Dynamical generation of CKM mixings by broken horizontal gauge interactions

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(Received 16 March 1995)

The fermion mass matrices are calculated in the framework of the dynamical mass generation by the broken horizontal gauge interactions. The nonproportional mass spectra between up, and down sectors and CKM mixings are obtained solely by radiative corrections due to the ordinary gauge interactions.

PACS number(s): 12.15.Ff, 11.30.Qc, 12.15.Hh, 12.60.−i

I. INTRODUCTION

The standard model offers a remarkably successful description of the gauge interactions of the particles thus far observed and accounts extremely well for the vast amount of high-energy particle experimental data. Nevertheless, it does not present any satisfactory understanding of matter parts, involving too many arbitrary parameters, particularly, in Higgs and Yukawa sectors. This means that the standard model itself has no answer for the origin of quark-lepton masses, Cabibbo-Kobayashi-Maskawa (CKM) mixings [1], and the number of generations. These fermion mass problems have been studied by many people with various ideas. The main purpose of these works is to elucidate two types of hierarchies of fermion masses, one of which is among generations and the other is among sectors (up, down, neutrino, and electron).

One of the most attractive scenarios is dynamical mass generation, for example, by extended technicolor [2] or the top-quark condensate model [3]. These models, however, do not explain the above hierarchy problems well in spite of their successes in symmetry breaking. We have so far been studying this problem with broken horizontal gauge symmetry [4–6], which is some extension of the top-quark condensate model. Horizontal gauge interactions have been studied to explain fermion mass matrices in other contexts [7]. In the previous papers, it was shown that the hypercharge gauge interaction $U(1)_Y$ plays an important role in generating a hierarchy between up and down sectors naturally. The other hierarchy among generations is explained by a suitable breaking pattern of horizontal gauge symmetry, which is, however, given by hand. One of the purposes of the present paper is to find out the underlying structure behind our model by studying the relation between the breaking pattern and induced fermion mass spectra.

It was pointed out that the hypercharge interaction does not generate well flavor mixings [8]; to be precise, the mass matrix of the up sector M_U is almost proportional to that of down sector M_D, since the effects of $U(1)_Y$ are so small. Here, we investigate whether or not Cabibbo-Kobayashi-Maskawa (CKM) mixings can occur in the above broken horizontal gauge model. The main task in the present paper is actually to show the breaking of the proportionality between M_U and M_D and to find the breaking pattern that causes CKM mixings by the $U(1)_Y$ radiative corrections. The plan of this paper is as follows. In Sec. II we review the previous papers in brief and present a model. In Sec. III we study the eigenvalue problem, which is equivalent to solving the mass gap equations approximately. In Sec. IV it is shown that nonlinear terms of the gap equations are essential for CKM mixings. In Sec. V a down-quark diagonalizing base is introduced. In Sec. VI we rewrite the down-sector gap equation as an eigenequation. In Sec. VII we show that CKM mixings can actually occur in particular cases. Some conclusions are given in Sec. VIII.

II. GAP EQUATION FOR FERMIONS

In this section, we shall review the mass gap equations for quarks and leptons induced by the horizontal gauge interactions, and investigate the general aspects of gap equations.

We introduce the horizontal gauge interactions

$$L_{\rm int} = f\bar{\psi}\gamma^\mu H_\mu^\kappa T_\kappa \psi, \tag{2.1}$$

in addition to the standard gauge interactions $[SU(3)_C \times SU(2)_L \times U(1)_Y]$, where T_κ's denote generators of horizontal gauge symmetry, say $SU(N)$, over N generations of fermions ψ.

It is assumed that the horizontal gauge symmetry breaks at the energy scale Λ with keeping the ordinary gauge symmetries, and the gauge fields H_μ^κ's acquire a real symmetric squared-mass matrix $\mu^2_{\kappa\kappa'}$. Considering an $SO(N^2-1)$ transformation $O_{\kappa\kappa'}$ which diagonalizes the $\mu^2_{\kappa\kappa'}$ with mass eigenvalues M_κ^2, the gauge interaction (2.1) is rewritten in terms of mass eigenmodes as

$$f\bar{\psi}\gamma^\mu \tilde{H}_\mu^\kappa \tilde{T}_\kappa \psi, \tag{2.2}$$

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where

$$\tilde{T}_\kappa = O_{\kappa\kappa'} T_{\kappa'}, \tag{2.3}$$

and

$$\tilde{H}^\kappa_\mu = O_{\kappa\kappa'} H^{\kappa'}_\mu . \tag{2.4}$$

The $\tilde{H}^\kappa_\mu$ denotes the mass eigenfield of the horizontal gauge boson with mass M_κ.

Before discussing the gap equation, we briefly comment on the number of generations N and horizontal symmetry breaking. As pointed out in the original works of the top-quark condensate scenario, the top quark must be much heavier than we expect in experiments in order to supply sufficient masses for the weak bosons. This problem can be avoided by introducing higher generations [9], which may be the dominant sources of the weak boson masses. This is the case for our model, which implies that we must consider $N \geq 4$ models. It is assumed, however, that the higher generations are nearly decoupled from the ordinary three generations, so that we take the $N = 3$ model and SU(3) horizontal symmetry, hereafter.

The second question is what type of breaking of horizontal symmetry should be considered. Here, we comment only general features of the breaking pattern. Consider a case that all horizontal gauge bosons have the same mass. It is easily seen that the fermion mass matrix is proportional to a unit matrix because the fermions have a global horizontal SU(3) symmetry. This suggests that we should consider some hierarchical structures of horizontal symmetry breaking for obtaining the realistic fermion mass matrices. We hope that some group theoretical structures underlie the hierarchical horizontal symmetry breaking, for example, a so-called sequential breaking.

Now, let us consider the mass gap equations for fermions. Since it is difficult to solve a Schwinger-Dyson (SD) equation, which has momentum-dependent solutions, in general, especially for the present broken gauge interactions, we adopt the following two approximations. One is the replacement of the intermediate horizontal gauge interactions by four-Fermi ones. The other is the introduction of some weights into the mass gap equations, which represent the effects of the gauge boson mass. Noting that our main purpose is the investigation of the texture of the fermion mass matrices, these approximations do not influence our results.

Let us start with a simple case that all horizontal gauge bosons have the same mass Λ. The intermediate horizontal gauge interactions are replaced by the four-Fermi interactions

$$L_{\rm int} = -\frac{f^2}{2\Lambda^2}(\bar{\psi}\gamma_\mu T_\kappa \psi)(\bar{\psi}\gamma^\mu T_\kappa \psi). \tag{2.5}$$

The mass gap equation indicated by the diagram in Fig.1 is

$$M = \frac{f^2}{4\pi^2}\sum_\kappa T_\kappa \left[1 + \frac{M^2}{\Lambda^2}\ln\frac{M^2}{\Lambda^2}\right] M T_\kappa, \tag{2.6}$$

where we take a horizontal breaking scale Λ for a cutoff. Note that T_κ's are already represented on the gauge boson mass diagonal base in this case.

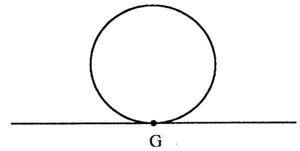

FIG. 1. A self-energy diagram.

The next step is to consider a more complicated case that the horizontal gauge bosons have different masses. Supposing that one of the horizontal gauge bosons has an infinite mass, its contribution to the gap equation drops out. This extreme case tells us that the contribution of a heavy gauge boson to the gap equation is small, and that of the light one is large. To incorporate this effect into the gap equation, we modify Eq.(2.6) by introduction of some weights ρ_κ, corresponding to the gauge bosons $\tilde{H}_\kappa$, which become small for the heavy gauge bosons:

$$M = \frac{f^2}{4\pi^2}\sum_\kappa \rho_\kappa \tilde{T}_\kappa \left[1 + \frac{M^2}{\Lambda^2}\ln\frac{M^2}{\Lambda^2}\right] M \tilde{T}_\kappa. \tag{2.7}$$

Here, we take Λ for the mass of the lightest gauge bosons. Note that the dominant part of Eq. (2.7), which comes from the lightest gauge bosons, is not modified, i.e., $\rho_{\rm\ lightest} = 1$, and for the other gauge bosons, $\rho_\kappa < 1$. Equation (2.6) is the case that all $\rho_\kappa = 1$.

Now, we consider quark mass matrices of up and down sectors. These two mass matrices satisfy the same Eq. (2.7), since the horizontal interactions are common to both sectors. We can see from Eq. (2.7) that the mass differences among the generations depend upon the breaking pattern of the horizontal symmetry, ρ_κ and $\tilde{T}_\kappa$. Equation (2.7) cannot, however, discriminate between upsector and downsector, which leads us to the same mass matrices.

This result, however, can be avoided by taking into account that the vertical gauge forces influence each sector in different way. In fact, the vertical gauge interaction $U(1)_Y$ can discriminate between up and down sectors, giving small corrections to the gap equations dominated by four-Fermi or horizontal gauge interactions. Many people [10–12] evaluate the effective coupling $G_{\rm eff}$:

$$\frac{G_{\rm eff}}{G_{\rm cr}} = \frac{G}{G_{\rm cr}} + \frac{3}{8\pi^2} g_1^2(\Lambda) Y_L Y_R, \tag{2.8}$$

where $g_1(\Lambda)$ is $U(1)_Y$ running coupling constant at Λ, say, $3g_1^2(10\ {\rm TeV})/8\pi^2 \sim 5\times 10^{-3}$, $Y_{L(R)}$ is the hypercharge of left- (right-)handed quarks, and

$$G = \frac{f^2}{4\pi^2}. \tag{2.9}$$

$G_{\rm cr}$ is a critical coupling constant for the dynamical mass generation of Eq. (2.7). One of the present authors evaluated [5] a similar expression in terms of horizontal gauge coupling f by calculating two loop diagrams with QED corrections.

At a glance, this small correction could not induce a large mass splitting between up and down sectors, especially between top and bottom. At this point, however, it is quite important to note that our model is a near critical system, that is, the horizontal gauge coupling constant is taken to be very close to the critical one. This type of fine-tuning is in general needed to relate a high energy scale theory to a low energy physics. In fact, if the coupling constant is not fine-tuned in our case, the gap equation (2.7) has a solution $M = O(\Lambda)$. As long as we expect the mass scale $\Lambda_{\rm SM}$ of the standard model, a fine-tuning of the order of $O(\Lambda_{\rm SM}/\Lambda)$ is needed.

It has been pointed out that a small perturbation as mentioned above may be enhanced under the fine-tuned system [4]. To make this point clear, we consider two systems with different coupling constants G_U and G_D, where G_U is a little larger than G_D. One example is a case that $G_U > G_{\rm cr} > G_D$, which implies that solutions of the gap equations are $M_U \neq 0$ and $M_D = 0$. Another example is a case that $G_U > G_D > G_{\rm cr}$ and $G_U - G_{\rm cr} \gg G_D - G_{\rm cr}$. We rewrite the gap equation Eq. (2.7) as

$$\frac{1}{G_{\rm cr}} - \frac{1}{G} \sim -\frac{M^2}{\Lambda^2}\ln\frac{M^2}{\Lambda^2}, \qquad (2.10)$$

where we neglect the matrix form for simplicity. Equation (2.10) indicates that $M_U \gg M_D$ in this example.

In this paper, we adopt the latter case, where $G_{U(D)}$ is a effective coupling for up(down) sectors. Indeed, the $\mathrm{U}(1)_Y$ interaction is attractive for up sector and repulsive for down sector. $M_U \gg M_D$ can be realized with a fine-tuning of G. To obtain the difference between m_t and m_b by $U(1)_Y$, we must take $\Lambda \sim 20$ TeV, which implies that $G_U - G_{\rm cr} \sim 10^{-3}$ and $m_t \sim 150$ TeV [10].

III. LINEARIZING APPROXIMATION AND EIGENVALUE PROBLEM

In this section, we shall investigate the relationship between breaking patterns of horizontal symmetry and mass matrices. In the preceding section, we have shown that solutions of Eq. (2.7) can provide a large difference between M_U and M_D. Noting that $M^2/\Lambda^2 \ll 1$, Eq. (2.7) is satisfied mainly by cancellation between linear terms of M on each hand side. This indicates that the matrix form of M is mainly determined by linear parts of Eq. (2.7), though the scale of M is determined by its nonlinear terms, as seen from Eq. (2.10). Therefore, we neglect nonlinear terms of the gap equation for a while in order to study forms of mass matrices.

The linearized gap equation is

$$M = G\sum_{\kappa}\rho_\kappa \tilde{T}_\kappa M \tilde{T}_\kappa. \qquad (3.1)$$

Note that this approximation is exact on the critical points. The essential point is that to solve Eq. (3.1) is nothing but an eigenvalue problem, in which the coupling constant G and mass matrix M correspond to an eigenvalue and an eigenvector, respectively. At a glance, only discrete and finite number of couplings would be allowed, because we consider now only linear parts of gap equation.

When Eq. (3.1) has several eigenvalues, which should we select? This is a problem of how to search for the most stable solutions. The answer is given by choosing the smallest eigenvalue for a near critical system. Because the fine-tuned solution is less stable than the others, it must be the one and only nontrivial solution. It means that the eigenvalue of the fine-tuned solution is the smallest. In fact, supposing that there are two positive eigenvalues G_1 and G_2 ($G_1 < G_2$), solutions of the full gap equation (2.7) with coupling constant G in the following five cases are conceivable:

(a) $G < G_1 < G_2$. There are no nontrivial solutions.

(b) $G_1 \lesssim G < G_2$. There is one fine-tuned solution, corresponding to the G_1 eigenmode.

(c) $G_1 < G < G_2$. There is one solution, corresponding to the G_1 eigenmode.

(d) $G_1 < G_2 \lesssim G$. There is one fine-tuned solution corresponding to the G_2 eigenmode beside another solution corresponding to G_1.

(e) $G_1 < G_2 < G$. There are two solutions.

By noting that the fine-tuned state is less stable than the others, the G_1 mode in case (d) turns out to be more stable than the fine-tuned G_2 mode and chosen. Then, (b) is the only case that we want.

We apply the above rule to simple examples, such as $\rho_\kappa = \{0,1\}$ and $\tilde{T}_\kappa = \frac{1}{2}\lambda_\kappa$, where λ_κ is the Gell-Mann matrix. These examples mean that the horizontal gauge bosons $\tilde{H}_\kappa$ corresponding to $\rho_\kappa = 0$ have very large masses and $\tilde{H}_\kappa$ corresponding to $\rho_\kappa = 1$ have small masses Λ. Moreover, some symmetries are assumed to be survived at Λ, for example, SU(3), SU(2) $\times$U(1), and U(1).

(i) SU(3) case. In this case, all $\rho_\kappa = 1$. Equation (3.1) has one positive eigenvalue:

$$G = \frac{3}{4}, \qquad M \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}. \qquad (3.2)$$

This result is natural since there is a global SU(3) horizontal symmetry. It is, however, undesirable phenomenologically.

(ii) SU(2)$\times$U(1) case. Here, we take $\rho_{1,2,3,8} = 1$ and $\rho_{4,5,6,7} = 0$. Equation (3.1) has two positive eigenvalues

$$G = \frac{6}{5}, \qquad M \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix}, \qquad (3.3)$$

$$G = 3, \qquad M \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}. \qquad (3.4)$$

The solution (3.4) is desirable phenomenologically, which means only one generation is massive. However, it is ruled out by the principle that the smallest eigenvalue must be selected. Then, we have the phenomenologically undesirable solution (3.3) in this case.

(iii) U(1) case. We take $\rho_8 = 1$ and the others are zero. Equation (3.1) has two positive eigenvalues

$$G = 3, \qquad M \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}, \tag{3.5}$$

$$G = 12, \qquad M \sim \begin{pmatrix} a & b & \\ c & d & \\ & & 0 \end{pmatrix}, \tag{3.6}$$

where a, b, c, and d are arbitrary parameters. The smallest eigenvalue solution (3.5) is phenomenologically desirable. The realization of the solution (3.5) is also understandable if we note that $\lambda_8 \sim \mathrm{diag}(1,1,-2)$, which implies that third generation feels the H_8 interaction twice as much as the others do.

IV. ORIGIN OF DIFFERENCE BETWEEN UP AND DOWN SECTORS

From now on, we shall study the origin of mixings. In the preceding section, we have shown that the linearized gap equation (3.1) well describes the matrix form of M. Starting from this linearizing approximation, we take account of the effects of nonlinear terms in the gap equation (2.7). For simplicity, we apply the linearizing approximation to down sector as in the preceding section, because G_D is closer to G_{cr} than G_U. In this sense, we deal with only up sector below.

The nonlinear terms of (2.7) play two important roles in our model. One is to determine the scale of mass matrix M, as mentioned above. The other is to generate mixing, which means that a solution M of Eq. (2.7) is not proportional to the solution of linearized Eq. (3.1). In order to understand this intuitively, we introduce an iteration method for solving the gap equation (2.7).

At first, it is assumed that the linearized gap equation (3.1) has eigenvalues G_i ($G_1 < G_2 < \cdots < G_n$) and corresponding eigenvectors M_i. We rewrite the gap equations (3.1) and (2.7) as

$$M = GA_0[M] \tag{4.1}$$

and

$$M = G\left(A_0[M] + A_1[M]\right), \tag{4.2}$$

respectively, where A_0 is a linear operation and A_1 is a nonlinear one. One operation of A_0 on M_i is

$$M_i \longrightarrow GA_0[M_i] = \frac{G}{G_i} M_i, \tag{4.3}$$

which means that, if $G > G_i$, the corresponding mode M_i grows with iteration, and if $G < G_i$, it dumps. The mode corresponding to the smallest eigenvalue G_1 is most dominant, as mentioned in the preceding section, since it has the largest factor G/G_1 in (4.3). If $G \neq G_i$, M_i diverges or vanishes by repetition of (4.1). This indicates that Eq. (4.1) demands that G is one of the eigenvalues and M is the corresponding eigenvector.

When G does not belong to eigenvalues, (4.2) must be considered. In neglecting the matrix form in (2.7), the nonlinear term $A_1[M]$ in (4.2) has negative contribution. If $G < G_i$, M_i dumps faster than (4.1) and the solution is $M = 0$. If $G > G_i$, the effects of A_1 weaken the degree of divergence of M in iteration, and the scale of the solutions is determined when effects of A_1 and A_0 cancel each other. Larger G/G_i requires larger scale of M since the effect of A_1 should be sufficiently large for canceling that of A_0.

A_1 has another effect in general, which rotates eigenmodes and generates mixings. Starting from M_1 with $G > G_1$, which is the most dominant mode, one operation of (4.2) leads us to

$$G\left(A_0[M_1] + A_1[M_1]\right) \longrightarrow \left(\frac{G}{G_1} + \delta\right) M_1 + (\text{other modes}), \tag{4.4}$$

where δ is $O(M_1^2/\Lambda^2)$. It is important to point out that other modes are smaller than M_1 with many iterations because $A_1[M]$ is $O(M^2/\Lambda^2)$ and G/G_i is smaller than G/G_1. The scale of other modes is determined by the balance of effects between A_0 and A_1. In general, when G/G_i is larger or A_1 generates M_i mode more, M_i mode is larger.

Experiments show non-proportionality between M_U and M_D, for example, $m_c/m_t : m_s/m_b \sim 1 : 3$. Can we realize such non-proportionality? The answer is that, if G_1 and G_2 are sufficiently close, it is possible. As shown above, when G/G_1 and G/G_2 are not so different, the suppression of the M_2 mode weakens and this mode survives at last.

V. DOWN-QUARK DIAGONALIZING BASE

The gap equations for up and down sectors are

$$\begin{aligned} M_D &= G_D \sum_\kappa \rho_\kappa \tilde{T}_\kappa M_D \tilde{T}_\kappa, \\ M_U &= G_U \sum_\kappa \rho_\kappa \tilde{T}_\kappa \left[1 + \frac{M_U^2}{\Lambda^2} \ln \frac{M_U^2}{\Lambda^2}\right] M_U \tilde{T}_\kappa. \end{aligned} \tag{5.1}$$

ρ_κ was given in our previous paper [4] as

$$\rho_\kappa = \ln \frac{\Lambda^2}{M_\kappa^2}, \tag{5.2}$$

with cutoff Λ and horizontal gauge boson masses M_κ.

Before entering into detailed discussion, we will briefly summarize the base transformations of the gap equations. Equation (5.1) contain multi-index quantities $(\tilde{T}_\kappa)_{ij}$ and M_{ij} where i, j denote the quark generations running 1–3 and κ the SU(3) generators running 1–8. Corresponding to these two types of indices, we deal with two different types of bases: (a) the quark base $(i, j, \ldots)$ and (b) the horizontal gauge (HG) boson base $(\kappa, \kappa', \ldots; \alpha, \beta, \ldots)$.

(a) The quark base is transformed by the SU(3) matrix. Quark mass matrices can always be diagonalized by this base transformation.

(b) The HG boson base is transformed by O(8) matrix. We have already used this type of rotation to diagonalize the HG boson mass matrix and $T_\kappa \equiv \frac{1}{2}\lambda_\kappa$ was replaced by $\tilde{T}_\kappa$.

Type (a) transformations U constitute a proper subset (subgroup) of the type (b) transformations. Indeed, type (b) transformations have greater degrees of freedom than (a). We can always rewrite any SU(3) matrix U as its adjoint representation $R_{\alpha\beta}[\in\mathrm{O}(8)]$ defined by

$$R_{\alpha\beta} = \frac{1}{2}\mathrm{tr}U^\dagger\lambda^\alpha U\lambda^\beta. \tag{5.3}$$

By virtue of the orthogonality of the Gell-Mann matrices $\mathrm{tr}\lambda^\alpha\lambda^\beta = 2\delta^{\alpha\beta}$, Eq. (5.3) can also be written as

$$R_{\alpha\beta}\lambda^\beta = U^\dagger\lambda^\alpha U. \tag{5.4}$$

However, we cannot express every O(8) rotation in the form of Eq. (5.3). Especially, horizontal mixing angles, i.e., a rotation matrix $O_{\kappa\alpha}$, which transforms the HG bosons from the standard Gell-Mann base into the mass diagonal base, cannot always be compensated for by the quark base transformation. In other words, $\tilde{T}_\kappa \equiv \frac{1}{2}O_{\kappa\alpha}\lambda^\alpha$ can never be written in the form of the right-hand side of Eq. (5.4) in general. We write down again the gap equations (5.1), indicating the horizontal mixing angles manifestly

$$M_D = x_\kappa O_{\kappa\alpha}O_{\kappa\beta}\lambda^\alpha M_D\lambda^\beta, \tag{5.5}$$

$$M_U = \xi x_\kappa O_{\kappa\alpha}O_{\kappa\beta}\lambda^\alpha\left[1+\frac{M_U^2}{\Lambda^2}\ln\frac{M_U^2}{\Lambda^2}\right]M_U\lambda^\beta, \tag{5.6}$$

where $x_\kappa = \frac{1}{4}G_D\rho_\kappa$ and $\xi = G_U/G_D$. ξ is evaluated from Eq. (2.8):

$$\xi = 1 + O(10^{-3}). \tag{5.7}$$

Now, let us define the down-quark diagonalizing base (DDB), which is selected to diagonalize the down-sector quark mass matrix. Suppose that we successfully solve Eq. (5.5) for given x_κ, $O_{\kappa\alpha}$ and obtain a solution M_D. There exists a unitary transformation U_D which diagonalize M_D. Transforming Eqs. (5.5) and (5.6) by U_D, we obtain

$$D_D = x_\kappa\tilde{O}_{\kappa\alpha}\tilde{O}_{\kappa\beta}\lambda^\alpha D_D\lambda^\beta, \tag{5.8}$$

$$M_U = \xi x_\kappa\tilde{O}_{\kappa\alpha}\tilde{O}_{\kappa\beta}\lambda^\alpha\left[1+\frac{M_U^2}{\Lambda^2}\ln\frac{M_U^2}{\Lambda^2}\right]M_U\lambda^\beta, \tag{5.9}$$

where we denote $\tilde{O} = OR$, using a rotation matrix R defined by

$$R_{\alpha\beta} = \frac{1}{2}\mathrm{tr}U_D^\dagger\lambda^\alpha U_D\lambda^\beta, \tag{5.10}$$

and redefine the quark mass matrices

$$U_D^\dagger M_D U_D \longrightarrow D_D = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}, \tag{5.11}$$

$$U_D^\dagger M_U U_D \longrightarrow M_U = V_{\mathrm{KM}}^\dagger\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}V_{\mathrm{KM}}. \tag{5.12}$$

Here, V_{KM} is the CKM matrix. Note that, in this expression, all ambiguous unphysical degrees of freedom are fixed; i.e., the mass matrices are written only by the quark masses and mixing angles.

VI. DOWN-SECTOR EQUATION

In the preceding section, we obtained the set of equations (5.8) and (5.9) in the DDB, which the quark mass parameters, i.e., the masses and the mixings, should satisfy. Our task is now to find out weight parameters x_κ and rotation matrix $\tilde{O}_{\kappa\alpha}$, which give rise to phenomenologically acceptable quark mass parameters. Note that x_κ and $\tilde{O}_{\kappa\alpha}$ are constrained by the DDB condition. In fact, the vanishing of the off-diagonal elements of D_D in Eq. (5.8) imposes six real conditions on 8 + 28 degrees of freedom of x_κ and $\tilde{O}_{\kappa\alpha}$. Moreover, taking account of the experimental values of the diagonal elements (down quark masses), two additional real conditions exist. Note that Eq. (5.8) does not determine an overall scale of solutions.

Since the down-sector equation (5.8) is a linear equation for D_D, we can rewrite it in the form of the nine-dimensional eigenvalue problem to see the above conditions in more detail. Let us define nine orthonormal matrix units σ_p of the Hermitian matrices:

$$\begin{aligned}
&\sigma_1 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad \sigma_4 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \sigma_7 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
&\sigma_2 = \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}, \quad \sigma_5 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \sigma_8 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\
&\sigma_3 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}, \quad \sigma_6 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \sigma_9 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\end{aligned} \tag{6.1}$$

satisfying $\mathrm{tr}\sigma_p\sigma_q = \delta_{pq}$. The down-sector mass matrix can be represented in terms of σ_p:

$$D_D = d_p\sigma_p, \qquad d_p = \mathrm{tr}\sigma_p D_D, \tag{6.2}$$

where d_p's are components corresponding to σ_p's. Taking trace of Eq. (5.8) multiplied with σ_p, we obtain the eigenequation for the nine-dimensional vector d_p:

$$d_p = A_{pq}d_q, \tag{6.3}$$

where A_{pq} is defined as

$$A_{pq} = x_\kappa \tilde{O}_{\kappa\alpha}\tilde{O}_{\kappa\beta}\mathrm{tr}\sigma_p\lambda_\alpha\sigma_q\lambda_\beta. \tag{6.4}$$

The 9×9 matrix A_{pq}, which is real symmetric and traceless by definition, contains all physical information of the mass matrix of the horizontal gauge boson corresponding to x_κ and $\tilde{O}_{\kappa\alpha}$.

Generally, A_{pq} has several eigenvalues. We introduce eigenvalue η explicitly into Eq. (6.3):

$$d_p = \eta A_{pq}d_q. \tag{6.5}$$

By construction of A_{pq}, Eq. (6.5) must have the solution of $d_p = (m_d, m_s, m_b, 0, 0, 0, 0, 0, 0)$ with $\eta = 1$, which corresponds to the down quark diagonalized solution (DDS) of (5.11). In addition to this, $\eta \neq 1$ solutions can also be realized for gauge coupling ηG_D. In order for DDS to be chosen, we require that the DDS (5.11) should be the most stable solution, which corresponds to the smallest eigenvalue and is realized for the weakest gauge coupling. If there exist solutions for $\eta < 1$, they will dominate as was seen in Sec. IV. Therefore, Eq. (6.5) should not have $\eta < 1$ solutions.

Consequently, we can summarize the following DDB conditions for the matrix A_{pq}: (I) A_{pq} should have an eigenvector $d_p = (m_d, m_s, m_b, 0, 0, 0, 0, 0, 0)$ with eigenvalue 1; (II) the eigenequation (6.5) has only $\eta \geq 1$ eigenvalues or negative.

VII. A TOY MODEL

Let us apply our formulation to some simple cases. We assume that only three HG bosons are light, which have only nonzero x_κ of x_3, x_6, x_8. We also constrain $\tilde{O}_{\kappa\alpha}$ to be a 3×3 matrix with κ and α being 3, 6, 8. The corresponding three Gell-Mann matrices are

$$\lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 2 \end{pmatrix}. \tag{7.1}$$

Under this assumption, the first generation does not couple with the other generations. In addition to the diagonal elements (2,2) and (3,3), only the real part of (2,3) element in Eq. (5.8) gives the nontrivial constraint:

$$m_s = x_\kappa\left[\left(\tilde{O}_{\kappa3}\tilde{O}_{\kappa3} + \frac{1}{3}\tilde{O}_{\kappa8}\tilde{O}_{\kappa8} - \frac{2}{\sqrt{3}}\tilde{O}_{\kappa3}\tilde{O}_{\kappa8}\right)m_s + \tilde{O}_{\kappa6}\tilde{O}_{\kappa6}m_b\right],$$
$$m_b = x_\kappa\left[\tilde{O}_{\kappa6}\tilde{O}_{\kappa6}m_s + \frac{4}{3}\tilde{O}_{\kappa8}\tilde{O}_{\kappa8}m_b\right], \tag{7.2}$$
$$0 = x_\kappa\left[\left(\frac{1}{\sqrt{3}}\tilde{O}_{\kappa6}\tilde{O}_{\kappa8} - \tilde{O}_{\kappa3}\tilde{O}_{\kappa6}\right)m_s - \frac{2}{\sqrt{3}}\tilde{O}_{\kappa6}\tilde{O}_{\kappa8}m_b\right].$$

Since m_d is negligible compared with m_s and m_b, and the first generation is decoupled from the other two generations, we take $m_d = 0$ by hand. It means that the constraint from the (1,1) element in Eq. (5.8) is trivial.

By setting m_s/m_b to its experimental value, we can calculate x_κ for given $\tilde{O}_{\kappa\alpha}$ using the above equations (7.2). We parametrize $\tilde{O}_{\kappa\alpha}$ by Euler-like angles:

$$\tilde{O} = \begin{pmatrix} \cos\theta_{36} & -\sin\theta_{36} & 0 \\ \sin\theta_{36} & \cos\theta_{36} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_{38} & 0 & -\sin\theta_{38} \\ 0 & 1 & 0 \\ \sin\theta_{38} & 0 & \cos\theta_{38} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{68} & -\sin\theta_{68} \\ 0 & \sin\theta_{68} & \cos\theta_{68} \end{pmatrix}. \tag{7.3}$$

We search all parameter space of these Euler angles and have found some allowed region (Figs. 2,3) which satisfies the DDB conditions (I) and (II), by the following procedures.

(1) Give the Euler angles, and calculate x_κ from Eq. (7.2). [For some peculiar values of the Euler angles, Eq. (7.2) is degenerate so that we cannot obtain x_κ from this equation. These values are presented by 'v' in the figures.]

(2) Check all x_κ to be positive. (The negative x_κ is unphysical. It is indicated as '−' in the figures.)

(3) Construct A_{pq} in Eq. (6.4) for this x_κ with the $\tilde{O}_{\kappa\alpha}$, and find its eigenvalues η [see Eq. (6.5)]. If there exists some η which is less than 1, DDS is less stable and

```
       -90   -60   -30    0   +30   +60   +90  θ68
        +     +     +     +     +     +     +
-90   + --------------------v----------------
        --------------------A----------------
        --------------------A----------------
-60   + --------------------A----------------
        --------------------x----------------
        ---------------------x---------------
-30   + -A-----------------------------------
        ------------------A------------------
        ------------------A------------------
0     + v-----------------A-----------------v
        ------------------A------------------
        ------------------A------------------
+30   + ------------------A----------------A-
        ------------------x------------------
        -----------------xx------------------
+60   + -------------------------------------
        ----------------A--------------------
        ----------------A-----------------A--
+90   + ----------------v--------------------
θ38
```

FIG. 2. Allowed regions of the $\theta_{68} - \theta_{38}$ plane with $\theta_{36} = 10°$, which are indicated as '**A**.'

```
                 10.0      13.0      16.0         θ68
            +    +    +    +    +    +    +
-100  + ---------A-----------------------------
        ---------A-----------------------------
        ---------A-----------------------------
        ---------------------------------------
        ---------A-----------------------------
-80   + ---------A-----------------------------
        ---------AA----------------------------
        --------AAA----------------------------
        --------AAAA---------------------------
        -------AAAAAAA-------------------------
-60   + -------AAAAAAAA------------------------
        ------xxxxxxxxxxx----------------------
        ------xxxxxxxxxxxxx--------------------
        ------xxxxxxxxxxxxxxxx-----------------
        --------xxxxxxxxxxxxxxxxx--------------
-40   + ----------xxxxxxxxxxxxxxxxxx-----------
        -----------------xxxxxxxxxxxxxxxx------
        ----------------------------xxxxxxxxxx-
        ---------------------------------------
        ---------------------------------------
-20   + ---------------------------------------
θ38
```

FIG. 3. A magnification of Fig. 2. ($\theta_{36} = 10°$, $\theta_{38} = -60° \pm 40°$, $\theta_{68} = 13° \pm 5.7°$.)

not realized (indicated as '**x**'). Only the case with η being not less than 1 is allowed by the DDB conditions. There exists at least one eigenvalue which is equal to 1. It is corresponding to DDS and indicated as '**A**'.

For example, Fig. 2 shows a $\theta_{68} - \theta_{38}$ plane with $\theta_{36} = 10°$, and an allowed region is magnified in Fig. 3.

We can now solve Eq. (5.9) for up sector with allowed Euler angles obtained above by iteration. We simply replace the matrix $(1/\Lambda^2)\ln[M_U^2/\Lambda^2]$ by $\zeta = (1/\Lambda^2)\ln[(M_U)^2_{33}/\Lambda^2]$, since this modification does not affect our result so much:

$$M_U = \xi x_\kappa \tilde{O}_{\kappa\alpha}\tilde{O}_{\kappa\beta}\lambda^\alpha \left[M_U + \zeta M_U^3\right]\lambda^\beta. \tag{7.4}$$

Here, we present an interesting solution M_U, which is not proportional to M_D, as expected, such as

$$\frac{M_U}{\Lambda} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 75.15\times 10^{-6} & -1.75\times 10^{-6} \\ 0 & -1.75\times 10^{-6} & 0.00876880 \end{pmatrix}, \tag{7.5}$$

with $(\theta_{36}, \theta_{38}, \theta_{68}) = (10°, -85.765°, 10°)$ and $\xi = 1.001$. It means that $m_t = 175$ GeV, $m_c = 1.5$ GeV, $m_u = 0$ when $\Lambda = 20$ TeV. The above Euler angles correspond to the weights $\rho_3 = 0.75008512$, $\rho_6 = 0.00080017$, and $\rho_8 = 0.72880273$. The eigenvalues and eigenvectors of corresponding A_{pq} is as shown in Table I.

Here, we arrange Table I in order of inverse of eigenvalues. Since A_{pq} is nonvanishing only for $p, q \le 6$, we regard A as a 6×6 matrix. Eigenvector $d^{(1)}$ with $\eta = 1$ corresponds to the down-sector solution.

Let us investigate features of the above solutions in brief. As shown in Sec. IV, up-sector solution M_U is formed by mixing a little $d^{(3)}$ with $d^{(1)}$. These two eigenvalues are close compared with others except for $\eta^{(2)}$, the eigenvector of which is decoupled in the gap equation (5.9).

The present solution can generate CKM mixings. From Eq. (5.12), the CKM matrix is given as

$$V_{\rm KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\frac{1}{2}\alpha^2 & -\alpha \\ 0 & \alpha & 1-\frac{1}{2}\alpha^2 \end{pmatrix} \quad \text{with} \quad \alpha = 0.0002, \tag{7.6}$$

off-diagonal elements of which arise from mixing the $d^{(3)}$ eigenvector. Since we have not searched all allowed regions, we could not here realize agreement between Eq. (7.6) and the experimental results.

TABLE I. The eigenvalues η^i and eigenvectors d^i of A_{pq}.

$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$	$\eta^{(4)}$	$\eta^{(5)}$	$\eta^{(6)}$
1.00000000	1.01981096	1.02345204	−2.10257169	−2.01046516	−1.98033194
$d^{(1)}$	$d^{(2)}$	$d^{(3)}$	$d^{(4)}$	$d^{(5)}$	$d^{(6)}$
0.00000000	1.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.03996804	0.00000000	0.99915918	0.00000000	−0.00913700	0.00000000
0.99920096	0.00000000	−0.03996637	0.00000000	0.00036548	0.00000000
0.00000000	0.00000000	0.00914431	0.00000000	0.99995819	0.00000000
0.00000000	0.00000000	0.00000000	−0.32998315	0.00000000	0.94398682
0.00000000	0.00000000	0.00000000	0.94398682	0.00000000	0.32998315

TABLE II. The ratio r of $d^{(1)}$ to $d^{(3)}$ for several couplings G.

G	1.0004	1.0006	1.0008	1.0010	1.0012
r	−0.0110	−0.0166	−0.0223	−0.0281	−0.0340

VIII. CONCLUSIONS AND DISCUSSIONS

We have discussed the dynamical mass generation and the possibility of CKM mixings by the broken horizontal gauge interactions. The essential point of generating the difference between M_U and M_D is that the fine-tuned system is perturbed by the radiative $\mathrm{U}(1)_Y$ corrections. The above results are induced by the mechanism that nonlinear terms of the gap equation mix eigenvectors of the linearized gap equation. We should emphasize that the above results are caused though the horizontal interactions themselves do not discriminate between up and down sectors.

In the above model, there arise no Cabibbo angle and CP-violating phase. In order to get realistic CKM matrix, we should consider the first generation and horizontal interactions corresponding to λ_κ with imaginary elements.

In this paper, we have replaced broken horizontal gauge interactions with four-Fermi interactions, which corresponds to some truncation of the full Schwinger-Dyson equations. In general, this approximation does not affect the above discussions so much. However, the first generation considered, this replacement may have huge effects since mass matrices have a hierarchical structure with a small eigenvalue.

In Sec. III, we adopted the linearized gap equation for down sector. This is justified by confirming that the solution of the nonlinear equation goes to that of the linearized equation in the limit of $G \to G_{\rm cr}$. We explain it by using the model in Sec. VII with $\theta_{36} = 10°$, $\theta_{38} = -85.75°$, and $\theta_{68} = 10°$. We define the eigenvectors in this case as $d^{(i)}$ like in Sec. VII. The solution of the linearized equation is $d^{(1)}$ and that of the nonlinear equation is a linear combination of $d^{(1)}$ and $d^{(3)}$ approximately. Table II shows the ratio r of $d^{(1)}$ to $d^{(3)}$ for the solutions of nonlinear equation with several coupling G.

Next, we discuss the horizontal breaking scale Λ. In the model of Sec. VII, $m_t = 175$ GeV demands that $\Lambda = 20$ TeV. It does not agree with the present experiments of flavor-changing neutral currents (FCNC's), which requires that $\Lambda > 1000$ TeV. However, the diagonal horizontal interactions are free from this constraint. Off-diagonal interactions do not satisfy this constraint in general. Fortunately, in the above solution, the small weight ρ_6 obtained above suppresses FCNC's due to H_6 interaction to some extent.

In the original top-quark condensation scenario, in which the cutoff scale is larger than the grand unified theory (GUT) scale, renormalization effects are not so small because of long evolution of renormalization group down to low energy. In the present case, the cutoff scale is much smaller than the GUT scale. The effect of the evolution is negligible as far as semiquantitative structure of mass matrices is considered.

ACKNOWLEDGMENT

The present authors wish to thank S. Tanaka for valuable discussions and a careful reading of the manuscript.

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