PHYSICAL REVIEW D VOLUME 52, NUMBER 8 15 OCTOBER 1995

Dynamical chiral symmetry breaking, Goldstone's theorem, and the consistency of the Schwinger-Dyson and Bethe-Salpeter equations

H. J. Munczek
Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045
(Received 21 February 1995)

A proof of Goldstone's theorem is given that highlights the necessary consistency between the exact Schwinger-Dyson equation for the fermion propagator and the exact Bethe-Salpeter equation for fermion-antifermion bound states. The approach is tailored to the case when a global chiral symmetry is dynamically broken. Criteria are provided for maintaining the consistency when the exact equations are modified by approximations. In particular, for gauge theories in which partial conservation of the axial vector current (PCAC) should hold, a constraint on the approximations to the fermion–gauge-boson vertex function is discussed, and a vertex model is given which satisfies both the PCAC constraint and the vector Ward-Takahashi identity.

PACS number(s): 11.30.Qc, 11.10.St, 11.30.Rd

I. INTRODUCTION AND MAIN RESULT

A large amount of work on dynamical chiral symmetry breaking and its application to low-energy strong-interaction physics [1–4] suggests a picture of the low-mass pseudoscalar mesons as "almost" Nambu-Goldstone (NG) bosons. The NG bosons would arise due to the spontaneous breaking of the flavor chiral symmetry present in the usual quark model when the weak and electromagnetic interactions are absent. For light quarks the small masses induced by the Higgs mechanism have been included in the formalism quite successfully by partially conserved axial vector current (PCAC) algebra, operator product, and renormalization-group techniques, which further support the above picture [2].

An alternative to these methods is to calculate the properties of the mesons as quark-antiquark bound states through the use of the Bethe-Salpeter (BS) equation, which can be derived rigorously from the underlying field theory and therefore preserves its symmetries. In this context another equation that plays an essential role is the Schwinger-Dyson (SD) equation for the fermion propagator, a quantity that appears explicitly in the BS equation. Spontaneous chiral symmetry breaking is signaled by the appearance of an otherwise absent scalar term in the quark propagator. Consequently this change in the propagator should be reflected in the BS equation through the appearance of a massless pseudoscalar solution. Since the early work of Nambu and Jona-Lasinio [1], this has been found to happen explicitly in a variety of chiral invariant models when use is made of the ladder approximation *both* for the SD and BS equations [1,5]. Also, in the ladder approximation it has been found numerically [6] that when a small quark mass term is added to the SD and BS equations, the successful results of current algebra and other general techniques mentioned above are preserved.

It is obvious that the ladder approximation to the SD equation for a fermion is not entirely satisfactory. The SD equation has been the subject of extensive research, particularly of studies directed to incorporate an improved structure for the vertex function, in order for it to satisfy the vector Ward-Takahashi (WT) identity and to exhibit correct infrared and ultraviolet behaviors. These studies include the use of the gauge technique [7] as well as the use of algebraic combinations of fermion propagators [8]. In this context, a natural consistency requirement, which is addressed here, is the following: given changes in the SD equation, which takes it beyond the ladder approximation, the BS equation should change in a way that preserves the appearance of NG bosons and other dynamically broken chiral symmetry features.

A procedure to guarantee this consistency requirement is given in what follows with the help of the effective action formalism for composite operators as developed by Cornwall, Jackiw, and Tomboulis (CJT), who showed that it can be applied to the analysis of the SD equation and of dynamical symmetry breakdown [9]. The CJT method was further extended to include a variational principle yielding SD equations for vertices as well as for propagators [10]. It has also been shown that the CJT formalism is a convenient framework for the exact treatment of the S matrix for bound states and of the Bethe-Salpeter equation [11]. Variational methods for composite operators, including the CJT method, have been used extensively before [12] to generate the ladder approximation SD and BS equations and to discuss their chiral properties. Extending those results beyond the ladder approximation, the variational formalism will be applied here to give a proof of Goldstone's theorem for the specific case of dynamical chiral symmetry breaking in a theory with fermions and to discuss several applications. As in any other formalism, Goldstone's theorem is implicitly given by the symmetry properties of the system. The advantage of the explicit proof given here is that it emphasizes the common origin of the exact SD and BS equations and thus provides prescriptions to maintain the validity of the theorem and associated chiral symmetry features such as PCAC, when subjecting the exact equations and related Green's function to truncations and other approximations.

I start by considering a situation in which there is a set of fermions $\psi_a(x)$ in interaction with some other fields, such that one can construct a CJT action $\Gamma[B]$, which is a functional of a "classical" bilocal, bispinor, field $B_a^b(xy)$, with each of the labels a, b indicating spinor as well as internal symmetry indices. In what follows, the indices will occasionally be left implicit. Repeated variables and indices are assumed to be integrated or summed over. The CJT action yields [9] the exact SD equation for the fermion propagator $S_F(x,y)$:

$$\left.\frac{\delta\Gamma}{\delta B(xy)}\right|_{B=S_F} = 0 \;, \tag{1}$$

where

$$iS_F(x,y) = \langle 0|T\psi(x)\overline{\psi}(y)|0\rangle \;. \tag{1a}$$

One can also see [11,13] that the exact BS equation for a bound state of mass M described by a wave function

$$\psi_a^b(x,y,p) = \chi_a^b(x-y,p)e^{ip[x\alpha+y(1-\alpha)]} \;, \tag{2}$$

where $0 < \alpha < 1$, and $p^2 = M^2$, has the form

$$\left.\frac{\delta^2\Gamma[B]}{\delta B_a^b(xy)\delta B_{a'}^{b'}(x'y')}\right|_{B=S_F} \psi_{a'}^{b'}(x',y',p) = 0 \;. \tag{3}$$

The action is now assumed to be obtained from a formally global chiral invariant Lagrangian field theory in which the fermions transform as

$$\psi'(x) = e^{i\gamma_5\theta\tau^l}\psi(x) \;, \quad \overline{\psi}'(y) = \overline{\psi}(y)e^{i\gamma_5\theta\tau^l} \;. \tag{4}$$

The parameter θ is real, and the τ^l are Hermitian matrix representations of the generators of the flavor group. I also assume that there are no anomalies in the axial vector currents associated with the indices l in Eq. (4). Under these circumstances, inspection of the CJT action shows that

$$\Gamma[B'] = \Gamma[B] \;, \quad B'(x,y) = e^{i\gamma_5\theta\tau^l}B(x,y)e^{i\gamma_5\theta\tau^l} \;. \tag{5}$$

Under an infinitesimal transformation with $\theta = \epsilon$, we then have

$$0 = \delta_5\Gamma[B] = \frac{\epsilon\delta\Gamma}{\delta B_{a'}^{b'}(x',y')}\{i\gamma_5\tau^l, B(x',y')\}_{a'}^{b'} \;. \tag{6}$$

From Eq. (6) we also obtain

$$0 = \frac{\delta}{\delta B_a^b(xy)}[\delta_5\Gamma[B]] \;, \tag{7}$$

or, in more detail,

$$0 = \frac{\delta^2\Gamma[B]}{\delta B_a^b(x,y)\delta B_{a'}^{b'}(x'y')}\{\gamma_5\tau^l, B(x'y')\}_{a'}^{b'} + \frac{\delta\Gamma}{\delta B_{a'}^{b}(xy)}(\gamma_5\tau^l)_{a'}^{a} + \frac{\delta\Gamma}{\delta B_a^{b'}(xy)}(\gamma_5\tau^l)_b^{b'} \;. \tag{8}$$

Setting $B(xy) = S_F(x,y)$ in (8), the last two terms vanish if the SD equation (1) holds. We see then that if the vacuum is not chiral invariant, that is, if

$$\{\gamma_5\tau^l, S_F(x,y)\} = \langle 0|\{\gamma_5\tau^l, T\psi(x)\overline{\psi}(y)\}|0\rangle \neq 0 \;, \tag{9}$$

then the BS equation has a pseudoscalar solution of vanishing four-momentum, a Nambu-Goldstone boson, since Eq. (8) becomes

$$\left.\frac{\delta^2\Gamma[B]}{\delta B_a^b(xy)\delta B_{a'}^{b'}(x'y')}\right|_{B=S_F}\{\gamma_5\tau^l, S_F(x',y')\}_{a'}^{b'} = 0 \;. \tag{10}$$

The results described above are true for the exact SD and BS equations. It is clear that any approximate treatment of either equation has to be accompanied by a treatment of the other equation, which maintains the validity of Goldstone's theorem [14]. From the derivation above we see that this will happen if both the approximated SD and BS equations are derived through Eqs. (1) and (3) from the same approximated, but chiral invariant, bilocal effective action satisfying Eq. (5). As an alternative, the situation in which an explicit form for the effective action is not available is discussed in Sec. II C, where the chirally compatible BS equation is obtained directly from the SD equation.

II. FORMALISM AND APPLICATIONS

A. Basic definitions and chiral properties

I now consider the specific case of the fermions interacting with vector gauge fields A_μ. The action for such a system can be written as

$$S(\psi,\overline{\psi},A_\mu) = \int\{\overline{\psi}(i\partial\!\!\!/_x - m)\psi + \mathcal{L}(A_\mu) + \mathcal{L}_I(\psi,\overline{\psi},A_\mu)\}d^4x \;. \tag{11}$$

$\mathcal{L}(A_\mu)$ is the Lagrangian for the gauge fields and includes gauge-fixing terms and ghost fields, when present. The A_μ fields do not change under global or local chiral transformations. The fermions transform as given by Eq. (4), and the interaction Lagrangian $\mathcal{L}_I(\psi,\overline{\psi},A_\mu)$ is assumed to be invariant under those transformations.

The CJT action $\Gamma[B]$ can be obtained as follows: $Z[J]$, the generating functional for fermion Green's functions, is given by

$$Z[J] = \frac{1}{N}\int D\psi\,D\overline{\psi}\,DA_\mu \times \exp\left[iS(\psi,\overline{\psi},A_\mu) - i\int\overline{\psi}(x)J(xy)\psi(y)dx\,dy\right] , \tag{12}$$

where $Z[0] = 1$. Then, $W[J]$, the generating functional for connected fermion Green's functions, can be expressed by

$$W[J] = -i\ln Z[J] \;. \tag{13}$$

The classical bilocal field $B(x,y)$ is defined as

$$B(x,y) \equiv -i\delta W[J]/\delta J(yx) \ , \tag{14}$$

with

$$B(xy)|_{J=0} = -i\langle 0|T\psi(x)\overline{\psi}(y)|0\rangle = S_F(xy) \ . \tag{14a}$$

Finally, the effective action is constructed as

$$\Gamma[B] = W[J] - iB(xy)J(yx) \ . \tag{15}$$

From (15) we see that

$$i\delta\Gamma[B]/\delta B(xy) = J(yx) \ , \tag{15a}$$

and, because of (14a), the SD equation (1) follows.

For the system of fields analyzed here, the effective action has the form [9]

$$\Gamma[B] = -i\,\mathrm{Tr}\{(i\not\partial - m)B\} + \overline{\Gamma}[B] \ , \tag{16}$$

where $\overline{\Gamma}[B]$ is invariant under local as well as global chiral transformations. If we perform an infinitesimal transformation

$$\delta_5 B(xy) = i\gamma_5\tau^l\epsilon(x)B(xy) + B(xy)i\gamma_5\tau^l\epsilon(y) \ , \tag{17}$$

Eq. (16) yields the basic expression

$$\mathrm{Tr}\left\{\frac{\delta\Gamma}{\delta B}\delta_5 B\right\} = -i\,\mathrm{Tr}\{(i\not\partial - m)\delta_5 B\} \ . \tag{18}$$

This equation can also be obtained directly from expression (12) by performing an infinitesimal chiral change in the integration variables and by use of the definitions (13)–(15).

If the chiral transformation is global and if the mass matrix m is zero, the right-hand side in Eq. (18) vanishes. Then Goldstone's theorem follows in the manner discussed in Sec. I.

If the chiral transformation is local, and because the infinitesimal $\epsilon(x)$ is otherwise an arbitrary function of x, an integration by parts in (18) yields

$$\mathrm{tr}\int[\tau^l\gamma_5\gamma\cdot\partial_x\{B(xy)\delta(x-y)\} + i\{m,\tau^l\}\gamma_5 B(xy)\delta(x-y)]dy - \mathrm{tr}\int\left\{\frac{\delta\Gamma}{\delta B(xy)}\tau^l\gamma_5 B(xy) + \frac{\delta\Gamma}{\delta B(yx)}B(yx)\gamma_5\tau^l\right\}dy = 0 \ . \tag{19}$$

In this expression tr indicates trace over discrete indices, and there is no integration over x. Equation (19) expresses, in the framework of the effective action, the contents of the partially conserved axial vector current (PCAC) relationships among Green's functions. For instance, taking the functional derivative of (19) with respect to $B_a^b(x'y')$, setting $B = S_F$, and applying the resulting operator to a pseudoscalar solution of the BS equation results in the exact relationship

$$\mathrm{tr}[\tau^l\not p\gamma_5\chi(0,p)] = \mathrm{tr}[\{m,\tau^l\}\gamma_5\chi(0,p)] \ , \tag{20}$$

where use has been made of Eqs. (1), (2), and (3).

Equation (20) can also be obtained through the use of field operator methods and gives [2], to first order in m, the PCAC formula of Gell-Mann, Oakes, and Renner for pseudoscalar masses [15]. Since the ladder approximation SD and BS equations can be obtained from an approximated CJT effective action of the form of Eq. (16) with $\overline{\Gamma}[B]$ locally chiral invariant, these equations should give solutions satisfying PCAC conditions such as that of Eq. (20). As discussed in Sec. I, this is the case both qualitatively and quantitatively [6].

B. SD equation and fermion-antifermion–gauge-boson vertex

In Eq. (16) $\overline{\Gamma}[B]$ can be written as the sum of free and interacting parts, each one locally chiral invariant, in the form [9]

$$\overline{\Gamma}[B] = i\,\mathrm{Tr}\,\ln B + \Gamma_2[B] \ . \tag{21}$$

We then have, with implicit discrete indices,

$$\delta\Gamma/\delta B(yx) = -iS_0^{-1}(x,y) + iB^{-1}(xy) + \delta\Gamma_2/\delta B(yx) \ , \tag{22}$$

$$S_0^{-1}(x,y) = -i\not\partial_y\delta(x-y) - m\delta(x-y) \ , \tag{22a}$$

where $S_0^{-1}(x,y)$ is the free inverse propagator. The SD equation is, setting $B = S_F$,

$$-S_F^{-1}(x,y) + S_0^{-1}(x,y) + i\delta\Gamma_2/\delta B(yx)|_{B=S_F} = 0 \ . \tag{23}$$

For simplicity, one can restrict the discussion to the case in which there is only one gauge vector field. The "self-mass" term in Eq. (23) can be written in the symmetrized form

$$\Sigma(x,y) = i\left.\frac{\delta\Gamma_2}{\delta B(yx)}\right|_{B=S_F} = -i\tfrac{1}{2}g^2\int d^4z\,d^4x'\{\gamma_\mu G_{\mu\nu}(x,z)S_F(x,x')\Gamma_\nu(z;x',y) + \Gamma_\mu(z;x,x')S_F(x',y)G_{\mu\nu}(z,y)\gamma_\nu\} \ , \tag{24}$$

where both the vector boson propagator $G_{\mu\nu}$ and the vertex function Γ_ν are functionals of S_F. Since $\Gamma_2[B]$ is invariant under a local chiral transformation, Eq. (24) shows that the self-mass $\Sigma(x,y)$ and the vertex function $\Gamma_\nu(z;x,y)$ should transform as $S_F^{-1}(x,y)$, that is [16],

$$\Sigma(x,y) \to e^{-i\gamma_5\tau^l\theta(x)}\Sigma(x,y)e^{-i\gamma_5\tau^l\theta(y)} , \quad (25)$$

$$\Gamma_\nu(z;x,y) \to e^{-i\gamma_5\tau^l\theta(x)}\Gamma_\nu(z;x,y)e^{-i\gamma_5\tau^l\theta(y)} , \quad (25a)$$

under the substitution

$$S_F(x,y) \to e^{i\gamma_5\tau^l\theta(x)}S_F(x,y)e^{i\gamma_5\tau^l\theta(x)} . \quad (26)$$

This requirement is compatible with the vector Ward-Takahashi (WT) identity

$$\frac{\partial}{\partial z_\mu}\Gamma_\mu(z;x,y) = i\{\delta(y-z) - \delta(x-z)\}S_F^{-1}(x,y) . \quad (27)$$

The vertex function $\Gamma_\mu(z;x,y)$ satisfies its own SD equation, which couples it to a four-particle vertex function. Then the extended composite field action [10] should permit an analysis of the properties of the four-particle vertex similar to the one above for $\Gamma_\nu(z;x,y)$. In a simplified approach, a large number of models [3,7,8] have been discussed for $\Gamma_\nu(z;x,y)$ with the requirement that it satisfy the WT identity (27) as well as appropriate symmetry properties and renormalization requirements. Many of the models, usually presented in momentum space, are linear in $S_F^{-1}(p)$ and $S_F^{-1}(p+k)$, which appear multiplied by functions of p and $p+k$, the momenta of the fermions at the vertex. In configuration space this involves derivatives of $S_F^{-1}(x,y)$ with respect to x and y, and therefore those models generally fail to satisfy condition (25a) for $\Gamma_\mu(z;x,y)$ as well as the PCAC equation (19), since these models imply an additional explicit local chiral symmetry breaking in the effective action.

A vertex model linear in S_F^{-1} and satisfying Eqs. (23) and (25a) can be constructed as

$$[\Gamma_\mu(z;x,y)]_a^b = [F_\mu(y-z;x-z)]_{aa'}^{bb'}[S_F^{-1}(x,y)]_{b'}^{a'} , \quad (28)$$

where the matrix structure of F_μ is such that it preserves condition (25a). F_μ should also satisfy

$$\frac{\partial}{\partial z_\mu}[F_\mu(z-y;z-=z)]_{aa'}^{bb'} = i[\delta(y-z) - \delta(x-z)]\delta_a^{b'}\delta_{a'}^b . \quad (29)$$

As an example, a simple choice for F_μ is

$$F_\mu = \delta_a^{b'}\delta_{a'}^b \int [e^{iq\cdot(z-y)} - e^{iq\cdot(z-x)}]\frac{(x_\mu - y_\mu)}{q\cdot(x-y)} \times \frac{d^4q}{(2\pi)^4} + F_\mu^T , \quad (30)$$

where $\partial F_\mu^T/\partial z_\mu = 0$. In momentum space, with

$$\Gamma_\mu(z;x,y) = \frac{1}{(2\pi)^8}\int e^{-ip'\cdot(x-z)-ip\cdot(z-y)} \times\Gamma_\mu(p',p)d^4p'd^4p \quad (31)$$

and

$$S_F^{-1}(x-y) = \frac{1}{(2\pi)^4}\int e^{-ip\cdot(x-y)}S_F^{-1}(p)d^4p , \quad (32)$$

the form (28) with the choice (30) translates into [17]

$$\Gamma_\mu(p+k,p) = \frac{\partial}{\partial p_\mu}\int_0^1 S_F^{-1}(p+\alpha k)d\alpha + \Gamma_\mu^T , \quad (33)$$

which, with $k_\mu\Gamma_\mu^T = 0$, satisfies the WT identity

$$k_\mu\Gamma_\mu(p+k,p) = S_F^{-1}(p+k) - S_F^{-1}(p) . \quad (34)$$

C. BS equation

As shown in Sec. I, taking a further derivative with respect to B in Eq. (22) and then setting $B = S_F$ allows one to write the BS equation (3). Approximations to the exact action $\Gamma[B]$, which maintain the chiral symmetry properties of the system can be obtained from the loop expansion [9] of the CJT action or, alternatively, by approximating the self-mass and vertex functionals in such a way that (25) and (25a) hold. If we make the assumption that their functional dependence on B is the same as their dependence on S_F, we can define, for use in the BS equation,

$$\left.\frac{\delta^2\Gamma_2[B]}{\delta B(x_1y_1)\delta B(xy)}\right|_{B=S_F} \equiv \frac{\delta\Sigma(x,y)}{\delta S_F(x_1,y_1)} . \quad (35)$$

If an explicit form is not readily available for $\Gamma_2[B]$, the chiral properties can be probed "on shell," that is, for $B = S_F$, by using the SD equation (23), which can be written as $\delta\Gamma/\delta S_F(x,y) = 0$. We then have, for $m = 0$, the identity

$$0 = \epsilon\left\{i\gamma_5\tau^l, \frac{\delta\Gamma[S_F]}{\delta S_F(x,y)}\right\} . \quad (36)$$

Using (22), (25), (26), and (35), we see that (36) is equivalent to

$$0 = \delta_5\left\{\frac{\delta\Gamma[S_F]}{\delta S_F(x,y)}\right\} = i\epsilon\frac{\delta^2\Gamma[S_F]}{\delta S_F(x,y)\delta S_F(x_1,y_1)}\{\gamma_5\tau^l, S_F(x_1y_1)\} , \quad (37)$$

which demonstrates Goldstone's theorem. If instead of (36) we consider the identity

$$0 = i\gamma_5\tau^l\epsilon(y)\frac{\delta\Gamma[S_F]}{\delta S_F(x,y)} + \frac{\delta\Gamma[S_F]}{\delta S_F(x,y)}i\gamma_5\tau^l\epsilon(x) , \quad (38)$$

we obtain

$$i\gamma_5\tau^l[\epsilon(y)-\epsilon(x)]\not\partial_x\delta(x-y)+\delta(x-y)\epsilon(x)\{\gamma_5\tau^l,m\} + \frac{\delta^2\Gamma[S_F]}{\delta S_F(x,y)\delta S_F(x_1,y_1)}\delta_5 S_F(x_1,y_1)=0\ . \quad (39)$$

Applying this expression to a pseudoscalar solution $\chi(x-y,p)e^{ip[x\alpha+y(1-\alpha)]}$ of the BS equation, the last term vanishes, and we have again, as in Eq. (20), the PCAC relationship

$$\mathrm{tr}[\tau^l\not p\gamma_5\chi(0,p)-\gamma_5\{\tau^l,m\}\chi(0,p)]\int d^4x\,\epsilon(x)e^{ipx}=0\ . \quad (40)$$

III. CONCLUSIONS

The results discussed in Sec. II A have shown that, beyond the usual ladder approximation, the variational formalism for composite fields is a very convenient tool for the study of exact bound-state fermion-antifermion equations and their chiral symmetries, both global and local. As shown there, the very close connection between the exact SD and BS equations allows a straightforward proof of Goldstone's theorem and the study of the effects of the explicit breaking of chiral symmetry (PCAC). In addition, the discussion provides constructive procedures to maintain the chiral consistency of approximated SD and BS equations.

The discussion in Sec. II B leads to the PCAC constraint Eq. (25) on the local chiral properties of the vertex function, namely, that it transforms as the inverse fermion propagator. It is also shown how the constraint can be implemented, along with the WT identity, when modeling the vertex function. The constraint is present in quantum electrodynamics and chromodynamics when studying spontaneous chiral symmetry breaking and its dependence on the coupling constant. The use, in this context, of vertex models that do not satisfy the PCAC constraint is therefore open to question.

Finally, it was shown in Sec. II C that, even in the absence of an explicit expression for $\Gamma[B]$, if a satisfactory model is given for the vertex function and therefore for the self-mass function, then on-shell approximated functionals $\delta\Gamma/\delta S$ and $\delta^2\Gamma/\delta S\delta S$ can be defined. The functionals can then be used to give SD and BS equations, which are chirally compatible, yielding Goldstone's theorem as well as the consequences of PCAC discussed in Sec. II A.

ACKNOWLEDGMENTS

The author thanks P. Jain, D. W. McKay, S. Pokorski, and J. P Ralston for useful comments. The work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-85-ER40214.

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); M. Baker, K. Johnson, and B. W. Lee, *ibid.* **133**, B209 (1964); H. Pagels, Phys. Rev. D **7**, 3689 (1973); R. Jackiw and K. Johnson, *ibid.* **8**, 2386 (1973); J. M. Cornwall and R. E. Norton, *ibid.* **8**, 3338 (1973); E. J. Eichten and F. L. Feinberg, *ibid.* **10**, 3254 (1974). More recent references and some well-known definitions, equations, and identities mentioned in the present paper can be found in [2], [3], and [4] below.

[2] V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993).

[3] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).

[4] S. Pokorski, *Gauge Field Theories* (Cambridge University Press, Cambridge, Great Britain, 1987).

[5] R. Delbourgo and M. D. Scadron, J. Phys. G **5**, 1621 (1979).

[6] P. Jain and H. J. Munczek, Phys. Rev. D **46**, 438 (1992).

[7] R. Delbourgo and A. Salam, Phys. Rev. **125**, 1398 (1964); R. Delbourgo and P. West, J. Phys. A **10**, 1049 (1977); Phys. Lett. **72B**, 96 (1977); R. Delbourgo, Nuovo Cimento **49A**, 484 (1979); J. E. King, Phys. Rev. D **27**, 1821 (1983); P. Rembiesa, *ibid.* **33**, 2333 (1986); B. Haeri, Jr., *ibid.* **43**, 2701 (1991).

[8] J. S. Ball and T. W. Chiu, Phys. Rev. D **22**, 2542 (1980); D. C. Curtis and M. R. Pennington, *ibid.* **42**, 4165 (1990); D. Atkinson, J. C. R. Bloch, V. P. Gusynin, M. R. Pennington, and M. Reenders, Phys. Lett. B **329**, 117 (1994); Z. Dong, H. J. Munczek, and C. D. Roberts, *ibid.* **333**, 536 (1994); A. Bashir and M. R. Pennington, Phys. Rev. D **50**, 7679 (1994).

[9] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[10] R. E. Norton and J. M. Cornwall, Ann. Phys. (N.Y.) **91**, 106 (1975).

[11] D. W. McKay and H. J. Munczek, Phys. Rev. D **40**, 4151 (1989).

[12] R. W. Haymaker, Riv. Nuovo Cimento **14**, 1 (1991).

[13] R. Fukuda, Prog. Theor. Phys. **87**, 1487 (1987).

[14] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 962 (1962).

[15] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[16] Since $S_F^{-1}(x,x')S_F(x',y)=\delta(x-y)$, we have that

$$\begin{aligned}\delta_5 S_F^{-1}(x,y) &= [\delta S_F^{-1}(x,y)/\delta S_F(x',y')]\delta_5 S_F(x',y')\\ &= -S_F^{-1}(x,x')\delta_5 S_F(x',y')S_F^{-1}(y',y)\\ &= -\{i\gamma_5\tau^l\epsilon(x)S_F^{-1}(x,y)+S_F^{-1}(x,y)i\gamma_5\tau^l\epsilon(y)\},\end{aligned}$$

which shows that

$$S_F^{-1}(x,y)\to e^{-i\gamma_5\tau^l\theta(x)}S_F^{-1}(x,y)e^{-i\gamma_5\tau^l\theta(y)}.$$

[17] H. J. Munczek, Phys. Lett. B **175**, 215 (1986). In this reference an expression identical to Eq. (33), but with $S_F^{-1}(p+\alpha k)$ replaced by $-S_F(p+\alpha k)$, was given for the dressed vertex $\Lambda_\mu(p+k,p)=S_F(p+k)\Gamma_\mu(p+k,p)S_F(p)$.