Confronting left-right symmetric models with electroweak precision data at the Z peak

Apostolos Pilaftsis*

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK

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In view of the recent and future electroweak precision data accumulated at the CERN LEP and the SLC, we systematically analyze possible new physics effects that may occur in the leptonic sector within the context of $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ theories. It is shown that nonobservation of flavor-violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of lepton asymmetries at the Z peak form a set of complementary observables, yielding severe constraints on the parameter space of these theories. Contributions of new-physics effects to $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ are found to give interesting mass relations for the flavor-changing Higgs scalars present in these models.

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I. INTRODUCTION

The Large Electron Positron collider (LEP) at CERN and the Stanford Linear Collider (SLC) are powerful e^+e^- machines operating at the Z peak, which can confront theoretical predictions of the minimal standard model (SM) with experimental results to a high accuracy. A full analysis of all the electroweak precision data including those of the year 1995 will either establish the SM up to the one-loop electroweak level or signal the onset of new physics. In this context, analyzing electroweak oblique parameters [1] has become a common strategy to test the viability of models beyond the SM. The electroweak oblique parameters are sensitive physical quantities, when the new-physics interactions couple predominantly to W and Z bosons. However, it is imperative to explore additional observables that could be particularly sensitive to other sectors of the SM.

In this paper, we will study a new set of *complementary* leptonic observables and explicitly demonstrate the severe limitations that they can impose on model building of three-generation extensions of the SM. The set of observables comprises flavor-changing leptonic decays of the Z boson [2, 3], universality-breaking parameters $U_{\rm br}$ for the diagonal decays $Z \to l\bar{l}$ [4], and universality-violating parameters $\Delta\mathcal{A}_{l_1 l_2}$ based on leptonic asymmetries measured at LEP and/or SLC [5]. For the sake of illustration, we will consider a minimal left-right symmetric model (LRSM) [6] described in Sec. II. Such a model can naturally generate vector–axial $(V-A)$ as well as $V + A$ flavor-dependent $Zl\bar{l}$ couplings leading to new physics effects that can be probed via the leptonic observables mentioned above. This set of observables will be discussed, in some detail, in Sec. III. Explicit calculations and related technical details will be relegated

to Appendixes A and B. In Sec. IV, we will present numerical estimates for the aforementioned leptonic observables within the framework of a minimal LRSM and investigate their potential of effectively constraining this model. Furthermore, attention will be paid to possible LRSM contributions to R_b. Section V contains our conclusions.

II. THE LRSM

Left-right symmetric theories based on the gauge group $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ [7, 6] were motivated from the fact that the spontaneous breakdown of gauge and discrete symmetries can be accomplished on the same footing. Such models can naturally arise from $SO(10)$ grand unified theories via the breaking pattern [7, 8]

$$SO(10) \to SU(4)_{\rm PS} \times SU(2)_R \times SU(2)_L$$
$$\to SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L} \to SM,$$

where the subscript PS denotes the Pati-Salam gauge group [7]. We will, however, focus our analysis on the LRSM described in [6].

In the LRSM, right-handed neutrinos together with the right-handed charged leptons form three additional weak isodoublets in a three-generation model. The classification of the quark sector proceeds in an analogous way. To be specific, the assignment of quantum numbers to fermions under the gauge group $SU_R(2) \times SU(2)_L \times U(1)_{B-L}$ is arranged as

$$L'_L = \begin{pmatrix} \nu'_l \\ l' \end{pmatrix}_L \quad : \quad (0, 1/2, -1),$$

$$L'_R = \begin{pmatrix} \nu'_l \\ l' \end{pmatrix}_R \quad : \quad (1/2, 0, -1), \qquad (2.1)$$

$$Q'_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \quad : \quad (0, 1/2, 1/3),$$

$$Q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \quad : \quad (1/2, 0, 1/3), \qquad (2.2)$$

*Electronic address: pilaftsis@v2.rl.ac.uk

where the prime on the fermionic fields simply denotes weak eigenstates. In order to break the left-right gauge symmetry down to $U(1)_{em}$ [6], we have to introduce one Higgs bidoublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \qquad (2.3)$$

that transforms as $(1/2^*, 1/2, 0)$ and two complex Higgs triplets

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix}$$

$$\text{and} \quad \Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}, \qquad (2.4)$$

with quantum numbers $(0, 1, 2)$ and $(1, 0, 2)$, respectively. For simplicity, we will consider that only $\langle \phi_1^0 \rangle = \kappa_1/\sqrt{2}$ and $\langle \delta_R^0 \rangle = v_R/\sqrt{2}$ acquire vacuum expectation values

(VEV's). In the Born approximation [9], this can be accomplished by imposing invariance of the general Higgs potential under a judicious discrete symmetry D of the bidoublet Φ and the Higgs triplets $\Delta_{L,R}$ [10]. In fact, we will concentrate on case (d) of Ref. [11], to which the reader is referred for more details. In case (d), it is $\langle \delta_L^0 \rangle = \langle \phi_2^0 \rangle = 0$ at the tree level, implying that the charged gauge bosons W_L and W_R represent also physical states with masses $M_L = M_W$ and M_R, respectively. The massive neutral gauge bosons Z_L and Z_R mix one another with a small mixing angle of order $\kappa_1^2/v_R^2 \sim 10^{-2}$. To a good approximation, we will therefore neglect Z_L-Z_R mixing effects in our calculations.

Such a minimal LRSM allows the presence of baryon-lepton- $[(B - L)$-]violating operators in the Yukawa sector. In fact, the $(B - L)$-violating interactions are introduced by the triplet fields $\Delta_{L,R}$ and give rise to Majorana mass terms $m_{M_{ij}}$ in the following way:

$$\mathcal{L}_{\text{int}}^{B-L} = -\frac{\sqrt{2}m_{M_{ij}}}{2v_R} \left(h_{ij} \bar{L}_{L_i}'^C \varepsilon_{ij} \Delta_L L_{L_j}' + \bar{L}_{R_i}'^C \varepsilon_{ij} \Delta_R L_{R_j}' \right) + \text{H.c.} \qquad (2.5)$$

Here, ε_{ij} stands for the usual Levi-Civita tensor and all the parameters $h_{ij} = 1$ if left-right symmetry is forced explicitly. However, a phenomenological analysis of muon and τ decays shows that $h_{ij} \ll 1$ [11]. As a result, δ_L^+ and δ_L^{++} loop effects have been found to be negligible [12].

In case (d), the neutrino mass matrix takes the general seesaw-type form

$$M^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix}, \qquad (2.6)$$

where M^ν is (6×6)-dimensional matrix. In Eq. (2.6), m_D is a Dirac mass term associated with the $SU(2)_L$ breaking and connects the left-handed neutrinos with the right-handed ones. Relevant theoretical and phenomenological aspects related to such neutrino mass models may be found in Ref. [13]. The matrix M^ν can always be diagonalized by a unitary 6×6 matrix U^ν according to the common prescription $U^{\nu T} M^\nu U^\nu = \hat{M}^\nu$. After diagonalization, one gets six physical Majorana neutrinos n_i through the transformations

$$\begin{pmatrix} \nu_L' \\ \nu_R'^C \end{pmatrix}_i = \sum_{j=1}^{2n_G} U_{ij}^{\nu*} \, n_{Lj} \quad \text{and} \quad \begin{pmatrix} \nu_L'^C \\ \nu_R' \end{pmatrix}_i = \sum_{j=1}^{2n_G} U_{ij}^\nu \, n_{Rj}. \qquad (2.7)$$

The first $n_G = 3$ neutral states ν_i ($\equiv n_i$ for $i = 1, \ldots, n_G$) are identified with the known n_G light neutrinos, while the remaining n_G mass eigenstates N_j ($\equiv n_{j+n_G}$ for $j = 1, \ldots, n_G$) are heavy Majorana neutrinos predicted by the model. In addition to the leptonic sector, the quark sector of such an extension contains non-SM couplings of the fermionic fields to the gauge and Higgs bosons. Part of the LRSM couplings has been listed in Refs. [9, 11, 14]. The relevant Feynman rules and an additional discussion

are given in Appendix A.

Adopting the conventions of Ref. [13], the interactions of the Majorana neutrinos n_i and charged leptons l_i with the gauge bosons $W_L^\pm$ ($\equiv W^\pm$) and Z_L are correspondingly mediated by the mixing matrices

$$B_{lj}^L = \sum_{k=1}^{n_G} V_{lk}^L U_{kj}^{\nu*} \quad \text{and} \quad C_{ij}^L = \sum_{k=1}^{n_G} U_{ki}^\nu U_{kj}^{\nu*}, \qquad (2.8)$$

with $l = 1, \ldots, n_G$ and $i, j = 1, \ldots, 2n_G$. By analogy, there exist mixing matrices B_{li}^R and C_{ij}^R given by

$$B_{lj}^R = \sum_{k=n_G+1}^{2n_G} V_{lk}^R U_{kj}^{\nu*} \quad \text{and} \quad C_{ij}^R = \sum_{k=n_G+1}^{2n_G} U_{ki}^\nu U_{kj}^{\nu*}, \qquad (2.9)$$

which are responsible for the couplings of $W_R^\pm$ and Z_R to charged leptons and Majorana neutrinos. In Eqs. (2.8) and (2.9), the unitary $n_G \times n_G$ matrices V^L and V^R are responsible for the diagonalization of the charged lepton mass matrix via biunitary transformations.

Because of the specific structure of M^ν in Eq. (2.6), the flavor-mixing matrices B^L and C^L satisfy a number of identities that may be found in [15]. These identities, which result from the requirement of unitarity and renormalizability of the theory, turn out to be very useful in deriving model-independent relations between the mixings B_{li}^L, C_{ij}^L and heavy neutrino masses. In a two-generation-mixing model, we have [16, 17]

$$B_{lN_1}^L = \frac{\rho^{1/4} s_L^{\nu_l}}{\sqrt{1 + \rho^{1/2}}}, \qquad B_{lN_2}^L = \frac{i s_L^{\nu_l}}{\sqrt{1 + \rho^{1/2}}}, \qquad (2.10)$$

where $\rho = m_{N_2}^2/m_{N_1}^2$ (≥ 1) is a mass ratio of the two heavy Majorana neutrinos N_1 and N_2 present in such

a model, and $s_L^{\nu_l}$ are light-heavy-neutrino lepton-flavor (LF) mixings defined as [18]

$$(s_L^{\nu_l})^2 \equiv \sum_{i=n_G+1}^{2n_G} |B_{li}^L|^2 \simeq \left(m_D^\dagger \frac{1}{m_M^2} m_D \right)_{ll}. \qquad (2.11)$$

Furthermore, the mixings $C_{N_i N_j}^L$ are determined by

$$C_{N_1 N_1}^L = \frac{\rho^{1/2}}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2,$$

$$C_{N_2 N_2}^L = \frac{1}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2,$$

$$C_{N_1 N_2}^L = -C_{N_2 N_1}^L = \frac{i\rho^{1/4}}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2. \qquad (2.12)$$

At this stage, it should be noted that M^ν of Eq. (2.6) takes the known seesaw form [19] in case $m_M \gg m_D$. Nevertheless, this mass-scale hierarchy can dramatically be relaxed in a two-family seesaw-type model, in which light-neutrino masses can radiatively be induced in agreement with experimental upper bounds [13]. The light-heavy neutrino mixings $s_L^{\nu_l}$ of such scenarios can, in principle, scale as $s_L^{\nu_l} \sim m_D/m_M$ rather than the known seesaw relation $s_L^{\nu_l} \sim \sqrt{m_{\nu_l}/m_N}$. This implies that

high Dirac SU(2)$_L$-breaking components are allowed to be present in M^ν and only the ratio m_D/m_M ($\sim s_L^{\nu_l}$) gets limited by a global analysis of low-energy data. Recently, such an analysis has been performed in Ref. [20], in which the combined effect of all possible effective operators of the charged- and neutral-current interactions is considered. Although a careful analysis can provide some model-dependent caveats, we will, however, consider the following conservative upper bounds for the LF mixings:

$$(s_L^{\nu_e})^2, \ (s_L^{\nu_\mu})^2 \ < \ 0.010, \qquad (s_L^{\nu_\tau})^2 \ < \ 0.040,$$

and

$$(s_L^{\nu_e})^2 (s_L^{\nu_\mu})^2 \ < \ 1 \times 10^{-8}. \qquad (2.13)$$

For example, the last constraint in Eq. (2.13) comes from the nonobservation of decays of the type $\mu \to e\gamma$, eee, or the absence of μ - e conversion events in nuclei.

In LRSM's, the mixing matrices B^L, C^L, B^R, and C^R obey the useful relations

$$\sum_{i=1}^{2n_G} B_{l_1 i}^L B_{l_2 i}^R = 0, \qquad \sum_{l=1}^{n_G} B_{li}^{R*} B_{lj}^R = C_{ij}^R,$$

$$C_{ij}^{L*} + C_{ij}^R = \delta_{ij}. \qquad (2.14)$$

In a two-generation-mixing model, Eq. (2.14) together with Eq. (2.9) can be used to obtain the mixings

$$B_{l_1 N_1}^R = \cos\theta_R \sqrt{1 - C_{N_1 N_1}^L}, \qquad B_{l_2 N_1}^R = -\sin\theta_R \sqrt{1 - C_{N_1 N_1}^L},$$

$$B_{l_1 N_2}^R = -\cos\theta_R \frac{C_{N_1 N_2}^L}{\sqrt{1 - C_{N_1 N_1}^L}} - i\sin\theta_R \left(\frac{(1 - C_{N_1 N_1}^L)(1 - C_{N_2 N_2}^L) - |C_{N_1 N_2}^L|^2}{1 - C_{N_1 N_1}^L} \right)^{1/2},$$

$$B_{l_2 N_2}^R = \sin\theta_R \frac{C_{N_1 N_2}^L}{\sqrt{1 - C_{N_1 N_1}^L}} - i\cos\theta_R \left(\frac{(1 - C_{N_1 N_1}^L)(1 - C_{N_2 N_2}^L) - |C_{N_1 N_2}^L|^2}{1 - C_{N_1 N_1}^L} \right)^{1/2}. \qquad (2.15)$$

Consequently, the leptonic sector of this two-generation scenario depends only on five free parameters: the masses of the two heavy Majorana neutrinos m_{N_1} and m_{N_2} (or equivalently m_{N_1} and ρ), the mixing angles $(s_L^{\nu_i})^2$, which are, however, constrained by upper limits given in Eq. (2.13), and an unconstrained Cabibbo-type angle θ_R.

III. SLC AND LEP OBSERVABLES

In this section, we will define more precisely the framework of our calculations. In the limit of vanishing charged lepton masses, the amplitude responsible for the decay $Z \to l_1 \bar{l}_2$ can generally be parametrized as

$$\mathcal{T}_l = \frac{ig_w}{2c_w} \varepsilon_Z^\mu \bar{u}_{l_1} \gamma_\mu [g_L^{l_1 l_2} P_L + g_R^{l_1 l_2} P_R] v_{l_2}, \qquad (3.1)$$

where g_w is the usual SU(2)$_L$ electroweak coupling constant, ε_Z^μ is the Z-boson polarization vector, u (v) is the

Dirac spinor of the charged lepton l_1 (l_2), $P_L(P_R) = [1 - (+)\gamma_5]/2$, and $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$. In Eq. (3.1), we have defined

$$g_{L,R}^{l_1 l_2} = g_{L,R}^l + \delta g_{L,R}^{l_1 l_2}, \qquad g_L^l = \sqrt{\rho_l}(1 - 2\bar{s}_w^2),$$

$$g_R^l = -2\sqrt{\rho_l}\bar{s}_w^2, \qquad (3.2)$$

where ρ_l, $\bar{s}_w$, $\delta g_{L,R}^{ll}$ ($\equiv \delta g_{L,R}^l$) are obtained beyond the Born approximation and are renormalization scheme dependent. It should be noted that ρ_l, $\bar{s}_w$ introduce universal oblique corrections [1], whereas $\delta g_{L,R}^{l_1 l_2}$ are flavor dependent. Obviously, an analogous expression will be valid for the decay $Z \to b\bar{b}$, as soon as b-quark mass effects can be neglected.

It is convenient to reexpress the flavor-dependent electroweak corrections in terms of the loop functions $\Gamma_{l_1 l_2}^L$ and $\Gamma_{l_1 l_2}^R$ as

$$\delta g_L^{l_1 l_2} = \frac{\alpha_w}{2\pi} \Gamma_{l_1 l_2}^L, \qquad \delta g_R^{l_1 l_2} = \frac{\alpha_w}{2\pi} \Gamma_{l_1 l_2}^R.$$

The nonoblique loop functions $\Gamma^L_{l_1 l_2}$ and $\Gamma^R_{l_1 l_2}$ depend on whether the underlying theory is of $V{-}A$ or $V{+}A$ nature. In Appendix B, we have analytically derived the loop functions $\Gamma^L_{l_1 l_2}$ and $\Gamma^R_{l_1 l_2}$ in the context of LRSM's. It is then straightforward to obtain the branching ratio for the possible decay of the Z boson into two different charged leptons through the expression

$$B(Z \to \bar{l}_1 l_2 + l_1 \bar{l}_2) = \frac{\alpha^3_w}{48\pi c^2_w}\frac{M_Z}{\Gamma_Z}\left[|\Gamma^L_{l_1 l_2}|^2 + |\Gamma^R_{l_1 l_2}|^2\right],$$

(3.3)

with $\alpha_w = g^2_w/4\pi$. Such an observable is constrained by LEP results to be, e.g., $B(Z \to e\tau) \lesssim 10^{-5}$ [21].

Another observable that has been introduced in [4] is the universality-breaking parameter $U^{(l_1 l_2)}_{\rm br}$. To leading order of perturbation theory, $U^{(l_1 l_2)}_{\rm br}$ is given by

$$U^{(l_1 l_2)}_{\rm br} = \frac{\Gamma(Z \to l_1 \bar{l}_1) - \Gamma(Z \to l_2 \bar{l}_2)}{\Gamma(Z \to l_1 \bar{l}_1) + \Gamma(Z \to l_2 \bar{l}_2)} - U^{(l_1 l_2)}_{\rm br}({\rm PS})$$

$$= \frac{g^l_L(\delta g^{l_1}_L - \delta g^{l_2}_L) + g^l_R(\delta g^{l_1}_R - \delta g^{l_2}_R)}{(g^l_L)^2 + (g^l_R)^2}$$

$$= U^{(l_1 l_2)}_{\rm br}(L) + U^{(l_1 l_2)}_{\rm br}(R),$$

(3.4)

where $U^{(l_1 l_2)}_{\rm br}({\rm PS})$ characterizes known phase-space corrections coming from the nonzero masses of the charged leptons l_1 and l_2 that can always be subtracted, and

$$U^{(l_1 l_2)}_{\rm br}(L) = \frac{g^l_L(\delta g^{l_1}_L - \delta g^{l_2}_L)}{(g^l_L)^2 + (g^l_R)^2}$$

$$= \frac{\alpha_w}{2\pi}\frac{g^l_L}{(g^l_L)^2 + (g^l_R)^2}\,{\rm Re}(\Gamma^L_{l_1 l_1} - \Gamma^L_{l_2 l_2}),$$

(3.5)

$$U^{(l_1 l_2)}_{\rm br}(R) = \frac{g^l_R(\delta g^{l_1}_R - \delta g^{l_2}_R)}{(g^l_L)^2 + (g^l_R)^2}$$

$$= \frac{\alpha_w}{2\pi}\frac{g^l_R}{(g^l_L)^2 + (g^l_R)^2}\,{\rm Re}(\Gamma^R_{l_1 l_1} - \Gamma^R_{l_2 l_2}).$$

(3.6)

To make contact with the corresponding observable given in [21], we derive the relation

$$\frac{\Gamma(Z \to l\bar{l})}{\Gamma(Z \to l'\bar{l}')} = 2U^{(ll')}_{\rm br} + 1.$$

The results of a combined analysis at LEP and/or SLC regarding lepton universality at the Z resonance can be summarized [22, 23] as follows:

$$|U^{(ll')}_{\rm br}| < 5 \times 10^{-3} \qquad ({\rm SM}:0),$$

$$\mathcal{A}_\tau(\mathcal{P}_\tau) = 0.143 \pm 0.010 \qquad ({\rm SM}:0.143),$$

$$\mathcal{A}_e(\mathcal{P}_\tau) = 0.135 \pm 0.011,$$

$$\mathcal{A}^{(0,l)}_{\rm FB} = 0.0170 \pm 0.0016 \qquad ({\rm SM}:0.0153),$$

$$\mathcal{A}_{LR}({\rm SLC}) = 0.1637 \pm 0.0075. \tag{3.7}$$

In parentheses, we quote the theoretical predictions obtained in the SM. Note that $\mathcal{A}_e$ should equal the left-right asymmetry $\mathcal{A}_{LR}$ measured at the SLC. From (3.7), one can deduce $\Delta\mathcal{A}_{\tau e} \simeq -10\%$, when comparing the measurement of $\mathcal{A}_\tau$ at LEP with that of $\mathcal{A}_e$ at SLC. On the other hand, if one assumes that the accuracy in the measurement of $\mathcal{A}_{LR}$ is correct, one could then interpret the experimental sensitivity to $\mathcal{A}_{LR}$ as a stronger upper bound on new physics with $|\Delta\mathcal{A}_{\tau e}| < 4\%$. Furthermore, ongoing SLC experiments are measuring the observable

$$A^{\rm FB}_{LR}(f) = \frac{\Delta\sigma(e^-_L e^+ \to f\bar{f})_{\rm FB} - \Delta\sigma(e^-_R e^+ \to f\bar{f})_{\rm FB}}{\Delta\sigma(e^-_L e^+ \to f\bar{f})_{\rm FB} + \Delta\sigma(e^-_R e^+ \to f\bar{f})_{FB}}$$

$$= \frac{3}{4}\mathcal{P}_e\mathcal{A}_f.$$

(3.8)

The forward-backward left-right asymmetry for individual flavors represents an interesting alternative either to constraining or establishing possible deviations from SM lepton universality.

Leptonic asymmetries (or equivalently forward-backward asymmetries) can also play a crucial role to constrain new physics. Here, we will be interested in experiments at LEP/SLC that measure the observable

$$\mathcal{A}_l = \frac{\Gamma(Z \to l_L \bar{l}) - \Gamma(Z \to l_R \bar{l})}{\Gamma(Z \to l\bar{l})} = \frac{(g^{ll}_L)^2 - (g^{ll}_R)^2}{(g^{ll}_L)^2 + (g^{ll}_R)^2}$$

$$= \frac{(g^l_L)^2 - (g^l_R)^2 + 2(g^l_L\delta g^l_L - g_R\delta g^l_R)}{(g^l_L)^2 + (g^l_R)^2 + 2(g^l_L\delta g^l_L + g^l_R\delta g^l_R)}.$$

(3.9)

In view of the recent discrepancy of more than 2σ between the experimental results of $\mathcal{A}_{LR}$ at SLC and $\mathcal{A}_e$ at LEP, we will study the nonuniversality parameter of leptonic asymmetries [5]:

$$\Delta\mathcal{A}_{l_1 l_2} = \frac{\mathcal{A}_{l_1} - \mathcal{A}_{l_2}}{\mathcal{A}_{l_1} + \mathcal{A}_{l_2}}$$

$$= \frac{1}{\mathcal{A}^{({\rm SM})}_l}\left(U^{(l_1 l_2)}_{\rm br}(L) - U^{(l_1 l_2)}_{\rm br}(R)\right) - U^{(l_1 l_2)}_{\rm br},$$

(3.10)

where $\mathcal{A}^{({\rm SM})}_l$ may be given by the SM value in Eq. (3.7). At this point, it should be stressed that requiring $U^{(l_1 l_2)}_{\rm br} = 0$ does *not necessarily* imply $\Delta\mathcal{A}_{l_1 l_2} = 0$. As we will later see, in LRSM's one can naturally encounter the possibility in which $U_{\rm br}(L) \simeq -U_{\rm br}(R)$ while $\Delta\mathcal{A}_{l_1 l_2}$ becomes sizable. Moreover, the physical quantities $U^{(l_1 l_2)}_{\rm br}$ and $\Delta\mathcal{A}_{l_1 l_2}$ do not depend explicitly on universal electroweak oblique parameters, especially when the latter ones may poorly constrain such three-generation scenarios [4].

Another observable which will still be of interest is

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to {\rm hadrons})}$$

$$= 0.2202 \pm 0.0020 \qquad ({\rm SM}:0.2158). \tag{3.11}$$

If the measurement at LEP is correct, R_b turns out to be about 2σ off from the theoretical prediction of the minimal SM. New physics contributions to R_b can be conveniently calculated through [24]

$$R_b = 0.22\left[1 + 0.78\nabla^{({\rm SM})}_b(m_t) - 0.06\Delta\rho^{({\rm SM})}(m_t)\right],$$

(3.12)

where $\nabla_b^{(SM)}(m_t)$ and $\Delta\rho^{(SM)}(m_t)$ contains the m_t-dependent parts of the vertex and oblique corrections, respectively. Practically, only $\nabla_b^{(SM)}(m_t)$ gives significant negative contributions to R_b, which behave, in the large top-mass limit, as [25]

$$\nabla_b^{(SM)}(m_t) \simeq -\frac{20\alpha_w s_w^2}{13\pi}\frac{m_t^2}{M_Z^2}. \tag{3.13}$$

If there are new physics effects contributing to $\nabla_b^{(SM)}(m_t)$, these can be estimated by

$$\nabla_b^{(new)}(m_t) = \frac{\alpha_w}{2\pi}\frac{g_L^b\mathrm{Re}[\Gamma_{bb}^L(m_t) - \Gamma_{bb}^L(0)] + g_R^b\mathrm{Re}[\Gamma_{bb}^R(m_t) - \Gamma_{bb}^R(0)]}{(g_L^b)^2 + (g_R^b)^2}, \tag{3.14}$$

where $g_L^b = 1-2s_w^2/3$ and $g_R^b = -2s_w^2/3$. In the next section, we will analyze numerically the size of new physics effects expected in LRSM.

IV. NUMERICAL RESULTS AND DISCUSSION

Since there is a large number of free parameters that could vary independently in the LRSM, we have fixed to typical values all of them except one each time and investigated the behavior of our observables as a function of the remaining kinematic variable. More explicitly, we have found that δ_R^{++} quantum corrections to the effective $Zl_R\bar{l}_R$ coupling shown in Fig. 1 are very small, since $M_{\delta_R^{++}} > 5$ TeV for phenomenological reasons [11]. The very same lower mass bound should obey the flavor-changing scalars ϕ_2^{0r} [$\equiv \mathrm{Re}(\phi_2^0)/\sqrt{2}$] and ϕ_2^{0i} [$\equiv \mathrm{Im}(\phi_2^0)/\sqrt{2}$] [11]. However, the mass difference between the two flavor-changing scalars should not be too large because the latter would lead to large negative contributions to R_b (we will discuss the consequences from a large mass-difference realization between the flavor-changing scalars at the end of this section). In our estimates, we have assumed that ϕ_2^{0r} and ϕ_2^{0i} are nearly degenerate and heavier than 5 TeV. In such a case, loop effects involving flavor-changing scalars are found to be vanishingly small.

In order to increase the predictability of our LRSM but still keep our analysis on a general basis, we shall consider a two-generation mixing scenario. Then, the free parameters of our minimal model are the LF mixings $(s_L^{\nu_i})^2$ [which are, however, constrained, to some extent, by upper bounds given in Eq. (2.13)], the two heavy neutrino masses m_{N_1} and m_{N_2} (which have been taken to be at the same mass scale m_N), the masses of the charged gauge boson M_R and its orthogonal associate scalar M_h, and a Cabibbo-type angle θ_R that rotates the right-handed charged leptons to the corresponding mass eigenstates.

In Figs. 2(a–d), we present plots of $B(Z \to e^-\tau^+ + e^+\tau^-)$ as a function of m_N, M_R, M_h, and θ_R while keeping fixed the remaining kinematic parameters each time. In Fig. 2(a), we see the characteristic *quadric*, m_N^4/M_W^4, dependence of the branching ratio [3, 15]. The dashed, dotted, and dash-dotted lines represent results coming purely from the $SU(2)_R$ sector for $(s_L^{\nu_r})^2 = 0.040, 0.030,$ and 0.020, respectively. The solid lines i, ii, and iii correspond to a complete computation for the three different LF mixings mentioned above. If we assume some typical values for the rest of the parameters, i.e., $M_R = 0.4$ TeV, $M_h = 30$ TeV, and $\theta_R = 0$, we find that $B(Z \to e^-\tau^+ + e^+\tau^-) \lesssim 2.\,10^{-6}$ for $m_N = 3$ TeV. Although the size of new physics effects may be probed at LEP, the reported value is still $B(Z \to e\tau) < 10^{-5}$, and it does not yet impose rather severe constraints on the parameter space of the theory. This conclusion is also supported by Figs. 2(b–d). In Fig. 2(c), it is worth observing the logarithmic dependence of the branching ratio on the mass ratio M_h/M_R, which can also render the decay channel $Z \to e\tau$ measurable. In Fig. 2(d), one can further see the strong dependence of $B(Z \to e^-\tau^+ + e^+\tau^-)$ on θ_R. However, a similar, though complementary, behavior will be found to be present in the observables U_{br} and $\Delta\mathcal{A}$.

We now proceed by examining numerically the dependence of the universality-breaking parameter U_{br} as a function of various kinematic variables shown in Figs. 3(a–d). Again, we observe the nondecoupling behavior of the heavy neutrino mass in the observable U_{br} [4], which is closely related to potentially large $SU(2)_L$ Dirac mass terms, m_D, present in our model [15]. The size of new physics becomes significant for $m_N \lesssim 3$ TeV, i.e., $U_{\mathrm{br}} \sim 4$–5×10^{-3}. In Fig. 3(b), we see that the value of U_{br} decreases rapidly as M_R increases. In Fig. 3(c), we remark again on the logarithmic dependence of U_{br} on M_h^2/M_R^2. In our estimates, we have used a Cabibbo-type angle $\theta_R = 45°$, which turns out to be a rather moderate value as is displayed in Fig. 3(d). As has been mentioned above, the electroweak corrections originating genuinely from the $SU(2)_R$ sector depend on the angle θ_R. Looking at Fig. 3(d), one can readily see that the choice $\theta_R = 45°$ gives smaller effects of nonuniversality in the leptonic partial widths of the Z boson. If we had chosen $\theta_R = -45°$, in such a case we would have obtained much stronger combined bounds on the mass parameters and LF mixings.

One may get the impression that new-physics effects can be minimized by selecting θ_R to lie in a specific range. This is, however, not true, since the universality-breaking parameter $\Delta\mathcal{A}_{l_1l_2}$ will play a complementary role as is shown in Figs. 4(a–d). In Fig. 4, we list the results after adding both contributions coming from $SU(2)_L$ and $SU(2)_R$ gauge sectors. Thus, we may be sensitive up to

$m_N \lesssim 1.5$ TeV for $(s_L^{\nu_\tau})^2 = 0.04$ and $(s_L^{\nu_e})^2 = 0.01$ [see Fig. 4(a)]. In Fig. 4(b), we display the decoupling effect of a very heavy W_R. In Fig. 4(c), we find again the logarithmic enhancement caused by the nondegeneracy between $W_R^\pm$ and $h^\pm$. In addition, it should be noted that interesting phenomenology can only arise for relatively light W_R bosons, i.e., $M_R \lesssim 1$ TeV. The latter observation can also be verified from Fig. 4(d), in which $\Delta\mathcal{A}$ is drawn as a function of θ_R for $M_R = 0.4$, 0.6, and 0.8 TeV. Furthermore, one can recognize the complementary role that $B(Z \to l_1 l_2)$, U_{br}, and $\Delta\mathcal{A}$ play as far as θ_R is concerned, when comparing Figs. 2 (d), 3(d), and 4(d). For example, the choice $\theta_R = -45°$ would make $\Delta\mathcal{A}$ more difficult

to observe, whereas U_{br} becomes larger for this value of θ_R. Of course, scenarios where M_R is at the TeV scale may not be compatible with K_L-K_S phenomenology if we assume an exact left-right symmetry in the Yukawa sector of the model. Nevertheless, in LRSM's that possess nonmanifest or pseudomanifest left-right symmetry, such a constraint is not valid any longer [26].

In the following, we will try to address the question of whether there exist possibilities of inducing positive contributions to R_b within our LRSM. As has already been noticed in Sec. III, only positive contributions to R_b are of potential interest, which will help to achieve a better agreement between theoretical prediction and the
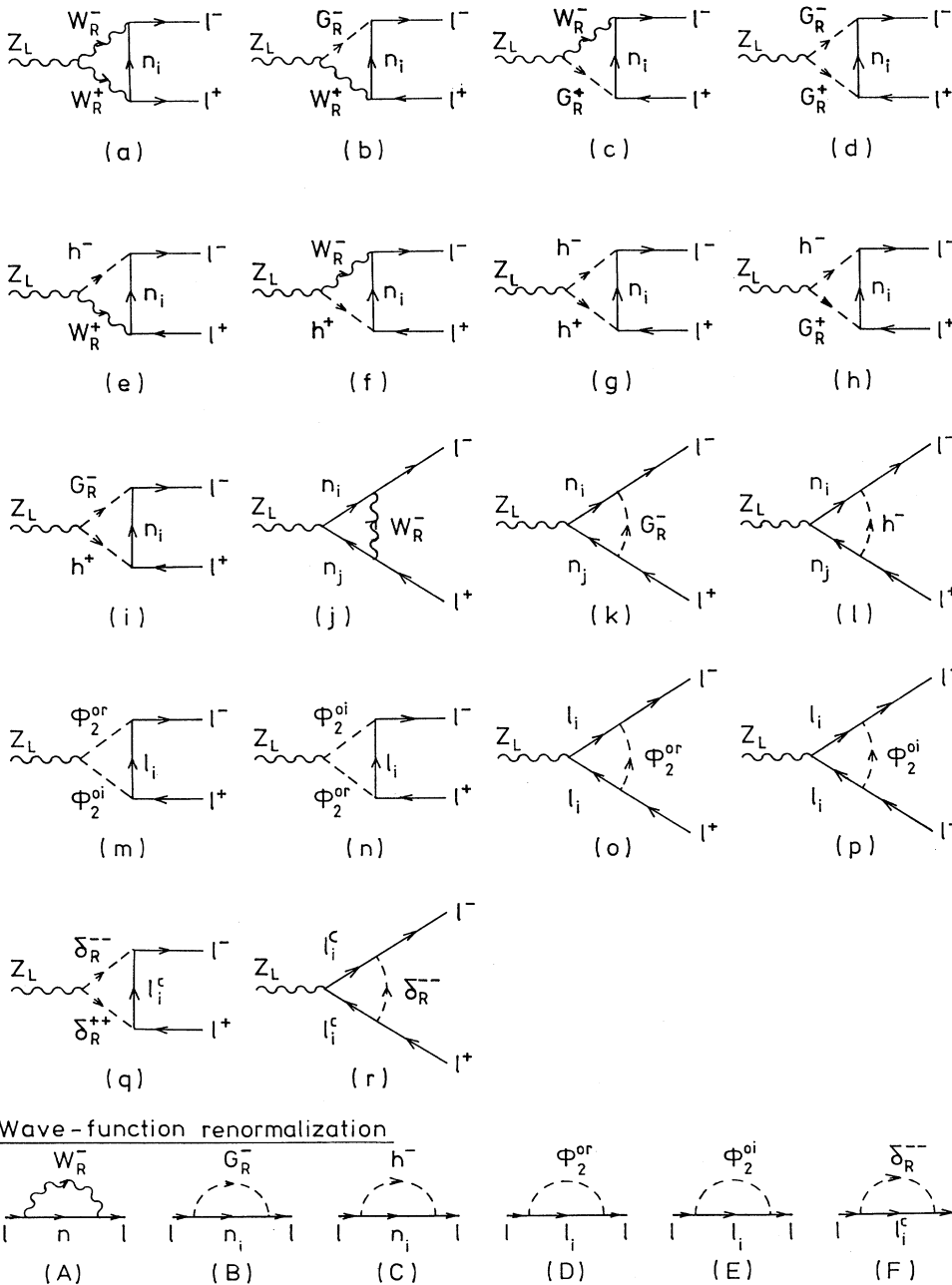

FIG. 1. Feynman graphs contributing to the effective nonoblique $Zl_R\bar{l}_R$ coupling in the LRSM.

experimental value of R_b. In the LRSM, we first consider the Feynman graphs $1(m)$ and $1(n)$, where the external leptons are replaced by b quarks and virtual down-type quarks are running in the place of charged leptons. The interaction Lagrangians of the flavor-changing scalars ϕ_2^{0r} and ϕ_2^{0i} with the d, s, b quarks can be obtained by Eq. (A17) after making the obvious replacements. These couplings are enhanced, as they are proportional to the top-quark mass. In fact, the flavor-changing scalars generate effective $Zb\bar{b}$ couplings of both $V-A$ and $V+A$ nature. In the limit $M_{\phi_2^{0r}}$, $M_{\phi_2^{0i}} \gg M_Z$, the effective $Zb\bar{b}$ couplings take the simple form

$$\mathrm{Re}[\Gamma_{bb}^R(m_t) - \Gamma_{bb}^R(0)]$$

$$= \frac{1}{8}|V_{tb}^R|^2 \frac{m_t^2}{M_W^2} \left(\frac{\lambda_S + \lambda_I}{2(\lambda_S - \lambda_I)} \ln \frac{\lambda_S}{\lambda_I} - 1 \right), \quad (4.1)$$

$$\mathrm{Re}[\Gamma_{bb}^L(m_t) - \Gamma_{bb}^L(0)]$$

$$= -\frac{1}{8}|V_{tb}^L|^2 \frac{m_t^2}{M_W^2} \left(\frac{\lambda_S + \lambda_I}{2(\lambda_S - \lambda_I)} \ln \frac{\lambda_S}{\lambda_I} - 1 \right), \quad (4.2)$$

where λ_S and λ_I are defined in Appendix B after Eq. (B1). The analytic function in the parentheses of the right-hand side (RHS) of Eqs. (4.1) and (4.2) is always positive and equals zero when the two scalars ϕ_2^{0r}, ϕ_2^{0i} are degenerate. Substituting Eqs. (4.1) and (4.2) into Eq. (3.14), one finds that the SM value of R_b gets further decreased. This leads automatically to the restriction

$$M_{\phi_2^{0r}} \simeq M_{\phi_2^{0i}}. \quad (4.3)$$

The mass relation (4.3) has been used throughout our numerical estimates.

Other quantum corrections that could help to produce positive contributions to R_b are due to diagrams similar
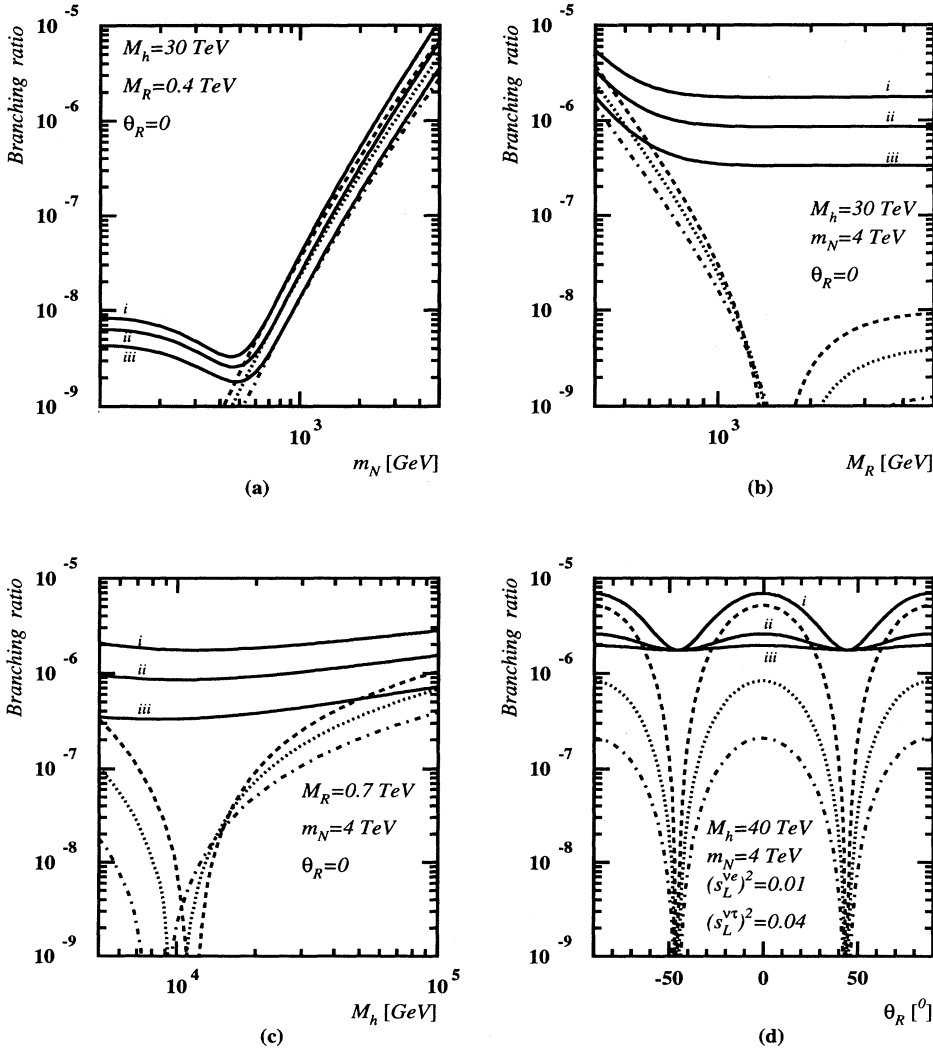

FIG. 2. $B(Z \rightarrow l_1 \bar{l}_2 + \bar{l}_1 l_2, l_1 \neq l_2)$ in a two-generation LRSM as a function of (a) the heavy neutrino mass $m_N (= m_{N_1} = m_{N_2})$, (b) W_R-boson mass M_R, (c) charged Higgs boson M_h [for the LF mixings $(s_L^{\nu_\tau})^2 = 0.04$ (curve i), 0.03 (curve ii), 0.020 (curve iii), and $(s_L^{\nu_e})^2 = 0.01$, $(s_L^{\nu_\mu})^2 = 0]$, and (d) a Cabibbo-type angle θ_R [for $M_R = 0.4$ TeV (curve i), 0.6 TeV (curve ii), and 0.8 TeV (curve iii)]. Numerical estimates coming solely from the SU(2)$_R$ sector are also shown. The results analogous to curves i, ii, and iii are correspondingly given by the dashed, dotted, and dash-dotted lines.

to Figs. 1(h) and 1(d), which involve the t and b quarks. Indeed, an analogous calculation gives

$$\mathrm{Re}[\Gamma_{bb}^{R}(m_t) - \Gamma_{bb}^{R}(0)] = -\frac{1}{4}|V_{tb}^{R}|^{2}\frac{m_t^{2}}{M_W^{2}}\,s_{\beta}^{2}c_{\beta}^{2}$$

$$\times\left(\frac{\lambda_h + \lambda_R}{2(\lambda_h - \lambda_R)}\,\ln\frac{\lambda_h}{\lambda_R} - 1\right).$$

$$(4.4)$$

However, the RHS of Eq. (4.4) is proportional to $s_{\beta}^{2} = M_W^{2}/M_R^{2}$ yielding a rather small effect. If we insist on canceling the negative SM vertex correction $\nabla_{b}^{SM}(m_t)$ through the contribution (4.4), we find the highly unnatural mass ratio

$$\frac{M_h}{M_R} \simeq e^{70}.$$

The latter also demonstrates the difficulty to obtain radiatively positive contributions to R_b within the LRSM. In Ref. [27], it has been indicated that Z_L-Z_R mixing

effects may produce contributions of either sign to R_b. We will not pursue this topic here. Instead, we review in short the theoretical situation for a possible positive increase of R_b, as it has been predicted in various extensions of the SM so far.

In the minimal supersymmetric (SUSY) SM, R_b can, in principle, receive a positive shift coming from the large Yukawa coupling of the charged Higgsino to the scalar t and b quarks [28]. R_b can also get enhanced from large $\tan\beta$ scenarios [28, 29]. However, considering a number of constraints originating from $B(b \to s\gamma)$ measurements, relic abundances of the lightest SUSY particle, the net SUSY effect on R_b is considerably reduced and R_b is found to be ~ 0.2166, which is still about 1.5σ below the experimental value given in Eq. (3.11) [30]. In this context, a wide class of extended techicolor theories gives rise to a negative shift to R_b [29]. An interesting exception to this feature is specific technicolor models, in which fermion masses are generated radiatively via mixing of technibaryons [31].
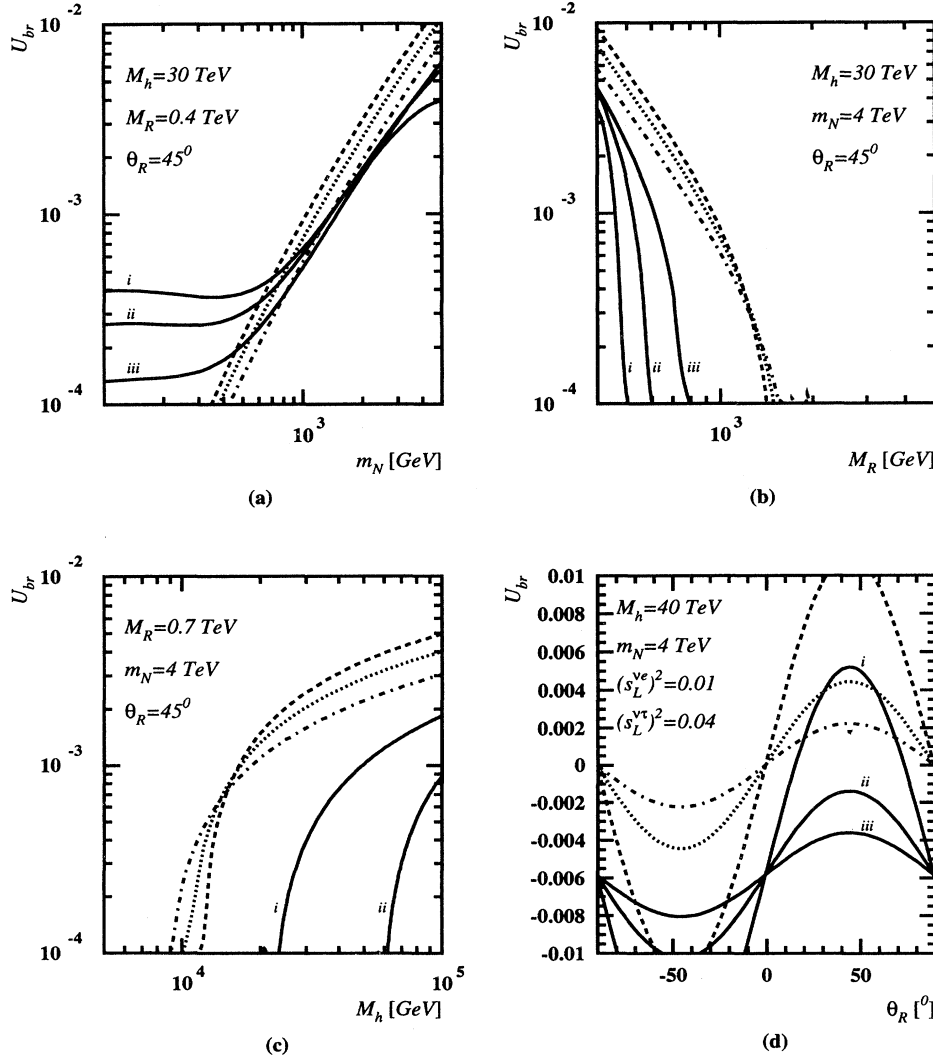

FIG. 3. Numerical estimates of $U_{br}^{l_1 l_2}$ for the same set of parameters as in Fig. 2.

V. CONCLUSIONS

We have shown that lepton-flavor-violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of lepton asymmetries at LEP/SLC represent a set of complementary observables and can hence impose severe limitations on model building in the leptonic sector. For our illustrative purposes, we have considered a LRSM with two-generation mixing. We have found that the observables $B(Z \to l_1 l_2)$, $U_{\mathrm{br}}^{l_1 l_2}$, and $\Delta \mathcal{A}_{l_1 l_2}$ are sensitive to different parameter-space regions of this minimal scenario. For instance, if $(s_L^{\nu_\tau})^2 = 0.03$, $(s_L^{\nu_e})^2 = 0.01$, $M_R = 0.4$ TeV, and $M_h = 30$ TeV, then heavy neutrinos are found to have masses that do not exceed 2 TeV for any value of the Cabibbo-type angle θ_R. On the other hand, constraints on new physics from R_b prefer scenarios in which flavor-changing scalars are degenerate in mass.

It may be worth remarking again the fact that LRSM's can naturally predict $U_{\mathrm{br}} \simeq 0$, for some choice of parameters, which could naively be interpreted that lepton universality is preserved in nature. As has been shown in this paper, universality violation can manifest itself in lepton asymmetries $\Delta \mathcal{A}_{l_1 l_2}$ as well. This is, however, not an accidental feature of the LRSM but may have a general applicability to unified models, such as supersymmetric extensions of the SM [5]. In general, such theories can naturally generate both nonuniversal $V-A$ and $V+A$ $Z f \bar{f}$ couplings, yielding effects that may be detected by current experiments at LEP and SLC.
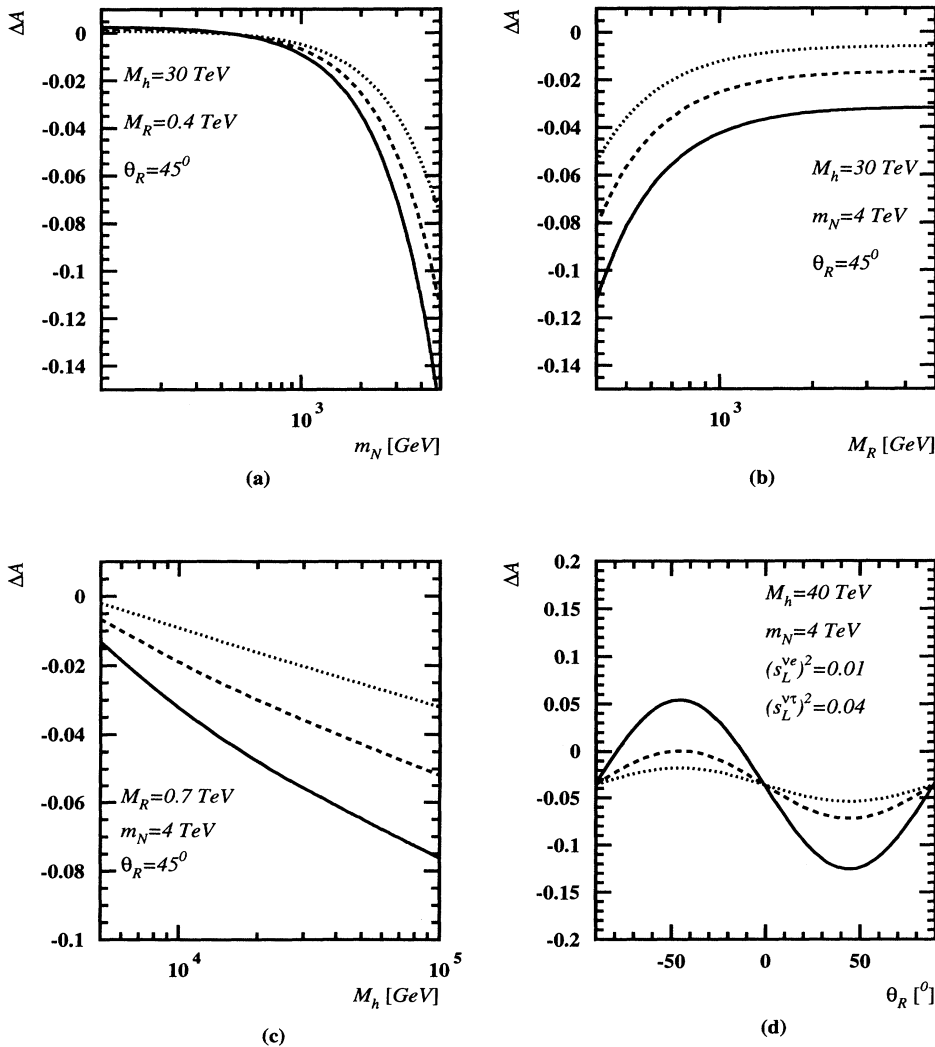
ACKNOWLEDGMENTS

FIG. 4. Numerical estimates of $\Delta \mathcal{A}_{l_1 l_2}$ for the same set of parameters as in Fig. 2. Only the total LRSM contribution to $\Delta \mathcal{A}_{l_1 l_2}$ is shown, where the corresponding curves i, ii, and iii are now given by the solid, dashed, and dotted lines, respectively.

APPENDIX A: FEYNMAN RULES IN THE LRSM

Although some of the Feynman rules required in our problem were given in Refs. [9, 11], we list all the relevant Feynman rules and Lagrangians governing the interactions of the gauge and Higgs bosons with leptons and neutrinos, as well as the trilinear couplings of the bosons. The covariant derivative acts on the Higgs multiplets as follows:

$$D_\mu \Phi = \partial_\mu \Phi + i \frac{g_L}{2} \vec{\sigma} \cdot \vec{W}_{L\mu} \Phi - i \frac{g_R}{2} \Phi \vec{\sigma} \cdot \vec{W}_{R\mu}, \quad (A1)$$

$$D_\mu \Delta_{R,L} = \partial_\mu \Delta_{R,L} + i \frac{g_{R,L}}{2} [\vec{\sigma} \cdot \vec{W}_{R,L\mu}, \Delta_{R,L}]$$
$$+ i g' B_\mu \Delta_{R,L}, \quad (A2)$$

where σ_i are the known 2×2 Pauli matrices, and g_L [g_R] are the $SU(2)_L$ [$SU(2)_R$] weak coupling constants which will be set equal to $g_w = g_L = g_R$ [g_w is the usual $SU(2)_L$ weak coupling constant in the SM]. To facilitate our computational task, we will further assume that the corresponding neutral gauge boson Z_L is the Z of the SM to a good approximation. Also, we will list the novel LRSM interactions together with the SM couplings in order to avoid possible ambiguities between relative signs.

The trilinear couplings of gauge, Higgs, and would-be Goldstone bosons may therefore be obtained by (all momenta flow into the vertex)

$$Z_{L\nu}(r)W^+_{L\lambda}(p)W^-_{L\mu}(q) : -i g_w c_w f_{\mu\nu\lambda}(q,r,p), \quad (A3)$$

$$Z_{L\nu}(r)W^+_{R\lambda}(p)W^-_{R\mu}(q) : i g_w \frac{s_w^2}{c_w} f_{\mu\nu\lambda}(q,r,p), \quad (A4)$$

$$Z_{L\nu}(r)G^+_L(p)W^-_{L\mu}(q) : -i g_w M_W \frac{s_w^2}{c_w} g_{\mu\nu}, \quad (A5)$$

$$Z_{L\nu}(r)G^+_R(p)W^-_{R\mu}(q) : i g_w M_W \frac{s_\beta^2 - s_w^2}{s_\beta c_w} g_{\mu\nu}, \quad (A6)$$

$$Z_{L\nu}(r)h^+(p)W^-_{R\mu}(q) : -i g_w M_W \frac{c_\beta}{c_w} g_{\mu\nu}, \quad (A7)$$

$$Z_{L\mu}(r)G^+_L(p)G^-_L(q) : -i \frac{g_w}{2c_w}(1 - 2s_w^2)(p - q)_\mu, \quad (A8)$$

$$Z_{L\mu}(r)G^+_R(p)G^-_R(q) : -i \frac{g_w}{2c_w}(s_\beta^2 - 2s_w^2)(p - q)_\mu, \quad (A9)$$

$$Z_{L\mu}(r)h^+(p)h^-(q) : -i \frac{g_w}{2c_w}(c_\beta^2 - 2s_w^2)(p - q)_\mu, \quad (A10)$$

$$Z_{L\mu}(r)G^\pm_R(p)h^\mp(q) : \mp i \frac{g_w}{2c_w} s_\beta c_\beta (p - q)_\mu, \quad (A11)$$

$$Z_{L\mu}(r)\phi^{0r}_2(p)\phi^{0i}_2(q) : \frac{g_w}{2c_w}(p - q)_\mu, \quad (A12)$$

$$Z_{L\mu}(r)\delta^{++}_R(p)\delta^{--}_R(q) : 2i \frac{g_w}{c_w} s_w^2 (p - q)_\mu. \quad (A13)$$

Here, we have defined $s_\beta = \sqrt{1 - c_\beta^2} = M_W/M_R$ and the Lorentz tensor $f_{\mu\nu\lambda}(q,r,p) = (r - q)_\lambda g_{\mu\nu} + (q - p)_\nu g_{\lambda\mu} + (p - r)_\mu g_{\nu\lambda}$.

The corresponding couplings of the gauge, Higgs, and would-be Goldstone bosons to the charged leptons and neutrinos can be read off from the Lagrangians

$$\mathcal{L}^{W_R}_{int} = -\frac{g_w}{\sqrt{2}} W^{-\mu}_R B^R_{li} \, \bar{l}\gamma_\mu P_R n_i \quad + \quad \text{H.c.,} \quad (A14)$$

$$\mathcal{L}^{G^-_R}_{int} = -\frac{g_w}{\sqrt{2}M_W} s_\beta \, G^-_R B^R_{li} \, \bar{l}\Big[m_l P_R - m_{n_i} P_L\Big] n_i \quad + \quad \text{H.c.,} \quad (A15)$$

$$\mathcal{L}^{h^-}_{int} = \frac{g_w}{\sqrt{2}M_W} c_\beta \, h^- \, \bar{l}\left[B^R_{li} m_l P_R - B^R_{lj}\left(\delta_{ji} - \frac{C^{R*}_{ji}}{c_\beta^2}\right) m_{n_j} P_L\right] n_i \quad + \quad \text{H.c.,} \quad (A16)$$

$$\mathcal{L}^{\phi^0_2}_{int} = -\frac{g_w}{2M_W}\phi^{0r}_2 \, \bar{l}_1\Big[B^L_{l_1 j} m_{n_j} B^{R*}_{l_2 j} P_R + B^R_{l_1 j} m_{n_j} B^{L*}_{l_2 j} P_L\Big] l_2 - \frac{i g_w}{2M_W}\phi^{0i}_2 \, \bar{l}_1\Big[B^L_{l_1 j} m_{n_j} B^{R*}_{l_2 j} P_R - B^R_{l_1 j} m_{n_j} B^{L*}_{l_2 j} P_L\Big] l_2, \quad (A17)$$

$$\mathcal{L}^{\delta^{++}_R}_{int} = \frac{g_w}{2\sqrt{2}M_W}\frac{s_\beta}{c_\beta}\delta^{++}_R \, \bar{l}^C_1 B^{R*}_{l_1 j} m_{n_j} B^{R*}_{l_2 j} P_R l_2 \quad + \quad \text{H.c.,} \quad (A18)$$

where the mixing matrices B^R and C^R are defined in Sec. II, $\phi^{0r}_2 = \text{Re}(\phi^0_2)/\sqrt{2}$, and $\phi^{0i}_2 = \text{Im}(\phi^0_2)/\sqrt{2}$. The couplings of Z_L ($\equiv Z$) to Majorana neutrinos may be found in Ref. [13].

APPENDIX B: THE NONOBLIQUE $ZL\bar{L}$ VERTEX

We analytically evaluate the loop amplitudes in the limit of vanishing external lepton masses. We adopt dimensional regularization in conjunction with the reduction algorithm of Ref. [32]. Unlike the metric notation of Ref. [32], we use the Minkowskian metric, $g^{\mu\nu} = \text{diag}(1, 1, ..., -1)$.

The nonoblique effective Zll' vertex function is similar to the one obtained in [3]. Its analytic form is given by (summation over repeated indices implied)

$$
\Gamma_{ll'}^L = B_{li}^L B_{l'j}^{L*} \Bigg(\delta_{ij} \Big\{ c_w^2 \lambda_Z \Big(C_{11}(\lambda_i, 1, 1) + C_{23}(\lambda_i, 1, 1) - C_{22}(\lambda_i, 1, 1) \Big)
$$

$$
+ 6 c_w^2 C_{24}(\lambda_i, 1, 1) - s_w^2 \lambda_i C_0(\lambda_i, 1, 1) + \frac{1}{2}(1 - 2 s_w^2)[\lambda_i C_{24}(\lambda_i, 1, 1)
$$

$$
+ \frac{1}{2}\lambda_i B_1(0, \lambda_i, 1) + B_1(0, \lambda_i, 1)] \Big\}
$$

$$
+ C_{ij}^L \Big\{ -C_{24}(1, \lambda_i, \lambda_j) - \frac{1}{2}\lambda_Z[C_0(1, \lambda_i, \lambda_j) + C_{11}(1, \lambda_i, \lambda_j)
$$

$$
+ C_{23}(1, \lambda_i, \lambda_j) - C_{22}(1, \lambda_i, \lambda_j)] - \frac{1}{4}\lambda_i \lambda_j C_0(1, \lambda_i, \lambda_j) \Big\}
$$

$$
+ \frac{1}{2} C_{ij}^{L*} \sqrt{\lambda_i \lambda_j} \Big\{ C_0(1, \lambda_i, \lambda_j) + \frac{1}{2}\lambda_Z[C_{23}(1, \lambda_i, \lambda_j) - C_{22}(1, \lambda_i, \lambda_j)] + C_{24}(1, \lambda_i, \lambda_j) \Big\}
$$

$$
+ \frac{1}{4} C_{ij}^R \sqrt{\lambda_i \lambda_j} \Big\{ 2 C_{24}(0, \lambda_S, \lambda_I) - C_{24}(\lambda_S, 0, 0) - C_{24}(\lambda_I, 0, 0) + \frac{1}{2}
$$

$$
+ s_w^2[B_1(0, 0, \lambda_S) + B_1(0, 0, \lambda_I)] - \frac{1}{2}\lambda_Z[C_{23}(\lambda_S, 0, 0) - C_{22}(\lambda_S, 0, 0) + C_{23}(\lambda_I, 0, 0) - C_{22}(\lambda_I, 0, 0)] \Big\} \Bigg), \quad \text{(B1)}
$$

where $\lambda_i = m_{n_i}^2 / M_W^2$, $\lambda_Z = M_Z^2 / M_W^2$, $\lambda_S = M_{\phi_2^0r}^2 / M_W^2$, and $\lambda_I = M_{\phi_2^0i}^2 / M_W^2$. Note that there is a contribution proportional to C_{ij}^R that originates solely from the Higgs sector of the LRSM. In the notation of [33], the first three of the six arguments of the C functions are always evaluated at $(0, \lambda_Z, 0)$.

In the LRSM, virtual neutrinos and Higgs scalars induce a nonuniversal Z boson coupling to right-handed charged leptons, Γ^R, as shown in Fig. 1. The contributions of the individual graphs to Γ^R are listed below:

$$
\Gamma_{ll'}^R(a) = B_{li}^R B_{l'i}^{R*} s_w^2 \Big\{ \lambda_Z[C_{22}(\lambda_i, \lambda_R, \lambda_R) - C_{23}(\lambda_i, \lambda_R, \lambda_R) - C_{11}(\lambda_i, \lambda_R, \lambda_R)] - 6 C_{24}(\lambda_i, \lambda_R, \lambda_R) \Big\}, \quad \text{(B2)}
$$

$$
\Gamma_{ll'}^R(b + c) = B_{li}^R B_{l'i}^{R*}(s_w^2 - s_\beta^2)\lambda_i C_0(\lambda_i, \lambda_R, \lambda_R), \quad \text{(B3)}
$$

$$
\Gamma_{ll'}^R(d) = \frac{1}{2} B_{li}^R B_{l'i}^{R*} s_\beta^2(s_\beta^2 - 2 s_w^2)\lambda_i C_{24}(\lambda_i, \lambda_R, \lambda_R), \quad \text{(B4)}
$$

$$
\Gamma_{ll'}^R(e + f) = -\frac{1}{2} B_{li}^R B_{l'j}^{R*}(\delta_{ij} s_\beta^2 \lambda_i - \sqrt{\lambda_i \lambda_j} C_{ij}^L)[C_0(\lambda_i, \lambda_R, \lambda_h) + C_0(\lambda_j, \lambda_R, \lambda_h)], \quad \text{(B5)}
$$

$$
\Gamma_{ll'}^R(g) = \frac{1}{2} B_{lk}^R B_{l'n}^{R*} \sqrt{\lambda_k \lambda_n} \left(1 - \frac{2 s_w^2}{c_\beta^2}\right)(s_\beta^2 \delta_{ki} - C_{ki}^L)(s_\beta^2 \delta_{in} - C_{in}^L) C_{24}(\lambda_i, \lambda_h, \lambda_h), \quad \text{(B6)}
$$

$$
\Gamma_{ll'}^R(h + i) = -\frac{1}{2} B_{li}^R B_{l'j}^{R*} \sqrt{\lambda_i \lambda_j} s_\beta^2(s_\beta^2 \delta_{ij} - C_{ij}^L)[C_{24}(\lambda_i, \lambda_R, \lambda_h) + C_{24}(\lambda_j, \lambda_R, \lambda_h)], \quad \text{(B7)}
$$

$$
\Gamma_{ll'}^R(j) = -\frac{1}{2} B_{li}^R B_{l'j}^{R*} \Bigg(C_{ij}^{L*} \Big\{ 1 - 2 C_{24}(\lambda_R, \lambda_i, \lambda_j) - \lambda_Z[C_0(\lambda_R, \lambda_i, \lambda_j) + C_{11}(\lambda_R, \lambda_i, \lambda_j)
$$

$$
+ C_{23}(\lambda_R, \lambda_i, \lambda_j) - C_{22}(\lambda_R, \lambda_i, \lambda_j)] \Big\} + C_{ij}^L \sqrt{\lambda_i \lambda_j} C_0(\lambda_R, \lambda_i, \lambda_j) \Bigg), \quad \text{(B8)}
$$

$$
\Gamma_{ll'}^R(k) = -\frac{1}{4} B_{li}^R B_{l'j}^{R*} s_\beta^2 \sqrt{\lambda_i \lambda_j} \Bigg(C_{ij}^L \Big\{ 2 C_{24}(\lambda_R, \lambda_i, \lambda_j) - \frac{1}{2} + \lambda_Z[C_{23}(\lambda_R, \lambda_i, \lambda_j)
$$

$$
- C_{22}(\lambda_R, \lambda_i, \lambda_j)] \Big\} - C_{ij}^{L*} \sqrt{\lambda_i \lambda_j} C_0(\lambda_R, \lambda_i, \lambda_j) \Bigg), \quad \text{(B9)}
$$

$$
\Gamma_{ll'}^R(l) = -\frac{1}{4} B_{lk}^R B_{l'n}^{R*} \sqrt{\lambda_k \lambda_n} \frac{1}{c_\beta^2}(s_\beta^2 \delta_{ki} - C_{ki}^L)(s_\beta^2 \delta_{nj} - C_{jn}^L) \Bigg(C_{ij}^L \Big\{ 2 C_{24}(\lambda_h, \lambda_i, \lambda_j) - \frac{1}{2}
$$

$$
+ \lambda_Z[C_{23}(\lambda_h, \lambda_i, \lambda_j) - C_{22}(\lambda_h, \lambda_i, \lambda_j)] \Big\} - C_{ij}^{L*} \sqrt{\lambda_i \lambda_j} C_0(\lambda_h, \lambda_i, \lambda_j) \Bigg), \quad \text{(B10)}
$$

$$\Gamma_{ll'}^R (m+n) = -\frac{1}{2} B_{li}^R B_{l'j}^{R*} C_{ij}^L \sqrt{\lambda_i \lambda_j} C_{24}(0, \lambda_S, \lambda_I),$$ (B11)

$$\Gamma_{ll'}^R (o+p) = \frac{1}{8} B_{li}^R B_{l'j}^{R*} C_{ij}^L (1 - 2s_w^2) \sqrt{\lambda_i \lambda_j} \left\{ 2C_{24}(\lambda_S, 0, 0) + 2C_{24}(\lambda_I, 0, 0) - 1 \right.$$

$$\left. + \lambda_Z [C_{23}(\lambda_S, 0, 0) - C_{22}(\lambda_S, 0, 0) + C_{23}(\lambda_I, 0, 0) - C_{22}(\lambda_I, 0, 0)] \right\},$$ (B12)

$$\Gamma_{ll'}^R (q) = \frac{1}{2} B_{li}^R B_{l'j}^{R*} C_{ij}^{R*} s_w^2 \frac{s_\beta^2}{c_\beta^2} \sqrt{\lambda_i \lambda_j} C_{24}(0, \lambda_\delta, \lambda_\delta),$$ (B13)

$$\Gamma_{ll'}^R (r) = -\frac{1}{8} B_{li}^R B_{l'j}^{R*} C_{ij}^{R*} s_w^2 \frac{s_\beta^2}{c_\beta^2} \sqrt{\lambda_i \lambda_j} \left\{ 2C_{24}(\lambda_\delta, 0, 0) - \frac{1}{2} + \lambda_Z [C_{23}(\lambda_\delta, 0, 0) - C_{22}(\lambda_\delta, 0, 0)] \right\},$$ (B14)

where $\lambda_h = M_{h+}^2 / M_W^2$ and $\lambda_\delta = M_{\delta_R^{++}}^2 / M_W^2$. In addition to the irreducible three-point functions, we should take wave-function renormalization constants into account [Figs. 1(A)–1(F)]. These additional nonuniversal corrections generated by the self-energies are calculated to give

$$\Gamma_{ll'}^R (A) = -\frac{1}{2} B_{li}^R B_{l'i}^{R*} s_w^2 [1 + 2B_1(0, \lambda_i, \lambda_R)],$$ (B15)

$$\Gamma_{ll'}^R (B) = -\frac{1}{2} B_{li}^R B_{l'i}^{R*} s_w^2 s_\beta^2 \lambda_i B_1(0, \lambda_i, \lambda_R),$$ (B16)

$$\Gamma_{ll'}^R (C) = -\frac{1}{2} B_{lk}^R B_{l'n}^{R*} \frac{s_w^2}{c_\beta^2} \sqrt{\lambda_k \lambda_n} (s_\beta^2 \delta_{ki} - C_{ki}^L)(s_\beta^2 \delta_{in} - C_{in}^L) B_1(0, \lambda_i, \lambda_h),$$ (B17)

$$\Gamma_{ll'}^R (D+E) = -\frac{1}{4} B_{li}^R B_{l'j}^{R*} C_{ij}^L s_w^2 \sqrt{\lambda_i \lambda_j} [B_1(0, 0, \lambda_S) + B_1(0, 0, \lambda_I)],$$ (B18)

$$\Gamma_{ll'}^R (F) = \frac{1}{8} B_{li}^R B_{l'j}^{R*} C_{ij}^{R*} s_w^2 \frac{s_\beta^2}{c_\beta^2} \sqrt{\lambda_i \lambda_j} B_1(0, 0, \lambda_\delta).$$ (B19)

The sum of Eqs. (B2)–(B19) should be free from UV divergences, when $l \neq l'$. This can easily be verified by employing the identities that the mixing matrices $B^{L,R}$ and $C^{L,R}$ obey (see also discussion in Sec. II). An ultimate check for the correctness of our analytic results is the vanishing of all terms involving s_w^2 in the limit $\lambda_Z \to 0$, due to a vectorlike Ward identity associated with the electromagnetic current.

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