.2/2	-0.03_{-2}^{+2}	10.4/8
(1,0,0)	(-1,0,0)	1.205_{-3}^{+3}	0.77_{-3}^{+3}	0.77_{-3}^{+3}	0.7/2	-0.01_{-2}^{+3}	3.0/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.062_{-2}^{+2}	0.91_{-1}^{+1}	0.93_{-1}^{+1}	3.1/2	0.11_{-2}^{+2}	23.4/5
(1,0,0)	(1,0,0)	0.994_{-4}^{+4}	0.95_{-4}^{+5}	0.95_{-4}^{+5}	1.3/2	0.00_{-0}^{+0}	5.1/6
(1,0,0)	(0,0,0)	1.062_{-2}^{+2}	0.87_{-1}^{+1}	0.88_{-1}^{+1}	0.4/2	-0.02_{-2}^{+3}	0.5/5
(1,0,0)	(0,1,0)	1.127_{-4}^{+4}	0.78_{-2}^{+2}	0.81_{-2}^{+2}	0.3/2	0.02_{-1}^{+2}	9.9/8
(1,0,0)	(-1,0,0)	1.261_{-4}^{+3}	0.68_{-2}^{+3}	0.72_{-2}^{+2}	3.1/2	0.04_{-2}^{+2}	6.0/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.088_{-2}^{+3}	0.94_{-1}^{+1}	0.90_{-1}^{+1}	5.2/2	-0.03_{-3}^{+3}	21.9/5
(1,0,0)	(1,0,0)	1.005_{-4}^{+4}	1.00_{-5}^{+5}	0.94_{-4}^{+5}	1.5/2	-0.06_{-27}^{+24}	4.3/5
(1,0,0)	(0,0,0)	1.037_{-1}^{+1}	0.96_{-1}^{+1}	0.91_{-1}^{+1}	0.2/2	-0.12_{-3}^{+3}	0.3/5
(1,0,0)	(0,1,0)	1.128_{-4}^{+4}	0.84_{-2}^{+2}	0.81_{-2}^{+2}	0.2/2	-0.08_{-2}^{+2}	7.7/8
(1,0,0)	(-1,0,0)	1.252_{-4}^{+4}	0.75_{-3}^{+3}	0.74_{-3}^{+3}	0.2/2	-0.04_{-2}^{+2}	3.3/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.088_{-2}^{+3}	0.92_{-1}^{+1}	0.90_{-1}^{+1}	3.7/2	0.06_{-3}^{+2}	23.7/5
(1,0,0)	(1,0,0)	0.996_{-5}^{+5}	0.97_{-4}^{+4}	0.94_{-4}^{+4}	2.1/2	0.06_{-65}^{+57}	5.0/5
(1,0,0)	(0,0,0)	1.062_{-2}^{+3}	0.90_{-1}^{+1}	0.88_{-1}^{+1}	0.2/2	-0.06_{-2}^{+3}	0.4/5
(1,0,0)	(0,1,0)	1.155_{-5}^{+5}	0.78_{-2}^{+2}	0.78_{-2}^{+2}	0.2/2	-0.02_{-2}^{+2}	7.4/8
(1,0,0)	(-1,0,0)	1.351_{-5}^{+4}	0.67_{-2}^{+2}	0.69_{-2}^{+2}	1.6/2	0.01_{-2}^{+2}	5.4/5

TABLE VII. (*Continued*).

$\kappa_Q = 0.125 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.047^{+2}_{-1}	0.93^{+1}_{-1}	0.95^{+1}_{-1}	3.4/2	0.12^{+2}_{-2}	24.2/5
(1,0,0)	(1,0,0)	0.994^{+3}_{-3}	0.96^{+4}_{-4}	0.96^{+4}_{-4}	0.9/2	0.00^{+0}_{-0}	6.5/6
(1,0,0)	(0,0,0)	1.047^{+2}_{-1}	0.88^{+1}_{-1}	0.89^{+1}_{-1}	0.7/2	-0.04^{+3}_{-2}	0.8/5
(1,0,0)	(0,1,0)	1.096^{+3}_{-3}	0.82^{+2}_{-2}	0.84^{+2}_{-2}	0.4/2	0.02^{+2}_{-2}	13.6/8
(1,0,0)	(-1,0,0)	1.197^{+3}_{-3}	0.74^{+3}_{-2}	0.77^{+3}_{-3}	2.4/2	0.04^{+2}_{-2}	4.5/5
$\kappa_Q = 0.133 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.047^{+2}_{-1}	0.98^{+1}_{-1}	0.94^{+1}_{-1}	2.2/2	0.23^{+2}_{-2}	19.4/5
(1,0,0)	(1,0,0)	1.000^{+4}_{-4}	1.00^{+5}_{-5}	0.95^{+4}_{-4}	0.1/2	-0.46^{+41}_{-33}	1.7/5
(1,0,0)	(0,0,0)	1.088^{+3}_{-2}	0.88^{+1}_{-1}	0.85^{+1}_{-1}	0.6/2	0.04^{+3}_{-2}	0.7/5
(1,0,0)	(0,1,0)	1.139^{+4}_{-4}	0.81^{+2}_{-2}	0.79^{+2}_{-2}	0.4/2	0.08^{+2}_{-2}	10.2/8
(1,0,0)	(-1,0,0)	1.278^{+4}_{-4}	0.69^{+3}_{-2}	0.70^{+3}_{-2}	6.4/2	0.09^{+1}_{-1}	8.4/5
$\kappa_Q = 0.125 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.088^{+3}_{-2}	0.93^{+1}_{-1}	0.90^{+1}_{-1}	4.6/2	0.01^{+2}_{-2}	23.4/5
(1,0,0)	(1,0,0)	1.000^{+4}_{-4}	0.99^{+5}_{-5}	0.94^{+4}_{-4}	1.8/2	-0.02^{+30}_{-36}	4.7/5
(1,0,0)	(0,0,0)	1.047^{+2}_{-1}	0.94^{+1}_{-1}	0.90^{+1}_{-1}	0.2/2	-0.09^{+3}_{-3}	0.3/5
(1,0,0)	(0,1,0)	1.139^{+4}_{-4}	0.82^{+2}_{-2}	0.80^{+2}_{-2}	0.2/2	-0.05^{+2}_{-2}	7.7/8
(1,0,0)	(-1,0,0)	1.278^{+4}_{-4}	0.72^{+3}_{-2}	0.72^{+3}_{-2}	0.7/2	-0.02^{+2}_{-2}	4.2/5
$\kappa_Q = 0.133 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14144$							
(0,0,0)	(1,0,0)	1.088^{+3}_{-2}	0.88^{+1}_{-1}	0.90^{+1}_{-1}	2.8/2	0.10^{+2}_{-2}	21.3/5
(1,0,0)	(1,0,0)	0.994^{+6}_{-6}	0.95^{+5}_{-5}	0.95^{+5}_{-5}	1.6/2	0.00^{+0}_{-0}	4.3/6
(1,0,0)	(0,0,0)	1.088^{+3}_{-2}	0.84^{+1}_{-1}	0.86^{+1}_{-1}	0.3/2	-0.02^{+3}_{-2}	0.6/5
(1,0,0)	(0,1,0)	1.184^{+5}_{-5}	0.73^{+2}_{-2}	0.75^{+2}_{-2}	0.2/2	0.01^{+2}_{-2}	6.4/8
(1,0,0)	(-1,0,0)	1.375^{+5}_{-5}	0.60^{+2}_{-2}	0.65^{+2}_{-2}	2.8/2	0.03^{+2}_{-1}	6.3/5

with initial and final meson momenta.

As evidenced by the low values in the first χ^2/N_{DF} column of all three tables, the fits which give h_{lat}^+ from the temporal component of R^μ are very good. The fact that the values in the second χ^2/N_{DF} column of these tables are generally larger may be due to the fact that spatial and temporal components of our three-point functions may have different discretization errors, as described in Sec. III. When we fit these components simultaneously to the asymptotic form of Eq. (25) while holding h_{lat}^+ fixed, we are not fitting to a form which takes into account these discrepancies and consequently obtain a larger χ^2/N_{DF} . As discussed in Sec. IIC, however, this fitting strategy is the only one that guarantees that h^+ does not suffer significantly from discretization errors.

Given the number of different mass combinations and momentum channels we have, our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ are remarkably consistent. Keeping the light-quark mass fixed we find that for recoils ω which are approximately the same, the values of $h^+(\omega)/[1 + \beta^+(\omega)]$ are equal within errors even when they are obtained from different momentum and/or heavy-quark mass combinations. This supports the validity of our procedure and is also an indication that the radiative corrections obtained using Neubert's results [7] are accurate. The fact that $h^+(\omega)/[1 + \beta^+(\omega)]$ does not appear to depend strongly on the mass of the heavy quarks is also an indication that the coefficients of the corrections proportional to inverse powers of the heavy-quark masses are not very large (see Sec. V).

There are two momentum combinations on which we wish to comment. The first is $\mathbf{p} = (\pi/12a, 0, 0)$ to $\mathbf{p}' = (\pi/12a, 0, 0)$ which, for degenerate transitions, has zero recoil. For such transitions, current conservation requires that $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0)$ equal 1. We find values of $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0)$ which are just barely consistent with 1 at the level of 1σ for $\kappa_q = 0.14144$. The situation deteriorates when the mass of the light quark decreases (see Tables VIII and IX). Since for given quark masses $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0)$ is extracted from a single three-point function [the one with $\mathbf{p}' = (\pi/12a, 0, 0)$ and $\mathbf{q} = (0, 0, 0)$], it is much more susceptible to statistical fluctuations than most other values of h^+ which are obtained from averages of three-point functions over many equivalent momentum combinations. To show that this slight discrepancy is statistical, we consider two measures of $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0)$ which use the same three-point function and normalization. The first is

$$h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0) = \frac{Z_V^{\text{eff}}(\kappa_Q; (0, 0, 0) \rightarrow (0, 0, 0))}{Z_V^{\text{eff}}(\kappa_Q; (\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0))} \quad (40)$$

with Z_V^{eff} defined in Eq. (28). The second is the expression above multiplied by the ratio $C_2(t_f; (\pi/12a, 0, 0))/\bar{C}_2(t_f; \pi/12a)$ where $\bar{C}_2(t_f; \pi/12a)$ is the average of the six $\mathbf{p} = (\pm\pi/12a, 0, 0), (0, \pm\pi/12a, 0)$ and

TABLE VIII. Results for $h^+(\omega)$, $h^+(\omega)/[1 + \beta^+(\omega)]$, and $h^-(\omega)$ obtained with the fitting procedure described in Sec. II C. The light-quark hopping parameter is fixed to $\kappa_q = 0.14226$ and all heavy-quark mass combinations are presented. Only transitions with initial and final meson momenta less or equal to $\pi/(12a)$ are included.

\mathbf{p}	\mathbf{p}'	ω	$h^+(\omega)$	$h^+(\omega)/[1 + \beta^+(\omega)]$	χ^2/N_{DF}	$h^-(\omega)$	χ^2/N_{DF}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.039^{+2}_{-2}	0.95^{+1}_{-1}	0.96^{+1}_{-1}	3.4/2	0.14^{+4}_{-4}	20.8/5
(1,0,0)	(1,0,0)	0.996^{+4}_{-4}	0.89^{+7}_{-7}	0.89^{+7}_{-7}	0.2/2	0.00^{+0}_{-0}	2.9/6
(1,0,0)	(0,0,0)	1.039^{+2}_{-2}	0.87^{+2}_{-2}	0.88^{+2}_{-2}	0.7/2	-0.04^{+6}_{-5}	0.7/5
(1,0,0)	(0,1,0)	1.080^{+4}_{-4}	0.84^{+4}_{-3}	0.86^{+4}_{-3}	0.2/2	0.04^{+4}_{-3}	8.6/8
(1,0,0)	(-1,0,0)	1.165^{+4}_{-4}	0.81^{+5}_{-4}	0.84^{+5}_{-4}	1.3/2	0.05^{+4}_{-4}	3.7/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.039^{+2}_{-2}	0.99^{+1}_{-1}	0.96^{+1}_{-1}	3.0/2	0.23^{+4}_{-3}	18.8/5
(1,0,0)	(1,0,0)	0.999^{+5}_{-4}	0.93^{+8}_{-7}	0.89^{+7}_{-7}	0.1/2	-0.72^{+83}_{-60}	0.6/5
(1,0,0)	(0,0,0)	1.067^{+3}_{-2}	0.87^{+2}_{-2}	0.85^{+2}_{-2}	0.3/2	0.01^{+5}_{-4}	0.6/5
(1,0,0)	(0,1,0)	1.109^{+5}_{-4}	0.83^{+4}_{-3}	0.83^{+4}_{-3}	0.5/2	0.06^{+4}_{-3}	8.1/8
(1,0,0)	(-1,0,0)	1.219^{+5}_{-4}	0.78^{+4}_{-4}	0.79^{+5}_{-4}	3.5/2	0.10^{+3}_{-3}	6.8/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.050^{+2}_{-2}	0.95^{+1}_{-1}	0.95^{+1}_{-2}	3.4/2	0.09^{+4}_{-4}	19.8/5
(1,0,0)	(1,0,0)	0.996^{+5}_{-4}	0.90^{+7}_{-7}	0.89^{+7}_{-7}	0.3/2	0.81^{+119}_{-162}	1.1/5
(1,0,0)	(0,0,0)	1.039^{+2}_{-2}	0.89^{+2}_{-2}	0.88^{+2}_{-2}	0.6/2	-0.04^{+5}_{-5}	0.6/5
(1,0,0)	(0,1,0)	1.091^{+4}_{-4}	0.84^{+4}_{-3}	0.85^{+4}_{-3}	0.1/2	0.01^{+4}_{-3}	6.3/8
(1,0,0)	(-1,0,0)	1.187^{+4}_{-4}	0.81^{+4}_{-4}	0.83^{+4}_{-4}	1.0/2	0.03^{+4}_{-3}	3.8/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.050^{+2}_{-2}	0.96^{+1}_{-1}	0.95^{+1}_{-1}	3.2/2	0.18^{+4}_{-3}	19.1/5
(1,0,0)	(1,0,0)	0.997^{+5}_{-5}	0.91^{+7}_{-7}	0.89^{+7}_{-7}	0.1/2	-0.89^{+143}_{-107}	0.8/5
(1,0,0)	(0,0,0)	1.067^{+3}_{-2}	0.86^{+2}_{-2}	0.85^{+2}_{-2}	0.3/2	0.00^{+5}_{-4}	0.7/5
(1,0,0)	(0,1,0)	1.120^{+5}_{-5}	0.81^{+4}_{-3}	0.81^{+4}_{-3}	0.2/2	0.04^{+3}_{-3}	6.0/8
(1,0,0)	(-1,0,0)	1.244^{+5}_{-5}	0.75^{+4}_{-4}	0.78^{+4}_{-4}	2.8/2	0.08^{+3}_{-3}	6.6/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.067^{+3}_{-2}	0.95^{+1}_{-1}	0.93^{+1}_{-1}	3.7/2	0.04^{+4}_{-4}	18.6/5
(1,0,0)	(1,0,0)	0.999^{+5}_{-4}	0.91^{+7}_{-7}	0.88^{+7}_{-7}	0.7/2	-0.04^{+57}_{-69}	1.9/5
(1,0,0)	(0,0,0)	1.039^{+2}_{-2}	0.91^{+2}_{-2}	0.89^{+2}_{-2}	0.3/2	-0.04^{+6}_{-5}	0.5/5
(1,0,0)	(0,1,0)	1.109^{+5}_{-4}	0.83^{+4}_{-3}	0.82^{+4}_{-3}	0.1/2	-0.02^{+4}_{-3}	4.4/8
(1,0,0)	(-1,0,0)	1.219^{+5}_{-4}	0.80^{+4}_{-4}	0.81^{+4}_{-4}	0.7/2	0.01^{+4}_{-4}	4.1/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.067^{+3}_{-2}	0.91^{+1}_{-1}	0.93^{+1}_{-1}	3.5/2	0.12^{+4}_{-3}	19.2/5
(1,0,0)	(1,0,0)	0.995^{+7}_{-5}	0.88^{+7}_{-7}	0.88^{+7}_{-7}	0.5/2	0.00^{+0}_{-0}	1.6/6
(1,0,0)	(0,0,0)	1.067^{+3}_{-2}	0.84^{+2}_{-2}	0.86^{+2}_{-2}	0.3/2	-0.01^{+5}_{-3}	0.8/5
(1,0,0)	(0,1,0)	1.138^{+6}_{-5}	0.77^{+3}_{-3}	0.79^{+4}_{-3}	0.1/2	0.02^{+4}_{-3}	4.2/8
(1,0,0)	(-1,0,0)	1.282^{+5}_{-5}	0.71^{+4}_{-4}	0.76^{+4}_{-4}	2.1/2	0.05^{+3}_{-2}	6.6/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.097^{+4}_{-3}	0.94^{+2}_{-2}	0.90^{+2}_{-1}	4.2/2	-0.02^{+4}_{-3}	17.8/5
(1,0,0)	(1,0,0)	1.009^{+7}_{-5}	0.91^{+7}_{-8}	0.85^{+7}_{-8}	1.7/2	-0.37^{+39}_{-43}	3.0/5
(1,0,0)	(0,0,0)	1.039^{+2}_{-2}	0.94^{+2}_{-2}	0.89^{+2}_{-2}	0.1/2	-0.06^{+6}_{-5}	0.8/5
(1,0,0)	(0,1,0)	1.141^{+6}_{-5}	0.82^{+3}_{-3}	0.80^{+3}_{-3}	0.1/2	-0.06^{+4}_{-3}	3.0/8
(1,0,0)	(-1,0,0)	1.273^{+5}_{-5}	0.79^{+4}_{-3}	0.79^{+4}_{-3}	0.4/2	-0.01^{+4}_{-4}	4.4/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14226$							
(0,0,0)	(1,0,0)	1.097^{+4}_{-3}	0.91^{+1}_{-1}	0.90^{+1}_{-1}	4.1/2	0.05^{+4}_{-4}	19.5/5
(1,0,0)	(1,0,0)	0.999^{+8}_{-6}	0.88^{+8}_{-8}	0.84^{+7}_{-8}	2.1/2	-0.63^{+92}_{-104}	3.3/5
(1,0,0)	(0,0,0)	1.067^{+3}_{-2}	0.88^{+2}_{-2}	0.86^{+2}_{-2}	0.2/2	-0.02^{+3}_{-3}	1.1/5
(1,0,0)	(0,1,0)	1.171^{+6}_{-6}	0.77^{+3}_{-3}	0.77^{+3}_{-3}	0.1/2	-0.02^{+4}_{-2}	2.7/8
(1,0,0)	(-1,0,0)	1.343^{+6}_{-6}	0.71^{+4}_{-3}	0.73^{+4}_{-4}	1.3/2	0.03^{+3}_{-3}	6.7/5

TABLE IX. Results for $h^+(\omega)$, $h^+(\omega)/[1 + \beta^+(\omega)]$, and $h^-(\omega)$ obtained with the fitting procedure described in Sec. II C. The light-quark hopping parameter is fixed to $\kappa_q = 0.14262$ and all heavy-quark mass combinations are presented. Only transitions with initial and final meson momenta less or equal to $\pi/(12a)$ are included.

\mathbf{p}	\mathbf{p}'	ω	$h^+(\omega)$	$h^+(\omega)/[1 + \beta^+(\omega)]$	χ^2/N_{DF}	$h^-(\omega)$	χ^2/N_{DF}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.041^{+2}_{-2}	0.95^{+2}_{-2}	0.96^{+2}_{-2}	2.7/2	0.12^{+6}_{-5}	12.8/5
(1,0,0)	(1,0,0)	0.997^{+4}_{-5}	0.79^{+11}_{-11}	0.79^{+11}_{-11}	0.1/2	0.00^{+0}_{-0}	1.1/6
(1,0,0)	(0,0,0)	1.041^{+2}_{-2}	0.84^{+3}_{-3}	0.85^{+3}_{-3}	0.8/2	-0.05^{+8}_{-7}	1.0/5
(1,0,0)	(0,1,0)	1.083^{+4}_{-4}	0.82^{+5}_{-5}	0.84^{+5}_{-5}	0.1/2	0.02^{+6}_{-5}	4.8/8
(1,0,0)	(-1,0,0)	1.170^{+4}_{-4}	0.84^{+6}_{-7}	0.88^{+6}_{-7}	0.5/2	0.04^{+6}_{-6}	2.5/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.041^{+2}_{-2}	0.99^{+2}_{-2}	0.96^{+2}_{-2}	3.0/2	0.22^{+7}_{-6}	11.0/5
(1,0,0)	(1,0,0)	1.001^{+5}_{-6}	0.82^{+12}_{-11}	0.79^{+11}_{-11}	0.1/2	-0.36^{+125}_{-100}	0.9/5
(1,0,0)	(0,0,0)	1.070^{+3}_{-3}	0.85^{+4}_{-3}	0.83^{+4}_{-3}	0.2/2	-0.01^{+7}_{-7}	1.1/5
(1,0,0)	(0,1,0)	1.114^{+5}_{-5}	0.82^{+5}_{-5}	0.82^{+5}_{-5}	0.2/2	0.05^{+5}_{-5}	5.0/8
(1,0,0)	(-1,0,0)	1.226^{+6}_{-5}	0.81^{+6}_{-7}	0.82^{+6}_{-7}	1.6/2	0.10^{+5}_{-4}	4.5/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.052^{+2}_{-2}	0.96^{+2}_{-3}	0.96^{+2}_{-3}	2.5/2	0.08^{+6}_{-5}	11.5/5
(1,0,0)	(1,0,0)	0.998^{+5}_{-5}	0.79^{+11}_{-12}	0.78^{+11}_{-12}	0.1/2	-0.25^{+187}_{-251}	1.0/5
(1,0,0)	(0,0,0)	1.041^{+2}_{-2}	0.86^{+3}_{-3}	0.86^{+3}_{-3}	0.8/2	-0.03^{+8}_{-7}	1.0/5
(1,0,0)	(0,1,0)	1.095^{+5}_{-5}	0.82^{+5}_{-5}	0.83^{+5}_{-5}	0.2/2	0.00^{+5}_{-5}	3.4/8
(1,0,0)	(-1,0,0)	1.192^{+4}_{-4}	0.85^{+6}_{-7}	0.88^{+6}_{-7}	0.4/2	0.03^{+6}_{-6}	2.6/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.052^{+2}_{-2}	0.96^{+2}_{-1}	0.94^{+2}_{-1}	3.0/2	0.16^{+5}_{-5}	10.6/5
(1,0,0)	(1,0,0)	0.998^{+6}_{-6}	0.80^{+12}_{-13}	0.79^{+11}_{-12}	0.2/2	-0.11^{+211}_{-177}	1.1/5
(1,0,0)	(0,0,0)	1.070^{+3}_{-3}	0.84^{+3}_{-3}	0.83^{+3}_{-3}	0.3/2	0.00^{+7}_{-6}	1.3/5
(1,0,0)	(0,1,0)	1.125^{+6}_{-6}	0.79^{+5}_{-5}	0.80^{+5}_{-5}	0.1/2	0.03^{+5}_{-5}	3.6/8
(1,0,0)	(-1,0,0)	1.252^{+6}_{-6}	0.78^{+5}_{-6}	0.81^{+6}_{-6}	1.3/2	0.07^{+5}_{-4}	4.4/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.070^{+3}_{-3}	0.96^{+2}_{-3}	0.94^{+2}_{-3}	2.4/2	0.03^{+6}_{-5}	10.3/5
(1,0,0)	(1,0,0)	1.001^{+5}_{-6}	0.79^{+11}_{-13}	0.76^{+11}_{-13}	0.2/2	-0.70^{+92}_{-106}	1.4/5
(1,0,0)	(0,0,0)	1.041^{+2}_{-2}	0.89^{+3}_{-3}	0.87^{+3}_{-3}	0.6/2	-0.02^{+8}_{-8}	0.8/5
(1,0,0)	(0,1,0)	1.114^{+5}_{-5}	0.81^{+5}_{-5}	0.81^{+5}_{-5}	0.3/2	-0.02^{+6}_{-5}	2.3/8
(1,0,0)	(-1,0,0)	1.226^{+6}_{-5}	0.85^{+6}_{-6}	0.86^{+6}_{-6}	0.3/2	0.01^{+6}_{-6}	2.8/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.070^{+3}_{-3}	0.91^{+2}_{-2}	0.92^{+2}_{-2}	3.2/2	0.10^{+5}_{-5}	10.4/5
(1,0,0)	(1,0,0)	0.998^{+7}_{-7}	0.77^{+11}_{-12}	0.77^{+11}_{-12}	0.1/2	0.00^{+0}_{-0}	1.5/6
(1,0,0)	(0,0,0)	1.070^{+3}_{-3}	0.82^{+3}_{-3}	0.83^{+3}_{-3}	0.3/2	0.01^{+6}_{-5}	1.3/5
(1,0,0)	(0,1,0)	1.145^{+6}_{-6}	0.75^{+5}_{-4}	0.77^{+5}_{-4}	0.2/2	0.01^{+4}_{-4}	2.5/8
(1,0,0)	(-1,0,0)	1.292^{+6}_{-6}	0.74^{+5}_{-5}	0.79^{+6}_{-6}	0.9/2	0.05^{+5}_{-4}	4.5/5
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.102^{+4}_{-4}	0.95^{+3}_{-3}	0.91^{+3}_{-3}	2.6/2	-0.03^{+6}_{-5}	9.5/5
(1,0,0)	(1,0,0)	1.012^{+6}_{-6}	0.77^{+13}_{-15}	0.73^{+12}_{-14}	0.9/2	-0.87^{+61}_{-64}	2.0/5
(1,0,0)	(0,0,0)	1.041^{+2}_{-2}	0.92^{+2}_{-3}	0.88^{+2}_{-3}	0.3/2	-0.01^{+8}_{-8}	0.7/5
(1,0,0)	(0,1,0)	1.147^{+6}_{-6}	0.80^{+5}_{-5}	0.78^{+4}_{-5}	0.2/2	-0.06^{+6}_{-4}	1.6/8
(1,0,0)	(-1,0,0)	1.283^{+6}_{-6}	0.84^{+6}_{-6}	0.83^{+6}_{-6}	0.2/2	-0.01^{+6}_{-6}	2.9/5
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = 0.14262$							
(0,0,0)	(1,0,0)	1.102^{+4}_{-4}	0.91^{+2}_{-2}	0.90^{+2}_{-2}	3.6/2	0.03^{+5}_{-5}	10.6/5
(1,0,0)	(1,0,0)	1.002^{+8}_{-8}	0.75^{+13}_{-13}	0.72^{+12}_{-13}	1.0/2	-1.65^{+149}_{-155}	2.0/5
(1,0,0)	(0,0,0)	1.070^{+3}_{-3}	0.86^{+3}_{-3}	0.84^{+2}_{-3}	0.2/2	0.00^{+4}_{-5}	1.4/5
(1,0,0)	(0,1,0)	1.179^{+7}_{-7}	0.75^{+4}_{-4}	0.75^{+5}_{-4}	0.2/2	-0.02^{+5}_{-4}	1.8/8
(1,0,0)	(-1,0,0)	1.357^{+8}_{-7}	0.74^{+6}_{-5}	0.76^{+6}_{-6}	0.6/2	0.03^{+5}_{-4}	4.7/5

$(0, 0, \pm\pi/12a)$ two-point functions.⁸ Using the values of Z_V^{eff} given in Table VI, the first procedure gives $h^+((\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0))$ equal to 1 to within 1%, even when the mass of the light spectator antiquark is reduced, while the second procedure gives results very much in line with the rather low results of Tables VII–IX. The reason why the first procedure is more precise is explained in footnote 7. Moreover, that the results given by the second procedure agree better with our standard procedure for obtaining h^+ should not be too surprising, as the latter also makes use of average two-point functions.

The second small inconsistency we wish to comment on is the one arising from the comparison of $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$ with $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$, for which ω is the same. To check the validity of our results, we have re-analyzed our data by fitting our three-point functions

$$C_3^4(t; \mathbf{p}', \mathbf{q})_{Q \rightarrow Q'} \underset{t, t_f \sim t \rightarrow \infty}{\sim} \frac{Z_P(\mathbf{p}^2) Z_{P'}(\mathbf{p}'^2)}{4E_P E_{P'}} e^{-(E_P - E_{P'} + \delta E)t - E_{P'} t_f} \langle P'(\mathbf{p}') | V_I^4(0) | P(\mathbf{p}) \rangle, \quad (41)$$

with an extra parameter δE , instead of to the form given in Eq. (21). The parameter δE is designed to absorb slight statistical differences in the time behavior of two- and three-point functions. One would worry about the consistency of adding this extra parameter if it were to be large compared to the values of the various energies which enter the exponential factor in Eq. (41) since it is inconsistent to allow for changes in the energies while holding wave-function factors fixed—the two quantities are extremely correlated—and it is inconsistent to claim that $E_P - E_{P'}$ is different for two- and three-point functions but that $E_{P'}$ is the same. However, we find values of δE which are on the order of 10^{-3} and consistent with zero.

In addition to reconciling the values for $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$ and $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$, this method increases the statistical errors on all values of $h^+(\omega)$ because of the additional freedom introduced by the new parameter. We do not use this new fitting method as our main one because of the potential inconsistencies mentioned above and because the introduction of the extra parameter δE is difficult to generalize sensibly to situations where one simultaneously fits more than one four-vector component of a three-point function.

The results given by all of these different methods of analyzing the data are consistent within statistical errors. This gives us faith that the results for h^+ in Tables VII–IX are valid representations of our data. The most likely reason, then, for the slight discrepancy between $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$ and $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$ is that it arises from the same statistical fluctuation that yields the low value for $h^+((\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_Q)$.

directly to the asymptotic form given in Eq. (21), fixing the energies and wave-functions factors which appear in this asymptotic form to their two-point function values, and normalizing the resulting $h_{\text{lat}}^+(\omega)$ according to Eq. (38). This procedure yields values for h^+ which are nearly identical to the ones given in Tables VII–IX. The only values that change significantly compared to the size of their error bars are those corresponding to $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$. In this different way of analyzing the data, the values we find for $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$ are lower, making them nearer the values for $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$. This partial bridging of the gap, however, comes at the expense of large χ^2/N_{DF} 's ranging from 2 to 5. One can fix both problems, bridging the gap completely and bringing the χ^2/N_{DF} down, by fitting the time component of our three-point functions to

Like the three-point function which gives $h^+((\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_Q)$ the one from which $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$ is obtained is not averaged with equivalent three-point functions. $h^+((0, 0, 0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$, on the other hand, is obtained from the average of the six three-point functions corresponding to the transitions $(0, 0, 0) \rightarrow (\pm\pi/12a, 0, 0), (0, \pm\pi/12a, 0), (0, 0, \pm\pi/12a)$.

As mentioned earlier, current conservation requires that $h^-(\omega) \equiv 0$ for degenerate transitions. In order to determine whether our results are consistent with this requirement, we must know how large the discretization errors on $h^-(\omega)$ might be. As suggested by the results for Z_V^{eff} (see Table VI), there may be discretization errors of the order of 10% which cause the spatial components of our three-point functions to be low compared to the temporal components. One can easily convince oneself, by considering the set of equations corresponding to different components of the vector current in Eq. (25), that such discretization errors would cause $|h^-(\omega)|$ to take on values up to about 0.1. This is indeed what we find. Thus, to the level of accuracy with which we can determine $|h^-(\omega)|$, we can conclude that $h^-(\omega)$ is consistent with zero for degenerate transitions.

For nondegenerate transitions, the results we obtain for $h^-(\omega)$ resemble very much those found in degenerate transitions. They are consistent with zero at the level of 2σ .⁹ Thus, as far as we can resolve, $h^-(\omega)$ is small for all ω , most probably less than about 0.1 to 0.2.

Using this information, we can now put a bound on the size of the error that we are making on $h^+(\omega)$ by neglecting the contribution of $h^-(\omega)$ to the temporal component

⁹ $h^-(\omega)$ is large with large errors when $\mathbf{p} = \mathbf{p}' = (\pi/12a, 0, 0)$, because its coefficient in the equation which determines it is $v^1 - v^{1'} = (\pi/12)(1/aM_p - 1/aM_{p'})$, a small number when $M_p \simeq M_{p'}$.

⁸This ratio of two-point functions should of course be 1 in the limit of infinite statistics.

of Eq. (25). Using the fact that the ratio of velocity factors, $r \equiv |v^0 - v^{0'}|/(v^0 + v^{0'})$, is at most 0.07, and that $h^+(\omega)$ is always greater than 0.6, we find that the error we make on $h^+(\omega)$ is at most $r_{\max} h^-(\omega)_{\max}/h^+(\omega)_{\min} \simeq 1\%$ to 2%. In most situations, if not all, it will be smaller than that. Thus, neglecting the contribution of $h^-(\omega)$ in obtaining $h^+(\omega)$ is a very good approximation indeed.

B. Chiral extrapolation

In the previous section we determined $h^+(\omega)$ for many different combinations of initial and final heavy quarks and for three light antiquarks whose masses straddle that of the strange quark. In the present section we describe the extrapolation of our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ to vanishing light-antiquark mass for which $\kappa_q = \kappa_{\text{crit}} = 0.143\,15(2)$ [29]. The chirally extrapolated results are relevant for the study of semileptonic decays of heavy-light mesons whose light antiquark is a \bar{u} or a \bar{d} . These results are summarized in Table X.

The extrapolations are covariant and linear in the improved, bare quark mass, $am_q^I = am_q[1 - (1/2)am_q]$, where $am_q = (1/2\kappa_q - 1/2\kappa_{\text{crit}})$. We fit $h^+(1 + \beta^+)$ and ω to the forms $\alpha_{h^+}(am_q^I) + \beta_{h^+}$ and $\alpha_\omega(am_q^I) + \beta_\omega$, respectively. Then, $h_{\text{crit}}^+/(1 + \beta^+) = \beta_{h^+}$ and $\omega_{\text{crit}} = \beta_\omega$. The χ^2/N_{DF} for these extrapolations are given in columns four and six of Table X. As evidenced by the small values of these χ^2/N_{DF} 's, the extrapolations are for the most part very smooth. The only extrapolation which has an anomalously large χ^2/N_{DF} is the one for $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0); (m_Q, m_{Q'})$. As mentioned in Sec. IV A, even though current conservation requires that $h^+(\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0); (m_q, m_{q'}) = 1$ when $Q = Q'$, our results do not quite satisfy this constraint due to a statistical fluctuation. As Tables VII–IX further indicate, this constraint is less and less well satisfied as the mass of the light quark is reduced. The correlated extrapolation appears to be correct for this downward trend in the data, but does so at the expense of a large χ^2/N_{DF} .

We do not extrapolate $h^-(\omega)$ because this form factor potentially suffers from rather large discretization errors as discussed in Sec. II C and is therefore not entirely physical.

C. Interpolation to the strange quark

In the present section we describe the interpolation of our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ in the light-antiquark mass to the mass of the strange quark ($\kappa_s = 0.1419(1)$ [29]). The interpolated results are relevant for the study of semileptonic decays of heavy-light mesons which contain a strange antiquark. The results are summarized in Table XI. They are obtained from the same covariant, linear fits as the chirally extrapolated results of Sec. IV B so that χ^2/N_{DF} are the same as in Table X. The only difference is that the interpolated results are $h_s^+/(1 + \beta^+) = \alpha_{h^+}(am_s^I) + \beta_{h^+}$ and $\omega_s = \alpha_\omega(am_s^I) + \beta_\omega$,

TABLE X. Results for ω and $h^+(\omega)/[1 + \beta^+(\omega)]$, for $\kappa_q = \kappa_{\text{crit}} = 0.143\,15(2)$, obtained from covariant, linear extrapolations of the results for $\kappa_q = 0.141\,44, 0.142\,26, 0.142\,62$. All heavy-quark mass combinations are presented. The first χ^2/N_{DF} column corresponds to the ω extrapolation; the second to the extrapolation of $h^+(\omega)/[1 + \beta^+(\omega)]$.

p	p'	ω	χ^2/N_{DF}	$h^+(\omega)/[1 + \beta^+(\omega)]$	χ^2/N_{DF}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.042^{+3}_{-3}	0.1/1	0.97^{+2}_{-2}	0.0/1
(1,0,0)	(1,0,0)	0.997^{+6}_{-5}	0.2/1	0.94^{+8}_{-9}	4.6/1
(1,0,0)	(0,0,0)	1.042^{+3}_{-3}	0.1/1	0.87^{+3}_{-2}	2.3/1
(1,0,0)	(0,1,0)	1.086^{+8}_{-5}	0.1/1	0.86^{+5}_{-4}	0.5/1
(1,0,0)	(-1,0,0)	1.175^{+6}_{-5}	0.1/1	0.85^{+6}_{-5}	1.1/1
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.042^{+3}_{-3}	0.1/1	0.97^{+2}_{-1}	0.0/1
(1,0,0)	(1,0,0)	1.001^{+7}_{-7}	0.3/1	0.96^{+8}_{-9}	4.1/1
(1,0,0)	(0,0,0)	1.073^{+4}_{-3}	0.3/1	0.84^{+3}_{-2}	0.3/1
(1,0,0)	(0,1,0)	1.118^{+6}_{-6}	0.2/1	0.83^{+4}_{-4}	0.1/1
(1,0,0)	(-1,0,0)	1.235^{+7}_{-6}	0.2/1	0.80^{+6}_{-5}	0.8/1
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.054^{+3}_{-3}	0.2/1	0.95^{+2}_{-2}	0.4/1
(1,0,0)	(1,0,0)	0.998^{+6}_{-6}	0.2/1	0.94^{+8}_{-9}	4.3/1
(1,0,0)	(0,0,0)	1.042^{+3}_{-3}	0.1/1	0.87^{+3}_{-2}	1.1/1
(1,0,0)	(0,1,0)	1.098^{+6}_{-5}	0.2/1	0.84^{+6}_{-4}	0.3/1
(1,0,0)	(-1,0,0)	1.199^{+6}_{-5}	0.1/1	0.83^{+6}_{-5}	1.7/1
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.054^{+3}_{-3}	0.2/1	0.95^{+1}_{-1}	0.2/1
(1,0,0)	(1,0,0)	0.998^{+8}_{-7}	0.3/1	0.95^{+8}_{-9}	4.1/1
(1,0,0)	(0,0,0)	1.073^{+4}_{-3}	0.3/1	0.85^{+3}_{-2}	1.3/1
(1,0,0)	(0,1,0)	1.130^{+7}_{-7}	0.3/1	0.81^{+6}_{-4}	0.4/1
(1,0,0)	(-1,0,0)	1.262^{+7}_{-6}	0.2/1	0.79^{+5}_{-5}	0.6/1
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.073^{+4}_{-4}	0.3/1	0.93^{+2}_{-2}	0.6/1
(1,0,0)	(1,0,0)	1.001^{+7}_{-7}	0.3/1	0.93^{+9}_{-9}	4.7/1
(1,0,0)	(0,0,0)	1.042^{+3}_{-3}	0.1/1	0.88^{+3}_{-2}	1.6/1
(1,0,0)	(0,1,0)	1.118^{+6}_{-6}	0.2/1	0.82^{+5}_{-3}	0.4/1
(1,0,0)	(-1,0,0)	1.235^{+7}_{-6}	0.2/1	0.81^{+6}_{-5}	1.7/1
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.073^{+4}_{-4}	0.3/1	0.93^{+2}_{-1}	0.1/1
(1,0,0)	(1,0,0)	0.997^{+9}_{-7}	0.4/1	0.93^{+9}_{-9}	4.0/1
(1,0,0)	(0,0,0)	1.073^{+4}_{-4}	0.3/1	0.85^{+3}_{-2}	1.9/1
(1,0,0)	(0,1,0)	1.150^{+8}_{-8}	0.3/1	0.79^{+5}_{-4}	0.6/1
(1,0,0)	(-1,0,0)	1.304^{+8}_{-8}	0.2/1	0.77^{+5}_{-5}	0.7/1
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.108^{+5}_{-5}	0.4/1	0.90^{+2}_{-2}	0.5/1
(1,0,0)	(1,0,0)	1.013^{+9}_{-7}	0.4/1	0.92^{+9}_{-9}	5.1/1
(1,0,0)	(0,0,0)	1.042^{+3}_{-3}	0.1/1	0.89^{+3}_{-2}	2.2/1
(1,0,0)	(0,1,0)	1.154^{+7}_{-7}	0.3/1	0.80^{+4}_{-3}	0.7/1
(1,0,0)	(-1,0,0)	1.296^{+7}_{-7}	0.3/1	0.79^{+5}_{-5}	1.7/1
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = \kappa_{\text{crit}}$					
(0,0,0)	(1,0,0)	1.108^{+5}_{-5}	0.4/1	0.90^{+2}_{-1}	0.0/1
(1,0,0)	(1,0,0)	1.002^{+11}_{-9}	0.4/1	0.90^{+8}_{-10}	4.0/1
(1,0,0)	(0,0,0)	1.073^{+4}_{-4}	0.3/1	0.86^{+2}_{-2}	1.6/1
(1,0,0)	(0,1,0)	1.188^{+9}_{-9}	0.4/1	0.77^{+5}_{-4}	0.6/1
(1,0,0)	(-1,0,0)	1.374^{+9}_{-9}	0.3/1	0.74^{+5}_{-4}	0.9/1

TABLE XI. Results for ω and $h^+(\omega)/[1 + \beta^+(\omega)]$, for $\kappa_q = \kappa_s = 0.1419(1)$, obtained from covariant, linear interpolations of the results for $\kappa_q = 0.14144, 0.14226, 0.14262$. All heavy-quark mass combinations are presented. The χ^2/N_{DF} are the same as for the chiral extrapolations (see Table X).

\mathbf{p}	\mathbf{p}'	ω	$h^+(\omega)/[1 + \beta^+(\omega)]$
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.039_{-2}^{+2}	0.96_{-1}^{+1}
(1,0,0)	(1,0,0)	0.996_{-3}^{+4}	0.97_{-5}^{+5}
(1,0,0)	(0,0,0)	1.039_{-2}^{+2}	0.90_{-1}^{+2}
(1,0,0)	(0,1,0)	1.079_{-3}^{+4}	0.87_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.161_{-4}^{+4}	0.82_{-3}^{+4}
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.121, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.039_{-2}^{+2}	0.96_{-1}^{+1}
(1,0,0)	(1,0,0)	0.999_{-4}^{+4}	0.97_{-5}^{+5}
(1,0,0)	(0,0,0)	1.065_{-2}^{+2}	0.86_{-1}^{+2}
(1,0,0)	(0,1,0)	1.106_{-4}^{+4}	0.83_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.213_{-4}^{+4}	0.76_{-3}^{+4}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.049_{-2}^{+2}	0.95_{-1}^{+1}
(1,0,0)	(1,0,0)	0.996_{-4}^{+4}	0.97_{-6}^{+5}
(1,0,0)	(0,0,0)	1.039_{-2}^{+2}	0.90_{-1}^{+2}
(1,0,0)	(0,1,0)	1.089_{-3}^{+4}	0.85_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.182_{-4}^{+4}	0.80_{-3}^{+4}
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.125, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.049_{-2}^{+2}	0.94_{-1}^{+1}
(1,0,0)	(1,0,0)	0.996_{-4}^{+4}	0.97_{-5}^{+5}
(1,0,0)	(0,0,0)	1.065_{-2}^{+2}	0.87_{-1}^{+2}
(1,0,0)	(0,1,0)	1.117_{-4}^{+4}	0.82_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.238_{-5}^{+4}	0.75_{-3}^{+3}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.065_{-2}^{+2}	0.93_{-1}^{+1}
(1,0,0)	(1,0,0)	0.999_{-4}^{+4}	0.96_{-6}^{+5}
(1,0,0)	(0,0,0)	1.039_{-2}^{+2}	0.90_{-1}^{+2}
(1,0,0)	(0,1,0)	1.106_{-4}^{+4}	0.84_{-3}^{+3}
(1,0,0)	(-1,0,0)	1.213_{-4}^{+4}	0.78_{-3}^{+4}
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.129, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.065_{-2}^{+2}	0.93_{-1}^{+1}
(1,0,0)	(1,0,0)	0.995_{-5}^{+5}	0.96_{-6}^{+5}
(1,0,0)	(0,0,0)	1.065_{-2}^{+2}	0.87_{-1}^{+2}
(1,0,0)	(0,1,0)	1.134_{-5}^{+5}	0.80_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.273_{-6}^{+4}	0.73_{-3}^{+3}
$\kappa_Q = 0.121 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.094_{-3}^{+3}	0.90_{-1}^{+1}
(1,0,0)	(1,0,0)	1.007_{-4}^{+5}	0.95_{-6}^{+6}
(1,0,0)	(0,0,0)	1.039_{-2}^{+2}	0.91_{-1}^{+2}
(1,0,0)	(0,1,0)	1.136_{-5}^{+4}	0.81_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.264_{-6}^{+4}	0.75_{-3}^{+3}
$\kappa_Q = 0.129 \rightarrow \kappa_{Q'} = 0.133, \kappa_q = \kappa_s$			
(0,0,0)	(1,0,0)	1.094_{-3}^{+3}	0.90_{-1}^{+1}
(1,0,0)	(1,0,0)	0.998_{-6}^{+6}	0.94_{-5}^{+5}
(1,0,0)	(0,0,0)	1.065_{-2}^{+2}	0.87_{-1}^{+2}
(1,0,0)	(0,1,0)	1.165_{-6}^{+5}	0.78_{-2}^{+3}
(1,0,0)	(-1,0,0)	1.332_{-7}^{+5}	0.70_{-2}^{+2}

where m_s^I is the improved, bare mass of the strange quark.

V. DEPENDENCE OF $h^+(\omega)$ ON HEAVY-QUARK MASS

Having obtained $h^+(\omega)$ to good accuracy, we can now attempt to determine the dependence of this quantity on the masses of the initial and final heavy quarks. For the purpose of this study, we have calculated $h^+(\omega)/[1 + \beta^+(\omega)]$ for additional heavy-quark combinations when $\kappa_q = 0.14144$. We concentrate on the results which correspond to our heaviest, light antiquark ($\kappa_q = 0.14144$) because these results have smaller statistical uncertainties and will therefore enable us to resolve the dependence on heavy-quark mass more accurately. We will assume, in the following, that our findings for $\kappa_q = 0.14144$ provide a good description of the dependence on heavy-quark mass of our results for smaller light-antiquark masses. That this assumption may be justified is confirmed by the mild dependence of $h^+(\omega)$ on light-antiquark mass (see Sec. VI).

The first indication that the dependence of $h^+(\omega)/[1 + \beta^+(\omega)]$ on heavy-quark mass must be very weak is shown in Fig. 4. In this figure, we plot together the form factors $h^+(\omega)[1 + \beta^+(\omega)]$ for each of our four $Q \rightarrow Q$, degenerate transitions with $\kappa_q = 0.14144$. It is natural to begin looking for small heavy-quark mass effects in this data because its normalization is free of uncertainties associated with radiative or power corrections (see Sec. III C).

The four sets of data lie very much on the same curve. To show this more precisely, we fit each set individually

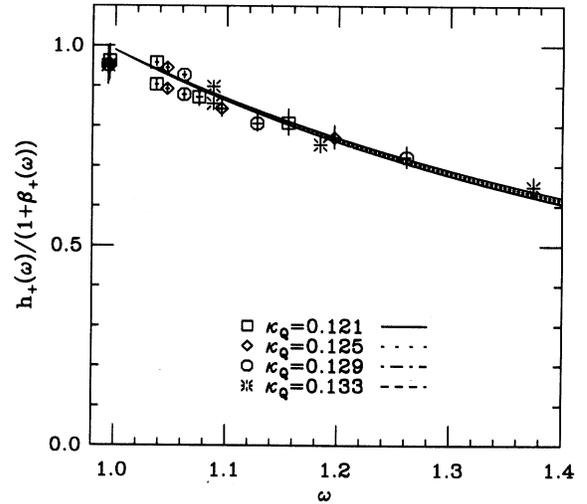


FIG. 4. $h^+(\omega)/[1 + \beta^+(\omega)]$ vs ω for all four elastic scattering reactions: $\kappa_Q = \kappa_{Q'} = 0.121, 0.129, 0.133$. The light-quark hopping parameter is fixed to $\kappa_q = 0.14144$. The curves are obtained by fitting each heavy-quark data set to $s\xi_{\text{NR}}(\omega)$. The data points as well as the curves corresponding to different heavy quarks are really indistinguishable. The $1/m_Q$ corrections to $h^+(\omega)/[1 + \beta^+(\omega)]$ cannot therefore be very large. (See text for details.)

to the parametrizations $\xi_{\text{NR}}(\omega)$ and $s\xi_{\text{NR}}(\omega)$,¹⁰ where

$$\xi_{\text{NR}}(\omega) \equiv \frac{2}{\omega+1} \exp\left(-2\rho^2 - 1\right) \frac{\omega-1}{\omega+1} \quad (42)$$

is a parametrization for the Isgur-Wise function suggested by Neubert and Rieckert in [30]. In Eq. (42), $\rho^2 = -\xi'(1)$. We have introduced the supplementary parameter s to absorb possible normalization errors. We summarize our findings in Table XII and plot the fit curves in Fig. 4. These results clearly show that the four different data sets are entirely compatible and suggest that the dependence of $h^+(\omega)/[1+\beta^+(\omega)]$ on heavy-quark mass is quite small over most of the range of experimentally accessible recoils.¹¹

Before interpreting this observation, let us quantify this heavy-quark-mass dependence more precisely. We will do so under the assumption that this small dependence is due to power corrections. We have also tested the assumption that it is due to am_Q -discretization errors but find that this assumption is less well satisfied by our data (i.e., it leads to higher χ^2/N_{DF}). According to Eq. (3), we have

$$\frac{h^+(\omega)}{1+\beta^+(\omega)} \simeq [1 + \gamma^+(\omega)]\xi(\omega). \quad (43)$$

Now, to leading order in the heavy-quark expansion,

$$\begin{aligned} \gamma^+(\omega; m_Q, m_{Q'}) &= g_Q(\omega, \alpha_s z)\epsilon_Q \\ &\quad + g_{Q'}(\omega, \alpha_s, z)\epsilon_{Q'} \\ &\quad + O(\epsilon_Q^2, \epsilon_{Q'}^2, \epsilon_Q\epsilon_{Q'}), \end{aligned} \quad (44)$$

where $\epsilon_{Q^{(\prime)}} = \bar{\Lambda}_{4144}/(2m_{Q^{(\prime)}})$,¹² the argument of α_s is $m_Q, m_{Q'}$ or some average of the two and $z = m_Q/m_{Q'}$. The functions g_Q and $g_{Q'}$ correspond to matrix elements of dimension-five operators in the HQET Lagrangian evaluated at order $O(\epsilon_{Q^{(\prime)}}^0)$. These two functions must be equal when $Q = Q'$. They must also be equal in the absence of radiative corrections as HQET cannot distinguish the flavor of a heavy quark at order $O(\epsilon_{Q^{(\prime)}}^0)$. In the presence of radiative corrections, however, the two functions will have different values when $Q \neq Q'$. The amount by which they differ will be partly governed by logarithms of the heavy-quark masses, as indicated by

$$\begin{aligned} R^+(\omega, x) &\equiv \frac{1}{\epsilon_{Q^1}} \left(1 - \frac{h^+(\omega; m_Q, m_{Q'})/[1+\beta^+(\omega; m_Q, m_{Q'})]}{h^+(\omega; m_{Q^1}, m_{Q'})/[1+\beta^+(\omega; m_{Q^1}, m_{Q'})]} \right) \\ &= g(\omega)(1-x) + O(\epsilon_Q, \epsilon_{Q^1}), \end{aligned} \quad (47)$$

¹⁰Since we are only interested in comparing $h^+(\omega)/[1+\beta^+(\omega)]$ for different heavy-quark mass, any reasonable parametrization will do.

¹¹The fact that the values of χ^2/N_{DF} are relatively high for all of these fits is explained after Eq. (50) in Sec. VI.

¹²Here, $\bar{\Lambda}_{4144}$ is the energy carried by the light degrees of freedom when $\kappa_q = 0.14144$. We take it to be $\bar{\Lambda}_{4144} = (a^{-1}/4)[3(M_V^{4144} - M_X^4) + (M_P^{4144} - M_X^4)] + \bar{\Lambda}_\chi = 0.63 \text{ GeV}$.

TABLE XII. Results of fits of our data for $h^+(\omega)/[1+\beta^+(\omega)]$ to the parametrizations $s\xi_{\text{NR}}(\omega)$ and $\xi_{\text{NR}}(\omega)$ described in the text. The four $\kappa_Q = \kappa'_Q (= 0.121, 0.125, 0.129, 0.133)$ transitions with $\kappa_q = 0.14144$ are considered in turn. Only transitions with initial and final meson momenta less or equal to $(\pi/12a)$ are included.

$\kappa_Q = \kappa'_Q$	$s\xi_{\text{NR}}(\omega)$		$\xi_{\text{NR}}(\omega)$	
	(ρ^2, s)	χ^2/N_{DF}	ρ^2	χ^2/N_{DF}
0.121	1.4_{-3}^{+3}	0.99_{-1}^{+1}	12.5/3	1.6_{-3}^{+2} 13.0/4
0.125	1.4_{-3}^{+2}	0.99_{-1}^{+1}	13.6/3	1.6_{-3}^{+2} 14.1/4
0.129	1.4_{-2}^{+1}	0.99_{-1}^{+1}	13.5/3	1.5_{-2}^{+2} 13.9/4
0.133	1.4_{-2}^{+1}	0.99_{-1}^{+1}	11.1/3	1.4_{-1}^{+2} 11.5/4

the presence of the running coupling constant in the functions' arguments. The way in which g_Q and $g_{Q'}$ depend on z will also be different. Nevertheless, since the difference between g_Q and $g_{Q'}$ is a difference of radiative corrections, it is very small. We will neglect this difference in what follows and assume that

$$\begin{aligned} \gamma^+(\omega; m_Q, m_{Q'}) &= g(\omega)(\epsilon_Q + \epsilon_{Q'}) \\ &\quad + O(\epsilon_Q^2, \epsilon_{Q'}^2, \epsilon_Q\epsilon_{Q'}, \alpha_s\epsilon_Q, \alpha_s\epsilon_{Q'}). \end{aligned} \quad (45)$$

It is worthwhile noting, at this point, that Luke's theorem requires

$$g(1) = 0. \quad (46)$$

To evaluate $g(\omega)$ we need $h^+(\omega; m_Q, m_{Q'})$ at a fixed ω for different Q or Q' . Because momenta on the lattice are quantized this is difficult to achieve. There is one kinematical situation, however, where we have enough measurements of $h^+(\omega)$ at fixed ω for different heavy quarks to determine $g(\omega)$. When the momentum of one of the mesons vanishes, ω become independent of that meson's mass. There are four values of ω for which this happens, corresponding to $|\mathbf{p}| = \pi/12a$ and $|\mathbf{p}'| = 0$ for $\kappa_Q = 0.121, 0.129$, and $|\mathbf{p}| = 0$ and $|\mathbf{p}'| = \pi/12a$ for $\kappa_{Q'} = 0.125, 0.133$. For each of these four points, we have four measurements of $h^+(\omega)$ corresponding to four different values of the mass of the meson which is at rest. We pick one of these four measurements and use it to normalize the remaining three. Thus, we construct the ratio

with $x \equiv m_{Q^1}/m_Q$. Here we have assumed that it is the initial meson which has vanishing momentum. We then fit the resulting three data points for R^+ at fixed ω to a straight line in x . The slope and intercept of this line is $g(\omega)$ [see Eq. (47)]. We summarize the details of these fits in Table XIII. In Fig. 5, we show this data with the corresponding fit (solid line) for each one of the four values of ω . The data for R^+ satisfies the parametrization of Eq. (47) surprisingly well. One should remember that all

TABLE XIII. Power corrections to $h^+(\omega)$ for four values of ω when $\kappa_q = 0.14144$. See text for definition of R^+ and $g(\omega)$.

pt	κ_Q	$\kappa_{Q'}$	\mathbf{P}_Q	$\mathbf{P}_{Q'}$	ω	x	R^+
1	0.121	0.121	(1,0,0)	(0,0,0)	1.037_{-1}^{+1}	1.00	0
		0.125				1.16	-0.015_{-15}^{+17}
		0.129				1.40	-0.031_{-23}^{+35}
		0.133				1.79	-0.052_{-35}^{+51}
		$g(\omega) = 0.073_{-81}^{+52}$ with $\chi^2/N_{\text{DF}} = 0.1/2$					
2	0.121	0.125	(0,0,0)	(1,0,0)	1.047_{-1}^{+2}	1.00	0
		0.125				1.16	0.002_{-7}^{+7}
		0.129				1.40	0.021_{-12}^{+15}
		0.133				1.79	0.031_{-17}^{+23}
		$g(\omega) = -0.041_{-29}^{+23}$ with $\chi^2/N_{\text{DF}} = 0.4/2$					
3	0.129	0.121	(1,0,0)	(0,0,0)	1.062_{-2}^{+2}	1.00	0
		0.125				1.16	-0.006_{-25}^{+13}
		0.129				1.40	-0.049_{-25}^{+25}
		0.133				1.79	-0.062_{-44}^{+54}
		$g(\omega) = 0.083_{-65}^{+51}$ with $\chi^2/N_{\text{DF}} = 0.8/2$					
4	0.121	0.133	(0,0,0)	(1,0,0)	1.088_{-2}^{+3}	1.00	0
		0.125				1.16	0.010_{-12}^{+11}
		0.129				1.40	0.013_{-26}^{+23}
		0.133				1.79	0.003_{-44}^{+37}
		$g(\omega) = -0.025_{-50}^{+53}$ with $\chi^2/N_{\text{DF}} = 0.5/2$					

power corrections are subtracted at $\omega = 1$ by our normalization procedure [see Eq. (39) and ensuing discussion]. However, this is not a problem if one is interested only in $O(\epsilon_Q, \epsilon_{Q'})$ power corrections to h^+ since these must vanish at zero recoil according [Eq. (46)].

In Fig. 6, we plot g as a function of ω . $g(\omega)$ is consistent with zero over the range of recoils ω that we can explore ($1 \leq \omega \leq 1.1$). Since $g(\omega)$ shows no trend over that range and since the functions $h^+(\omega)/[1 + \beta^+(\omega)]$ plotted in Fig. 4 exhibit no mass dependence over a range of recoils from 1 to 1.4, we conclude that $g(\omega)$ ought to remain small (less than about 0.2) over the full range of experimentally interesting recoils ($1 \leq \omega \leq 1.55$). We believe that these results indicate that the $1/m_Q$ corrections to $h^+(\omega)$ and the remaining am_Q discretization errors in $h^+(\omega)$ are genuinely small because we explore a non-negligible range of heavy-quark masses—from about 1 to 2 GeV. It seems quite unlikely that discretization errors or higher-order power corrections would cancel the leading power corrections over such a range.

Because $g(\omega)$ appears to be less than about 0.2 over the full range of recoils, we predict that power corrections to the form factor h^+ corresponding to physical $\bar{B} \rightarrow D l \bar{\nu}$ decays must be less than about $O(\epsilon_c^2) \simeq 3\%$ to $O[0.2 \times (\epsilon_b + \epsilon_c), \epsilon_c^2] \sim 5 - 10\%$ over the full range of recoils for $m_b = 4.80$ GeV, $m_c = 1.45$ GeV, and $\bar{\Lambda} = 0.50$ GeV [9].¹³ This is significantly smaller than the

$O(\epsilon_c) \simeq 15\%[O(\epsilon_c^2)$ and $O(\epsilon_b)$ may each contribute an additional 5%] that one may naively have expected. It appears, then, that the protection Luke's theorem provides a zero recoil extends over the full range of recoils and that for the particular combination $h^+(\omega)/[1 + \beta^+(\omega)]$ the flavor component of the heavy-quark symmetry is well satisfied in the charmed sector. This is in stark contrast with our findings for the decay constant, f_D , of the pseudoscalar D meson [12]. In [12], we find that the $O(\epsilon_c)$ corrections to the heavy-quark limit prediction for this decay constant are of the order of 30%.

These results for $g(\omega)$ also mean that our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ are, to a good approximation, infinite heavy-quark-mass results. Thus, the functions $h^+(\omega)/[1 + \beta^+(\omega)]$ that we measure are effectively Isgur-Wise functions and we can consistently combine data corresponding to different initial and final heavy quarks. This is what we do in the following.

In principle one could also try to quantify power corrections to $h^-(\omega)$. In the absence of radiative corrections, we find from the results of [9] that these power corrections are given by

$$\gamma^-(w; m_Q, m_{Q'}) = [1 - 2\eta(w)](-\epsilon_Q + \epsilon_{Q'}), \quad (48)$$

where, like $g(\omega)$ defined in Sec. V, $\eta(\omega)$ is a subleading, universal form factor.¹⁴ Equation (48) indicates that power corrections proportional to ϵ_Q are equal and opposite to those proportional to $\epsilon_{Q'}$. This prediction is consistent with the mass dependence we observe in our results. However, because our normalization procedure is optimized for determining $h^+(\omega)$ and not $h^-(\omega)$, it is not clear to what extent the mass dependence due to power corrections can be resolved from that due to discretization errors and to higher-order power corrections coming from our normalization procedure [see Eq. (38)].

VI. DEPENDENCE OF $h^+(\omega)$ ON LIGHT-QUARK MASS: ISGUR-WISE FUNCTIONS

We established in the previous section that, for fixed light-antiquark mass, we can combine the results for $h^+(\omega)/[1 + \beta^+(\omega)]$ corresponding to different values of the initial and final heavy-quark mass. We further established that the resulting function is an Isgur-Wise function: $\xi_{u,d}(\omega)$ when the mass of the light antiquark vanishes; $\xi_s(\omega)$ when the light antiquark is given the mass of the strange quark.

We plot $\xi_{u,d}(\omega)$ and $\xi_s(\omega)$ in Figs. 7 and 8, respectively. We fit the corresponding data to the parametrizations $s\xi_{\text{NR}}(\omega)$, $\xi_{\text{NR}}(\omega)$. The parameter s is added to absorb possible uncertainties in the normalization of these form factors. Because the parametrization $\xi_{\text{NR}}(\omega)$ is only one of many possible parametrizations, we also fit our results to $s\xi_{\text{lin}}(\omega)$, $\xi_{\text{lin}}(\omega)$, $\xi_{\text{quad}}(\omega)$, and $s\xi_{\text{quad}}(\omega)$ where

¹³We have included, in this estimate, potential higher-order corrections that may have been subtracted by our normalization procedure.

¹⁴Luke's theorem does not constrain $\eta(\omega)$ at $\omega = 1$ as it did $g(\omega)$.

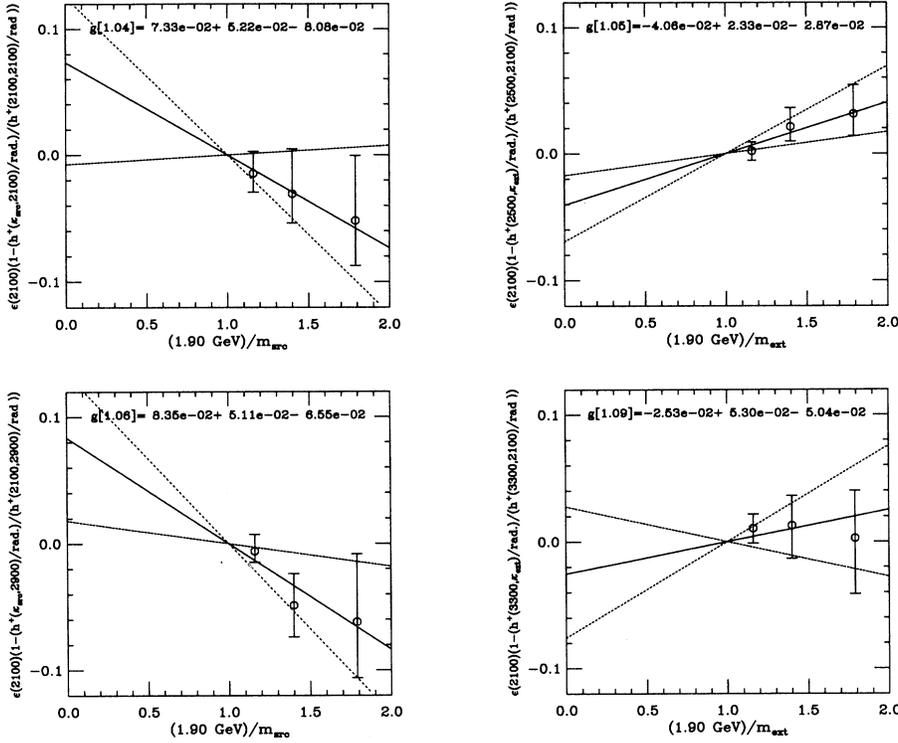


FIG. 5. R^+ vs $x = m_Q^1/m_Q$ at fixed ω for four values of ω . The solid lines are obtained by fitting these results to the parametrization given in Eq. (47) and the dotted lines represent errors. The slope and intercept of this line is the subleading form factor $g(\omega)$ [Eqs. (45) and (43)]. The light-quark hopping parameter is $\kappa_q = 0.14144$.

$$\xi_{\text{lin}}(\omega) = 1 - \rho^2(\omega - 1) \quad (49)$$

is a simple linear parametrization, and

$$\xi_{\text{quad}}(\omega) = 1 - \rho^2(\omega - 1) + \frac{c}{2}(\omega - 1)^2 \quad (50)$$

is a quadratic parametrization. The parameter c in Eq. (50) is, of course, the curvature of the Isgur-Wise function at $\omega = 1$. We tabulate the results of these different fits in Table XIV. In this table, we also present the results of performing these fits on the data corresponding

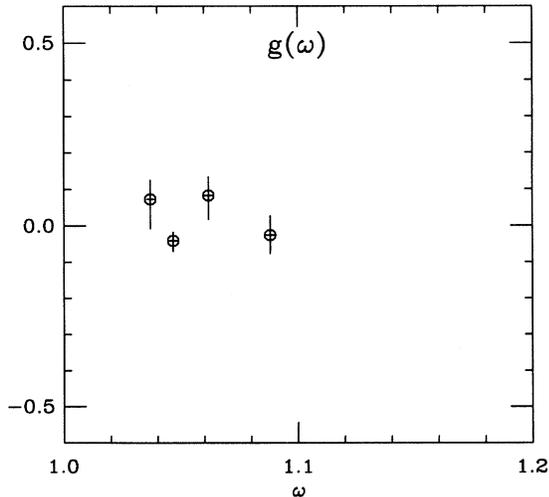


FIG. 6. The subleading form factor $g(\omega)$ [Eqs. (45) and (43)]. The light-quark hopping parameter is $\kappa_q = 0.14144$.

to $\kappa_q = 0.14144, 0.14226, \text{ and } 0.14262$.

The fact that the values of χ^2/N_{DF} are relatively high for all of these fits should not in itself be taken as an indication that the parametrizations of Eqs. (42), (49), and (50) are poor representations of the Isgur-Wise functions. These large values of χ^2/N_{DF} are due to the discrepancy

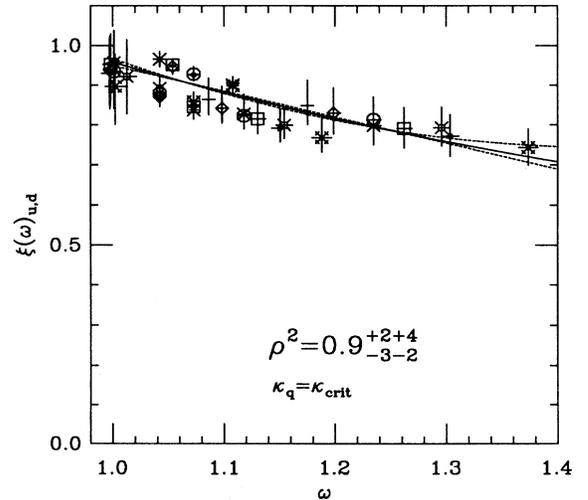


FIG. 7. $\xi_{u,d}(\omega) = h^+(\omega)/[1 + \beta^+(\omega)]$ vs ω for $\kappa_q = \kappa_{\text{crit}}$. The different symbols correspond to different values of initial and final heavy-quark mass. The solid curve is obtained by fitting these results to $s\xi_{\text{NR}}(\omega)$ while the dashed curve corresponds to a fit to $s\xi_{\text{lin}}(\omega)$ and the dotted curve corresponds to a fit to $s\xi_{\text{quad}}(\omega)$. The value of ρ^2 shown on the plot is the one given in Eq. (51).

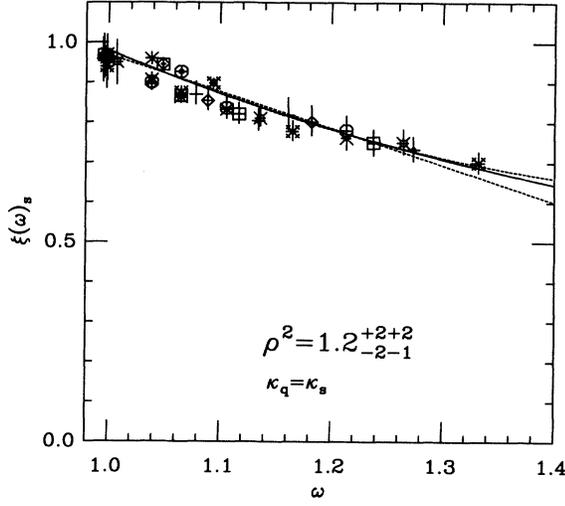


FIG. 8. $\xi_s(\omega) = h^+(\omega)/[1 + \beta^+(\omega)]$ vs ω for $\kappa_q = \kappa_s$. The different symbols correspond to different values of initial and final heavy-quark mass. The solid curve is obtained by fitting these results to $s\xi_{\text{NR}}(\omega)$ while the dashed curve corresponds to a fit to $s\xi_{\text{lin}}(\omega)$ and the dotted curve corresponds to a fit to $s\xi_{\text{quad}}(\omega)$. The value of ρ^2 shown on the plot is the one given in Eq. (52).

that we mentioned in Sec. IV A between our measurements of $h^+((0,0,0) \rightarrow (\pi/12a, 0, 0); m_Q, m_{Q'})$ and of $h^+((\pi/12a, 0, 0) \rightarrow (0, 0, 0); m_{Q'}, m_Q)$. Because of this discrepancy, no parametrization can fit our data with a good value of χ^2/N_{DF} . χ^2/N_{DF} 's nevertheless seem to favor the use of the extra parameter s but do not seriously

discriminate between $s\xi_{\text{NR}}(\omega)$, $s\xi_{\text{min}}(\omega)$, and $s\xi_{\text{quad}}(\omega)$. We have tried fitting our data to yet other parametrizations and of all the fitting functions, $s\xi_{\text{lin}}(\omega)$ yields the lowest values for ρ^2 . The reason for this is that $s\xi_{\text{lin}}(\omega)$ is the only parametrization which does not have positive curvature. Since $s\xi_{\text{lin}}(\omega)$ is in that sense an exception, we will not use it as our standard fitting function but because it is a valid parametrization for these Isgur-Wise functions we will make certain that our results have errors which encompass the values it gives for the slope. Furthermore, since both $s\xi_{\text{NR}}(\omega)$ and $s\xi_{\text{quad}}(\omega)$ give nearly identical fits (see Figs. 7 and 8), we will use $s\xi_{\text{NR}}(\omega)$ as our standard in the following because it has one less parameter and yields better χ^2/N_{DF} 's.

Having already argued in Sec. V that mass-dependent discretization errors are small, we turn to momentum-dependent lattice artifacts. To quantify these momentum-dependent discretization errors we resort to the following procedure. We fit the data for $\xi_{u,d}(\omega)$ and $\xi_s(\omega)$ for fixed initial and final meson momentum and all heavy-quark combinations, to the parametrization $s\xi_{\text{NR}}(\omega)$. The variation in the results of fits to these different momentum sets should give us some indication of how large these momentum-dependent lattice artifacts are. Some of this variation, of course, may be due to statistical fluctuations of the sort we mentioned in Sec. IV A.

We summarize the results of the fits to the different momentum sets in Table XV. It is reassuring that the value of s for the case $(0, 0, 0) \rightarrow (\pi/12a, 0, 0)$ is very close to 1, because the corresponding data are our best points. They are the points for which our normalization procedure is optimal because they are obtained from three-

TABLE XIV. Results of fits of our data for $h^+(\omega)/[1 + \beta^+(\omega)]$ to the parametrizations $\xi_{\text{NR}}(\omega)$, $\xi_{\text{lin}}(\omega)$, and $\xi_{\text{quad}}(\omega)$ with and without the additional parameter s , as described in the text. For fixed κ_q , all heavy-quark mass combinations are used. Only transitions with initial and final meson momenta less or equal to $\pi/(12a)$ are included. Here $\kappa_{\text{crit}} = 0.14315(2)$ and $\kappa_s = 0.1419(1)$.

κ_q		$s\xi_{\text{NR}}(\omega)$		χ^2/N_{DF}	$\xi_{\text{NR}}(\omega)$		
	ρ^2	s	ρ^2		χ^2/N_{DF}		
0.141 44	1.3^{+2}_{-1}	0.98^{+1}_{-1}		109/38	1.5^{+2}_{-2}	121/39	
κ_s	1.2^{+2}_{-2}	0.98^{+1}_{-1}		95/38	1.4^{+2}_{-2}	106/39	
0.142 26	1.0^{+2}_{-3}	0.96^{+2}_{-1}		113/38	1.4^{+2}_{-3}	140/39	
0.142 62	0.7^{+3}_{-3}	0.94^{+2}_{-2}		100/38	1.4^{+3}_{-4}	134/39	
κ_{crit}	0.9^{+2}_{-3}	0.96^{+2}_{-2}		69/38	1.3^{+3}_{-3}	88/39	
κ_q		$s\xi_{\text{lin}}(\omega)$		χ^2/N_{DF}	$\xi_{\text{lin}}(\omega)$		
	ρ^2	s	ρ^2		χ^2/N_{DF}		
0.141 44	1.0^{+1}_{-1}	0.97^{+1}_{-1}		111/38	1.3^{+1}_{-1}	159/39	
κ_s	0.9^{+1}_{-1}	0.97^{+1}_{-1}		97/38	1.2^{+1}_{-1}	139/39	
0.142 26	0.8^{+1}_{-2}	0.96^{+2}_{-1}		114/38	1.2^{+2}_{-2}	170/39	
0.142 62	0.6^{+2}_{-2}	0.93^{+2}_{-2}		100/38	1.1^{+2}_{-3}	155/39	
κ_{crit}	0.7^{+1}_{-2}	0.95^{+2}_{-2}		71/38	1.1^{+2}_{-2}	111/39	
κ_q		$s\xi_{\text{quad}}(\omega)$		χ^2/N_{DF}	$\xi_{\text{quad}}(\omega)$		
	ρ^2	c	s		ρ^2	c	χ^2/N_{DF}
0.141 44	1.2^{+2}_{-2}	$1.6^{+1.2}_{-1.3}$	0.98^{+2}_{-1}	108/37	1.6^{+2}_{-2}	$3.9^{+1.4}_{-1.4}$	115/38
κ_s	1.2^{+2}_{-2}	$2.0^{+1.2}_{-1.5}$	0.98^{+2}_{-2}	94/37	1.5^{+2}_{-2}	$4.0^{+1.4}_{-1.3}$	99/38
0.142 26	1.0^{+2}_{-3}	$1.4^{+1.3}_{-1.7}$	0.96^{+2}_{-1}	113/37	1.6^{+2}_{-3}	$5.1^{+1.6}_{-1.6}$	125/38
0.142 62	0.7^{+4}_{-4}	$1.1^{+2.2}_{-2.6}$	0.94^{+2}_{-2}	100/37	1.7^{+4}_{-4}	$6.9^{+2.1}_{-2.2}$	114/38
κ_{crit}	1.0^{+3}_{-3}	$2.3^{+1.5}_{-2.0}$	0.97^{+2}_{-3}	69/37	1.5^{+3}_{-3}	$5.0^{+1.5}_{-1.6}$	74/38

TABLE XV. The different momentum sets for $\xi_{u,d}(\omega)$ and $\xi_s(\omega)$ are fit to the parametrization $s\xi_{\text{NR}}(\omega)$ [Eq. (42)]. A momentum set comprises all combinations of initial and final heavy quarks with fixed initial and final momenta. These different fits are used to estimate remaining systematics (see text).

\mathbf{p}	\mathbf{p}'	$\xi_{u,d}(\omega)$			$\xi_s(\omega)$		
		ρ^2	s	χ^2/N_{DF}	ρ^2	s	χ^2/N_{DF}
(0,0,0)	(1,0,0)	1.1^{+3}_{-2}	1.01^{+2}_{-2}	0.3/6	1.2^{+2}_{-1}	1.00^{+2}_{-1}	0.1/6
(1,0,0)	(0,0,0)	1.3^{+5}_{-4}	0.93^{+4}_{-3}	0.8/6	1.4^{+3}_{-3}	0.95^{+3}_{-2}	0.7/6
(1,0,0)	(0,1,0)	1.2^{+4}_{-3}	0.95^{+8}_{-6}	0.1/6	1.4^{+2}_{-2}	0.96^{+4}_{-3}	0.1/6
(1,0,0)	(-1,0,0)	0.7^{+4}_{-2}	0.95^{+11}_{-8}	0.2/6	1.1^{+2}_{-2}	0.95^{+7}_{-5}	0.4/6

point functions which are much more correlated with the three-point function which yields the normalization factor $h_{\text{lat}}^+(1)$.¹⁵

Furthermore, these data have the smallest statistical errors and should have the smallest discretization errors, because the momenta of the incoming and outgoing mesons are less than or equal to the initial and final momenta of other momentum sets.

To accommodate the spread in the values in the slope parameters $\rho_{u,d}^2$ and ρ_s^2 corresponding to $\xi_{u,d}(\omega)$ and $\xi_s(\omega)$, we assign systematic errors to these parameters which encompass all the central values given in Table XV. The central value and statistical errors that we quote are given by fitting $s\xi_{\text{NR}}(\omega)$ to all momentum sets put together (see Table XIV). Thus, our final results for the slope at $\omega = 1$ are

$$\rho_{u,d}^2 = 0.9^{+2}_{-3}(\text{stat})^{+4}_{-2}(\text{syst}) \quad (51)$$

for $\xi_{u,d}(\omega)$ and

$$\rho_s^2 = 1.2^{+2}_{-2}(\text{stat})^{+2}_{-1}(\text{syst}) \quad (52)$$

for $\xi_s(\omega)$. Even though the exact values of these slope parameters are slightly different if different parametrizations for the Isgur-Wise functions are used, these differences are well within our error bars.

In Table XVI we compare our predictions for the slope of the Isgur-Wise functions $\xi_{u,d}$ and ξ_s with those of other authors. We find that our predictions for ρ^2 lie safely above the lower bound of Bjorken [31] and below the upper bound of de Rafael and Taron [32]. Our results for ρ_s^2 also agree with the lattice result of Bernard *et al.* [6] obtained with Wilson fermions for a light spectator antiquark with mass $m_q \sim m_s$, although the details and systematics of the two calculations are different. The authors of [6] do not quote a value of $\rho_{u,d}^2$ for vanishing light-quark mass.

Also for comparison, we quote an average experimental value for the slope of the Isgur-Wise function compiled by Neubert [33] from very recent results of the ALEPH [16] and CLEO [17] Collaborations as well as older data from the ARGUS Collaboration [34]:

$$\rho_{u,d}(\text{exp}) = 0.87(12)(20), \quad (53)$$

where the second error is theoretical and accounts for the uncertainty in the size of $1/m_c$ corrections [33]. Equations (51) and (53) agree remarkably well.

As can be inferred from Eqs. (51) and (52) and from Table XIV our results are compatible with the statement that ρ^2 is constant with light-quark mass, possibly decreasing slightly as this mass decreases. Such a decrease is consistent with one's intuition that it is more difficult to make the light degrees of freedom recoil, the heavier these degrees of freedom are. Furthermore, Høgaasen

TABLE XVI. Comparison of our lattice results for $-\xi'_{u,d}(1)$ and $-\xi'_s(1)$ to the theoretical predictions of various authors.

Reference	$-\xi'_{u,d}(1)$	$-\xi'_s(1)$
UKQCD	$0.9^{+2}_{-3}(\text{stat})^{+4}_{-2}(\text{syst})$	$1.2^{+2}_{-2}(\text{stat})^{+2}_{-1}(\text{syst})$
Bernard, Shen, and Soni [6]		$1.24(26)(\text{stat})(26)(\text{syst})$
de Rafael and Taron [32]	$\rho^2 < 1.42$	
Close and Wambach [37]	1.40	1.64
Narison [42]	1.00(2)	
Neubert [9]	0.66(5)	
Voloshin [43]	1.4(3)	
Bjorken [31]		$\rho^2 > 0.25$
Blok and Shifman [44]	$0.35 < \rho^2 < 1.15$	
Høgaasen and Sadzikowski [35]	0.98	1.135
Rosner [45]	1.59(43)	
Burdman [46]	1.08(10)	
Dai, Huang, and Jin [47]	1.05(20)	

¹⁵For readers familiar with the methods used to calculate three-point functions, the reason why the three-point functions corresponding to $(0,0,0) \rightarrow (\pi/12a,0,0)$ and $(0,0,0) \rightarrow (0,0,0)$ are strongly correlated is because they are built up from the same exponentiated propagator. Indeed, the initial momentum in our notation is the momentum of the exponentiated propagator.

and Sadzikowski [35] find a decrease in slope which is very similar to the trend we observe in the central value of ρ^2 when we include the extra parameter s in our fits. In fact, our predictions for ρ^2 itself are in excellent agreement with theirs. Their prediction is based on an improved bag model calculation and is an extension of earlier work by Sadzikowski and Zalewski [36]. A similar decrease in slope with spectator quark mass is observed by Close and Wambach [37] though the values they quote for $\rho_{u,d}^2$ and ρ_s^2 are slightly larger than the ones we find.

To test the robustness of our predictions for ρ^2 , we have explored many different procedures for obtaining $h^+(\omega)$, two of which we have already described in Sec. IV A. To obtain $h^+(\omega)$ for degenerate transitions, we have in addition tried normalizing our lattice results for $h_{\text{lat}}^+(\omega)$ by $h_{\text{lat}}^+((\pi/12a, 0, 0) \rightarrow (\pi/12a, 0, 0))$ instead of by $h_{\text{lat}}^+((0, 0, 0) \rightarrow (0, 0, 0))$ [see Eq. (38)]. When fitted to the $s\xi(\omega)$ parametrizations, the results obtained using all of these methods give very similar values for the slope parameter ρ^2 . They only differ in the value of s they predict, i.e., in their overall normalization. Thus, we are quite confident that our predictions for the slope are reliable but believe that it is important to allow for the extra normalization parameter s .

$$\frac{d\Gamma(\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu})}{d\omega} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (M_{B_{(s)}} + M_{D_{(s)}})^2 M_{D_{(s)}}^3 (\omega^2 - 1)^{3/2} \left| h^+(\omega) - \frac{M_{B_{(s)}} - M_{D_{(s)}}}{M_{B_{(s)}} + M_{D_{(s)}}} h^-(\omega) \right|^2. \quad (54)$$

So, in principle, we could obtain $|V_{cb}|$ by comparing our theoretical prediction for $d\Gamma(\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu})/d\omega$ to an experimental measurement of this rate. A major problem with this approach is that the rate $d\Gamma(\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu})/d\omega$ is helicity suppressed, as evidenced by the factor $(\omega^2 - 1)^{3/2}$, so that it is very difficult to get accurate experimental measurements close to $\omega = 1$ where the predictions of HQET are most reliable. Another problem with obtaining $|V_{cb}|$ from $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ decays is that one must know $h^-(\omega)$ to better accuracy than is given by our calculation: an error of 0.1 on h^- leads to an uncertainty of about 10% in the rate. We should

VII. EXTRACTION OF V_{cb}

In Sec. IV, we obtained $h^+(\omega)$ for a variety of $P \rightarrow P'$ transitions where $P(P')$ is a pseudoscalar meson composed of a heavy quark $Q(Q')$ and a light antiquark \bar{q} . In our study, both Q and Q' are quarks with masses around that of the charm quark. In Sec. V, however, we showed that our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ are independent of heavy-quark mass for masses around the charm quark mass or larger. This means, modulo the issue of power corrections at zero recoil, that our results can be used to describe not only $P \rightarrow P'$ transitions with $Q(Q') \sim c$ but $\bar{B}_q \rightarrow D_q$ decays as well, where the subscript q labels the flavor of the light antiquark. In Sec. VI, we studied the dependence of $h^+(\omega)/[1 + \beta^+(\omega)]$ on the mass of the light, spectator quark, m_q , and obtained results for $\xi_{u,d}(\omega)$ and $\xi_s(\omega)$. All of this means that our result for $\xi_{u,d}$, once multiplied by $(1 + \beta_{b \rightarrow c}^+)$, is in fact the form factor h^+ relevant for $\bar{B}_{u,d} \rightarrow D_{u,d}$ transitions, while $(1 + \beta_{b \rightarrow c}^+) \xi_s$ is the form factor h^+ relevant for $\bar{B}_s \rightarrow D_s$ transitions.

Now, the differential decay width for $\bar{B} \rightarrow D l \bar{\nu}$ is, in the limit of zero lepton mass [9],

mention, however, that Neubert [9] has estimated $h^-(\omega)$ using perturbation theory and sum rules in HQET and has found that its magnitude does not exceed 0.04 over the whole range of recoils ω . If this is true, its contribution to the rate of Eq. (54) should not exceed 4%.

We have not exhausted the predictions of heavy-quark symmetry. We have yet to exploit the spin component of this symmetry. Using a combination of the spin and flavor symmetry, we can relate our predictions for $\xi(\omega) \simeq h^+(\omega)/[1 + \beta^+(\omega)]$ to the form factors required to describe $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays. These form factors are defined by [9]

$$\begin{aligned} \frac{\langle D_{(s)}^*(\mathbf{p}', \epsilon) | \bar{c} \gamma^\mu b | \bar{B}_{(s)}(\mathbf{p}) \rangle}{\sqrt{M_{B_{(s)}} M_{D_{(s)}^*}}} &= i h^V(\omega) \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* v'_\alpha v_\beta, \\ \frac{\langle D_{(s)}^*(\mathbf{p}', \epsilon) | \bar{c} \gamma^\mu \gamma^5 b | \bar{B}_{(s)}(\mathbf{p}) \rangle}{\sqrt{M_{B_{(s)}} M_{D_{(s)}^*}}} &= (\omega + 1) \epsilon^{*\mu} h^{A_1}(\omega) - \epsilon^* \cdot v (v^\mu h^{A_2}(\omega) + v'^\mu h^{A_3}(\omega)), \end{aligned} \quad (55)$$

where $v = p/M_{B_{(s)}}$ and $v' = p'/M_{D_{(s)}^*}$. In the heavy-quark limit, these four form factors can be expressed in terms of the single Isgur-Wise function, $\xi_{u,d(s)}$. There are, of course, radiative and power corrections to these heavy-quark symmetry predictions. Thus, one has

$$h^i(\omega) = [\alpha^i + \beta^i(\omega) + \gamma^i(\omega)] \xi_{u,d(s)}(\omega), \quad (56)$$

with $i = V, A_1, A_2, A_3$ and

$$\begin{aligned} \alpha^{A_1} &= \alpha^{A_3} = \alpha^V = 1, \\ \alpha^{A_2} &= 0. \end{aligned} \quad (57)$$

Luke's theorem [10] further guarantees that, at zero recoil, h^{A_1} is free of $O(\epsilon_b, \epsilon_c)$ corrections, i.e., $\gamma^{A_1}(1) =$

$O(\epsilon_b^2, \epsilon_c^2)$. Because h^{A_1} is the only form factor to contribute to the differential decay rate for $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays at zero recoil, Luke's theorem implies that the leading nonperturbative corrections to this rate must be small at $\omega = 1$. More precisely, in the limit of zero lepton mass,

$$\frac{d\Gamma(\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu})}{d\omega} = \frac{G_F^2}{48\pi^3} M_{D_{(s)}^*}^3 (M_{B_{(s)}} - M_{D_{(s)}^*})^2 [1 + \beta^{A_1}(1)]^2 \sqrt{\omega^2 - 1} (\omega + 1)^2 |V_{cb}|^2 \xi_{u,d(s)}^2(\omega) \times \left[1 + 4 \left(\frac{\omega}{\omega + 1} \right) \frac{M_{\bar{B}_{(s)}}^2 - 2\omega M_{B_{(s)}} M_{D_{(s)}^*} + M_{D_{(s)}^*}^2}{(M_{B_{(s)}} - M_{D_{(s)}^*})^2} \right] K(\omega), \quad (58)$$

where $\beta^{A_1}(1) = -0.01$ [7] and $K(\omega)$ incorporates the radiative corrections, $\beta_{A_1}(\omega)$, away from $\omega = 1$, the nonperturbative power corrections, $\gamma^{A_1}(\omega)$, and the contributions of the three other form factors to the rate. From what we have said above, it should be clear that $K(1) = 1 + O(\epsilon_b^2, \epsilon_c^2)$. One can also show [9] that in the limit of exact heavy-quark symmetry, $K(\omega) = 1$ for all ω . Moreover, since we have factored out $\xi_{u,d(s)}(\omega)$ in the expression for the rate, the contributions of the three other form factors will be normalized by $\xi_{u,d(s)}$. This means that $K(\omega)$ is a collection of radiative and power corrections [see Eqs. (56) and (57)], many of which are kinematically suppressed: deviations of $K(\omega)$ from 1 ought to remain small. Neubert estimates [9], using perturbation theory and sum rules in HQET, that $K(\omega)$ may reduce the slope parameter, ρ^2 by 0.09 which corresponds to an enhancement of the rate by about 10% at maximum recoil and by a smaller amount for smaller ω . However, we cannot estimate yet how the physical $K(\omega)$ deviates from its values in the heavy-quark limit from our lattice calculation. For that we need to study $Q\bar{q}(0^-) \rightarrow Q'\bar{q}(1^-)$ decays, which we are currently analyzing. We also need to determine the $1/m_c^2$ corrections to $h^{A_1}(1)$, which as discussed in Sec. III C, we cannot get with a procedure analogous to the one presented in this paper. Hence, we will assume that $K(\omega) = K(1)$ for all ω which is a reasonable assumption given the size of our errors on the slope parameter ρ^2 . We can then use our lattice determination of $\xi_{u,d}(\omega)$ to extract V_{cb} from the experimentally measured differential decay rate for $\bar{B} \rightarrow D^* l \bar{\nu}$ ($\bar{B}_s \rightarrow D_s^* l \bar{\nu}$ has not yet been measured). This analysis differs from Neubert's extraction of $|V_{cb}|$ [15] in that we fix the ω dependence of the differential decay rate using our calculation instead of fitting it from experiment. This enables us not only to extract $|V_{cb}|$ with no free parameters, but also to check the validity of nonperturbative QCD against experiment. We find that the ω dependence predicted by our calculation agrees very well with the results of the ALEPH [16] and CLEO [17] collaborations.

In Figs. 9–11 we show least χ^2 fits to experimental data for $|V_{cb}|[1 + \beta^{A_1}(1)]K(\omega)\xi_{u,d}(\omega)$ from ALEPH [16], ARGUS [34], and CLEO [17], respectively. The only parameter is $|V_{cb}|$. The slope of the Isgur-Wise function is constrained to the value given by our lattice calculation [see Eq. (51)] and the functional form for the Isgur-Wise

function that is used is ξ_{NR} of Eq. (42).¹⁶ The results of these fits are summarized in Table XVII. Our results favor ALEPH and CLEO data over that of ARGUS. Using the data from CLEO, for instance, we find

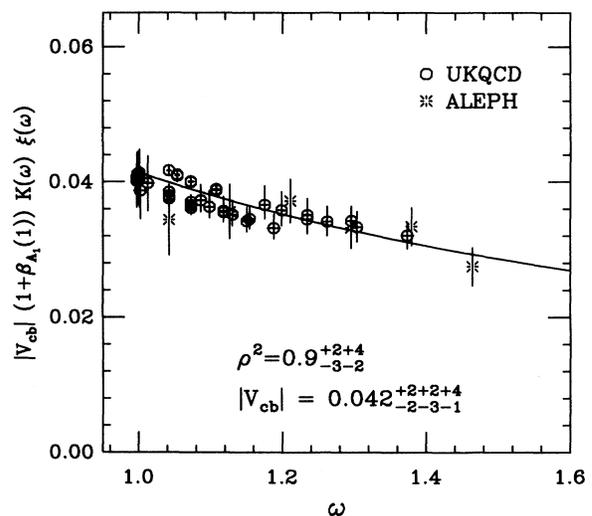


FIG. 9. Least χ^2 fit to experimental data for $|V_{cb}|[1 + \beta^{A_1}(1)]K(\omega)\xi(\omega)$ from ALEPH [16] assuming $K(\omega) = K(1)$. In this fit, the slope of the Isgur-Wise function is constrained to the value given by our lattice calculation [see Eq. (51)] and the functional form for the Isgur-Wise function that is used is ξ_{NR} of Eq. (42). The first set of errors on $|V_{cb}|$ is due to experimental uncertainties, the second set of errors results from the lattice statistical errors on ρ^2 , and the third, from the lattice systematic errors on ρ^2 . The experimental points were obtained from a measurement of the rate $d\Gamma(\bar{B} \rightarrow D^* l \bar{\nu})/d\omega$. Also shown are our appropriately scaled, chirally extrapolated results (octagons).

¹⁶As can be seen from Fig. 7 other parametrizations give very similar curves when fit to our results for $\xi_{u,d}$. Therefore, results for $|V_{cb}|$ obtained with these other parametrizations will be well within our quoted error bars.

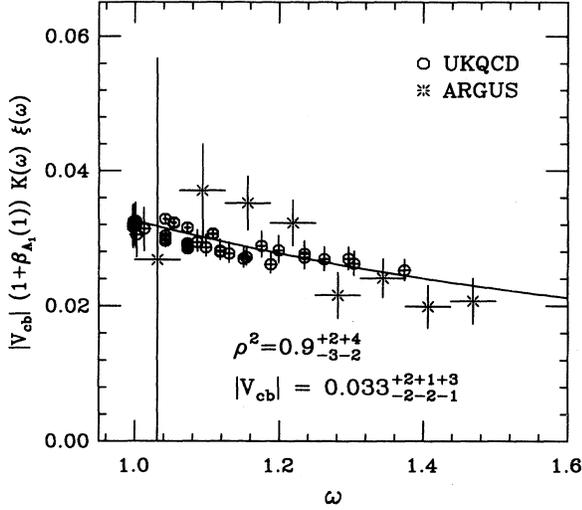


FIG. 10. Same fit as in Fig. 9 but for experimental data from the ARGUS Collaboration [34].

$$|V_{cb}| = 0.037^{+1+2+4}_{-1-2-1} \left(\frac{0.99}{1 + \beta^{A_1}(1)} \right) \frac{1}{1 + \delta_{1/m_c^2}}, \quad (59)$$

where δ_{1/m_c^2} are the power corrections proportional to $1/m_c^2$ in $K(1)$ which have been the subject of much controversy of late [33,38].

For comparison, we present recent experimental predictions for $|V_{cb}|K(1)$ obtained from a linear fit to the data¹⁷

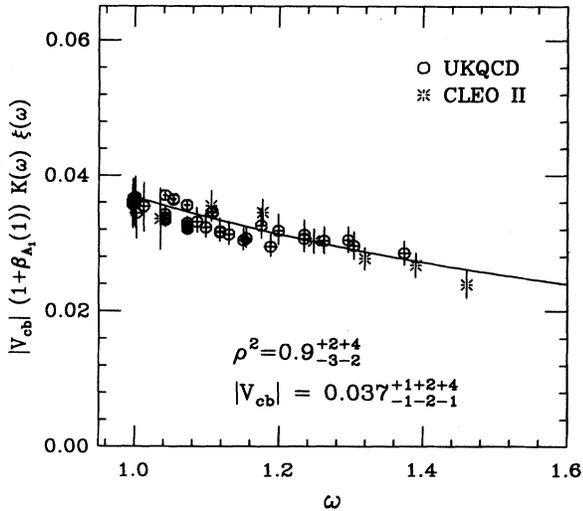


FIG. 11. Same fit as in Fig. 9 but for experimental data from the CLEO Collaboration [17].

¹⁷The ARGUS result has been corrected for the new D branching fractions [39].

$$|V_{cb}|K(1) \left(\frac{1 + \beta^{A_1}(1)}{0.99} \right) = \begin{cases} 0.0351 \pm 0.0019 \pm 0.0020 & \text{CLEO [17],} \\ 0.0385 \pm 0.0044 \pm 0.0035 & \text{ALEPH [16],} \\ 0.0392 \pm 0.0043 \pm 0.0025 & \text{ARGUS [34],} \end{cases} \quad (60)$$

where the first error is statistical and the second systematic. These results have been rescaled by Neubert [33] using the new lifetime values $\tau_{B^0} = 1.61(8)$ ps and $\tau_{B^+} = 1.65(7)$ ps [40]. These new lifetimes reduce $|V_{cb}|[1 + \beta^{A_1}(1)]K(1)$ by approximately 1%. Our results compare very well with these experimental measurements, especially in the case of the CLEO result. This is due to the fact that our Isgur-Wise function has an ω dependence which agrees very well with that of the CLEO data.

VIII. EXCLUSIVE DECAY RATES

Having determined the Isgur-Wise functions $\xi_{u,d}$ and ξ_s , we can evaluate exclusive branching ratios. For $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ decays, all we have to do is integrate Eq. (54) and multiply the results by the $\bar{B}_{(s)}$ meson lifetime. We approximate h^+ in Eq. (54) by $(1 + \beta^+) \xi_{NR}$ with ρ^2 given by Eq. (51) or Eq. (52) depending on whether the light antiquark is a \bar{u} , \bar{d} , or an \bar{s} (see Sec. VII). We neglect the $O(\epsilon_b, \epsilon_c)$ power corrections to h^+ since they appear to be small (see Sec. V) but add a 10% error to account for possible higher-order power corrections. We further neglect the contribution of h^- in accordance with Neubert's findings that this form factor is smaller than 0.04 over the whole range of ω [see discussion after Eq. (54)] but add 20% to our errors since an $|h^-(\omega)| \sim 0.2\xi(\omega)$ is consistent with radiative corrections and order of magnitude estimates of power corrections.

The branching ratios for $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays are equally simple to obtain. Here it is Eq. (58) that we must integrate over the range $1 \leq \omega \leq (M_{B_{(s)}}^2 + M_{D_{(s)}^*}^2)/2M_{B_{(s)}}M_{D_{(s)}^*}$. The Isgur-Wise functions used are the same as for $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ decays. As discussed after Eq. (58), we assume $K(\omega) = K(1)$. We further assume $K(\omega) = 1$ which leads to an uncertainty of the order of $O(2\epsilon_c^2) \sim 5 - 10\%$ in the branching ratios.

We summarize our results for $\mathcal{B}(\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu})$ and $\mathcal{B}(\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu})$ in Table XVIII. The first set of errors is obtained by adding our lattice statistical and systematic errors in quadrature. The second set of errors corresponds to the uncertainty due to deviations from the heavy-quark limit. For comparison, we list the experimentally measured values for these branching ratios. Agreement with our predictions is very satisfactory.

Finally, we give a prediction for the ratio of the rates for $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ and for $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$. In this ratio, the factors of $|V_{cb}|$ cancel and lifetimes do not appear. This ratio is thus a purely theoretical prediction. We find

TABLE XVII. Results for $|V_{cb}|$ from a fit of $|V_{cb}||[1 + \beta^{A_1}(1)]K(\omega)\xi_{\text{NR}}(\omega)$ to experimental data with $\xi_{\text{NR}}(\omega)$ fixed by our lattice computation [i.e., ρ^2 is given by Eq. (51)] and $K(\omega) = K(1)$. The experimental data are obtained from the differential branching ratio for $\bar{B} \rightarrow D^*l\bar{\nu}$ decays. In the $|V_{cb}|$ column, the first set of errors is due to experimental uncertainties, the second set of errors results from the lattice statistical errors on ρ^2 , and the third, from the lattice systematic errors on ρ^2 .

Experiment	$ V_{cb} \left(\frac{1+\beta^{A_1}(1)}{0.99} \right) (1 + \delta_{1/m_c^2})$	χ^2/N_{DF}
ALEPH	$0.042(2)_{-3-1}^{+2+4}$	3.0/5
ARGUS	$0.033(2)_{-2-1}^{+1+3}$	9.9/7
CLEO II	$0.037(1)_{-2-1}^{+2+4}$	4.5/6

$$\frac{\Gamma(\bar{B} \rightarrow D^*l\bar{\nu})}{\Gamma(\bar{B} \rightarrow Dl\bar{\nu})} = 3.2_{-2}^{+3}(\text{lat}) \pm 1.0(\text{hqs}) \quad (61)$$

and

$$\frac{\Gamma(\bar{B}_s \rightarrow D_s^*l\bar{\nu})}{\Gamma(\bar{B}_s \rightarrow D_s l\bar{\nu})} = 3.3_{-1}^{+2}(\text{lat}) \pm 1.0(\text{hqs}) \quad (62)$$

where the first set of errors was obtained by adding our lattice statistical and systematic errors in quadrature and the second set of errors, denoted by hqs, quantifies the uncertainty due to neglected power and radiative corrections. For comparison, the experimental result for $\Gamma(\bar{B} \rightarrow D^*l\bar{\nu})/\Gamma(\bar{B} \rightarrow Dl\bar{\nu})$ is 2.1(1) [41]. Though low compared to our prediction, this result is consistent with ours within errors.

IX. CONCLUSIONS

We have presented an extensive study of semileptonic $\bar{B}_{(s)} \rightarrow D_{(s)}l\bar{\nu}$ decays where we evaluate the matrix element, $\langle D|\bar{c}\gamma^\mu b|\bar{B}\rangle$, for many different values of m_b and m_c around the physical charm mass and three values of the light antiquark mass around that of the strange. Because the charm quark has a bare mass which is almost $\frac{1}{3}$ the inverse lattice spacing, mass-dependent discretization errors are a problem that we must contend with. To reduce these errors we use an $O(a)$ -improved quark action in which the leading such errors are no longer $O(am_Q)$ but rather $O(\alpha_s am_Q, (am_Q)^2)$. This reduces discretization errors from $O(40\%)$ to $O(5-15\%)$ at the mass of the charm. To reduce them even further we describe, in Sec. III C, a procedure for subtracting them at least partially. Only those discretization errors which have the

same dependence on ω as $h^+(\omega)$ will be fully subtracted. We believe, however, that the observation in Sec. V of $h^+(\omega)$'s lack of dependence on heavy-quark mass indicates that a fairly large proportion of discretization errors are eliminated with our procedure.

The fact that we obtain $h^+(\omega)$ and $h^-(\omega)$ for many values of the initial and final heavy-quark masses enables us to study their heavy-quark-mass dependence. We find that the residual dependence of $h^+/[1 + \beta^+(\omega)]$ on the heavy-quark mass is consistent with zero. Given our errors, we conclude that power corrections to the form factor h^+ for physical $\bar{B} \rightarrow D$ transitions are less than 10%. This is much smaller than the 25% corrections one is entitled to expect for form factors not protected by Luke's theorem. It is also in stark contrast with our findings for the decay constant, f_D , of the pseudoscalar D meson [12]. In [12], we find that the $O(\epsilon_c)$ corrections to the heavy-quark limit prediction for this constant are on the order of 30%. Thus, it appears that the protection from $O(\Lambda_{\text{QCD}}/m_c)$ effects that Luke's theorem provides at zero recoil extends to some extent over the full range of experimentally accessible ω . Our results for $h^+(\omega)/[1 + \beta^+(\omega)]$ are then, to a good approximation, the corresponding Isgur-Wise function.

Having obtained the Isgur-Wise function from $h^+(\omega)/[1 + \beta^+(\omega)]$ for three values of the mass of the light, spectator antiquark, we can study its dependence on light-quark mass. Interpolating the light antiquark to the strange, we obtain an Isgur-Wise function relevant for $\bar{B}_s \rightarrow D_s^{(*)}l\bar{\nu}$ decays which has a slope $-\xi'_s = 1.2_{-2}^{+2}(\text{stat})_{-1}^{+2}(\text{syst})$ at zero recoil when fit to a parametrization proposed by Neubert and Rieckert [30]. Extrapolating to a massless light antiquark yields an Isgur-Wise function relevant for $\bar{B} \rightarrow D^{(*)}l\bar{\nu}$ decays.

TABLE XVIII. Our predictions for various branching ratios compared to the experimentally measured values for these ratios. Our results are obtained assuming $|V_{cb}| = 0.038$ [41] $\tau_{B^0} = 1.53$ ps, $\tau_{B^0} = 1.54$ ps [48], $M_{B^0} = 5.28$ GeV, $M_{B^0} = 5.38$ GeV, $M_{D^+} = 1.87$ GeV, $M_{D^+} = 1.97$ GeV, $M_{D^{*+}} = 2.01$ GeV, and $M_{D^{*+}} = 2.11$ GeV [49]. Our errors are explained in the text. We only consider here semileptonic \bar{B}^0 and \bar{B}_s^0 decays because the experimental data for charged B meson decays are much less precise. The quoted experimental numbers were taken from [41].

	$\bar{B} \rightarrow Dl\bar{\nu}$	$\bar{B}_s \rightarrow D_s l\bar{\nu}$	$\bar{B} \rightarrow D^*l\bar{\nu}$	$\bar{B}_s \rightarrow D_s^*l\bar{\nu}$
UKQCD	$1.5_{-4}^{+4} \pm 0.3$	$1.3_{-2}^{+2} \pm 0.3$	$4.8_{-9}^{+8} \pm 0.5$	$4.4_{-5}^{+4} \pm 0.4$
ARGUS	$2.1 \pm 0.7 \pm 0.6$		$4.7 \pm 0.6 \pm 0.6$	
CLEO I	$1.8 \pm 0.6 \pm 0.3$		$4.1 \pm 0.5 \pm 0.7$	
CLEO II			$4.50 \pm 0.44 \pm 0.44$	

This function has a slope $-\xi'_{u,d} = 0.9_{-3}^{+2}(\text{stat})_{-2}^{+4}(\text{syst})$ at zero recoil. We observe a slight decrease in the magnitude of the central value of the slope as the mass of the light antiquark is reduced in accordance with one's understanding that more massive degrees of freedom have more inertia. Given the errors, however, the significance of this observation is limited.

We also use these functions, in conjunction with heavy-quark effective theory, to extract V_{cb} from the experimentally measured $\bar{B} \rightarrow D^* l \bar{\nu}$ decay rate. Our procedure for extracting $|V_{cb}|$ differs from that proposed by Neubert [15] in that we fix the ω dependence of the differential decay rate using our calculation instead of fitting it from experiment. This enables us not only to extract $|V_{cb}|$ with no free parameters, but also to check the validity of non-perturbative QCD against experiment. We find that the ω dependence predicted by our calculation agrees very well with the results of the ALEPH [16] and CLEO [17] collaborations. Using the data from CLEO, for instance, we find

$$|V_{cb}| = 0.037_{-1-2-1}^{+1+2+4} \left(\frac{0.99}{1 + \beta^{A_1}(1)} \right) \frac{1}{1 + \delta_{1/m_c^2}},$$

where δ_{1/m_c^2} is the power correction proportional to $1/m_c^2$ at zero recoil and $\beta^{A_1}(1)$, the relevant radiative correction. Here, the first set of errors is due to experimental uncertainties, the second due to statistical errors and the third to systematic errors in our evaluation of the Isgur-Wise function. We also use our Isgur-Wise functions and heavy-quark effective theory to calculate branching ratios for $\bar{B}_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ and $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays. Agreement with experiment is very good. Finally, we compute the following ratios of rates:

$$\frac{\Gamma(\bar{B} \rightarrow D^* l \bar{\nu})}{\Gamma(\bar{B} \rightarrow D l \bar{\nu})} = 3.2_{-2}^{+3}(\text{lat}) \pm 1.0(\text{hqs}) \quad (63)$$

and

$$\frac{\Gamma(\bar{B}_s \rightarrow D_s^* l \bar{\nu})}{\Gamma(\bar{B}_s \rightarrow D_s l \bar{\nu})} = 3.3_{-1}^{+2}(\text{lat}) \pm 1.0(\text{hqs}), \quad (64)$$

where the first set of errors was obtained by adding our lattice statistical and systematic errors in quadrature and the second set of errors, denoted by hqs, quantifies the

uncertainty due to neglected power and radiative corrections. In these ratios, the factors of $|V_{cb}|$ cancel and B -meson lifetimes are absent: they are purely theoretical predictions.

We are currently extending our study to the matrix elements relevant for $\bar{B}_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}$ decays. This will enable us not only to check our predictions for the various Isgur-Wise functions but also to test the heavy-quark spin symmetry. We are also undertaking a study of semileptonic $\Lambda_b \rightarrow \Lambda_c$ and $\Xi_b \rightarrow \Xi_c$ decays, where the $\Lambda_{b(c)}$ is a $J^P = \frac{1}{2}^+$ baryon composed of a $b(c)$ quark and two light quarks coupled to spin and isospin 0 and the $\Xi_{b(c)}$, another $J^P = \frac{1}{2}^+$ baryon composed of a $b(c)$ quark and two light quarks but this time with spin 0, isospin $\frac{1}{2}$, and strangeness -1 . That study should provide many interesting phenomenological predictions which are at the limit of current experimental knowledge as well as many tests of the heavy-quark symmetry. Finally, we are planning to repeat these studies on lattices with different lattice spacing in order to remove discretization errors in a more systematic way.

ACKNOWLEDGMENTS

L.L. would like to thank Southampton University's theory group for its hospitality and financial support during much of this project. We would also like to thank G. Martinelli and members of the APE Collaboration for stimulating discussions. This research was supported by the UK Science and Engineering Research Council under Grant Nos. GR/G 32779 and GR/J 21347, by the European Union under Contract No. CHRX-CT92-0051, by the University of Edinburgh, and by Meiko Limited. We are grateful to Edinburgh University Computing Service and, in particular, to Mike Brown for maintaining service on the Meiko i860 Computing Surface. C.T.S., D.G.R., and N.M.H. acknowledge the support of the Particle Physics and Astronomy Research Council. J.N. acknowledges the European Union for financial support through Contract No. CHBICT920066. Centre de Physique Théorique is Unité Propre de Recherche 7061.

[1] For a comprehensive review of B decays see, for example, *B Decays*, edited by S. Stone, 2nd ed. (World Scientific, Singapore, 1994).
 [2] S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987); M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); **47**, 511 (1988).
 [3] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
 [4] H. Georgi, Phys. Lett. B **240**, 447 (1990).
 [5] UKQCD Collaboration, S. P. Booth *et al.*, Phys. Rev. Lett. **72**, 462 (1994).
 [6] C. W. Bernard, Y. Shen, and A. Soni, Phys. Lett. B **317**, 164 (1993).

[7] M. Neubert, Phys. Rev. D **46**, 2212 (1992).
 [8] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).
 [9] M. Neubert, Phys. Rep. **245**, 259 (1994).
 [10] M. E. Luke, Phys. Lett. B **252**, 447 (1990).
 [11] UKQCD Collaboration, C. R. Allton *et al.*, Nucl. Phys. **B407**, 331 (1993).
 [12] UKQCD Collaboration, R. M. Baxter *et al.*, Phys. Rev. D **49**, 1594 (1994).
 [13] B. Sheikholeslami and R. Wohlert, Nucl. Phys. **B259**, 572 (1985).
 [14] G. Heatlie, G. Martinelli, C. Pittori, G. C. Rossi, and C. T. Sachrajda, Nucl. Phys. **B352**, 266 (1991).

- [15] M. Neubert, Phys. Lett. B **264**, 455 (1991).
- [16] ALEPH Collaboration, I. Scott, in *Proceedings of the 27th International Conference on High Energy Physics*, Glasgow, Scotland, 1994, edited by P. J. Bussey and I. G. Knowles (IOP, London, 1995).
- [17] CLEO Collaboration, T. Browder, in *Proceedings of the 27th International Conference on High Energy Physics* [16]; CLEO Collaboration, B. Barish *et al.*, Phys. Rev. D **51**, 1014 (1995).
- [18] UKQCD Collaboration, C. R. Allton *et al.*, Phys. Rev. D **47**, 5128 (1993).
- [19] C. W. Bernard and A. Soni, in *Quantum Fields on the Computer*, edited by M. Creutz (World Scientific, Singapore, 1992); A. S. Kronfeld and P. B. Mackenzie, Ann. Rev. Nucl. Part. Sci. **43**, 793 (1993).
- [20] G. Martinelli, in *Lattice '88*, Proceedings of the International Symposium, Batavia, Illinois, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 139 (1989)].
- [21] B. Efron, SIAM Rev. **21**, 460 (1979).
- [22] M. Beneke and V. M. Braun, Nucl. Phys. **B426**, 301 (1994).
- [23] I. Bigi *et al.*, Phys. Lett. B **339**, 160 (1994).
- [24] UKQCD Collaboration, D. S. Henty, R. D. Kenway, B. J. Pendleton, and J. I. Skullerud, Phys. Rev. D **51**, 5323 (1995).
- [25] A. Borrelli, R. Frezotti, E. Gabrielli, and C. Pittori, Nucl. Phys. **B409**, 382 (1993).
- [26] G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **48**, 2250 (1993).
- [27] UKQCD Collaboration, D. Henty, in *Lattice '93*, Proceedings of the International Symposium, Dallas, Texas, edited by T. Draper *et al.* [Nucl. Phys. B (Proc. Suppl.) **34**, 468 (1994)].
- [28] G. P. Lepage, in *Lattice '91*, Proceedings of the International Symposium, Tsukuba, Japan, edited by M. Fukugita *et al.* [Nucl. Phys. B (Proc. Suppl.) **26**, 45 (1992)]; G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **48**, 2250 (1993); A. S. Kronfeld, in *Lattice '92*, Proceedings of the International Symposium, Amsterdam, The Netherlands, edited by J. Smit and P. van Baal [Nucl. Phys. B (Proc. Suppl.) **30**, 445 (1993)]; P. Mackenzie, in *The Fermilab Meeting*, Proceedings of the Meeting of the Division of Particles and Fields of the APS, Batavia, Illinois, 1992, edited by C. Albright *et al.* (World Scientific, Singapore, 1993); in *Lattice '92* (this reference), p. 35.
- [29] UKQCD Collaboration, C. R. Allton *et al.*, Phys. Rev. D **49**, 474 (1994).
- [30] M. Neubert and V. Rieckert, Nucl. Phys. **B382**, 97 (1992).
- [31] J. D. Bjorken, in *Results and Perspectives in Particle Physics*, Proceedings of the 4th Rencontres de Physique de la Vallée D'Aoste, Lg Theiele, Italy, 1990, edited by M. Greco (Editions Frontieres, Gif-Sur-ivette, France, 1990), p. 583.
- [32] E. de Rafael and J. Taron, Phys. Rev. D **50**, 373 (1994).
- [33] M. Neubert, Phys. Lett. B **338**, 84 (1994).
- [34] ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **57**, 533 (1993).
- [35] H. Høgaasen and M. Sadzikowski, Z. Phys. C **64**, 427 (1994).
- [36] M. Sadzikowski and K. Zalewski, Z. Phys. C **59**, 677 (1993).
- [37] F. E. Close and A. Wambach, Phys. Lett. B **348**, 207 (1995).
- [38] M. Shifman, Minnesota Report No. TPI-MINN-94/31-T, hep-ph/9409358 (unpublished).
- [39] R. Patterson, *Proceedings of the 27th International Conference on High Energy Physics* [16].
- [40] F. DeJongh, to appear in the Proceedings of the 27th International Conference on High Energy Physics, Glasgow, Scotland, July 1994.
- [41] S. Stone, in *B Decays*, [1].
- [42] S. Narison, Phys. Lett. B **325**, 197 (1994).
- [43] M. B. Voloshin, Phys. Rev. D **46**, 3062 (1992).
- [44] B. Blok and M. Shifman, Phys. Rev. D **47**, 2949 (1993).
- [45] J. L. Rosner, Phys. Rev. D **42**, 3732 (1990).
- [46] G. Burdman, Phys. Lett. B **284**, 133 (1992).
- [47] Y. B. Dai, C. S. Huang, and H. Y. Jin, Z. Phys. C **56**, 707 (1992).
- [48] W. Venus (private communication).
- [49] Particle Data Group. K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).