Cubic scattering amplitudes for all massless representations of the Poincaré group in any space-time dimension

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Using the language of highest weight representations and the light cone formalism we construct a full list of cubic amplitudes of scattering for all bosonic massless representations of the Poincaré group in any even space-time dimension.

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The theory of massless higher spin fields is one of the promising branches of modern quantum field theory (see [1,2]). At present because of Refs. [3–6], a wide class of cubic interaction vertices for massless higher spin fields in flat space is known. These vertices turn out to be nonminimal for the case of interactions of higher spin fields with gravity. Cubic vertices for massless higher spin fields living in anti–de Sitter space-time were constructed in [7]. References [3–7] were devoted to the analysis of interactions in $D = 4$. A full list of cubic interaction vertices in the case of $D = 4$ flat space was given in [6]. In this paper we continue the investigation of the problem introducing the interaction for massless higher spin fields in higher space-time dimensions ($D > 4$) which was initiated in [8]. In Ref. [8] a full list of cubic interaction vertices for totally symmetric massless on-mass-shell representations of the Poincaré group for any D was obtained. Notice the results of Ref. [8] as well as of this paper do not contradict those of Refs. [9–12] because in our vertices for the case of interactions of higher spins with a graviton there are higher derivatives only. As is well known various types of symmetry properties of massless representations classify according to unitary irreps of the transversal $SO(D-2)$ group; i.e., in addition to the totally symmetric representations there are mixed symmetry ones [13–19]. It is likely that in studies of the self-consistency of higher (fourth,...) order interactions the former representations should be taken into consideration. We think that the higher order interaction vertices will be effectively nonlocal. The aim of this paper is to construct cubic interaction vertices for all massless representations of the Poincaré group in any even space-time dimension $D = 2n + 2$. To solve the problem it has been suggested [20] that the language of highest weight representations of the $SO(2n)$ group be used, and it was demonstrated how the procedure of construction of cubic interaction vertices works in the simplest cases $D = 5, 6$. The language of highest weight turns out to be extremely efficient for the analysis of cubic interaction vertices of any symmetry representations. Of course, use of this language leads to breaking of the manifest $SO(2n)$ transversal invariance. It is the price we pay in solving the problem under consideration. Efficiency of the different way of breaking of the manifest transversal invariance $[SO(8) \to SO(6) \times SO(2)]$ has been demonstrated in Ref. [21] by constructing the cubic vertices in the $D = 10$ superstring theory.

Before we proceed let us comment on the self-consistency of higher (fourth, ...) order interaction vertices. It is known (see Ref. [4]) that cubic vertices do not guarantee self-consistency of fourth order interactions. However due to Ref. [22] the possibilities for constructing consistent higher order vertices increased. Namely, in [22] for the case of massless higher spin fields living in $D = 4$ anti–de Sitter space-time the following remarkable fact was discovered: in order to solve the problem of higher order vertices it is necessary to use the spectrum of massless particles that contains every massless representation just once. Taking that into account one can suppose that the problem of higher order vertices in a flat space-time may be also solved by using the spectrum which contains every massless representation of the Poincaré group just once. Another fact that supports this point of view is provided by the following reasoning. There exists the so-called most singular representation (the oscillator representation) of the orthosymplectic algebra osp(8) that has the following highly interesting property: when it is restricted to the Poincaré or anti–de Sitter algebra then it turns out to contain every massless bosonic representation of corresponding algebra just once (see [23]). In other words the above-mentioned spectra can be obtained from one and the same representation of the osp(8). Because of that it is strongly believed that the difficulties of higher order vertices may be solved by incorporating an infinite number of different massless particles (every massless representation of the Poincaré algebra just once). The investigation of fourth order to interaction carried out in [24] supports this point of view.

Starting from the commutation relations (CR's) of the Poincaré algebra

$$[J^{\mu\nu},\, J^{\rho\sigma}] = \imath\eta^{\mu\rho}J^{\sigma\nu} + \cdots,\ [P^{\mu},\, J^{\rho\sigma}] = \imath\eta^{\mu\rho}P^{\sigma} + \cdots,$$
$$[P^{\mu},\, P^{\nu}] = 0\,,\ \eta^{\mu\nu} = \mathrm{diag}(+1, -1, \dots, -1)\,, \qquad (1)$$

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we introduce the light cone coordinates $x^{\pm} = (x^0 \pm x^{2n+1})/\sqrt{2}$, x^I, $I = 1,\ldots,2n$ and consider x^+ as the evolution parameter. Without loss of generality we analyze (1) for $x^+ = 0$. In the light cone coordinates the Poincaré algebra splits into a kinematical part spanned by the generators P_-, P^I, J^{IK}, J^{+-}, J^{+I}, which are usually realized quadratically in the physical fields, and a dynamical part spanned by the generators $H \equiv P_+$ and J^{-I} realized nonlinearly.

Since we use extensively the constructions of Ref. [25], we would like to introduce the complex extension of the rotation group, denoted as $\mathrm{SO}(2n,C)$. After that we transform the quadratical form $(x,x) = x^I x^I$ to the antidiagonal form $(x,x) = x^I s^{IJ} x^J$, where $s^{IJ} \equiv \delta^{I,2n-J+1}$. It will be convenient to introduce the index $i = 1,\ldots,n$ and split all vectors as $A^I = (A^i, A^{2n-i+1})$, after that we have the scalar product $(A,B) = \sum_{i=1}^{n}(A^i B^{2n-i+1} + A^{2n-i+1}B^i)$. For the case of small transversal indices $i,j,\ldots$, we drop the summation over repeated indices. The reality condition can be written as $(A^I)^* = s^{IJ}A^J$.

Differential realization $r(G)$ of the on-shell representation $[p_+ = p^I s^{IK} p^K/(2p_-)]$ of the Poincaré algebra has the form (see Ref. [26])

$$r(P_-) = \beta,\ r(P_+) = h,\ r(P^I) = p^I,\ r(J^{+-}) = -\hat{x}^-\beta\,,$$

$$r(J^{IK}) = \hat{x}^I p^K - \hat{x}^K p^I + M^{IK}\,,\ r(J^{+I}) = -\hat{x}^I\beta\,,$$

$$r(J^{-I}) = \hat{x}^- p^I - \hat{x}^I h - M^{IK} s^{KL} p^L/\beta\,, \qquad (2)$$

where $\beta \equiv p_-$, $h \equiv p^I s^{IK} p^K/(2\beta)$, $\hat{x}^I \equiv \imath s^{IK}\partial/\partial p^K$, $\hat{x}^- \equiv -\imath\partial/\partial\beta$, and M^{IK} are the generators of the $\mathrm{SO}(2n)$ group.

To use the standard framework of the highest weight representation we should convert generators of $\mathrm{SO}(2n)$ from the tensor form to the Cartan-Weyl form. To do that let us introduce anti-Hermitean generators of $\mathrm{SO}(2n)$: $\mathcal{M}^{I,J} \equiv \imath M^{IJ}$ which satisfy the following CR's: $[\mathcal{M}^{I,J}, \mathcal{M}^{K,L}] = s^{I,K}\mathcal{M}^{L,J} + 3$ perms. Now we link the $\mathcal{M}^{I,J}$ with elements of $\mathrm{so}(2n)$ algebra taken in the Cartan-Weyl form in the following fashion:

$$S^+_{l,k} \equiv \mathcal{M}^{2n-l+1,l+k}\,,\ S^-_{l,k} \equiv \mathcal{M}^{2n-l-k+1,l}\,,$$

$$\tilde{S}^+_{l,k} \equiv \mathcal{M}^{n+k+1,l+k}\,,\ \tilde{S}^-_{l,k} \equiv \mathcal{M}^{2n-l-k+1,n-k}\,, \qquad (3)$$

$$S^0_{l,k} \equiv \frac{1}{2}(M_l - M_{l+k})\,,\ \tilde{S}^0_{l,k} \equiv \frac{1}{2}(M_{n-k} + M_{2n-l-k+1})\,,$$

where $M_l \equiv \mathcal{M}^{2n-l+1,l}$. Throughout this paper, unless otherwise specified, the indices l,k for $S^{0\pm}_{l,k}$ and $\tilde{S}^{0\pm}_{l,k}$ run over $l = 1,\ldots,n-k-1$, $k = 1,\ldots,n-1$, and $l = n-k,\ldots,n$, $k = 1,\ldots,n-1$, respectively. The generators $S^{0\pm}_{l,k}$ form the su(n) subalgebra of so$(2n)$ while $\tilde{S}^{\pm 0}_{n,1}$ and $S^{\pm 0}_{l,1}$ form the Chevalley basis of so$(2n)$. The Cartan subalgebra is spanned by $\tilde{S}^0_{n,1}$ and $S^0_{l,1}$. The highest weight vector $|\mathbf{m}\rangle$ [where $\mathbf{m} = (m_1,\ldots,m_n)$] is defined in the following way:

$$[S^+_{l,1} = \tilde{S}^+_{n,1} = (2S^0_{l,1} - p_l) = (2\tilde{S}^0_{n,1} - p_n)]|\mathbf{m}\rangle = 0\,, \quad (4)$$

where $p_l \equiv m_l - m_{l+1}$, $l = 1,\ldots,n-1$, $p_n \equiv m_{n-1} + m_n$. The numbers $m_1,\ldots,m_n$, the so-called weight of the representation, satisfy the inequalities $m_1 \ge m_2 \ge \cdots \ge m_{n-1} \ge |m_n|$. To carry out the calculations it is necessary to have a certain realization of the representation. Let us briefly describe the realization we use (for more details see Refs. [25,27]). First, for any $g^c \in\mathrm{SO}(2n,C)$ we have the Gauss decomposition

$$g^c = g_- g_0^{(\delta)} g_+^{(\eta,\xi)}\,,\ g_+^{(\eta,\xi)} = \begin{pmatrix} \eta & \eta\xi s \\ 0 & s\eta^{(t)-1}s \end{pmatrix}\,, \qquad (5)$$

where

$$g_0^{(\delta)} = \mathrm{diag}(\delta_1,\ldots,\delta_n,\delta_n^{-1},\ldots,\delta_1^{-1})\,,$$

$\eta_{i,j} = 0$, for $i > j$, $\eta_{i,i} = 1$, $\xi_{i,j} = -\xi_{j,i}$, $s_{i,j} = \delta_{i,n-j+1}$, and g_- is a lower triangular matrix with units on the diagonal. The superscripts (t) and -1 denote a matrix transposition and inversion respectively. Now the representation of the $\mathrm{SO}(2n)$ group of weight $\mathbf{m}$ is constructed on the space of the functions $\Phi(\eta,\xi)$ in the following way: for all $g \in\mathrm{SO}(2n)$ one defines the operator T_g as

$$(T_g\Phi)(\eta,\xi) = \delta_1^{m_1}\cdots\delta_n^{m_n}\Phi(\eta\cdot g,\xi\cdot g)\,, \qquad (6)$$

where $\eta\cdot g$, $\xi\cdot g$, as well as δ_i are defined from the decomposition $g_+^{(\eta,\xi)}g = g_- g_0^{(\delta)} g_+^{(\eta\cdot g,\xi\cdot g)}$. Using (5) and (6) one can calculate the generators of a representation. In the quite extensive literature we find expressions for the Chevalley basis or for the S^+ part of the algebra only. Since we will need manifest expressions for all generators let us write down the results of our calculations (here and henceforth α designates the set $\{\eta,\,\xi\}$):

$$
\begin{aligned}
S^+_{l,k}(\alpha) &= \sum_j \left(\eta_{j,l}\frac{\partial}{\partial\eta_{j,l+k}} - \xi_{j,l+k}\frac{\partial}{\partial\xi_{j,l}}\right), \\
\tilde S^+_{l,k}(\alpha) &= \frac{\partial}{\partial\xi_{n-k,2n-l-k+1}}, \\
S^-_{l,k}(\alpha) &= \sum_j m_j\eta^{-1}_{l,j}\eta_{j,l+k} - \sum_j \xi_{j,l}\frac{\partial}{\partial\xi_{j,l+k}} + \sum_{i,j,b}\varepsilon^{i,b}\eta^{-1}_{l,b}\eta_{b,j}\eta_{i,l+k}\frac{\partial}{\partial\eta_{i,j}}, \\
\tilde S^-_{l,k}(\alpha) &= \sum_j (\xi_{j,2n-l-k+1}S^-_{n-k,j+k-n} - \xi_{j,n-k}S^-_{2n-l-k+1,j+l+k-2n-1}) \\
&\quad + \sum_{i<j}(\xi_{i,n-k}\xi_{j,2n-l-k+1} - \xi_{j,n-k}\xi_{i,2n-l-k+1})\frac{\partial}{\partial\xi_{i,j}}, \\
M_l(\alpha) &= m_l + \sum_j\left(\varepsilon^{j,l}\eta_{j,l}\frac{\partial}{\partial\eta_{j,l}} - \varepsilon^{l,j}\eta_{l,j}\frac{\partial}{\partial\eta_{l,j}} - \xi_{l,j}\frac{\partial}{\partial\xi_{l,j}}\right),
\end{aligned}
\tag{7}
$$

where $\varepsilon^{i,j} = 1$ (0) for $i < j$ ($i \geq j$). In the expression for $\tilde S^-_{l,k}$ there are terms $S^-_{i,j}$ with $j \leq 0$ which should be rewritten according to the rules $S^-_{i+j,-j} = S^+_{i,j}$, $S^-_{i,0} = M_i$ for $i, j > 0$. In (7) and below, unless otherwise specified, the indices i, j, b run over $1, \ldots, n$.

From now on we consider η_{ij} for $i < j$ and ξ_{ij} as the creation operators, η_{ii} as the unit operator and make, in (7), the substitutions $\partial/\partial\eta_{i,j} \to \bar\eta_{i,j}$ for $i < j$, $\partial/\partial\xi_{ij} \to \bar\xi_{ij}$, where $\bar\eta_{i,j}$ and $\bar\xi_{i,j}$ are the annihilation operators ($\bar\eta_{i,j}|\mathbf{m}\rangle = 0 = \bar\xi_{i,j}|\mathbf{m}\rangle$) with CR's:

$$
[\bar\eta_{i,j}, \eta_{i',j'}] = \varepsilon^{i,j}\delta_{i,i'}\delta_{j,j'},
$$
$$
[\bar\xi_{i,j}\xi_{i',j'}] = \delta_{i,i'}\delta_{j,j'} - \delta_{i,j'}\delta_{j,i'}.
$$

Using the method of Gelfand and Zetlin one can introduce the orthogonal basis system of the representation [in this work we rely only on the existence of such a basis (see Ref. [28])]: $|\mathbf{m}, \{\lambda\}\rangle = N(\mathbf{m}, \{\lambda\})\alpha^{\{\lambda\}}|\mathbf{m}\rangle$ where the set of numbers $\{\lambda\}$ designate the basis and N is a normalization factor. Now we introduce the generating function $|\Phi_{\mathbf{m}}(p,\alpha)\rangle \equiv \sum_{\{\lambda\}}\Phi^{\{\lambda\}}_{\mathbf{m}}(p)|\mathbf{m}, \{\lambda\}\rangle$, where, by definition, the fields $\Phi^{\{\lambda\}}_{\mathbf{m}}(p)$ (the dependence of fields on β is not shown explicitly) realize the highest weight representation of the SO($2n$) group. Writing the CR's for $\Phi_{\mathbf{m}}$ in the form

$$
[\Phi^{\{\lambda\}}_{\mathbf{m}}(p), \Phi^{\{\lambda'\}}_{\mathbf{m}'}(p')]|_{\text{equal } x^+} = \delta_{\mathbf{m},\lambda;\mathbf{m}',\lambda'}\frac{\delta^{(2n+1)}(p+p')}{2\beta},
\tag{8}
$$

one can construct quadratic level generators of the Poincaré algebra:

$$
G_{\mathbf{m}} = \int \beta d\beta d^{(2n)}p\langle\Phi_{\mathbf{m}}|r(G_{\mathbf{m}})|\Phi_{\mathbf{m}}\rangle.
$$

Now let us study the cubic corrections to H and J^{-I}. Analyzing the CR's of H and J^{-I} with P_-, P^I, J^{+I} and the CR's $[H, J^{-I}]$ and $[J^{-I}, J^{-K}]$ in the cubic approximation we get the following structure for H and J^{-I} describing the interaction of the fields $\Phi_{\mathbf{m}_a}$ (here and henceforth $a = 1,2,3$ labels the three interacting fields):

$$
H_3 = \int d\Gamma_3\,\langle\Phi_3|h_3\rangle,
\tag{9a}
$$

$$
J^{-I}_3 = \int d\Gamma_3\left[\langle\Phi_3|\,j^{-I}_3\rangle - \frac{1}{3}\left(\sum_a \hat x^I_a\langle\Phi_3|\right)|h_3\rangle\right],
\tag{9b}
$$

$$
|h_3\rangle = h_3(\alpha, \mathbf{P}, \beta)\prod_a|\mathbf{m}_a\rangle,
\tag{10a}
$$

$$
|\,j^{-I}_3\rangle = -(h^{(3)})^{-1}\sum_a r(J^{-I}_a)^\dagger|h_3\rangle,
\tag{10b}
$$

where $\langle\Phi_3| \equiv \prod_a\langle\Phi_{\mathbf{m}_a}(p_a, \bar\alpha_a)|$, $h^{(3)} \equiv \sum_a h_a$,

$$
d\Gamma_3 \equiv \delta^{(2n)}\left(\sum_a p^I_a\right)\delta\left(\sum_a\beta_a\right)\prod_a d\beta_a d^{(2n)}p_a,
\tag{11}
$$

$$
h_a \equiv \frac{p^I_a s^{IJ}p^J_a}{2\beta_a},\quad h^{(3)} = -\frac{\mathbf{P}^I s^{IJ}\mathbf{P}^J}{2\beta_1\beta_2\beta_3},\quad \mathbf{P}^I \equiv \frac{1}{3}\sum_a\check\beta_a p^I_a,
\tag{12}
$$

$$
r(J^{-I}_a)^\dagger = p^I_a\hat x^-_a - h_a\hat x^I_a - M^{IK}_a s^{KL}p^L_a/\beta_a,
\tag{13}
$$

and $\check\beta_a \equiv \beta_{a+1} - \beta_{a+2}$, $\beta_4 \equiv \beta_1$, $\beta_5 \equiv \beta_2$. The CR of H with J^{+-} tells us that $|h_3\rangle$ is a zero-degree homogeneity function with respect to β (the dependence of $\mathbf{P}$ on β should also be taken into account):

$$
\left(\sum_a\beta_a\frac{\partial}{\partial\beta_a} + \Delta(\mathbf{P})\right)h_3(\alpha, \mathbf{P}, \beta) = 0,
\tag{14a}
$$

$$\Delta(\mathbf{P}) \equiv \sum_i \left(\mathbf{P}^i \frac{\partial}{\partial \mathbf{P}^i} + \mathbf{P}^{2n-i+1} \frac{\partial}{\partial \mathbf{P}^{2n-i+1}} \right) . \qquad (14b)$$

From CR's of H with J^{IK} one gets the equations $\sum_a r(J^{IK}_a)|h_3\rangle = 0$ which can be rewritten in terms of $h_3(\alpha, \mathbf{P}, \beta)$ [(10a)]:

$$\left(\mathcal{L}^{I,K}(\mathbf{P}) + \sum_a \mathcal{M}^{I,K}_a(\alpha) \right) h_3(\alpha, \mathbf{P}, \beta) = 0 , \qquad (15a)$$

$$\mathcal{L}^{I,K}(\mathbf{P}) \equiv \mathbf{P}^I \frac{\partial}{\partial \mathbf{P}^{2n-K+1}} - (K \leftrightarrow I) . \qquad (15b)$$

To simplify our equations we do not write the Fock vacuum explicitly, and we consider $\mathcal{M}$, S, and $\tilde{S}$ as differential operators (7). The operator $\mathcal{L}^{I,K}(\mathbf{P})$ is an anti-Hermitean form of the orbital part of the angular moment and can be obtained from the equation $\mathcal{L}^{I,K}(\mathbf{P})|h_3\rangle = \imath \sum_a (\hat{x}^I_a p^K_a - \hat{x}^K_a p^I_a)|h_3\rangle$ where we take into account that $\mathbf{P}$ variables exhaust all p^I_a dependence of $|h_3\rangle$. The procedure of introducing the Cartan-Weyl basis for $\mathcal{L}$, denoted as $L^{\pm 0}_{l,k}$ and $\tilde{L}^{\pm 0}_{l,k}$, is identical to the one for $\mathcal{M}^{I,K}$ [in (3) it is necessary to make the substitutions L for S and $\mathcal{L}$ for $\mathcal{M}$]. In the case of a massless fields we can restrict ourselves to the interaction having κth power derivatives in the vertex. The corresponding equation on the vertex is $[\Delta(\mathbf{P}) - \kappa] h_3(\alpha, \mathbf{P}, \beta) = 0$.

In fact, the set of vertices h_3 satisfying Eqs. (14) and (15) is the solution to all CR's of the Poincaré algebra. However, from the set of solutions of physical interest are the vertices satisfying the additional condition which we call the locality condition. Let us formulate it. Expressions $|h_3\rangle$ and $|j^{-I}_3\rangle$ (10) must be regular in the limit $h^{(3)} \to 0$, and to have a nontrivial 3-point amplitude of scattering it is supposed that $h_{3(0)} \not\equiv 0$ [for the definition of $h_{3(0)}$ see (16b)]. From (10b) it is clear that $|j^{-I}_3\rangle$ will have a singularity when $h^{(3)} \to 0$ due to $h_{3(0)} \not\equiv 0$, and this singularity can be cancelled by an appropriate selection of $h_{3(0)}$. Thus we have the conditions

$$\sum_a r(J^{-I}_a)^\dagger h_3(\alpha, \mathbf{P}, \beta) \Big|_{h^{(3)} \to 0} = 0 , \qquad (16a)$$

$$h_{3(0)} \equiv h_3(\alpha, \mathbf{P}, \beta) \Big|_{h^{(3)} \to 0} \not\equiv 0 . \qquad (16b)$$

Equations (14)–(16) are the complete system of equations on the tree-level cubic interaction vertices which we are going to solve. We analyze Eqs. (14)–(16) in the following way.

(a) First, we are solving the "+" part of Eq. (15a):

$$\left(L^+_{l,k}(\mathbf{P}) + \sum_a S^{+(a)}_{l,k}(\alpha) \right) h_3(\alpha, \mathbf{P}, \beta) = 0 ,$$
$$\left(\tilde{L}^+_{l,k}(\mathbf{P}) + \sum_a \tilde{S}^{+(a)}_{l,k}(\alpha) \right) h_3(\alpha, \mathbf{P}, \beta) = 0 . \qquad (17)$$

To solve Eqs. (17) it is convenient, in place of P^I, to introduce new "momentum" variables: $q_{i,i+1}$, $i = 1, \ldots, n-1$; ρ_i, $i = 1, \ldots, n$, and $\mathbf{P}^n$ where

$$q_{ij} \equiv \theta^{i,j} \frac{\mathbf{P}^i}{\mathbf{P}^j} , \ \rho_j \equiv \sum_i \mathbf{P}^{2n-i+1} q_{ij} , \qquad (18)$$

and $\theta^{i,j} \equiv \delta^{i,j} + \varepsilon^{i,j}$. As a result we take the following solution of Eqs. (17):

$$h_3 = h_3(\overset{(1)}{\alpha}, \rho_1, \tilde{\rho}_n, \beta) , \qquad (18a)$$

$$\overset{(1)(a)}{\xi}_{ij} \equiv (\tau^{-1} q^{-1} \xi^{(a)} \overset{(t)}{q}{}^{-1} \overset{(t)}{\tau}{}^{-1})_{ij} - \frac{\mathbf{P}^n}{\rho_1} (\delta_{i,1}\delta_{j,n} - \delta_{i,n}\delta_{j,1}) , \qquad (18b)$$

$$\overset{(1)(a)}{\eta}_{ij} \equiv (\eta^{(a)} q \tau)_{ij} , \qquad (18c)$$

where

$$\tau_{i,j} \equiv \delta_{i,j} - \frac{\rho_j}{\rho_1} \delta_{i,1} \varepsilon^{1,j} , \ \tilde{\rho}_n \equiv \rho_n \mathbf{P}^n .$$

Recall that α designates the set $\{\eta, \xi\}$. Furthermore, there are additional equations on the $|h_3\rangle$ [they are the remains of Eqs. (17)], which we would like to write after introducing the dimensionless variable $\bar{\rho}_n \equiv \rho_n \mathbf{P}^n / (\rho_1)^2$ (notice that the variables $\overset{(1)}{\alpha}$ are also dimensionless) and rewriting expression (18a) in the form $h_3 = (\rho_1)^\kappa h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta)$. We have written down the dimensionfull factor $(\rho_1)^\kappa$ explicitly. Now the relevant equations can be written as

$$\sum_a S^{+(a)}_{l,k}(\overset{(1)}{\alpha}) h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta) = 0 , \qquad (19a)$$

$$\sum_a \tilde{S}^{+(a)}_{n+i-j+1,n-i}(\overset{(1)}{\alpha}) h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta) = 0 , \qquad (19b)$$

where $l = 2, \ldots, n-k$, $k = 1, \ldots, n-1$, and $i, j = 2, \ldots, n-1$.

(b) Second, we rewrite the "−" and "0" parts of Eq. (15a) in terms of the variables $\overset{(1)}{\alpha}$ and $\bar{\rho}_n$. The equations obtained are

$$\sum_a [S^{-(a)}_{l,k}(\overset{(1)}{\alpha}) + \delta_{l,1} \bar{\rho}_n \tilde{S}^{+(a)}_{n+1-k,n-1}(\overset{(1)}{\alpha})] \times h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta) = 0, \quad (20a)$$

$$\sum_a [\tilde{S}^{-(a)}_{n+i-j+1,n-i}(\overset{(1)}{\alpha}) + \bar{\rho}_n (\delta_{i,1} \varepsilon^{1,j} S^{+(a)}_{1,j-1}(\overset{(1)}{\alpha}) - (i \leftrightarrow j))] \times h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta) = 0 , \quad i, j = 1, \ldots, n-1, \quad (20b)$$

$$\left[\sum_a M^{(a)}_l(\overset{(1)}{\alpha}) + \delta_{l,1} \left(\kappa - 2\bar{\rho}_n \frac{\partial}{\partial \bar{\rho}_n} \right) \right] h_{3\kappa}(\overset{(1)}{\alpha}, \bar{\rho}_n, \beta) = 0 \qquad (20c)$$

[in Eq. (20c) $l = 1, \ldots, n$].

(c) Third, writing Eqs. (16a) in terms of the same variables we obtain

$$\sum_a \check{\beta}_a S^{-(a)}_{1,j-1}(\overset{(1)}{\alpha})h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)=0\,, \tag{21a}$$

$$\sum_a \check{\beta}_a \tilde{S}^{-(a)}_{j,n-1}(\overset{(1)}{\alpha})h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)=0\,, \tag{21b}$$

$$\sum_a \check{\beta}_a\left(\beta_a\frac{\partial}{\partial\beta_a}-M_1^{(a)}(\overset{(1)}{\alpha})\right)h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)=0\,, \tag{21c}$$

where $j=2,\ldots,n$. The $h_{3\kappa(0)}$ is defined by (16b) making there the substitutions $h_{3(0)}\to h_{3\kappa(0)}$ and $h_3\to h_{3\kappa}$. The set of Eqs. (19)–(21) supplemented by Eq. (14) is the complete system equations for $h_{3\kappa}(\overset{(1)}{\alpha},\,\bar{\rho}_n,\,\beta)$. Since $h_{3\kappa(0)}$, which is a dimensionless 3-point amplitude of scattering, is the object of prime physical interest we restrict ourselves to solutions of Eqs. (14), (19)–(21) for $h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)$; i.e., from now on we analyze the relevant equations in $(\bar{\rho}_n)^0$ approximation.

Equations (19) and (20) express the invariance conditions with respect to the transversal rotations. In the approaches based on the tensor realization of the representations of the transversal group one would first analyse these equations and then the rest ones (see Refs. [8,29]), while in the approach based on the highest weight [20] it turns out to be more convenient to use the following procedure in solving (19)–(21).

(i) Multiplying the $l=1$ part of Eq. (20a) and the $i=1$ part of Eq. (20b) by $\check{\beta}_1$, subtracting Eqs. (21a) and (21b) respectively from the resulting expressions, and repeating this procedure for cyclic permutations of the particle labels (1,2,3), one obtains the equations

$$\Big(\frac{S^{-(1)}_{1,j-1}(\overset{(1)}{\alpha})}{\beta_1}=\frac{S^{-(2)}_{1,j-1}(\overset{(1)}{\alpha})}{\beta_2}=\frac{S^{-(3)}_{1,j-1}(\overset{(1)}{\alpha})}{\beta_3}\Big)h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)\,, \tag{22a}$$

$$\Big(\frac{\tilde{S}^{-(1)}_{j,n-1}(\overset{(1)}{\alpha})}{\beta_1}=\frac{\tilde{S}^{-(2)}_{j,n-1}(\overset{(1)}{\alpha})}{\beta_2}=\frac{\tilde{S}^{-(3)}_{j,n-1}(\overset{(1)}{\alpha})}{\beta_3}\Big)h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)\,, \tag{22b}$$

where $j=2,\ldots,n$. The general solution of Eqs. (22) is

$$h_{3\kappa(0)}(\overset{(1)}{\alpha},\,\beta)=\prod_a\prod_{\nu=1}^{n-1}(\overset{(2)}{\xi}{}^{(a)}_{\nu}\overset{(1)}{\eta}{}^{1,\nu+1}_{(a)})^{m_\nu^{(a)}-m_{\nu+1}^{(a)}}\times(\overset{(2)}{\xi}{}^{(a)}_{n-1})^{2m_n^{(a)}}\,\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)\,, \tag{23}$$

where we have introduced the variables

$$X_\sigma\equiv\sum_a\beta_a/\overset{(3)}{\eta}{}^{(a)}_{\sigma}\,,\quad X_{2n-\sigma-1}\equiv\sum_a\beta_a/\overset{(3)}{\xi}{}^{(a)}_{\sigma}\,,$$

$$\overset{(3)}{\eta}_{\sigma,\gamma}\equiv\sum_\nu\Theta_{\sigma,\nu}\overset{(2)}{\eta}_{\nu,\gamma}\,,\quad \overset{(3)}{\xi}_{\sigma,\gamma}\equiv\overset{(2)}{\xi}_{\sigma,\gamma}\,, \tag{24}$$

$$\frac{1}{\overset{(3)}{\eta}_\sigma}\equiv\frac{1}{\overset{(2)}{\eta}_\sigma}-\sum_\nu\frac{1}{\overset{(2)}{\xi}_1}\overset{(2)}{\xi}_{\sigma,\nu}\overset{(2)}{\eta}_{1,\nu}\,,\quad \overset{(3)}{\xi}_\sigma\equiv\frac{1}{\overset{(2)}{\eta}_{1,\sigma}}\overset{(2)}{\xi}_1\,,$$

$$\overset{(2)}{\xi}_{\sigma,\gamma}\equiv\overset{(1)}{\xi}_{\sigma+1,\gamma+1}-\frac{\overset{(1)}{\xi}_{1,\sigma+1}}{\overset{(2)}{\eta}_\gamma}+\frac{\overset{(1)}{\xi}_{1,\gamma+1}}{\overset{(2)}{\eta}_\sigma}\,, \tag{25}$$

$$\overset{(2)}{\eta}_{\sigma,\gamma}\equiv\sum_\nu\varepsilon^{\sigma,\nu}\frac{\overset{(1)}{\eta}{}^{1,\nu}}{\overset{(1)}{\eta}{}^{1,\sigma+1}}\overset{(1)}{\eta}_{\nu,\gamma+1}\,,\quad \overset{(2)}{\xi}_\sigma\equiv\sum_\nu\overset{(2)}{\eta}_{\sigma,\nu}\overset{(1)}{\xi}_{1,\nu+1}\,,$$

$$\frac{1}{\overset{(2)}{\eta}_\sigma}\equiv-\sum_\nu\frac{\overset{(2)}{\eta}{}^{\sigma,\nu}}{\overset{(1)}{\eta}{}^{1,\nu+1}}\,,\quad \Theta_{\sigma,\nu}\equiv\delta_{\sigma,\nu}-\delta_{\sigma,\nu-1}\frac{\overset{(2)}{\xi}_{\nu-1}}{\overset{(2)}{\xi}_\nu} \tag{26}$$

and $\overset{(r)}{\eta}{}^{\sigma,\beta}$ is the inversion of $\overset{(r)}{\eta}_{\sigma,\beta}$. In (24)–(26) the indices σ,γ,ν run over $1,\ldots,n-1$. Let us make of a comment on the variables introduced. To solve Eqs. (22) we diagonalize the generators $S^{-(a)}_{1,k}$ first and then $\tilde{S}^{-(a)}_{l,n-1}$, i.e., we split the solution procedure in two stages. The superscripts (2) and (3) are used to indicate these stages.

(ii) Now we rewrite Eqs. (19) and the $l=1$ part of Eqs. (20c) in terms of the new variables (24). The result of this procedure is

$$\left(L^{\pm 0}_{\sigma,\gamma}(X)+\sum_a S^{\pm 0(a)}_{\sigma,\gamma}(\overset{(3)}{\alpha})\big|_{\bar{m}}\right)\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=0\,, \tag{27a}$$

$$\left(\tilde{L}^{\pm 0}_{\sigma,\gamma}(X)+\sum_a \tilde{S}^{\pm 0(a)}_{\sigma,\gamma}(\overset{(3)}{\alpha})\big|_{\bar{m}}\right)\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=0, \tag{27b}$$

$$\left(\Delta(X)+\kappa-\sum_a m_1^{(a)}\right)\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=0\,, \tag{27c}$$

where $\Delta(X)$ and $L^{\pm 0}_{l,k}(X)$ can be obtained from (14b) and (15b) respectively making there the substitutions $n\to n-1$ and $\mathbf{P}\to X$. The $S^{\pm 0}_{\sigma,\gamma}|_{\bar{m}}$ and $\tilde{S}^{\pm 0}_{\sigma,\gamma}|_{\bar{m}}$ are obtained from (7), making there the shift $n\to n-1$ and then substituting $m_\sigma\to\bar{m}_\sigma=\varepsilon^{\sigma,n-1}m_{\sigma+1}-\delta_{\sigma,n-1}m_n$, $\sigma=1,\ldots,n-1$. In Eqs. (27) the indices σ,γ for $L^{\pm 0}_{\sigma,\gamma}$, $S^{\pm 0}_{\sigma,\gamma}|_{\bar{m}}$ and $\tilde{L}^{\pm 0}_{\sigma,\gamma}$, $\tilde{S}^{\pm 0}_{\sigma,\gamma}|_{\bar{m}}$ run over $\sigma=1,\ldots,n-\gamma-2$, $\gamma=1,\ldots,n-2$ and $\sigma=n-\gamma-1,\ldots,n-1$, $\gamma=1,\ldots,n-2$ respectively.

(iii) Rewriting Eq. (21c) and Eq. (14) in terms of (24) we get

$$\sum_a\check{\beta}_a(\beta_a\frac{\partial}{\partial\beta_a}+m_1^{(a)})\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=0\,, \tag{28a}$$

$$\sum_a(\beta_a\frac{\partial}{\partial\beta_a}+m_1^{(a)})\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=0\,. \tag{28b}$$

It should be emphasized that at this stage only Eq. (28a) expresses the locality condition while Eqs. (27), and (28b) reflect rotation invariance and homogeneity conditions. Thus to this stage we have reduced the $2n-1$ locality condition Eqs. (21) to the single Eq. (28a). The general solution of Eqs. (27) and (28) is

$$\bar{h}(\overset{(3)}{\alpha},\,X,\,\beta)=\prod_a\beta_a^{-m_1^{(a)}}G(\overset{(3)}{\alpha},\,X)\,, \tag{29}$$

where $G(\overset{(3)}{\alpha},\,X)$ is the generating function of the Clebsch-

Gordan coefficients describing the coupling of four representations of the SO$(2n-2)$ group: three representations with highest weights $\overline{\mathbf{m}}^{(a)}$ and one vector representation (X). The $G(\overset{(3)}{\alpha}, X)$ satisfies the equations obtained from Eqs. (27) making there the substitution $\bar{h} \to G(\overset{(3)}{\alpha}, X)$.

Now collecting all steps in derivation of $h_{3(0)}$ we have our final result:

$$h_{3(0)} = (\bar{\rho}_1)^{\kappa} \prod_a \beta_a^{-m_1^{(a)}} \prod_a \prod_{\nu=1}^{n-1} (\overset{(2)}{\xi}{}_{\nu}^{(a)} \overset{(1)}{\eta}{}^{1,\nu+1}_{(a)})^{m_\nu^{(a)} - m_{\nu+1}^{(a)}}$$

$$\times (\overset{(2)}{\xi}{}^{(a)}_{n-1})^{2m_n^{(a)}} G(\overset{(3)}{\alpha}, X) . \tag{30}$$

One can make sure that the $h_{3\kappa(0)}$ obtained satisfies the remainder of Eqs. (19)–(21).

For $h_{3\kappa(0)}$ to be well defined it must be a polynomial in primary operators α (5), or polynomial in $\overset{(1)}{\alpha}$ (18). As seen from (24)–(26) the nonpolynomial $\overset{(1)}{\alpha}$ dependence of $\overset{(3)}{\alpha}$ is the reason why the $h_{3\kappa(0)}$ is not polynomial generally. It turns out that, for the $h_{3\kappa(0)}$ to be polynomial, we must impose certain restrictions on the $\overline{\mathbf{m}}^{(a)}$ and κ [we must choose $\overline{\mathbf{m}}^{(a)}$ and κ in such a way that $\overset{(1)}{\eta}{}^{1,\gamma}$ and $\overset{(2)}{\xi}{}_{\sigma}$, coming from the prefactor of Eq. (30), suppress the maximal negative power of these variables coming from the G]. It is precisely the restrictions which leave no place for the minimal gravitational interaction of massless higher spin fields (see Refs. [20,29]).

Now let us briefly describe the advances made in this paper. We have started with Eq. (15) which expresses the invariance condition with respect to rotations of the SO$(2n)$ group and with Eq. (16) which expresses locality condition. As a result, we are finishing with expression (30) where the G is the generating function of the Clebsch-Gordan coefficients. Thus we have satisfied the locality condition. The expression (30) is the main result of the paper.

As an illustration of (30) let us consider particular cases $D = 5, 6$. Five and six dimensions is an arena where the complex structure of (30) can be stripped of complications of higher dimensions while hopefully retaining many of the physical features of the problem of interest. Notice (30) is valid for even dimensions. It can be straightforwardly generalized to odd dimensions by using the method above demonstrated.

The case of $D = 5$. For the case $D = 5$ the transversal group is SO(3). As is known its bosonic representations are labeled by integer j. Let $\Phi_j^{\lambda}(p)$ be $(2j+1)$ components of the massless spin-j field: $\lambda = -j, -j+1, \dots, j$. Now introducing one creation operator we collect all $\Phi_j^{\lambda}(p)$ in one Fock vector:

$$|\Phi_j(p\,,\,\alpha)\rangle = \sum_{\lambda=-j}^{j} \Phi_j^{\lambda}(p) \frac{\alpha^{j-\lambda}}{[(j+\lambda)!(j-\lambda)!]^{1/2}} |0\rangle . \tag{31}$$

For the cubic vertex describing the interaction of massless fields Φ_{j_a} (recall that $a = 1,2,3$) and having κth power of derivatives we derive the expression

$$h_{3(0)} = \bar{\mathbf{P}}^{\kappa} \prod_a \beta_a^{-j_a} \prod_a \zeta_a^{2j_a} \left(\sum_a \frac{\beta_a}{\zeta_a} \right)^{J-\kappa} , \tag{32}$$

where we have introduced the notation

$$\zeta_a = \alpha_a + \frac{1}{\sqrt{2}} \frac{\mathbf{P}^3}{\bar{\mathbf{P}}} , \quad \bar{\mathbf{P}} = \frac{\mathbf{P}^1 - \imath \mathbf{P}^2}{\sqrt{2}} , \tag{33a}$$

$$J = \sum_a j_a , \tag{33b}$$

[for the definition of $\mathbf{P}$, see (12)]. For $h_{3(0)}$ to be well defined it must a polynomial of ζ_a. As readily seen from (32) this requirement leads to the following constraints on the allowed values of j_a and κ:

$$J \geq \kappa , \tag{34a}$$

$$2j_a \geq J - \kappa . \tag{34b}$$

Now, having the inequalities (34) at our hand we are ready to discuss the no-go theorem. The minimal gravitational interaction of massless higher spin fields corresponds to $\kappa = 2$ with one of j_a, say j_1, taking the value $j_1 = 2$ (graviton), while $j_2, j_3 > 2$ (higher spins). It is easily seen that these values κ, j_a do not satisfy (34b). Since (32) and (34) form a complete list of interactions we conclude immediately that the above-mentioned interaction does not exist at all, a statement of no go. This statement for the case of four dimensions has been obtained in [9–12]. In these works it was shown that the "naive" gravitational interaction of massless higher spin fields, introduced via covariantization of their free actions in the flat space-time turned out to be inconsistent as this led to the loss of the higher spin gauge symmetries. The concrete obstacle was that the variation of the higher spin actions in a curved background contained the gravitational Weyl tensor which could not be compensated by some variation of the gravitation metric. In our approach it is the inequality (34b) that leave no place for the minimal gravitational interaction of massless higher spin fields. Since the no-go theorem has been rigorously established in four dimensions, our result can be considered as a generalization of no go to higher $D = 5$ dimension. Thus, (32) and (34) describe all possible Pauli-like interactions of massless higher spin fields. Note that spin-j massless field Φ_j can be associated with totally symmetric covariant tensor field $\Phi^{\mu_1,\dots,\mu_j}$. As is well known in four dimensions all massless fields can also be described by using totally symmetric tensor fields. However for the case $D \geq 6$, in addition to the totally symmetric massless representations, there exist so-called mixed symmetry ones. The latter can be associated with mixed symmetry covariant tensor fields only. It will be interesting to consider the question about minimal gravitational interactions for such fields. To do that we move to the $D = 6$ case.

The case of $D = 6$. Now the transversal group is SO(4). Its bosonic representations are labeled by two integers j_γ, $\gamma = 1,2$. The subscript $\gamma = 1,2$ refers to the two distinct $SU(2)_\gamma$ subgroups of the SO(4) group, while j_1 and j_2 denote the Casimirs of $SU(2)_1$ and $SU(2)_2$ respectively. The j_γ are expressible in terms of highest weight $[\mathbf{m} = (m_1, m_2)]$ of the representation SO(4) group as $j_1 = (m_1 + m_2)/2$, $j_2 = (m_1 - m_2)/2$, where m_1, m_2 satisfy the inequality $m_1 \geq |m_2|$. To describe $(2j_1+1)(2j_2+1)$ components of massless field we use $\Phi^{\lambda_1,\lambda_2}_{j_1,j_2}(p)$, where $\lambda_\gamma = -j_\gamma, -j_\gamma+1, \ldots, j_\gamma$. Introducing two creation operators α_γ we collect all $\Phi^{\lambda_1,\lambda_2}_{j_1,j_2}(p)$ in one Fock vector:

$$|\Phi_{j_1,j_2}(p\,,\alpha)\rangle = \sum_{\lambda_1=-j_1}^{j_1} \sum_{\lambda_2=-j_2}^{j_2} \Phi^{\lambda_1,\lambda_2}_{j_1,j_2}(p) \prod_{\gamma=1,2} \frac{\alpha^{j_\gamma-\lambda_\gamma}}{[(j_\gamma+\lambda_\gamma)!(j_\gamma-\lambda_\gamma)!]^{1/2}}|0\rangle\,. \tag{35}$$

The variables α_γ are expressible in terms of the variables η_{12}, ξ_{12} provided by Gauss decomposition [see Eq. (5)] as $\alpha_1 = \eta_{12}$, $\alpha_2 = \xi_{12}$. Note that massless representation Φ_{j_1,j_2} can be associated with the mixed symmetry covariant tensor field. This covariant tensor field has the structure of a Young tableaux whose first and second row have length equal to m_1 and $|m_2|$, respectively. The case $m_2 = 0$ (i.e., $j_1 = j_2$) corresponds to totally symmetric fields while $m_2 \neq 0$ (i.e., $j_1 \neq j_2$) to mixed symmetry ones.

Now the $h_{3(0)}$ from (30) can be changed to the form

$$h_{3(0)} = (\mathbf{P}^{\bar{1}})^\kappa \prod_a \beta_a^{-J_a} \prod_{a,\gamma} \zeta_{a\gamma}^{2j_{a\gamma}} \prod_\gamma \left(\sum_a \frac{\beta_a}{\zeta_{a\gamma}}\right)^{J_\gamma-\kappa/2}, \tag{36}$$

where we introduce the notation

$$\zeta_{a1} = \alpha_{a1} - \frac{\mathbf{P}^{\bar{2}}}{\mathbf{P}^{\bar{1}}}\,,\quad \zeta_{a2} = \alpha_{a2} - \frac{\mathbf{P}^{2}}{\mathbf{P}^{\bar{1}}}\,, \tag{37}$$

$$J_a = \sum_\gamma j_{a\gamma}\,, \tag{38a}$$

$$J_\gamma = \sum_a j_{a\gamma}\,. \tag{38b}$$

Note that $J_a = m_1^{(a)}$. The momentum $\mathbf{P}^I$ is defined in complex coordinates $z^1 = (x^1 + \imath x^2)/\sqrt{2}$, $z^2 = (x^3 + \imath x^4)/\sqrt{2}$, $z^{\bar{1}} = (z^1)^*$, $z^{\bar{2}} = (z^2)^*$: $\mathbf{P}^{\bar{1}} = (\mathbf{P}^1)^*$, $\mathbf{P}^{\bar{2}} = (\mathbf{P}^2)^*$. An asterisk is used to denote the complex conjugate. Again, for $h_{3(0)}$ to be well defined it must be a polynomial of $\zeta_{a\gamma}$. As readily seen from (36) this requirement leads to the following constraints on the allowed values of $j_{a\gamma}$ and κ:

$$J_\gamma \geq \frac{1}{2}\kappa\,, \tag{39a}$$

$$2j_{a\gamma} \geq J_\gamma - \frac{1}{2}\kappa\,. \tag{39b}$$

Now we are ready to discuss the no-go theorem for the case of totally as well as mixed symmetry massless fields. Recall that the ath massless representation corresponds to totally (mixed) symmetric field if $j_{a1} = j_{a2}$ ($j_{a1} \neq j_{a2}$). The minimal gravitational interaction of massless higher spin fields corresponds to $\kappa = 2$ with one of $j_{a\gamma}$, say $j_{1\gamma}$, taking the values $j_{11} = j_{12} = 2$ (graviton), while $j_{2\gamma}, j_{3\gamma} > 2$ (higher spins). It is easily seen that these values κ, $j_{a\gamma}$ do not satisfy (39b). In other words totally symmetric as well as mixed symmetry massless higher spin fields do not have minimal gravitational interaction, i.e., (36) and (39) describe Pauli-like interactions of massless higher spin fields for $D = 6$. As to totally symmetric fields, the above-mentioned generalization of no go to higher dimensions was expected.

We consider the fact that no go works for the case of mixed-symmetry massless fields one of the essential new results obtained in this paper. Note that it is hard to put the mixed-symmetry fields in the context of commonly used covariant procedures of deriving the no go, as even at that level of free actions there is no tractable covariant procedure to consider such fields.

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