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Charmonium in a weakly coupled quark-gluon plasma

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We present a model of charmonium as two heavy quarks propagating classically in a weakly coupled quark-gluon plasma. The quarks interact via a static, color-dependent potential and also suffer collisions with the plasma particles. We calculate the radiation width of the color octet state (for fixed, classical $q\bar{q}$ separation) and find that the state is long lived provided a finite gluon mass is used to establish a threshold energy.

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Theoretical studies of quarkonium in the quark-gluon plasma have generally treated the $q\bar{q}$ bound state as a unit, a color-singlet meson propagating in the medium [1–4], evolving slowly [5–8], and colliding with the plasma's constituents [9,10]. Paradoxically, the interaction binding the $q\bar{q}$ is taken to be a static, screened Coulomb potential [11,12], which implies that the density of plasma particles is high enough to allow construction of such a smooth potential—a weakly coupled plasma [13,14]. In this Brief Report we adopt the second point of view and construct a model accordingly. We consider [15] a picture of quarkonium as two heavy particles propagating classically, subject to a screened potential; these particles collide *individually* with plasma particles, changing momentum and color thereby. Such a model can be used to study in detail the diffusion of unbound $c\bar{c}$ pairs and the formation and destruction of charmonium in the plasma.

For simplicity, we specify the potential in the plasma rest frame, and neglect the associated color-magnetic interaction. In writing the $q\bar{q}$ interaction we keep track of the color state c of the pair. The potential takes the form

$$V_c(r) = \zeta_c \frac{\alpha_c}{r} \exp(-r/r_D) \ , \tag{1}$$

where $\zeta_c = -4/3$ and $1/6$ for the singlet and octet states, respectively. We distinguish *a priori* three regimes in r (see Fig. 1).

(1) For large r the interaction is negligible and the system is color-blind—the two quarks diffuse independently in the plasma [16], and their color state (which is irrelevant to their motion) is determined statistically.

(2) For intermediate r one must keep track of the color state in detail, and propagate the quarks according to the potential appropriate to the instantaneous color state.

(3) For small r the large gap between singlet and octet might induce a large transition rate due to gluon radiation. If the width Γ_8 of the octet state is larger than the gap ΔV, the octet ceases to exist as an excited state of the pair, vanishing into the $(q\bar{q})_{\text{singlet}} + g$ continuum.

The boundaries of these regions must be determined by explicit calculation of transition rates.

Before presenting the formal kinetic theory, we address the question of the existence of region 3. Transitions between the color states are caused by collisions of the individual heavy particles with plasma particles, and by radiation and absorption of gluons. Only radiation contributes to the octet state's width in the lowest order in α_s. The radiating quark moves in the potential field of its partner, which is otherwise a spectator to the process (see Fig. 2). For an initial quark momentum p relative to the plasma, the radiation rate is given by

$$\Gamma(p) = \frac{1}{2E_p} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3k}{(2\pi)^3 2k} \tilde{g}(k) (2\pi)^4 \delta^3(\mathbf{p}' + \mathbf{k} + \mathbf{p}) \times \delta(k + E_{p'} + V_1(r) - E_p - V_8(r)) \sum |\mathcal{M}|^2 \ . \tag{2}$$

Here $\tilde{g}(k)$ is the statistical factor for the outgoing gluon; the matrix element squared is averaged over initial spin and summed over final spins of the gluon and the participating quark. The summation over colors must be done carefully because of the condition that the initial state be an octet and the final state a singlet. We define a basis of

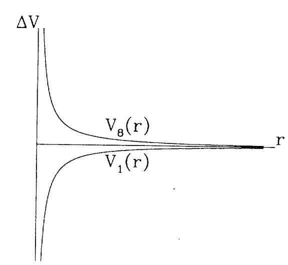

FIG. 1. The $q\bar{q}$ potential in color-singlet and color-octet channels, not drawn to scale.

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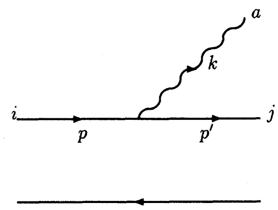

FIG. 2. Feynman diagram for gluon emission from a quark. i, j, and a are color indices. The lower line represents the spectator antiquark.

color state vectors $|ij\rangle$, where $i, j = 1,2,3$ are the color indices of the quark and antiquark, respectively. Then the singlet state is

$$|1\rangle = \frac{1}{\sqrt{3}} \sum_i |ii\rangle \tag{3}$$

while a member of the octet is[1]

$$|8r\rangle = \sqrt{2} \sum_{ij} T^r_{ij} |ij\rangle \ . \tag{4}$$

The Feynman matrix element then contains the color factor

$$C^{ar} = \langle 1|T^a|8r\rangle = \frac{1}{\sqrt{6}} \delta^{ar} \ , \tag{5}$$

where the color operator T^a acts on either the quark or the antiquark state only. Squaring the matrix element, summing over the color a of the emitted gluon, and averaging over r gives

$$\frac{1}{8} \sum_{a,r} |C^{ar}|^2 = \frac{1}{6} \ . \tag{6}$$

The result is

$$\sum |\mathcal{M}|^2 = \frac{8\pi\alpha_s}{3}(2m^2 - p \cdot p') \ . \tag{7}$$

Of course, either the quark or the antiquark may radiate; if their momenta are p_1 and p_2, then the total radiation rate from the pair is $\Gamma_8 = \Gamma(p_1) + \Gamma(p_2)$.

Calculating $\Gamma(p)$ from (2) gives the nonsensical result that $\Gamma(p) \neq 0$ in the limit $\Delta V \to 0$. In this limit

$$\Gamma(p = 0) \sim \frac{\alpha_s}{3m} \int \frac{kdk}{k + \sqrt{k^2 + m^2}} \times \sum |\mathcal{M}|^2 \tilde{g}(k) \delta(k - \Delta V) \ . \tag{8}$$

The source of the difficulty is that the Bose-Einstein factor contributes $1/k$ to counteract the vanishing phase space [17]. It is evidently a bad approximation to take the radiated gluon to be massless; the mass induced by the plasma must be taken into account. The simplest thing to do is to take (2) and to replace k in the measure, in $\tilde{g}(k)$, and in the energy δ function by $\sqrt{k^2 + m_g^2}$. This makes Γ vanish for gap values below the threshold $\Delta V = m_g$, as shown in Fig. 3. A more correct calculation would replace the energy δ function by the one-loop spectral density of transverse gluons [18,19], and include as well the radiation of longitudinal plasmons, similarly treated.

The qualitative result indicated by Fig. 3 is that there is no distinct small-r regime as we supposed. The octet state is meaningful even when ΔV is large. Of course, Γ_8 will be increased by the effects of collisions, and so it is still possible that the octet disappears at some value of r.

The radiation and collision rates are ingredients in the Boltzmann-Vlasov equation, that governs the evolution of the $q\bar{q}$ distribution. This distribution is defined as a function $f_c(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2; t)$ on the two-particle phase space, carrying as well the index c denoting the color state of the pair. It satisfies the transport equation

$$\frac{D}{Dt} f_c = \left(\frac{\partial f_c}{\partial t}\right)_{\text{collision}} \ . \tag{9}$$

The left-hand side is the convective derivative in phase space:

$$\frac{D}{Dt} f_c \equiv \left[\frac{\partial}{\partial t} + \frac{\mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{x}_1} + \frac{\mathbf{p}_2}{m} \cdot \frac{\partial}{\partial \mathbf{x}_2} + \mathbf{F}_c(\mathbf{x}_1 - \mathbf{x}_2) \cdot \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)\right] f_c \ , \tag{10}$$

where $\mathbf{F}_c(\mathbf{r}) = -\nabla V_c(r)$ is the color force. The right-hand side of (9) represents gain and loss through collisions and radiation. In the small- and intermediate-distance regimes, we distinguish collisions which change the color state from those which do not. The former (like the radiation process) carry momentum transfer of

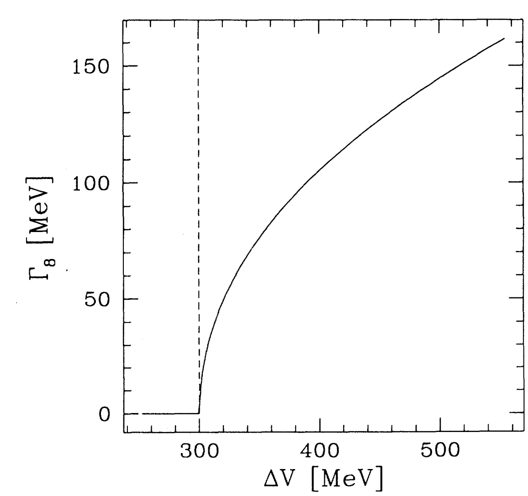

FIG. 3. Radiative width $\Gamma_8 = \Gamma(p_1) + \Gamma(p_2)$ of the octet state at $p_1 = p_2 = 0$ as a function of the gap ΔV. We set $\alpha_s = 0.6$ and the gluon mass is 300 MeV.

[1] We normalize the color matrices so that $\mathrm{Tr} T^a T^b = \frac{1}{2}\delta^{ab}$.

at least $\Delta V(r)$, and hence are hard collisions; the latter may carry very small momentum transfer, and hence may be approximated as soft. Thus we write

$$\left(\frac{\partial f_c}{\partial t}\right)_{\text{collision}} = R_{\text{soft}} + R_{\text{hard}} \ . \tag{11}$$

We can approximate the soft collision term by Fokker-Planck terms [16]:

$$R_{\text{soft}} = -\frac{\partial}{\partial \mathbf{p}_1} \cdot [\mathbf{G}_c(\mathbf{p}_1) f_c] + \frac{\partial^2}{\partial p_{1i} \partial p_{ij}} [N^c_{ij}(\mathbf{p}_1) f_c] + (1 \to 2) \ . \tag{12}$$

$\mathbf{G}_c$ and N^c_{ij} are drag and diffusion coefficients, calculated in the Landau approximation as moments of transition rates [20,16]. Here, as for the radiation rate calculated above, one has to be careful in projecting out the proper color channel. This makes $\mathbf{G}_c$ and N^c_{ij} dependent on c, and implies that only a subset of the scattering diagrams contributes to a given color channel.

We have calculated the drag and diffusion coefficients in the two color channels [15]. These can come only from soft collision processes, since radiation necessarily flips color. The octet coefficients are similar to those for the color-blind scattering that a single quark undergoes [16], reduced by roughly a factor 2–3 because of the restriction of the final states. The coefficients for the singlet state are smaller by an additional factor of 100, again because of the reduced number of final states, and also because of the smaller set of diagrams that contribute.

The hard collision terms cannot be approximated, and must be written as full-blown collision integrals. We have the loss and gain terms:

$$\begin{aligned} R_{\text{hard}} = &-[\Gamma_c(\mathbf{p}_1) + \Gamma_c(\mathbf{p}_2)] f_c(\mathbf{x}_a, \mathbf{p}_a; t) \\ &+ \int d^3k\, w(\mathbf{p}_1 - \mathbf{k} \to \mathbf{p}_1; \bar{c} \to c) f_{\bar{c}}(\mathbf{x}_a, \mathbf{p}_1 - \mathbf{k}, \mathbf{p}_2; t) \\ &+ \int d^3k\, w(\mathbf{p}_2 - \mathbf{k} \to \mathbf{p}_2; \bar{c} \to c) f_{\bar{c}}(\mathbf{x}_a, \mathbf{p}_1, \mathbf{p}_2 - \mathbf{k}; t) \ . \end{aligned} \tag{13}$$

Here $\Gamma_c(\mathbf{p})$ is the total rate of radiation- and collision-induced transitions for each quark, along the lines of Γ defined in (2). The functions w are differential transition rates into the phase space element at $(\mathbf{x}_a, \mathbf{p}_a)$ from other elements; since these are hard collisions, they involve perforce a color flip into c from the complementary state $\bar{c}$, and depend on position through the value of $\Delta V(r)$.

Our result for the radiative width of the octet state gives a lifetime of several fm/c for reasonable $q\bar{q}$ separations. If the collision rates do not affect this result strongly, then the evolution of the pair in phase space requires detailed numerical solution of the Boltzmann equation. If, on the other hand, the collisional width is large, then it may be reasonable to assume that the singlet and octet states are in equilibrium. Then a simplified Boltzmann equation may be studied, in which $f_c \propto \exp(-V_c/T)$. If the collisional width turns out to exceed the gap ΔV at some values of r, then the octet state ceases to exist and we are back to the consideration of three distinct regimes in r as described above.

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