Fermion masses, mixing angles, and supersymmetric SO(10) unification

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We reanalyze the problem of fermion masses in supersymmetric SO(10) grand unified models. In the minimal model, both low energy Higgs doublets belong to the same **10** representation of SO(10), implying the unification not only of the gauge but also of the third generation Yukawa couplings. These models predict large values of $\tan\beta \sim 50$. In this paper we study the effects of departing from the minimal conditions in order to see if we can find models with a reduced value of $\tan\beta$. In order to maintain predictability, however, we try to do this with the addition of only one new parameter. We still assume that the fermion masses arise from interactions of the spinor representations with a single **10** representation, but this **10** now only contains a part of the two light Higgs doublets. This enables us to introduce one new parameter $\omega = \lambda_b/\lambda_t$. For values of $\omega \ll 1$ we can in principle reduce the value of $\tan\beta$. In fact, ω is an overall factor which multiplies the down quark and charged lepton Yukawa matrices. Thus the theory is still highly constrained. We show that the first-generation quark masses and the CP violation parameter ϵ_K are sufficient to yield strong constraints on the phenomenologically allowed models. In the end, we find that large values of $\tan\beta$ are still preferred.

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I. INTRODUCTION

The standard model describes with a great degree of precision the observed elementary particle interactions. It provides, however, no answer to the fundamental questions about the origin of the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, the structure of fermion masses and mixing angles and their quantum numbers. Grand unified theories (GUT's) have the power to fill the gap between theory and experiment [1]. Indeed, within this framework the low energy group proceeds from the spontaneous breakdown of a single compact group. The simplest and most attractive grand unified theories are based on the unitary group SU(5) or the orthogonal group SO(10). Remarkably, all low energy fermion quantum numbers find a natural explanation within these theories. For instance, the 15 Weyl fermions in a standard model family, with their correct quantum numbers under the standard model gauge group, are contained in a **10** and a **5** representation of SU(5). Most notably, they are contained in a single spinor representation of SO(10), the extra state having the quantum numbers of a right-handed neutrino and leading therefore to the possibility of including neutrino masses in a natural way.

If the grand unified group breaks at very high energies to the standard model gauge group, an essential requirement is that the theory should be supersymmetric [2]. Not only does supersymmetry stabilize the hierarchy between the grand unified scale and the weak scale, but also the predictions coming from gauge coupling unification within supersymmetric theories are in remarkably good agreement with the precise measurements of the weak mixing angle performed at the CERN e^+e^- collider LEP [3–5]. Moreover, supersymmetry provides the natural framework for the construction of a theory of quantum gravity, and hence for the unification of all forces observed in nature. Supersymmetric grand unified theories also provide a simple theoretical framework for the understanding of fermion masses. The condition of bottom-τ Yukawa coupling unification implies, for instance, a large value of the top quark Yukawa coupling at the grand unification scale [6, 7], the low energy value of the top quark mass being governed, in general, by the infrared fixed point structure of the theory [8–10]. Hence, supersymmetric GUT's provide an understanding of the large value of the top quark mass [4, 5, 11, 12]. Moreover, in the minimal SO(10) model, the three Yukawa couplings of the third generation unify at the grand unification scale. This yields predictions not only for the top quark mass, but also for the ratio of Higgs vacuum expectation values (VEV's), $\tan\beta$, which becomes naturally large [13]. Large values of $\tan\beta$ are also associated with large corrections to the bottom mass [14, 15], which depend on the supersymmetric spectrum and which should be computed in a consistent way in order to obtain phenomenologically correct predictions for the top quark mass [16].

II. MINIMAL SO(10) MODELS

The hierarchy between the third- and the first- and second-generation quark masses, as well as the intergeneration mixing angles, may be explained by assuming that only the third-generation quarks couple to the **10** of the Higgs fields by renormalizable interactions, while

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the other mass terms are induced through higher order operators. A systematic search for this class of models within the framework of minimal SO(10) was done in Ref. [17], under the assumption that the model includes only three spinor representations, containing the three low energy families, a few extra heavy spinor representations, the **10** Higgs multiplet and some **45**'s, necessary for the correct breakdown of the gauge symmetry and for the generation of the fermion mass operators. All higher order operators are of the form

$$O_{ij} = 16_i \; \frac{M_G^k \; 45_{k+1} \cdots 45_m}{M_P^l \; 45_X^{m-l}} \; 10 \; \frac{M_G^n \; 45_{n+1} \cdots 45_p}{M_P^q \; 45_X^{p-q}} \; 16_j, \tag{1}$$

where the 45 VEV's in the numerator can be in any of the four directions $\mathbf{X}, \mathbf{Y}, \mathbf{B}-\mathbf{L}, \mathbf{T_{3R}}$ (discussed below) and the 45 in the denominator can only be in the $\mathbf{X}$ direction which breaks SO(10) down to the subgroup $\mathrm{SU}(5)\times\mathrm{U}(1)_X$. This occurs at a scale M_{10} which is assumed to lie between the GUT scale $M_G \sim 10^{16}$ GeV and the Planck scale M_P.

The adjoint **45**'s may be labeled according to the direction of their vacuum expectation values. There are four special directions [17]. The X direction, necessary for the breakdown of SO(10) to $\mathrm{SU}(5)\times\mathrm{U}(1)_X$ at the scale M_{10}. The $\mathbf{45_X}$ in the denominator can arise when integrating out heavy **16** and $\overline{\mathbf{16}}$ states with mass from the $\mathbf{45_X}$ VEV. Of course, this only makes sense if $M_{10} > M_G$. Other directions are the Y and $B-L$, which break SU(5) to the standard model gauge group. The presence of the latter is required for a natural solution of the doublet-triplet splitting problem in this theory [18]. Finally, there is another, linearly dependent direction $\mathbf{T_{3R}}$, which, as we shall explain below, may be useful to achieve low values of $\tan\beta$ within this model.

Taking into account the experimental constraints on the lowest generation fermion masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles, the authors of Ref. [17] identified nine potentially acceptable models, in which the up and down quark and lepton mass matrices are of the form

$$\lambda_a = \begin{pmatrix} 0 & z_a^{'}C & 0 \\ z_a C & y_a E e^{i\phi} & x_a^{'} B \\ 0 & x_a B & A \end{pmatrix}, \tag{2}$$

where z_a, $z_a^{'}$, x_a, $x_a^{'}$, and y_a are Clebsch factors, while A, B, C, E, and ϕ are arbitrary parameters, which respect the hierarchy $A \gg B, E \gg C$ and must be adjusted in order to obtain predictions in agreement with the present data. The Higgs sector provides an additional free parameter, which is the ratio of vacuum expectation values, $\tan\beta$. Using the presently best known low energy parameters m_e, m_μ, m_τ, m_c, m_b, and $|V_{cd}|$ as input, the values of M_t, $\tan\beta$, $|V_{cb}|$, $|V_{ub}|$, m_u, m_d, m_s, and the CP-odd Jarlskog invariant J [19] are predicted (we shall denote physical and running masses by capital and small letters, respectively). This leads, hence, to eight low energy predictions, which should be compared with the present experimental values.

There are several properties, which are shared by all these models. First of all, they maintain the Georgi-Jarlskog relation [20] of the y_a Clebsch factors: $|y_e|:|y_d|:|y_u| \equiv 3:1:0$. This relation of Clebsch factors appears in a natural way, for example, through the operator

$$O_{22} = 16_2 \; \frac{45_X}{M} \; 10 \; \frac{45_{B-L}}{45_X} \; 16_2, \tag{3}$$

and it is important in order to derive correct predictions for the first two generations of quark masses. In fact,

$$\frac{m_s}{m_d} \simeq \left(\frac{y_d}{y_e}\right)^2 \frac{m_\mu}{m_e} \left|\frac{z_e z_e^{'}}{z_d z_d^{'}}\right|. \tag{4}$$

Hence, as long as the equality $z_e z_e^{'} = z_d z_d^{'}$ holds, the ratio of lepton and quark masses is in good quantitative agreement with the observed experimental values.

Another important property of these models is the unification of the three Yukawa couplings of the third generation and, in particular, the unification of the bottom and top Yukawa couplings, which requires large values of $\tan\beta$. Such large values of $\tan\beta$ are associated with three effects.

(1) Potentially large corrections to the down quark mass matrix (these radiative corrections are discussed in detail in the Appendix).

(2) With some fine-tuning of GUT scale soft supersymmetry- (SUSY-) breaking parameters in order to obtain radiative electroweak symmetry breaking at the weak scale. The range of parameters which satisfy the second constraint (*when universal scalar masses are imposed at* M_G), in fact, requires the corrections to down quark masses to be large.

(3) The proton decay rate resulting from dimension-five baryon-violating interactions is enhanced.

It has recently been shown that the first two consequences of large $\tan\beta$ are ameliorated when the constraint of universal scalar masses is removed [21]. The corrections to the down quark masses can be small and the amount of fine-tuning is greatly reduced. The problem of an enhanced proton decay rate is unaffected. On the other hand, these strong constraints become weaker for smaller values of $\tan\beta$. It becomes an important question whether the prediction for large $\tan\beta$ can be altered without destroying the predictability of the theory.

III. TRYING TO REDUCE $\tan\beta$ IN MINIMAL SO(10) MODELS

Lower values of $\tan\beta$ can easily be achieved by assuming that only one **10** of Higgs couples to fermions, but this **10** contains only a piece of the two Higgs doublets, the other components coming, for instance, from an additional **10**. The overall effect is to multiply the down and lepton mass matrices by a factor ω, which is the ratio of the relative components of the two Higgs doublets in the **10** which couples to fermions. The minimal model would hence be obtained for $\omega = 1$.

Such a situation can come about as follows: Consider the superpotential

$$W = \mathbf{10}\,\mathbf{45_{B-L}}\mathbf{10}' + \left[M_1\mathbf{10}' + (M_2 + \mathbf{45_X})\,\mathbf{10}\right]\mathbf{10}'' , \tag{5}$$

where M_1 and M_2 are of order $M_{\rm GUT}$; **10**, **10**$'$, and **10**$''$ are decouplets and only **10** participates in the fermion mass operators.

The first term in W implements the Dimopoulos-Wilczek mechanism [18] and yields four light doublets: **2**, $\bar{\mathbf{2}}$, **2**$'$, and $\bar{\mathbf{2}}'$; the color triplets get a mass of order $M_{\rm GUT}$. The second term gives a mass to a linear combination of **2** and **2**$'$ (by pairing it with $\bar{\mathbf{2}}''$) and a different linear combination of $\bar{\mathbf{2}}$ and $\bar{\mathbf{2}}'$ (by pairing it with $2''$). Explicitly, the light states are given by

$$2_L = \frac{M_1\, 2 - (M_2 + \mathbf{v})\, 2'}{\sqrt{M_1^2 + (M_2 + \mathbf{v})^2}} \tag{6}$$

and

$$\bar{2}_L = \frac{M_1\, \bar{2} - (M_2 - \mathbf{v})\, \bar{2}'}{\sqrt{M_1^2 + (M_2 - \mathbf{v})^2}}, \tag{7}$$

where $\langle \mathbf{45_X} \rangle = \mathbf{v} \times X$, with $X = +\ (-)$ when it acts on the **5** ($\bar{\mathbf{5}}$) of a **10** representation, respectively. Since **2** couples to the up quarks and $\bar{\mathbf{2}}$ couples to the down quarks, in this example we have

$$h_t = \lambda \frac{M_1}{\sqrt{M_1^2 + (M_2 + \mathbf{v})^2}},$$
$$h_b = \lambda \frac{M_1}{\sqrt{M_1^2 + (M_2 - \mathbf{v})^2}}, \tag{8}$$

and

$$\omega = \frac{\sqrt{M_1^2 + (M_2 + \mathbf{v})^2}}{\sqrt{M_1^2 + (M_2 - \mathbf{v})^2}}. \tag{9}$$

Notice that, in this simple example, M_1 (or $M_2 \pm \mathbf{v}$) cannot be too small, or else a pair of light triplets **3** and $\bar{\mathbf{3}}$ would appear in the spectrum, affecting the prediction for $\sin^2\theta_W$. Hence, ω cannot become too small in this case.

From now on, we shall discuss the consequences of the departure from the minimal conditions, taking values of ω lower than one. Values of ω lower than one decreases the bottom to top Yukawa coupling ratio but still requires bottom-τ Yukawa coupling unification.

A. SO(10) models with moderate values of $\tan\beta$: The second and third generations

We have introduced the parameter $\omega \leq 1$ in an attempt to lower $\tan\beta$. In this section we discuss the results for the second and third generations with the additional parameter ω. In general, taking into account variations of ω and reasonable assumptions on radiative corrections to down quark masses, we find that, in order to avoid a very heavy top quark, with mass larger then 190 GeV, the value of $\tan\beta$ should be either larger than 20 or very close to 1.

As a general feature, in order to obtain unification of the bottom and τ Yukawa couplings, the third generation Yukawa couplings must partially compensate the strong gauge coupling renormalization group effects. For $\omega = 1$, this is partially achieved by large values of the bottom Yukawa coupling. Indeed, the relation between the bottom quark and τ masses is given by

$$\frac{m_b}{m_\tau} = \mathbf{G}\exp(-I_t - 3I_b + 3I_\tau), \tag{10}$$

where $\mathbf{G}$ includes the ω independent, gauge coupling dependent factors, $I_a = \int (h_a/4\pi)^2 dt$ with h_a the corresponding Yukawa coupling and $t = \ln(Q/M_Z)$. In the following, we shall always assume that the right-handed neutrinos acquire large Majorana masses of order $M_{\rm GUT}$ and hence decouple from the renormalization group equations. Although there is a partial cancellation of the bottom and τ Yukawa coupling contributions at scales close to the unification scale, due to the factor 3 and the relation $I_b > I_\tau$, the bottom contribution becomes important for $\omega = 1$. For values of $\omega < 1$, for which only the bottom and τ Yukawa couplings unify, the top Yukawa coupling must increase in order to compensate for the smaller contribution of the bottom Yukawa coupling. For smaller values: $\omega < 0.5$, associated with moderate or small values of $\tan\beta$, and in the absence of supersymmetric threshold corrections, the top quark Yukawa coupling must acquire large values at the grand unification scale, being driven towards its infrared fixed point value at low energies. The convergence of the top quark mass to its fixed point value is naturally weaker for $\omega \simeq 1$.

For $\tan\beta \geq 5$, the fixed point value of the pole top quark mass reads $M_t \simeq$ 190–210 GeV, which is somewhat large in comparison to the current experimentally preferred value $M_t \simeq 180 \pm 12$ GeV [22]. The convergence to the fixed point for moderate values of ω may be softened by the presence of large bottom mass corrections, which become particularly relevant for values of $\tan\beta > 10$. For values of $\tan\beta \leq 5$, the bottom mass corrections are generically small, but the infrared fixed point value of the top quark mass, $M_t \simeq \sin\beta \times 200$ GeV, is lowered by the $\sin\beta$ factor (see Fig. 1). Indeed, values of $\tan\beta \lesssim 3$ are required for the fixed point solution to be in the range of phenomenologically preferred values. As we shall discuss below, these small values of $\tan\beta$ demand very small values of ω.

Figure 1 shows the dependence of the pole top quark mass on $\tan\beta$ (and also on ω) for three different values of the strong gauge coupling $\alpha_s(M_Z)$ and different values of the coefficient K_c parametrizing the bottom mass corrections, $\delta m_b = -m_b K_c \tan\beta$. Values of $K_c \geq 0.005$ lead to significant corrections to the predicted top quark mass values and, as was shown in [16], may appear in the presence of universal soft supersymmetry-breaking mass parameters at the grand unification scale. We concentrate on positive values of K_c, since for negative and large K_c either the top quark mass is above its experimentally preferred values or a Landau pole in the top quark Yukawa coupling appears at scales below $M_{\rm GUT}$. In Fig.

1 we have chosen a representative value of $m_b(m_b) = 4.15$ GeV. Larger (lower) values of m_b within the experimentally allowed range $m_b = 4.25 \pm 0.25$ GeV, would lead to somewhat lower (larger) values of M_t [5], without changing the general properties of the solutions.

It is interesting to note that, for large values of K_c and low values of $\alpha_s(M_Z)$, the top quark mass predictions in model 6 differ from the ones obtained in model 9 for the same values of ω. This reflects the effect of the mixing between the second and third generations on the predictions for the third-generation masses. It is easy to prove that, although this effect is generically small, the τ mass in model 9 receives a significant correction due to the mixing, which for values of $\alpha_s(M_Z) = 0.115$ and $K_c = 0.006$ becomes of order 15%. Because of the condition of bottom-τ Yukawa unification, large τ mass corrections also imply large variations in the top quark mass predictions.

To summarize, we observe that depending on the size of the one-loop supersymmetric corrections to the down quark masses, successful top quark mass predictions may be obtained for the minimal models with $\omega = 1$, but also for moderate and small values of $\tan\beta$ (associated with moderate or very small values of ω). It is hence important to know if the same is true for the first two generations of quark masses and mixing angles. Since the relation

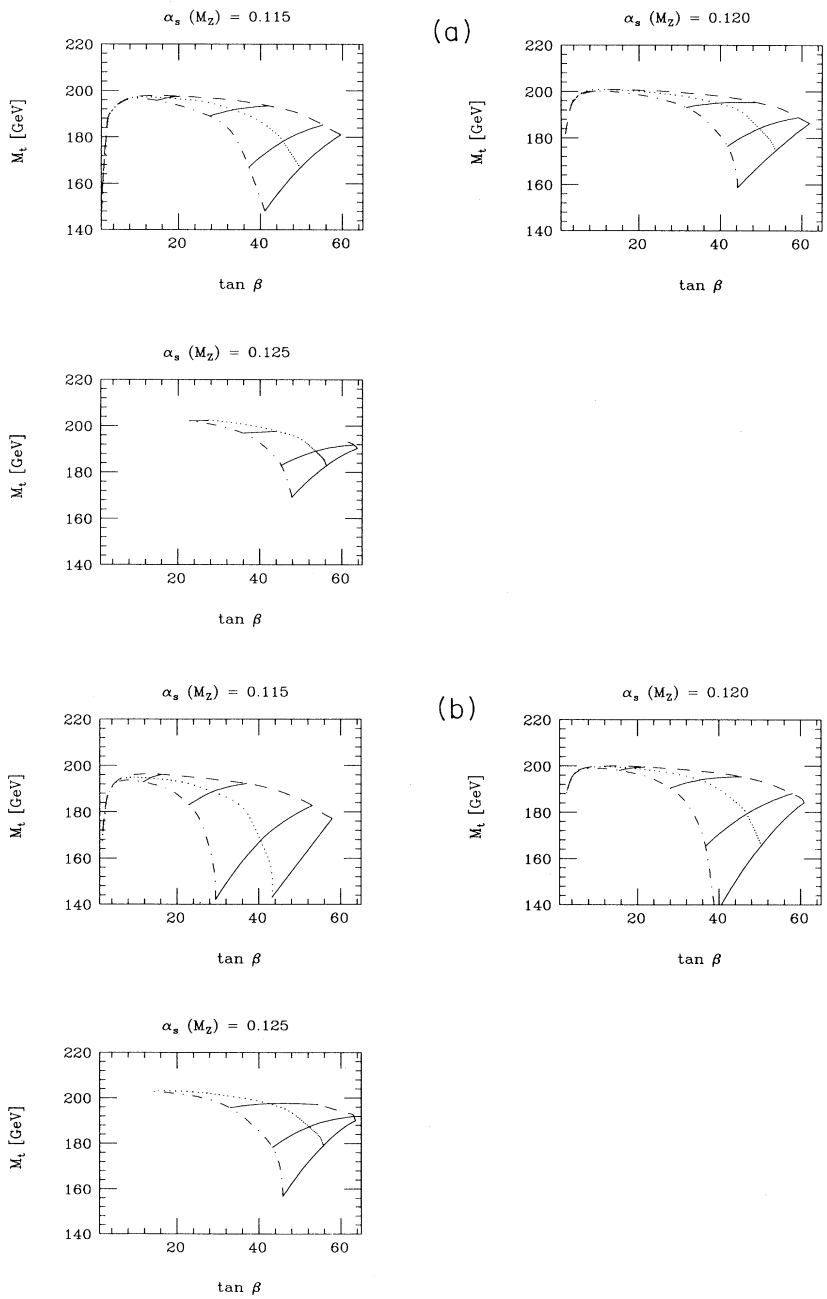

FIG. 1. The pole top quark mass as a function of $\tan\beta$ for a running bottom mass $m_b(m_b) = 4.15$ GeV and three different values of the strong gauge coupling, $\alpha_s(M_Z) = 0.115$, 0.120, and 0.125, respectively, for (a) model 6 and (b) model 9. The coefficient K_c parametrizing the down quark mass corrections takes values, $K_c = 0$ (dashed line), $K_c = -0.003$ (dotted line) and $K_c = -0.006$ (dot-dashed line). The solid lines represent, from right to left, values of $\omega = 1$, 0.6, 0.2, and 0.06, respectively. For large values of $\alpha_s(M_Z)$ the curves are cutted at the point at which the top Yukawa coupling becomes strong at high energy scales, $h_t^2(M_{\rm GUT})/4\pi \geq 1$.

between the down quark and lepton masses is only weakly dependent on ω, we should maintain the Georgi-Jarlskog relation even for values of ω different from one. Moreover, the values of V_{cb} can also be successfully accommodated for lower values of ω. This can be easily seen writing its dependence in terms of the top and charm quark masses:

$$|V_{cb}| \simeq \chi\sqrt{\frac{m_c}{\eta_c m_t}}\exp\left(\frac{I_b - I_t}{2}\right) , \tag{11}$$

where $\chi = |x_d - x_u|/\sqrt{|x_u x'_u|}$, and η_a are the ω independent, renormalization group factors relating the running masses at the scale M_Z with the on-shell ones (for the u, d, and s quarks, the scale of definition of the running masses is taken to be 1 GeV). For $\alpha_s(M_Z) \simeq 0.12$, for which the value of $\eta_c \simeq 2.2$, it follows that the phenomenologically preferred values of $|V_{cb}| = 0.040 \pm 0.005$, require values of $\chi < 1$ [23]. The different values of χ are the basis for the classification of models performed in Ref. [17], where the best fit to the data was achieved by two models: model 6, with $x_u = x'_u = -4$, $x_d = x'_d = -2/3$, and $x_e = x'_e = 6$, and model 9 with $x_u = x'_u = 1$, $x_d = 1/9$, $x_e = 9$ and $x'_d = x'_e = 1$. These models have $\chi = 5/6$ and $8/9$, respectively, and both lead to somewhat large values of V_{cb}. Model 4, with $\chi = 2/3$, leads to a better prediction for V_{cb}, but yields insufficient CP violation.

For lower values of ω, the dependence of Eq. (11) on I_b and I_t is such that the values of V_{cb} tend to decrease. Moreover, in the absence of down quark mass matrix corrections, for $\tan\beta \geq 4$, the value of V_{cb} decreases due to larger top quark mass values, which, as we discussed before, may become too large in comparison with the experimentally preferred ones. As shown in Fig. 1, lower values of the top quark mass may be obtained through large bottom mass corrections. Lowering the top quark mass enhances the value of V_{cb}, but the total effect of the down quark mass corrections on V_{cb} cannot be determined *a priori*; it depends on the relative size of the gluino corrections, which affect the value of V_{cb} due to their effect on the predicted top quark mass value, and the chargino corrections, which modify not only the top quark mass value through the bottom mass corrections, but they have also a direct effect on the CKM matrix elements [24] (see the Appendix).

Figure 2 shows the predictions for V_{cb} for models 6 and 9, as a function of $\tan\beta$, for three different bottom mass corrections and three different values of $\alpha_s(M_Z)$, under the assumption that $(\delta m_b/m_b)^{\tilde{g}} = -3(\delta m_b/m_b)^{ch}$ (which is reasonable in view of the running of the soft breaking parameters and the structure of the bottom mass corrections when the squark mass matrices are approximately three by three block diagonal [16]). We see that, independently of the bottom mass corrections, the predictions for V_{cb} may be significantly improved for moderate values of ω. Indeed, apart from the solutions with $\tan\beta$ very close to one, $\omega = 1$ leads to the largest values of V_{cb} for each fixed Δm_b correction. Observe as well that for values of $\tan\beta \leq 2$, V_{cb} increases, due to the lower values of m_t appearing in this regime. Furthermore, for the present case, for any fixed value of ω, there is an effective cancellation of the chargino and gluino-induced one loop corrections to V_{cb} and the total effect of the down quark mass corrections on V_{cb} is small. Consequently, since for a fixed value of ω large down quark mass corrections lower the value of $\tan\beta$, as can be seen from Fig. 2, they also yield larger values of V_{cb} for a given fixed value of $\tan\beta$. From Fig. 2 we also observe that the predictions for V_{cb} improve for larger values of $\alpha_s(M_Z)$. In fact, for moderate values of ω, if large values of $\alpha_s(M_Z)$ and large bottom mass corrections are present, the predictions for V_{cb} in model 6 may actually be below the preferred experimental values, but these solutions are associated with values of M_t which are generally too large.

From the discussion above, we see that the second- and third-generation fermion masses and mixing angles can be consistently described within an SO(10) GUT with $\omega \leq 1$. However, as we shall show in the following, the constraint coming from the predictions for the first-generation quark masses rule out values of $\omega \leq 0.5$ within the minimal model. In Sec. IV we show how to overcome this difficulty at the expense of adding one new operator and two more parameters, in addition to ω.

B. The first generation

The operator O_{12} is necessary to achieve acceptable predictions for the lowest generation quark masses. Indeed, within the minimal model, there is a "unique" operator,

$$O_{12} = 16_1\left(\frac{45_X}{M_P}\right)^n 10\left(\frac{45_X}{M}\right)^m 16_2, \tag{12}$$

with $n = m = 3$, which yields acceptable ratios for the masses of the up, down, and strange quarks. This operator determines the equality of the Clebsch factors z_a and z'_a and the ratios of the Clebsch factors $z_d/z_u = 27$ and $z_d/z_e = 1$ (the ratio of Clebsch factors z_d/z_u increases by a factor 3 for each power of 45_X). In addition the ratio, appearing in Eq. (4), $(z_e z'_e)/(z_d z'_d) = 1$. The above operator is of dimension ten, meaning that the absence of any lower dimensional operators should be insured by some symmetry of the theory.

For $\omega = 1$, one might think that the large ratio of Clebsch factors, z_d/z_u, arising from the above relation, Eq. (12), is necessary in order to compensate the $\tan\beta$ ($\simeq m_t/m_b$) dependence of the up-type quark masses with respect to the down-type quark ones. It is interesting to investigate then if lower values of ω, and hence of $\tan\beta$, can serve to relax the restrictions on the Clebsch factors and hence, to lower the dimensionality of the above O_{12} operator. This, however, is not the case, as can be easily shown considering the relation

$$\frac{m_u}{m_d} = \frac{m_s}{m_c}\left(\frac{m_t}{m_b}\right)^2 \frac{\eta_u\eta_c\eta_b^2}{\eta_d\eta_s}\left|\frac{z_u z'_u}{z_d z'_d}\right| \exp[4(I_t - I_b)]. \tag{13}$$

From Eq. (13) it follows that independent of the source of the hierarchy between the top and bottom quark masses, large ratios of Clebsch factors are necessary in order to obtain the phenomenologically preferred values for the ratio of the up to down quark masses, $0.2 \leq m_u/m_d \leq 0.8$

[25]. The additional dependence on the integral factors I_b and I_t does not help to lower this ratio. On the contrary, since for lower values of ω, the integral factor I_b decreases, while I_t changes only slightly, for the same values of the second- and third-generation quark masses the ratio of the up to the down quark masses increases. This means that, in order to keep phenomenologically allowed values of the first-generation quark masses, the ratio of the Clebsch factors z_d/z_u should actually increase, implying that the dimensionality of the operator O_{12} should correspondingly increase for lower values of ω.

It is therefore apparent that, keeping the same operator structure as before, the range of possible values of ω, that is to say of $\tan\beta$, will be restricted. Indeed, Fig. 3 shows the dependence of the ratio m_u/m_d as a function of $\tan\beta$ (ω) for models 6 and 9 and for different values of the down quark mass corrections, under the same assumptions discussed for Fig. 1. It follows that the down quark mass corrections help only marginally in getting phenomenologically allowed values for m_u/m_d, and values of $\omega \lesssim 0.5$ are disfavored for all these models. Indeed, for larger values of $\alpha_s(M_Z) \geq 0.12$, even larger values of ω are necessary in order to achieve good predictions for the first generation masses.

Increasing by one the dimensionality of the operator O_{12} keeps the equality $z_e z_e' = z_d z_d'$, necessary to achieve

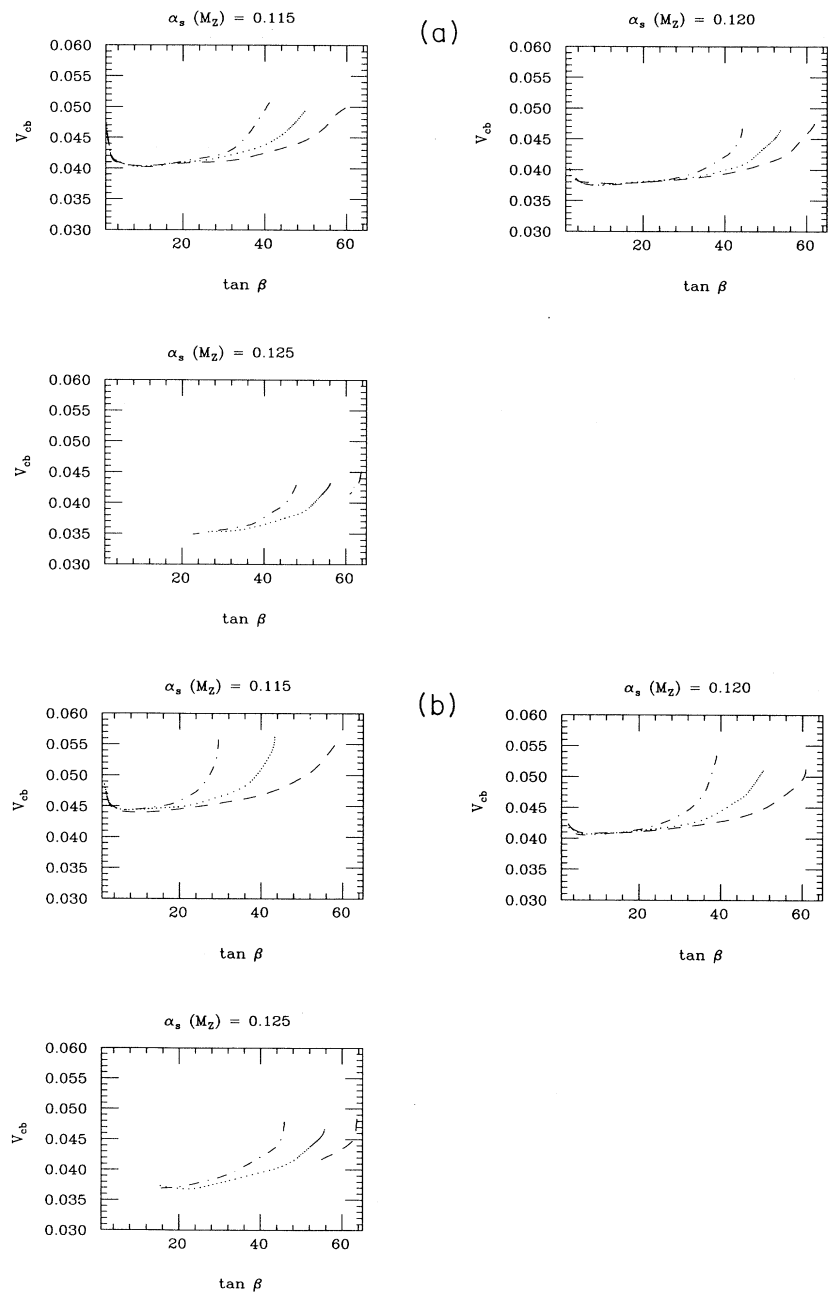

FIG. 2. The same as in Fig. 1 but for the Cabibbo-Kobayashi-Maskawa matrix element V_{cb} as a function of $\tan\beta$.

the proper ratio of first- and second-generation quark and lepton masses, Eq. (4), but leads to wrong predictions for the Cabibbo angle [17]. Indeed, ignoring small factors, the Cabibbo angle is approximately given by

$$s_c \simeq \sqrt{\frac{m_d}{m_s}}\sqrt{\left|\frac{z_d}{z_d'}\right|}. \tag{14}$$

Relaxing the equality $z_d = z_d'$ by increasing by one the power of one of the 45_X in O_{12} would change s_c by a factor $\sqrt{3}$, what would lead to predictions in conflict with present data.

Therefore, for low values of ω, if the dimension of the operator O_{12}, Eq. (12), is changed, to obtain correct values for the ratio of the first- and second-generation quark masses, it should be increased by two units. Once more, however, the variation in the dimensionality of this operator has an additional effect, which is related to the behavior of the Jarlskog CP-odd invariant $J = \mathrm{Im}[V_{ud}V_{tb}V_{td}^*V_{ub}^*]$. Ignoring again small, inessential factors, it is straightforward to show that

$$J \simeq \chi^2 \frac{|z_u z_d|}{|z_e z_e'|}\left|\frac{y_e}{y_d}\right|\frac{m_e}{m_\tau}\exp\left(2I_t + 2I_b - 3I_\tau\right). \tag{15}$$

Thus, increasing the dimension of the operator O_{12} in

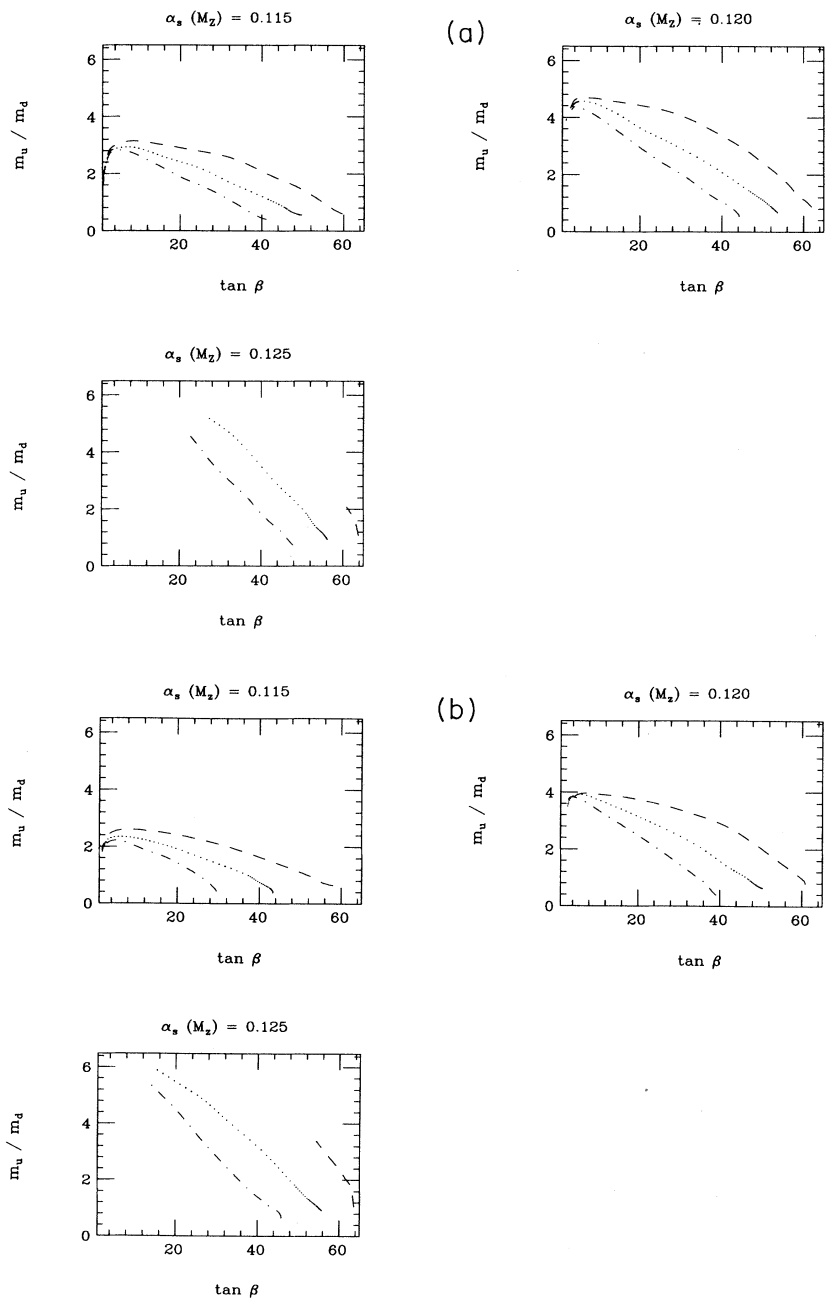

FIG. 3. The same as in Fig. 1 but for the ratio of the first-generation masses m_u/m_d as a function of $\tan\beta$.

two units implies a decrease in the CP-odd invariant J in a factor 3. Since the observed CP violation in the K system is well described by models 6 and 9 before the modification of the operator O_{12}, a factor 3 suppression of the Jarlskog invariant would imply that the amount of CP violation associated with the Cabibbo-Kobayashi-Maskawa is insufficient to explain the experimental data. The possible variations of V_{cb} (or equivalently of V_{td}) due to supersymmetric threshold corrections in the down quark sector, which we have discussed above, are not sufficiently large to compensate this type of effect. Numerically, we observe the effect of increasing the dimension of the operator O_{12} through the prediction for the bag parameter B_K [26]:

$$B_K \simeq \epsilon_K \frac{|z_e z_e^{'}|}{\sqrt{|z_d z_d^{'}||z_u|}} \frac{m_\tau}{m_e} \sqrt{\frac{m_d}{m_s}} \frac{\exp(2I_t + 2I_b - 3I_\tau)}{\chi^2 \ \sin\phi} \tag{16}$$

which tends to be larger than one in all models, and, hence, unacceptable since the phenomenologically preferred values are $B_K = 0.8 \pm 0.2$ [27, 23].

The general conclusion of this study is that, keeping the same operator structure as in Ref. [17], values of $\omega \leq 0.5$ cannot be accommodated, without spoiling the predictions for either the first-generation quark masses, the Cabibbo angle, or the CP-violation sector of the theory. Hence, the preferred value of $\omega \simeq 1$ restricts us to be close to the minimal SO(10) model and the values of $\tan\beta$ and M_t which lead to acceptable predictions are also quite restricted (see the discussion in Sec. III A). In the next section we show that it is still possible to obtain acceptable predictions for the first-generation with small values of $\tan\beta$. However, this solution requires the addition of one new operator and thus one more complex parameter in addition to the free parameter ω discussed above.

IV. EXTENDING MINIMAL SO(10) AND $\tan\beta \sim 1$

One could think of improving the agreement between the theoretical predictions and the experimentally observed values of the first generation masses, or the ϵ_K parameter, by assuming very large supersymmetric threshold corrections to these variables. In Fig. 3 we have shown that if the down quark mass corrections have the structure which naturally appears when the squark matrices are block diagonal (see the Appendix), only slight changes of the predictions for the first-generation masses are obtained through such threshold corrections. In the general case, however, the squark mass matrices may be far from being three by three block diagonal and first-generation down quark mass corrections, proportional to the second- or even third-generation masses, as shown in the appendix [Eq. (A1)], may be present.

Persuing this direction however opens up a Pandora's box of new possibilities and new problems. It is interesting to note that, if the supersymmetry breaking is transferred to the observable sector through gravitational effects, a nontrivial intergeneration squark mixing, generated through renormalization effects at scales of the order of the grand unification scale, is unavoidable [28–31]. A reliable computation of this effect demands, however, the knowledge of the precise physics beyond the grand unification scale. In general, a large squark mixing would also involve large flavor-changing neutral current effects. Barring unnatural cancellations, large flavor violations in the fermion sector can only be consistent with the experimental constraints on flavor-changing neutral currents and the neutron electric dipole moment if the characteristic scale of the squark masses is larger or of order 1 TeV. A large squark mixing also implies significant couplings of these heavy squarks to the Higgs sector of the theory (unless the third-generation squarks do not mix with the first- and second-generation ones), this will in turn imply a significant fine-tuning in order to preserve the stability of the weak scale. The presence of large supersymmetric corrections to the ϵ_K parameter have similar consequences. In this work, we assume the presence of a super Glashow-Iliopoulos-Maiani (GIM) mechanism and avoid the discussion of nonuniversal squark and slepton masses at the GUT scale. Note that in order to reduce the fine-tuning and large corrections associated with large values of $\tan\beta$ it is only necessary to have nonuniversal Higgs boson masses.

To improve the agreement between the theoretical and experimental predictions for small values of $\tan\beta$, a possible alternative is the modification of the structure of the operators discussed above. Since, as shown in Sec. III, low values of ω are perfectly consistent with the second- and third-generation quark and lepton masses and mixing angles, any modification should concentrate on the form of the "12" elements. In Ref. [17], it was argued that if O_{12} proceeds from a single operator, its form is uniquely determined. This conclusion is based on the analysis of the associated Clebsch factors and the relations given in Eqs. (4), (13), and (14). However, since O_{12} has a large dimension, the relaxation of the assumption that the "12" elements come from a single operator seems natural. If the effect of two operators had to add in an unnatural way in order to lead to the correct phenomenological predictions, the predictive power of the theory would be spoiled. Therefore, the additional operator should not modify the equality of $z_e z_e^{'}$ and $z_d z_d^{'}$ and should give no relevant corrections to the ratio of $z_d/z_d^{'}$. On the other hand, we want to modify the ratio of m_u/m_d without affecting the CP-odd sector in a relevant way. It is crucial to notice that there is a very important difference between the dependence of J and that of m_u/m_d on the Clebsch factors. While m_u/m_d depends on the product $z_u z_u^{'}$, the CP-odd invariant J depends on z_u, but is independent of $z_u^{'}$. Hence, we are searching for an operator which modifies $z_u^{'}$, leaving z_u invariant. There is only one combination of operators which satisfies all the above criteria: namely,

$$O_{12} = 16_1 (45_X)^n 10 (45_X)^n 16_2 + K 16_1 (45_X)^m (45_{T_{3R}})^l 10 (45_X)^m 16_2, \tag{17}$$

where K is of order 1. The predictions for z_a and $z_a^{'}$ within this framework are

$$\begin{aligned} z_u &= 1, \quad z_u^{'} = 1 + f, \\ z_d &= (-3)^n, \quad z_d^{'} = (-3)^n + (-1)^l(-3)^m f\ , \\ z_e &= (-3)^n, \quad z_e^{'} = (-3)^n + (-1)^l(-3)^m f, \end{aligned} \tag{18}$$

where f is the coefficient characterizing the relative weights of the two contributions, and it is computable from K and the vacuum expectation values above. For simplicity, we shall assume that f is a real number. In that case, for values of f of order 1 and values of m smaller than n by at least two units, it is easy to see that the only prediction which will be modified considerably is m_u/m_d. One can therefore achieve low values of $\tan\beta$ with a correct prediction for m_u/m_d. This demands very low values of ω and values of f close to -1. For instance, for model 6, $m_b(m_b) \simeq 4.2$ GeV, and $\alpha_s(M_Z) \simeq 0.12$, the value of ω which leads to $\tan\beta \simeq 1.5$ is as small as 0.004. In this case, values of $f \simeq -0.8$, $n = 3$, and $m = 0$ lead to good predictions for the CKM matrix and the quark masses.

V. CONCLUSIONS

We have analyzed the fermion mass problem within the context of supersymmetric SO(10) unification, studying not only the minimal case, but also the departure from the minimal conditions assuming that the fermion masses arise from interactions of the spinor representations with a single **10** representation, but this **10** only contains a part of the two light Higgs doublets. Moreover, we studied the implications of the down quark mass corrections, under the assumption that, within a good approximation, a super GIM mechanism is in effect. We have shown that, for $\omega < 1$ (moderate values of $\tan\beta$), and considering the simplest operator structure, large bottom mass corrections are helpful in accommodating the experimentally preferred values for M_t, yielding also acceptable values for V_{cb}. However, moderate or low values of $\tan\beta$ lead to wrong predictions either for the first-generation quark masses or for the CP-odd sector of the theory, a property that is not changed by the presence of supersymmetric threshold corrections. We have also shown that the operator structure may be extended to yield proper values for all fermion masses and mixing angles for low values of $\tan\beta \leq 3$. This extension requires, however, the presence of additional **45** states in the theory, one new operator contributing to the first-generation masses and another complex parameter.

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APPENDIX: SUPERSYMMETRIC THRESHOLD CORRECTIONS

Let us discuss the down quark mass corrections induced by supersymmetric particle loops in more detail. The dominant corrections to the down quark mass matrix are given by chargino-up squark and gluino-down squark one loop contributions and they read

$$\begin{aligned} (\delta m_d)_{IL} = &-\frac{2\alpha_s}{3\pi} M_{\tilde g} \sum_{j=2}^{6} \left[D_{Ij} D^*_{(L+3)j} \left(m^2_{\tilde d_j} - m^2_{\tilde d_1} \right) I\left(m^2_{\tilde d_j}, m^2_{\tilde d_1}, M^2_{\tilde g} \right) \right] \\ &+ \sum_{j=2}^{6} \sum_{\alpha=1}^{2} \left[\frac{d_L}{16\pi^2} Z^+_{2\alpha} Z^-_{2\alpha} m_\alpha C^*_{KL} U^*_{Kj} u_M U_{(M+3)j} C_{MI} \left(m^2_{\tilde u_j} - m^2_{\tilde u_1} \right) I\left(m^2_{\tilde u_j}, m^2_{\tilde u_1}, m^2_\alpha \right) \right]. \end{aligned} \tag{A1}$$

The above expression has also been obtained in Ref. [24]. In the above, U and D are the unitary matrices diagonalizing the six by six up and down squark mass matrices (D_{1i} and D_{4i} denote, for example, the component of the mass eigenstate $\tilde d_i$ in the left- and right-handed down squark, respectively), $Z^{\pm}_{\alpha\beta}$ are the unitary matrix which diagonalize the two by two chargino matrix, m_α are the chargino mass eigenstates, C_{IJ} are the CKM matrix elements, d_I and u_I are the down and up quark Yukawa couplings, respectively, and $M_{\tilde g}$ is the gluino mass. The integral $I(a,b,c)$ is given by

$$I(a,b,c) = \frac{ab\ln(a/b) + bc\ln(b/c) + ac\ln(c/a)}{(a-b)(b-c)(a-c)}. \tag{A2}$$

All indices denoted by capital letters run from 1 to 3 and a summation over the indices K and M is implicit. The state $\tilde d_1$ ($\tilde u_1$) denotes any particular eigenstate, which may be chosen, for example, as the heaviest one. A dependence of the above expression on the quark mass matrices is implicit in the necessary left-right-mixing term, which is only generated by terms proportional to the quark masses.

Furthermore, if the up and down squark matrices are three by three block diagonal, implying the existence of a super GIM mechanism in the theory, the following property is satisfied:

$$\begin{aligned} D_{Kj} D^*_{(M+3)j} &= \pm\delta_{KM} \frac{d_K(A_{d_K} - \mu\tan\beta)v_1}{m^2_{\tilde d_K} - m^2_{\tilde d_{(K+3)}}}, \\ U_{Kj} U^*_{(M+3)j} &= \pm\delta_{KM} \frac{u_K(A_{u_K} - \mu\cot\beta)v_2}{m^2_{\tilde u_K} - m^2_{\tilde u_{(K+3)}}}, \end{aligned} \tag{A3}$$

where A_K are the conventionally defined trilinear soft

supersymmetry-breaking terms and μ is the supersymmetric Higgs boson mass parameter appearing in the superpotential. The positive sign in the above expression corresponds to the case $j = K$, while the negative sign corresponds to the case $j = K + 3$. Keeping the dominant terms in the large $\tan\beta$ regime, the down quark mass corrections take, hence, a very simple form

$$(\delta m_d)_{IL} = \frac{2\alpha_s}{3\pi}\delta_{IL}(d_L v_1)\tan\beta\,\mu M_{\tilde g} I\left(m^2_{\tilde d_L}, m^2_{\tilde d_{(L+3)}}, M^2_{\tilde g}\right) + \sum_{j=2}^{6}\sum_{\alpha=1}^{2}\left[\frac{(d_L v_1)\tan\beta}{16\pi^2} Z^+_{2\alpha} Z^-_{2\alpha} m_\alpha C^*_{ML}|u_M|^2 \times A_{u_M} C_{MI} I\left(m^2_{\tilde u_M}, m^2_{\tilde u_{(M+3)}}, m^2_\alpha\right)\right]. \quad \text{(A4)}$$

The above expression reproduces the one obtained in Ref. [24] under similar assumptions. In the present limit, the gluino corrections affect only the values of the mass eigenstates, while the chargino corrections give also corrections to the off-diagonal terms. Studying the renormalization group evolution of the soft supersymmetry-breaking mass parameters one can show that the gluino contributions are generally dominant and opposite in sign to the chargino contributions [16]. Moreover, because of the hierarchy between the up quark masses, only the term proportional to $|u_3|^2$ becomes important in the chargino contributions. Hence the chargino-induced corrections to the down and strange masses are very small. Furthermore, as has been shown in Ref. [24], the most relevant corrections to the CKM matrix elements are given by $\delta V_{cb}/V_{cb} \simeq -(\delta m_b/m_b)^{ch.} \simeq \delta V_{td}/V_{td}$, where $(\delta m_b/m_b)^{ch}$ represents only the chargino contributions to the total bottom mass corrections.

[1] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[2] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); L. Ibañez and G.G. Ross, Phys. Lett. **105B**, 439 (1981).

[3] J. Ellis, S. Kelley, and D.V. Nanopoulos, Phys. Lett. B **260**, 131 (1991); P. Langacker and M.X. Luo, Phys. Rev. D **44**, 817 (1991); U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 817 (1991); F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cimento A **104**, 817 (1991).

[4] P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993); **49**, 1454 (1994).

[5] M. Carena, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. **B406**, 59 (1993).

[6] H. Arason, D.J. Castaño, B. Keszthelyi, S. Mikaelian, E.J. Piard, P. Ramond, and B.D. Wright, Phys. Rev. Lett. **67**, 2933 (1991). For earlier work on this subject, see L. Ibañez and Lopez, Nucl. Phys. **B233**, 511 (1984).

[7] S. Dimopoulos, L. Hall, and S. Raby, Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. D **45**, 4192 (1992).

[8] C.T. Hill, Phys. Rev. D **24**, 691 (1981); C.T. Hill, C.N. Leung, and S. Rao, Nucl. Phys. **B262**, 517 (1985).

[9] L. Alvarez Gaume, J. Polchinski, and M.B. Wise, Nucl. Phys. **B221**, 495 (1983); J. Bagger, S. Dimopoulos, and E. Masso, Phys. Rev. Lett. **55**, 920 (1985).

[10] M. Carena, T.E. Clark, C.E.M. Wagner, W.A. Bardeen, and K. Sasaki, Nucl. Phys. **B369**, 33 (1992).

[11] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. D **47**, 1093 (1993); V. Barger, M.S. Berger, P. Ohmann, and R.J.N. Phillips, Phys. Lett. B **314**, 351 (1993).

[12] W.A. Bardeen, M. Carena, S. Pokorski, and C.E.M. Wagner, Phys. Lett. B **320**, 110 (1994).

[13] See, for example, M. Olechowski and S. Pokorski, Phys. Lett. B **214**, 393 (1988); B. Anantharayan, G. Lazarides, and Q. Shafi, Phys. Rev. D **44**, 1613 (1991); W. Majerotto and B. Mösslacher, Z. Phys. C **48**, 273 (1990); D. Kapetanakis, M. Mondragon, and G. Zoupanos, *ibid.* **60**, 181 (1993).

[14] L.J. Hall, R. Rattazzi, and U. Sarid, Phys. Rev. D **50**, 7048 (1994).

[15] R. Hempfling, Phys. Rev. D **49**, 6168 (1994).

[16] M. Carena, M. Olechowski, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. **B426**, 269 (1994).

[17] G. Anderson, S. Dimopoulos, L.J. Hall, S. Raby, and G. Starkman, Phys. Rev. D **49**, 3660 (1994).

[18] S. Dimopoulos and F. Wilczek, in *The Unity of Fundamental Interactions*, Proceedings of the International School of Subnuclear Physics, Erice, Italy, 1981, edited by Z. Zichichi, Subnuclear Series Vol. 19 (Plenum, New York, 1983).

[19] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).

[20] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979); H. Georgi and D.V. Nanopoulos, Nucl. Phys. **B159**, 16 (1979); J. Harvey, P. Ramond, and D.B. Reiss, Phys. Lett. **92B**, 309 (1980); Nucl. Phys. **B199**, 223 (1982).

[21] R. Rattazzi, U. Sarid, and L.J. Hall, Stanford University Report No. SU-ITP-94/15, 1994 (unpublished); R. Hempfling, DESY Report No. 94-078 1994 (unpublished); M. Carena and C.E.M. Wagner, CERN Report No. CERN-TH.7321/94, 1994 (unpublished); D. Mattalliotakis and H. P. Nilles, Nucl. Phys. **B435**, 115 (1995); M. Olechowski and S. Pokorski, Phys. Lett. B **344**, 201 (1995).

[22] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **50**, 2966 (1994); Phys. Rev. Lett. **73**, 225 (1994); **74**, 2626 (1995); D0 Collaboration, S. Abachi *et al.*, *ibid.* **72**, 2138 (1994); **74**, 2422 (1995); **74**, 2632 (1995).

[23] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1195 (1994).

[24] T. Blazek, S. Raby, and S. Pokorski, Ohio State University Report No. OHSTPY-HEP-T-95-007, 1995 (unpublished).

[25] D. Kaplan and A. Manohar, Phys. Rev. Lett. **56**, 2004 (1986); H. Leutwyler, Nucl. Phys. **B337**, 388 (1991).

[26] J. M. Flynn, Mod. Phys. Lett. A **5**, 877 (1990); I. Dunietz, Ann. Phys. (N.Y.) **184**, 350 (1988); G. Buchalla, A. Buras, and M. Harlander, Nucl. Phys. **B337**, 313 (1990).

[27] S. Sharpe, in *Lattice '91*, Proceedings of the International Symposium, Tsukuba, Japan, edited by M. Fukugita *et al.* [Nucl. Phys. B (Proc. Suppl.) **26**, 197 (1992)].

[28] L.J. Hall, V.A. Kostelecky, and S. Raby, Nucl. Phys. **B267**, 415 (1986).

[29] H. Georgi, Phys. Lett. **169B**, 231 (1986).

[30] R. Barbieri and L.J. Hall, Phys. Lett. B **338**, 212 (1994).

[31] S. Dimopoulos and L.J. Hall, Phys. Lett. B **344**, 185 (1995).