Finite supersymmetric threshold corrections to CKM matrix elements in the large tanβ regime

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We evaluate the finite one-loop threshold corrections, proportional to tanβ, to the down quark mass matrix. These result in corrections to down quark masses and to Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The corrections to CKM matrix elements are the novel feature of this paper. For grand unified theories with large tanβ these corrections may significantly alter the low-energy predictions of four of the CKM matrix elements and the Jarlskog parameter J, a measure of CP violation. The angles α, β, and γ of the unitarity triangle and the ratio $|V_{ub}/V_{cb}|$, however, are not corrected to this order. We also discuss these corrections in the light of recent models for fermion masses. Here the corrections may be useful in selecting among the various models. Moreover, if one model fits the data, it will only do so for a particular range of SUSY parameters.

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I. INTRODUCTION

Minimal supersymmetric (SUSY) grand unified theories (GUT's) based on the gauge group SO(10) require tanβ (the ratio of the vacuum expectation values of the two Higgs scalar doublets present in the low-energy theory) to be of order $M_{\rm top}/M_{\rm bottom} \approx 50$. This follows from the unification of the top, bottom, and τ Yukawa couplings at the GUT scale, $M_{\rm GUT}$, and the necessity to fit the large top-to-bottom mass ratio at the weak scale [1]. Recent results using a general SO(10) operator analysis for fermion masses and mixing angles seem to be in significant agreement with experiments [2]. It was shown in [3–5], however, that there are potentially large finite one-loop corrections (proportional to tanβ) to the masses of the down-type quarks at the supersymmetric threshold. Note that these corrections were not included in the analysis of Ref. [2]. They may be as large as several tens of percent dependent on the sparticle spectrum [4]. Thus, they must be included when analyzing any SUSY theory with large tanβ. In this paper we emphasize that the nondiagonal elements of the down quark mass matrix also get potentially large corrections, thus leading to significant corrections to some Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the Jarlskog parameter J. Our main results are given in Eqs. (9), (16)–(18), and (22) [or in approximate form in Eqs. (25) and (28)]. Note that the Cabibbo angle and the CP-violating angles α, β, and γ are not significantly corrected to this order.

II. ONE-LOOP CORRECTIONS TO THE DOWN QUARK MASS MATRIX

When one integrates superpartners out of the minimal supersymmetric standard model (MSSM), there are significant $O(\tan\beta)$ one-loop corrections to the mass matrix of the down-type quarks originating in the diagrams with gluino–d-type-squark and chargino–u-type-squark loops yielding [see Figs. 1(a)–1(c), for the notation and conventions used and see the Appendix for a short derivation[1]]

$$\mathbf{m}_d = (V_d^{L0})^\dagger (1 + \epsilon \mathbf{\Gamma}_d + \epsilon V_{\rm CKM}^{0\dagger} \mathbf{\Gamma}_u V_{\rm CKM}^0) \mathbf{m}_d^{0{\rm diag}} V_d^{R0} \, , \tag{1}$$

with $\epsilon = (1/16\pi^2)\tan\beta$ and

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[1] We would like to thank Uri Sarid for bringing our attention to the second chargino diagram, Fig. 1(c).

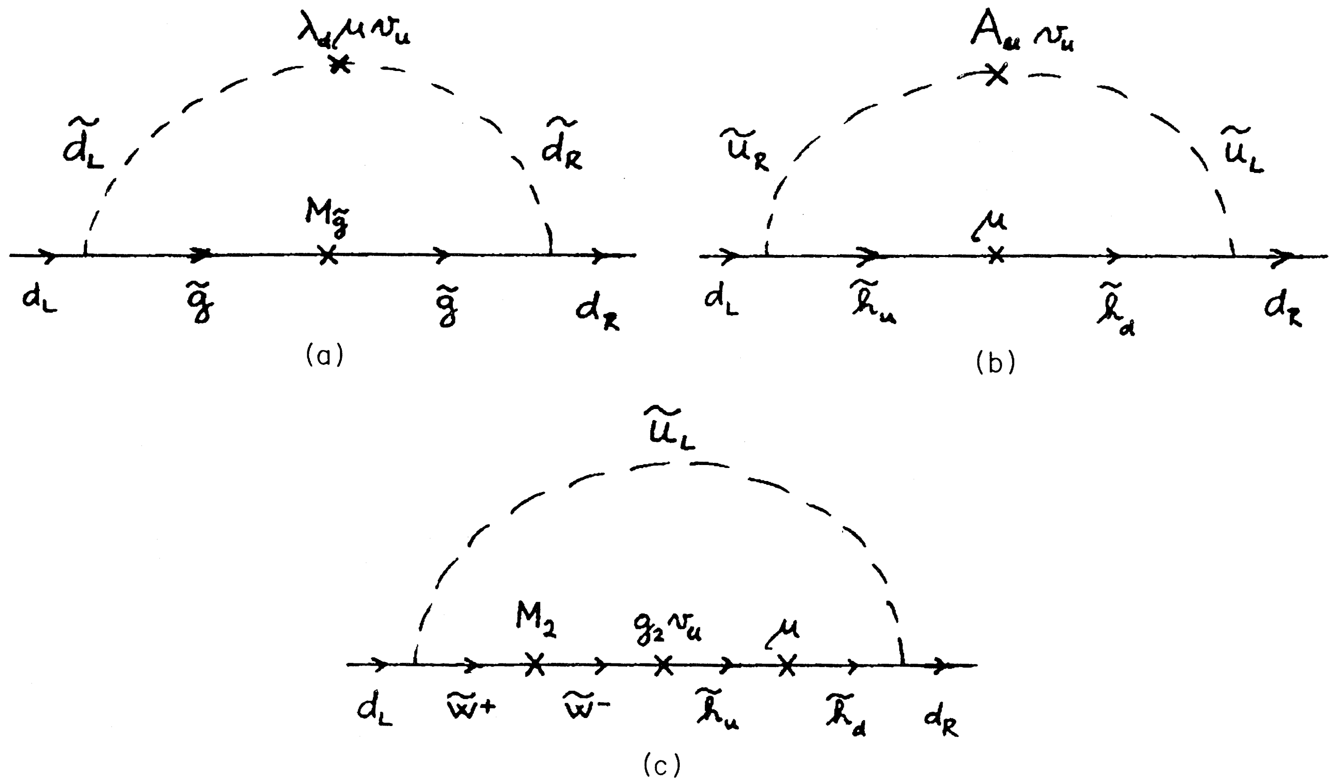

FIG. 1. Feynman diagrams for the one-loop corrections.

$$(\mathbf{\Gamma}_d)_{ij} = \frac{8}{3} g_3^2 (\Gamma^\dagger_{dL})_{i\alpha} \int dk \frac{M_{\tilde g}}{(k^2+M_{\tilde g}^2)(k^2+m^2_{\tilde d_\alpha})} (\Gamma_{dR})_{\alpha l} \frac{(\mathbf{m}_d^{0\mathrm{diag}})^{-1}_{lj}}{\tan\beta} \,, \tag{2}$$

$$(\mathbf{\Gamma}_u)_{ij} = -\lambda^{0\mathrm{diag}}_{u_i} (\Gamma^\dagger_{uR})_{i\alpha} \int dk \frac{U^{*\dagger}_{2A} m_{\chi_A} V_{A2}}{(k^2+m^2_{\chi_A})(k^2+m^2_{\tilde u_\alpha})} (\Gamma_{uL})_{\alpha j} (v_u)^{-1}$$
$$+ g_2 (\Gamma^\dagger_{uL})_{i\alpha} \int dk \frac{U^{*\dagger}_{2A} m_{\chi_A} V_{A1}}{(k^2+m^2_{\chi_A})(k^2+m^2_{\tilde u_\alpha})} (\Gamma_{uL})_{\alpha j} (v_u)^{-1} \,. \tag{3}$$

Uncorrected mass and mixing matrices are labeled with a superscript zero. The (6×3)-dimensional matrices Γ_{qL} and Γ_{qR} $(q = u, d)$ correspond to the additional transformations necessary to diagonalize squark mass matrices in a SUSY basis where quark mass matrices are diagonalized. Expressions for the $\mathbf{\Gamma}$'s are rather complex, since they involve the summation over the six-dimensional squark space. It has to be stressed, though, that despite the explicit $\tan\beta$ term in the denominator of (2) and a similar $(v_u)^{-1} = (v_d \tan\beta)^{-1}$ term in (3), there will not be any actual $\tan\beta$ suppression in the elements of the $\mathbf{\Gamma}$'s. In the interaction basis [see the Feynman diagrams, Fig. 1(a)–1(c)] one can easily recognize the R-L mixings among squarks in the loop and the mass insertions or mixings on the fermionic line in the diagram. Since in each diagram these mixings and mass insertions introduce a $\tan\beta$ unsuppressed quantity, the result represents a $\tan\beta$ unsuppressed correction. This correction is significant, since it corrects a $\tan\beta$ suppressed mass matrix. To emphasize this fact, the large ratio of Higgs vacuum expectation values (VEV) was pulled out into the ϵ's in (1). As a net effect, one can expect (at least some) terms in the $\mathbf{\Gamma}$'s to be of order (0.1–1). These terms are then enhanced by a factor of $\tan\beta$ [multiplying a standard small loop factor $(16\pi^2)^{-1}$ in our definition of ϵ] and thus lead to significant mass matrix corrections.

Concentrating on the above-mentioned diagrams, one has to mention that there are also neutralino diagrams, which contribute by finite $O(\tan\beta)$ terms to the d quark mass matrix. However, we have checked that (assuming degenerate gauginos at $M_{\rm GUT}$) these contributions are less than the leading gluino corrections roughly by a factor 16, as a result of smaller couplings, gaugino masses, and group factors. Therefore these diagrams will not be discussed separately in this paper, although they are included in our numerical analysis in Sec. IV, where we discuss their effects.

In order to gain some intuition for the $\mathbf{\Gamma}$'s, one can find an explicit form for them in the following approximation. First, neglect the second chargino diagram [Fig. 1(c)], since it is suppressed by a smaller coupling constant compared to the diagrams in Figs. 1(a) and 1(b). Then, in the evaluation of the remaining two diagrams assume that squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid assuming universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions at the low-energy SUSY scale.

That means that in this approximation the matrices Γ_{qL} and Γ_{qR} ($q = d, u$) are diagonal in generation space and are not completely trivial only because of the mixing between squarks of the same generation. The integrations in (2) and (3) are then easy to do along with the summation over $\alpha = 1, \ldots, 6$, i.e., over the squark mass eigenstates. The $\mathbf{\Gamma}$'s are then proportional to the off-diagonal term of the down (up) squark mass matrix for each individual generation separately. We find

$$(\mathbf{\Gamma}_d)_{ij} = \tfrac{8}{3} g_3^2 M_{\tilde g} \mu I_3(M_{\tilde g}^2, m^2_{\tilde d_{i_1}} m^2_{\tilde d_{i_2}})\delta_{ij} \ , \tag{4}$$

$$(\mathbf{\Gamma}_u)_{ij} = U^{*\dagger}_{2A} m_{\chi_A} V_{A2} A_0 I_3(m^2_{\chi_A}, m^2_{\tilde u_{i_1}}, m^2_{\tilde u_{i_2}})(\lambda_u^{0\rm diag})^2_{ij} \ , \tag{5}$$

with the function I_3 given by

$$I_3(a,b,c) = \frac{ab\,\ln(a/b) + bc\,\ln(b/c) + ac\,\ln(c/a)}{(a-b)(b-c)(a-c)} \ .$$

Terms suppressed by $\tan\beta$ have been neglected in these expressions. In this approximation both $\mathbf{\Gamma}$ matrices are diagonal, which makes calculations of the corrections to the masses and mixing angles in terms of mass eigenstates simple. In addition to that, note the large hierarchy in $\mathbf{\Gamma}_u$, and a much milder hierarchy in $\mathbf{\Gamma}_d$ based just on the nonequality of the squark masses. Note, if $\mathbf{\Gamma}_d$ were completely proportional to the identity matrix (i.e., the case of complete squark degeneracy), the gluino loop would not contribute to quark mixing corrections at all.[2]

One knows though, that the approximation used to derive (4) and (5) is not correct. The initial conditions at $M_{\rm GUT}$ need not be universal and, even if they were, squark masses and trilinear couplings run between the GUT (or string) and the low-energy SUSY scales and violate our assumptions. As a result, the explicit form of the potentially significant (i.e., $\tan\beta$ unsuppressed) elements in the $\mathbf{\Gamma}$'s is clouded by the fact that they no longer remain diagonal in generation space. In order to evaluate these effects we have performed a numerical analysis. The results are found in Sec. IV. We also show that our naive approximation, Eqs. (4) and (5), when suitably modified to take into account nonuniversal squark masses gives results that agree to within 25% with the two-loop numerical analysis.

In order to figure out the explicit form of the one-loop threshold corrections to the CKM matrix elements as well as to quark masses in terms of the $\mathbf{\Gamma}$ matrix elements one can define an unknown Hermitian matrix $\mathbf{B}$ as

$$V_d^L = (1 + i\epsilon \mathbf{B}) V_d^{L0} \ , \tag{6}$$

where, again, V_d^{L0} is the matrix diagonalizing down quarks in the absence of the SUSY corrections. Since there are no large [i.e., $O(\tan\beta)$] corrections to the up quark mass matrix,

$$\begin{aligned} V_{\rm CKM} \equiv V_u^L V_d^{L\dagger} &= V_u^{L0}(V_d^{L0})^\dagger (1 - i\epsilon\mathbf{B}) \\ &= V^0_{\rm CKM}(1 - i\epsilon\mathbf{B}) \ . \end{aligned} \tag{7}$$

$\mathbf{B}$ is determined through the diagonalization condition

$$(\mathbf{m}_d^{\rm diag})^2 \equiv {\rm diag}(m^2_{d_1}, m^2_{d_2}, m^2_{d_3}) = V_d^L \mathbf{m}_d \mathbf{m}_d^\dagger V_d^{L\dagger} \ , \tag{8}$$

where both V_d^L and $\mathbf{m}_d$ on the right-hand side (RHS) are to be expanded to first order in ϵ according to (6) and (1).

A. Corrections to down quark masses

Diagonal elements of this matrix equation (8) specify the corrections to the masses of the d, s, and b (d_1, d_2, and d_3) quarks. Note that the terms containing unknown $\mathbf{B}$ elements drop out of these equations:

$$\frac{\delta m_{d_i}}{m_{d_i}} = \epsilon\,{\rm Re}(\mathbf{\Gamma}_d)_{ii} + \epsilon[V^{0\dagger}_{\rm CKM}\,{\rm Re}(\mathbf{\Gamma}_u) V^0_{\rm CKM}]_{ii} \ . \tag{9}$$

This is an exact formula where the effects of squark rotations are fully included in the $\mathbf{\Gamma}$'s. Since the $\mathbf{\Gamma_u}$ matrix has some generation hierarchy (for more discussion on this see Sec. IV) because of the Yukawa couplings in the chargino loop the dominant correction from the chargino diagram goes to the b quark mass correction:

$$\left(\frac{\delta m_b}{m_b}\right)_{\chi^+} = \epsilon\,{\rm Re}(\mathbf{\Gamma}_u)_{33} + \epsilon O(10^{-3}) \ . \tag{10}$$

The suppression in the second term above is caused by the hierarchies present in (9). The largest next-to-leading correction indicated above results, for example, from the term $(V^{0\dagger}_{\rm CKM})_{32}\,{\rm Re}(\mathbf{\Gamma}_u)_{23}(V^0_{\rm CKM})_{33}$, where two orders

[2] Also note that the same analysis could be done for the corrections to the up quark mass matrix, and the above-mentioned approximation would show that the relevant $\mathbf{\Gamma}$ matrices [analogous to (4) and (5)] become suppressed by $\tan\beta$ after the up (instead of down) quark mass matrix is pulled out of the expression analogous to (1). Thus, in this case, there are no corrections proportional to $\tan\beta$. There are, however, corrections to charged lepton masses proportional to $\tan\beta$. These are smaller than those for down quarks but are still significant and must be included in any fermion mass analysis.

come from V_{cb}^* and at least one order from $(\mathbf{\Gamma}_u)_{32}$.

Note that the corrections to the masses of the s and d quarks can easily be as significant, or even larger than the correction to the b quark mass. While the gluino correction (which is the largest correction to each quark mass) to the b quark mass is larger due to the smaller b squark mass (in a universal-like scenario where one starts with all soft squark masses equal at the GUT scale), the chargino correction may invert the net effect, since it is always of opposite sign to the gluino correction and its contribution to the two lighter quarks is small.

B. Corrections to CKM matrix elements

The non-diagonal equations, i.e., those with zeros on the left-hand side (LHS) of the matrix equation (8), lead to

$$-i\epsilon\mathbf{B}_{ij} = \epsilon(\mathbf{\Gamma}_d)_{ij} + \epsilon(V^{0\dagger}_{\rm CKM}\mathbf{\Gamma}_u V^0_{\rm CKM})_{ij}\left[1+O\left(\frac{m^2_{d_i}}{m^2_{d_j}}\right)\right] , \qquad (11)$$

where ij indices correspond to the 12, 13, or 23 combinations and the transposed elements (for $i>j$), are obtained by the Hermiticity of $\mathbf{B}$. The diagonal elements of $\mathbf{B}$ remain undetermined by this procedure but, to the first order in the ϵ expansion they can be removed by phase redefinitions of the b, s, and d fields. We thus set the diagonal elements of $\mathbf{B}$ to zero.

Then from (7) we can easily derive[3]

$$\begin{aligned}\delta V_{cb} =\ & \epsilon[V_{cd}(\mathbf{\Gamma}_d)_{13} + V_{cs}(\mathbf{\Gamma}_d)_{23}] \\ & +\epsilon[\{\delta_{2j} - (V_{\rm CKM})_{23}(V^\dagger_{\rm CKM})_{3j}\}(\mathbf{\Gamma}_u)_{jk}(V_{\rm CKM})_{k3}] ,\end{aligned} \qquad (12)$$

$$\begin{aligned}\delta V_{ub} =\ & \epsilon[V_{ud}(\mathbf{\Gamma}_d)_{13} + V_{us}(\mathbf{\Gamma}_d)_{23}] \\ & +\epsilon[\{\delta_{1j} - (V_{\rm CKM})_{13}(V^\dagger_{\rm CKM})_{3j}\}(\mathbf{\Gamma}_u)_{jk}(V_{\rm CKM})_{k3}] ,\end{aligned} \qquad (13)$$

$$\begin{aligned}\delta V_{td} =\ & -\epsilon[V_{ts}(\mathbf{\Gamma}^\dagger_d)_{21} + V_{tb}(\mathbf{\Gamma}^\dagger_d)_{31}] \\ & -\epsilon[\{\delta_{3j} - (V_{\rm CKM})_{31}(V^\dagger_{\rm CKM})_{1j}\}(\mathbf{\Gamma}^\dagger_u)_{jk}(V_{\rm CKM})_{k1}] ,\end{aligned} \qquad (14)$$

$$\begin{aligned}\delta V_{ts} =\ & \epsilon[V_{td}(\mathbf{\Gamma}_d)_{12} - V_{tb}(\mathbf{\Gamma}^\dagger_d)_{32}] \\ & +\epsilon[(V_{\rm CKM})_{31}(V^\dagger_{\rm CKM})_{1j}(\mathbf{\Gamma}_u)_{jk} \\ & -(V_{\rm CKM})_{33}(V^\dagger_{\rm CKM})_{3j}(\mathbf{\Gamma}^\dagger_u)_{jk}](V_{\rm CKM})_{k2} .\end{aligned} \qquad (15)$$

As already mentioned, the numerical analysis shows that both $\mathbf{\Gamma}_u$ and $\mathbf{\Gamma}_d$ matrices keep track of the generation hierarchy from the Yukawa sector with the 33 element of the order of 0.1–1 and the relevant 12, 13, and 23 elements of small magnitude. Together with the hierarchy in the CKM matrix this implies that in each of the previous equations the dominant correction is the one containing the $(\mathbf{\Gamma}_u)_{33}$ term. However, the corrections due to the other terms are non-negligible resulting in a 25% effect. Thus, a good approximation to the exact results of Eqs. (12)–(15) is given by (for more detail, see Sec. IV)

$$\frac{\delta V_{cb}}{V_{cb}} \approx -\epsilon[(\mathbf{\Gamma}_u)_{33} - (\mathbf{\Gamma}_u)_{22} - \frac{V_{cs}}{V_{cb}}(\mathbf{\Gamma}_d)_{23}] \equiv -\epsilon\Delta , \qquad (16)$$

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx -\epsilon\Delta , \qquad (17)$$

$$\frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\epsilon\Delta^* . \qquad (18)$$

The results of Eqs. (17) and (18) follow directly from the unitarity of the CKM matrix and the fact that these are the only terms that receive significant corrections.

Note that as a consequence the ratio V_{ub}/V_{cb} remains *unchanged.* In addition, the numerical analysis shows that $\mathrm{Re}(\mathbf{\Gamma}_u)_{33} \gg \mathrm{Im}(\mathbf{\Gamma}_u)_{33}$; thus these dominant corrections to the CKM elements are roughly equal in magnitude, but opposite in sign, to the chargino corrections to the b quark mass, Eq. (10).

The other five CKM elements get the corrections of the form similar to (12)–(15). However, large $\mathbf{\Gamma}$ elements are always in the product with a small CKM matrix element, and the terms containing large diagonal CKM matrix elements are in the same way pushed down by small $\mathbf{\Gamma}$ elements in these corrections. Hence the actual numerical values of the corrections to V_{ud}, V_{us}, V_{cd}, V_{cs}, and V_{tb} are not significant, at least not at the present level of experimental accuracy. As an example, the dominant correction to, let us say V_{us} is

$$\delta V_{us} \sim \epsilon V_{ub}V_{ts}V^*_{tb}(\mathbf{\Gamma}^*_u)_{33} < 0.001 . \qquad (19)$$

III. *CP*-VIOLATING PARAMETERS

The Jarlskog parameter, which measures CP violation, can be obtained from the four CKM-matrix elements left after crossing out any row and any column of this matrix [6]:

$$J\sum_{\gamma,l}\epsilon_{\alpha\beta\gamma}\epsilon_{jkl} = \mathrm{Im}[V_{\alpha j}V_{\beta k}V^*_{\alpha k}V^*_{\beta j}] . \qquad (20)$$

Consider the product

$$J = \mathrm{Im}[V_{cs}V_{tb}V^*_{cb}V^*_{ts}] . \qquad (21)$$

Using the formulas (16) and (18) from the preceding section, it is easy to obtain the leading correction

$$\delta J \approx -2\epsilon\,\mathrm{Re}[\Delta]J . \qquad (22)$$

[3]Zero superscripts are dropped from now on, since they make no difference in the following expressions.

This threshold correction to J may *significantly* alter the prediction for ϵ_K in SUSY GUT models with large $\tan\beta$.

Note, it is not obvious how this result is obtained for other equivalent definitions of J. For example, at first glance one might guess that $\delta J \approx 0$ for J defined by $J = \mathrm{Im}[V_{ud}V_{cs}V_{us}^*V_{cd}^*]$. However, such a guess does not take into account that we have the imaginary part of the product in (20), and imaginary parts are small for every CKM matrix element, even if its absolute value is close to one. In this case the small corrections to the large CKM matrix elements become important, and, in fact, it is corrections to V_{cs} and V_{us} that lead to the result (22).

Finally, we note that although J changes, the angles of the unitarity triangle remain *uncorrected* to this order. This is easily understood from a geometrical point of view. For the "standard" choice of its sides, $|V_{ud}V_{ub}^*|$, $|V_{cd}V_{cb}^*|$, and $|V_{td}V_{tb}^*|$, each side contains one element that gets a significant correction, and (as a consequence of the unitarity of the CKM matrix discussed earlier) these corrections are identical in magnitude [see (16)–(18)]. Hence, the sides are contracted (or stretched) by the same multiplicative factor and the angles stay the same. The area of the triangle gets corrected, of course, twice as much as the sides, and that is the reason for the factor of 2 in (22) (recall that J measures the area of the triangle).

IV. NUMERICAL ANALYSIS AND CONCLUSIONS

In our numerical analysis we took the initial conditions (values at the GUT scale) for the dimensionless couplings from the SO(10) models [2], which give predictions for the low-energy data in good agreement with experiment. The initial values for the dimensionful soft SUSY-breaking parameters were taken from Refs. [4,7] in order to guarantee the radiative electroweak symmetry breaking at the weak scale. We focused mainly on simple nonuniversal cases. The numerical results presented below were obtained for $m_{H_1}^2 = 2.0m_0^2$, $m_{H_2}^2 = 1.5m_0^2$, and all other scalar masses equal m_0^2. Next, we used two-loop renormalization-group equations [8] to run all the couplings and mass parameters to the low-energy scale. Leading corrections to the CKM matrix elements have appeared practically independent of the exact value of the low-energy SUSY scale between M_Z and 500 GeV (changes were within 1% of the mass or the CKM element in question). In the actual numerical analysis the $\mathbf{\Gamma}$ matrices have been evaluated according to the following formulas [note that there are no divergent pieces from the integrals in (2) and (3) and that the chargino summation is easy to do]:

$$(\mathbf{\Gamma}_d)_{ij} = \frac{8}{3}g_3^2(\Gamma_{dL}^\dagger)_{i\alpha}\,\frac{-m_{\tilde{d}_\alpha}^2}{m_{\tilde{d}_\alpha}^2 - M_{\tilde{g}}^2}\,\ln\frac{m_{\tilde{d}_\alpha}^2}{M_{\tilde{g}}^2}(\Gamma_{dR})_{\alpha j}\frac{M_{\tilde{g}}}{m_{d_j}\tan\beta}\ , \tag{23}$$

$$\begin{aligned}(\mathbf{\Gamma}_u)_{ij} = {}& \lambda_{u_i}(\Gamma_{uR}^\dagger)_{i\alpha}(m_{\tilde{u}_\alpha}^2 - |M_2|^2)I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}_\alpha}^2)(\Gamma_{uL})_{\alpha j}\frac{\mu}{v_u} \\ & - g_2^2(\Gamma_{dL}^\dagger)_{i\alpha}I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}_\alpha}^2)(\Gamma_{uL})_{\alpha j}M_2\mu\ .\end{aligned} \tag{24}$$

M_2 is the W-ino mass parameter, and terms suppressed by $\tan\beta$ were dropped. The summation is only over $\alpha = 1,\ldots,6$ (there is no summation over i,j on the RHS of these equations). This summation could be done analytically in terms of the mass eigenvalues; however, the expressions are long and do not provide much insight, so we keep rather the compact forms above.

Typical values for these matrices at the weak scale follow:

$$\epsilon\mathbf{\Gamma}_d \approx \begin{pmatrix} 0.180+i & 3\times10^{-8} & -3\times10^{-6}-i & 9\times10^{-7} & 8\times10^{-5}+i & 2\times10^{-5} \\ -3\times10^{-6}+i & 7\times10^{-7} & 0.180-i & 4\times10^{-8} & -4\times10^{-4}+i & 6\times10^{-6} \\ -9\times10^{-5}+i & 8\times10^{-5} & 3\times10^{-5}+i & 1\times10^{-5} & 0.218-i & 3\times10^{-9} \end{pmatrix}\ ,$$

$$\epsilon\mathbf{\Gamma}_u \approx \begin{pmatrix} -0.022+i & 5\times10^{-16} & -9\times10^{-7}+i & 3\times10^{-7} & -3\times10^{-6}+i & 6\times10^{-6} \\ -9\times10^{-7}-i & 3\times10^{-7} & -0.022-i & 3\times10^{-11} & -1\times10^{-4}+i & 5\times10^{-9} \\ 1\times10^{-5}+i & 3\times10^{-5} & 5\times10^{-4}-i & 1\times10^{-8} & -0.116+i & 7\times10^{-10} \end{pmatrix}\ ,$$

where, in this case, we used the GUT scale values, $M_{1/2} = 400$ GeV, $m_0 = 250$ GeV, and $A_0 = -1100$ GeV, the weak scale value, $\mu = 270$ GeV, and model 4 of [2] for the Yukawa matrices (with the weak scale values $\lambda_t = 1.01$, $\tan\beta = 53$, and $V_{cb}^0 = 0.038$ as output). With these inputs we find $M_{\tilde{g}} = 1029$ GeV, $A_t = (A_u)_{33} = -(736 + i6\times10^{-7})$ GeV, up-squark mass eigenvalues (in GeV) (976,976,951,951,869,695) and down-squark mass eigenvalues (in GeV) (980,979,949,948,820,757). To gain some intuition for the size of the corrections, these particular values lead to $\delta m_b/m_b = 10.2\%$, $\delta m_s/m_s = 15.7\%$, $\delta m_d/m_d = 15.7\%$, $\delta V_{cb}/V_{cb} = \delta V_{ub}/V_{ub} = \delta V_{ts}/V_{ts} = \delta V_{td}/V_{td} = 8.0\%$, and $\delta J/J = 16.6\%$. As we discussed earlier, the approximation of retaining only the $(\mathbf{\Gamma}_u)_{33}$ term in Eq. (16) does not work extremely well, since it predicts an 11.6% correction. However, this leading correction is then lowered by about 1.5% coming from the $\mathbf{\Gamma}_d$ term in (12)–(15) and by an additional 2% from the subleading $\mathbf{\Gamma}_u$ terms. V_{ud}, V_{us}, V_{cd}, V_{cs}, and V_{tb} gets a relative correction less than 1%, e.g., $\delta V_{us}/V_{us} = 0.01\%$. Similarly, the corrections to the angles α, β, and γ of the unitarity triangle are much below 1%. Neutralino corrections have been included in the above numerical analysis. Their effects are as follows: the b mass is reduced by 1.6% and the masses of s and d are reduced by 1.3%. Integrating out neutralinos has less than a 1% impact on the CKM elements and CP-violating parameter J. We would like to emphasize that such corrections are generic for a large subspace of the allowed parameter space.

A. Approximate formulas for mass and mixing angle corrections

In Eqs. (4) and (5), we presented the results of a naive approximation, which assumes that squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid in the case of universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions when, in addition, one also neglects the renormalization group (RG) running from m_G to the low-energy SUSY scale. If one now includes the effect of RG running, quark and squark mass matrices can no longer be diagonalized in generation space by the same unitary transformations and the A parameters are no longer universal. We have checked that a simple approximation for the corrections to down quark masses and the CKM matrix elements (valid to 25%) can be obtained by using the results of Eqs. (4) and (5) with the values of squark and gluino masses obtained by RG running as input and by replacing A_0 with A_t (for the third generation) and the chargino mass with the low-energy value of μ. This approximation has been widely used in previous papers [3,5,4,7,9], where large bottom mass corrections have been recognized. In particular, in this improved approximation

$$\delta m_d \approx [\tilde{\epsilon}_{d1} + \tilde{\epsilon}_{d2} O(10^{-6})] m_d \; ,$$

$$\delta m_s \approx [\tilde{\epsilon}_{s1} + \tilde{\epsilon}_{s2} O(10^{-4})] m_s \; , \qquad (25)$$

$$\delta m_b \approx \{\tilde{\epsilon}_{b1} + \tilde{\epsilon}_{b2}[|V_{tb}|^2 + O(10^{-6})]\} m_b \; ,$$

where

$$\tilde{\epsilon}_{d_i 1} = \frac{2\alpha_s}{3\pi} \mu M_{\tilde{g}} I_3(M_{\tilde{g}}^2, m_{\tilde{d}_{i1}}^2, m_{\tilde{d}_{i2}}^2) \tan\beta \; , \qquad (26)$$

$$\tilde{\epsilon}_{d_i 2} = \frac{1}{16\pi^2} \mu A_{u_i} \lambda_{u_i}^2 I_3(\mu^2, m_{\tilde{u}_{i_1}}^2, m_{\tilde{u}_{i_2}}^2) \tan\beta \; . \qquad (27)$$

The analogous corrections for CKM matrix elements, also valid to about 25%, are given by

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx \frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\tilde{\epsilon}_{b2} \; , \qquad (28)$$

where $\tilde{\epsilon}_{b2}$ is defined in Eq. (27).

An important feature of the b quark mass correction is that the gluino and chargino contributions are of the opposite signs, and thus there is a partial cancellation between them. This effect with its consequences has been carefully studied in [4,7,9]. In these papers it was shown that the magnitude of the gluino contribution is always two to three times larger than the chargino contribution and can be as large as 50% for universal scalar masses at M_G. For nonuniversal scalar masses the corrections can be smaller.

B. Consequences for models of fermion masses

It is interesting to see what effect these corrections have for recent models of fermion masses and mixing angles. In the model of Ref. [10] the value of $|V_{cb}|$ is of order 0.054. This is large compared to the latest experimental values. In this model, tanβ can be either small or large. We would have to be in the large-tanβ regime for these corrections to be significant. In addition, consider models 4, 6, and 9 of Ref. [2]. In these models tanβ is expected to be large. Recall that the model-independent experimental value of $|V_{cb}|$ is 0.040 ± 0.003 according to [11] or 0.040 ± 0.005 based on [12]. For models 6 and 9, the predicted value of $V_{cb} \sim 0.048$–0.052 is at the upper end of the experimentally allowed range. For all these models we would choose $\mu M_{\tilde{g}} < 0$ so that the chargino correction to the b quark mass is positive and hence $\delta V_{cb}/V_{cb} < 0$. As a consequence the gluino correction is negative, which gives $\delta m_b < 0$. This has the effect of decreasing the prediction for m_t, since a smaller top Yukawa coupling is now needed to fit the experimental ratio m_b/m_τ. These corrections apparently improve the predictions of the above models. However, the corrections to the strange and down quark masses, which are equal and negative, may be a problem, since both ratios m_u/m_d and m_s/m_d were rather large and now the first one becomes even larger, while the second one stays the same. This problem is exacerbated by the fact that the authors in [9] find no solutions for $|\delta m_b/m_b| < 10\%$ with $\mu M_{\tilde{g}} < 0$ consistent with both the experimental rate for $b \to s\gamma$ and the cosmological constraint on the energy density of the Universe. There are solutions for larger values of $|\delta m_b/m_b|$, but this range of parameters may be seriously constrained by the ratios m_u/m_d and m_s/m_d.

For model 4 of Ref. [2], however, the situation may be better. In this model $|V_{cb}|$ is acceptably small ($V_{cb} \sim 0.038$–0.044). However, J is too small, and thus the bag constant B_K needed to fit ϵ_K is too large, i.e., greater than 1. In this case we need $\delta J/J > 0$. This would also increase $|V_{cb}|$ by half as much, which may be acceptable. In this case the chargino correction to the b quark mass is negative. Thus, the gluino correction to m_b is positive, and $\delta m_b > 0$. As a result, the top quark mass prediction increases. This restricts the magnitude of the effect to values of $|\delta m_b/m_b| < 10\%$. In this case both m_s and m_d increase, which improves the agreement with experiment in the $m_s/m_d - m_u/m_d$ plane, Finally, the $b \to s\gamma$ decay rate and the cosmological constraint can be satisfied [9].

Note that in either scenario the angles α, β, and γ of the unitarity triangle and the ratio V_{ub}/V_{cb} remain unchanged. These correlations of quark mass and mixing angle predictions with the sign of $\mu M_{\tilde{g}}$ are very intriguing, especially since this sign may be determined independently once SUSY particles are observed. In a particular model the allowed maximal corrections to masses and mixing angles may represent new constraints on the magnitude and sign of the SUSY parameters.

In summary, finite SUSY corrections to the masses of the down-type quarks may be significant in the limit of large $\tan\beta$. In this paper we have shown that the CKM matrix elements V_{cb}, V_{ub}, V_{ts}, and V_{td} receive similar corrections, while the correction to the Jarlskog parameter is enhanced by a factor of 2. The other elements of the CKM matrix and the angles of the unitarity triangle receive only small corrections, down by a factor of $\tan\beta$ or suppressed by the generation hierarchy present in Yukawa, CKM, or $\mathbf{\Gamma}$ matrices.

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APPENDIX

Conventions of the standard model are fixed by $\mathcal{L}_{\text{Yukawa}} = H_q \bar{Q}_L \lambda_q q_R$, quark mass matrix rotations by $m_q^{\text{diag}} = V_q^L m_q V_q^{R\dagger}$, and the CKM matrix is defined as $V_{\text{CKM}} = V_u^L V_d^{L\dagger}$. In the MSSM the relevant term in the superpotential is then $W = \bar{\hat{q}} \lambda_q^\dagger \hat{Q} \hat{H}_q$.

Looking closely at the SUSY threshold corrections to the d quark masses, there are the following one-loop diagrams contributing significantly in the large-$\tan\beta$ limit.

(i) Gluino diagram. Using Dirac notation, the quark-squark-gluino interaction, relevant for this paper, reads

$$\mathcal{L}_{\text{int}} = -\sqrt{2} g_3 \left(\frac{\lambda^A}{2}\right)_{ab} \{+(\bar{d}_a P_R \tilde{g}^A) \tilde{d}_{Lb} - \tilde{d}_{Ra}^\dagger (\bar{\tilde{g}}^A P_R d_b)\} + \text{H.c.} \tag{A1}$$

The *squark* interaction eigenstates are turned into the mass eigenstates according to

$$(\tilde{d}_R)_i = (V_d^{R0\dagger})_{ij} (\Gamma_{dR}^\dagger)_{j\alpha} \tilde{d}_\alpha \ , \tag{A2}$$

$$(\tilde{d}_L)_i = (V_d^{L0\dagger})_{ij} (\Gamma_{dL}^\dagger)_{j\alpha} \tilde{d}_\alpha \ . \tag{A3}$$

As indicated in these equations, the V matrices rotate squarks the same way as they do with quarks. The additional rotations are then performed by the 6×3 matrices $\Gamma_{dL,R}$. Rules for the Feynman diagrams using this notation can be found in [13]. Note that the indices $i, j, \ldots$ denote generation indices 1,2,3, the greek letters denote squark indices 1–6, and the implicit summation over the repeating indices is assumed. The diagram with the gluino and d-type squarks in the loop contributes to the *quark* self-energy matrix (amputated two-point function) as

$$-i\Sigma = (-i\sqrt{2} g_3)^2 C_2(\mathbf{3}) \int \frac{d^d k}{(2\pi)^d} (-) P_R \langle \tilde{g} \bar{\tilde{g}} \rangle P_R \times V_d^{L0\dagger} \Gamma_{dL}^\dagger \langle \tilde{d} \tilde{d}^\dagger \rangle \Gamma_{dR} V_d^{R0} + P_L \cdots P_L + \not{p} \text{ terms} \ . \tag{A4}$$

Only the term with the two right-handed projectors corrects the mass matrix. The term indicated as $P_L \cdots P_L$ contains similar corrections to $\mathbf{m}^\dagger$. To obtain formula (1) in the text one performs the rotation to Euclidean space and integrates out angular variables. The integral measure dk in (2) stands for $k^2 d(k^2)$, and the integration limits are assumed to be zero and infinity. Note that in the main text $\mathbf{m}_d^0$ was appended to these equations in a not very elegant way, but that is for later convenience.

(ii) Chargino diagram. Quark-squark-chargino interaction that is relevant for this paper reads

$$\mathcal{L}_{\text{int}} = (\bar{d} P_R (V^{*\dagger})_{2A} \tilde{\chi}_A^c) \lambda_{\mathbf{u}} \tilde{u}_R + \tilde{u}_L^\dagger \lambda_{\mathbf{d}} ((U^{*\dagger})_{2A} \bar{\tilde{\chi}}_A^c P_R d) - g_2 (\bar{d} P_R (V^{*\dagger})_{1A} \tilde{\chi}_A^c) \tilde{u}_L + \text{H.c.} \tag{A5}$$

u squarks are rotated to their mass eigenstates in exactly the same way as the d squarks above, defining the $\Gamma_{uR,L}$ matrices. Contribution to the d quark self-energy from this interaction reads

$$-i\Sigma = (i)^2 \int \frac{d^d k}{(2\pi)^d} (U_{A2} V_{B2} P_R \langle \tilde{\chi}_A^c \bar{\tilde{\chi}}_B^c \rangle P_R \lambda_{\mathbf{u}} \times V_u^{R0\dagger} \Gamma_{uR}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} V_u^{L0} \lambda_{\mathbf{d}} - U_{A2} V_{B1} P_R \langle \tilde{\chi}_A^c \bar{\tilde{\chi}}_B^c \rangle P_R g_2 V_u^{L0\dagger} \Gamma_{uL}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} \times V_u^{L0} \lambda_{\mathbf{d}}) + P_L \cdots P_L + \not{p} \text{ terms} \ .$$

U and V diagonalize the chargino mass matrix. The fact that one of their indices is 1 (2), traces back to the W-ino (higgsino) interaction in the quark-squark-chargino vertex of the loop. Summation over $A, B = 1, 2$ is assumed. Explicit forms of the U and V matrices and further details about the notation can be found in Ref. [14]. In order to derive Eq. (1), one has to use the relations between the diagonalized and nondiagonalized mass and λ matrices, briefly mentioned at the beginning of this appendix. The VEV of the scalar Higgs H_d is added in order to pull out the mass matrix on the RHS for future convenience, and when combined with $\tan\beta$ (which is pulled out into the ϵ) it yields $(v_u)^{-1}$ in the final expression (3) given in the text.

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