Magnetic moments and charge radii of decuplet baryons in a field-theoretic quark model

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(Received 24 June 1994; revised manuscript received 9 March 1995)

We make nonrelativistic as well as relativistic estimations of the magnetic moments and charge radii of decuplet baryons in a field-theoretic quark model, where translationally invariant decuplet baryon states are described by constituent quark field operators and harmonic oscillator wave functions. The relativistic estimations of the magnetic moments made are, however, $O(|\vec{p}|/m)$ corrections over the nonrelativistic contribution where the higher order corrections for the ground states treated here, being small, are neglected. The constituent quark field operators here with a particular ansatz satisfy the equal time algebra and are also Lorentz boosted in a definite manner to describe the baryons in motion. The estimations for the magnetic moments and their ratios, with a single harmonic oscillator radius parameter for the octet as well as decuplet baryons, show a reasonable agreement with the most recent experimental measurements for Δ^{++} and Ω^-, which have constrained different models of hadrons to explain both. We feel that the final results in this regard, expected in the near future in succeeding experiments, can further constrain different theoretical models. However, because of the lack of experimental data, the estimated charge radii in the present model stand as model predictions which may be verified in future experiments.

PACS number(s): 13.40.Em, 12.39.Ki, 14.20.Gk

I. INTRODUCTION

Though the structure of the magnetic moments and charge radii of a baryon decuplet are of much interest, the experimental observations in this regard are still sparse. The recent measurements of the magnetic moment of Δ^{++} by pion bremsstrahlung analysis [1] and that of Ω^- by the E756 Collaboration [2] have generated fresh interest, resulting in several publications based on the framework of different theoretical models [3–6]. However, being low-energy phenomena, these cannot be studied from first-principles applications of QCD [7], despite its being the correct theory for hadrons. For this, one resorts to phenomenological models of hadrons incorporating the fundamental ingredients of QCD. Thus we investigate here the magnetic moments and charge radii of a baryon decuplet in a field-theoretic quark model of composite hadrons [8], where baryons are assumed to consist of constituent quarks occupying fixed energy levels [9]. The constituent quarks are described by four component quark field operators satisfying the equal time algebra. Further, the SU(6) symmetric states of decuplet baryons in the rest frame are described in the present model in terms of constituent quark field operators and a ground-state harmonic oscillator wave function. For example, we describe one such normalized state for Δ^{++} as

$$|\Delta^{++}(\vec{0})\rangle = \frac{\epsilon_{ijk}}{6}\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{U}_\Delta(\vec{k}_1,\vec{k}_2,\vec{k}_3)[u^{i\dagger}_{I1/2}(\vec{k}_1)u^{j\dagger}_{I1/2}(\vec{k}_2)u^{k\dagger}_{I1/2}(\vec{k}_3)]|\text{vac}\rangle \ , \tag{1.1}$$

where the normalized ground-state harmonic oscillator wave function $\tilde{U}_B(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ for any decuplet baryon B is written as

$$\tilde{U}_B(\vec{k}_1,\vec{k}_2,\vec{k}_3) = (3R_B^4/\pi^2)^{3/4}\exp\left[-(R_B^2/6)\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right] \ , \tag{1.2}$$

with R_B^2 as the harmonic oscillator radius. Also, in the present model the constituent quark field operators describing hadrons at rest are Lorentz boosted in a particular manner [9] to describe the hadrons in motion. Again, for example, taking this into account the translationally invariant state for Δ^{++} is written as [9]

$$\begin{aligned}|\Delta^{++}(\vec{p})\rangle = \frac{\epsilon_{ijk}}{6}(p^0/M_{\Delta^{++}})\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{U}_{\Delta^{++}}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\\ \times[u^{i\dagger}_{I1/2}(L(p)k_1)u^{j\dagger}_{I1/2}(L(p)k_2)u^{k\dagger}_{I1/2}(L(p)k_3)]|\text{vac}\rangle \ ,\end{aligned} \tag{1.3}$$

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with $L(p)$ as the familiar Lorentz boosting matrix and $Q_{Ir}(L(p)k)$ as the Lorentz boosted quark field operator of flavor Q. Thus, in a similar manner, within the framework of the present model one can also write the states for other decuplet baryons by taking their appropriate spin-isospin and flavor configurations into account.

The present model was earlier applied to different coherent [10] and incoherent [11] hadronic phenomena with a reasonable amount of success. Recently the model has been applied to explain the strong CP violation problem in the context of the electric dipole moment of neutron [12], weak mixing problem in the context of K_L-K_S and B_d^0-$\bar{B}_d^0$ mixing [13], leptonic decay of vector mesons [14], and weak leptonic decay of pseudoscalar mesons [15]. Most recently the model has also successfully been applied to explain static properties of octet baryons [16] and weak radiative decays of charmed mesons [17]. In view of the success of the present field-theoretic quark model we attempt to estimate here, within its framework, the magnetic moments and charge radii of decuplet baryons.

We organize the paper as follows. In Sec. II we describe the calculations of magnetic moments and charge radii of decuplet baryons. In Sec. III we estimate the results and compare them with the experiments, wherever available, and other theoretical calculations.

II. MAGNETIC MOMENTS AND CHARGE RADII OF DECUPLET BARYONS

We estimate here the magnetic moments and charge radii of decuplet baryons based on the framework of the earlier mentioned field-theoretic quark model [8,9]. However, for magnetic moments we do the estimations in nonrelativistic as well as relativistic frames whereas for the charge radii for the sake of simplicity of their calculations we do so only in the relativistic Breit frame.

A. Magnetic moments of decuplet baryons

The nonrelativistic estimation of magnetic moments of decuplet baryons needs the electromagnetic current in quark space which is written as [8]

$$\mathcal{H}_I(x) = \sum_{Q,\alpha} e_Q \bar{Q}^\alpha(x)\gamma^\mu A_\mu Q^\alpha(x) , \tag{2.1}$$

where α is the color index and $Q(x)$ is the quark field operator of the model [8] which when substituted in Eq. (2.1) with $A^0 = 0$ yields

$$\mathcal{H}_I^{\mathrm{em}}(x) = \sum_{Q,\alpha} e_Q g_Q Q_I^{\alpha\dagger}(x)[f_Q \vec{\sigma}\cdot(\vec{\nabla}\times\vec{A})]Q_I^\alpha(x) , \tag{2.2}$$

where f_Q and g_Q are the parameters entering through the constituent quark field operators of the model [8,9] which are related to each other by the constraint of equal time algebra and were in fact parametrized during the earlier applications of the model [11–17]. Further, when the effective Hamiltonian in Eq. (2.2) is sandwiched between the static decuplet baryon states $|B_{3/2}(\vec{0})\rangle$ one obtains a nonrelativistic expression for the magnetic moment of a decuplet baryon B, in general, as

$$\mu_B = (2\pi)^3 \left\langle B_{3/2}(\vec{0}) \middle| \sum_Q e_Q g_Q Q_I^{\alpha\dagger}(\vec{0}) \times (f_Q\sigma_3) Q_I^\alpha(\vec{0}) \middle| B_{3/2}(\vec{0}) \right\rangle , \tag{2.3}$$

where the factor $(2\pi)^3$ arises due to translational invariance. Equation (2.3) also when written in momentum space becomes

$$\mu_B = \left\langle B_{3/2}(\vec{0}) \middle| \sum_Q e_Q g_Q \int d\vec{k}'\, d\vec{k}\, Q_I^{\alpha\dagger}(\vec{k}') \times f_Q(\vec{k}')\sigma_3 Q_I^\alpha(\vec{k}) \middle| B_{3/2}(\vec{0}) \right\rangle . \tag{2.4}$$

Next using the normalized spin flavor SU(6) states explicitly for the decuplet baryons as described in the earlier section and the two component quark field operators of the model [8] one can obtain a general expression for magnetic moment of decuplet baryon B as

$$\mu_B = \sum_{i=1}^{3} e_{Qi} g_{Qi} \left[1 - \frac{g_{Qi}^2}{2R_B^2}\right] , \tag{2.5}$$

where Q_i's are the flavor contents of the baryon.

We next extend the present formalism to calculate the magnetic moments of decuplet baryons in the relativistic framework, where again the electromagnetic current is sandwiched between the baryon states in motion. However, while doing so, one identifies the magnetic moment of the decuplet baryon in the Breit frame as [9]

$$i(2\pi)^3 u_{I1/2}^{i\dagger}[\vec{\sigma}\times(-2\vec{p})]^m u_{I1/2}^i \mu_B = \langle B_{3/2}(-\vec{p})|J^m(\vec{0})|B_{3/2}(\vec{p})\rangle , \tag{2.6}$$

where the space part of the electromagnetic current $J^m(\vec{0})$ in momentum space is written as [9]

$$J^m(\vec{0}) = (2\pi)^3 \int d\vec{k}' d\vec{k} \times \sum_{Q,\alpha} e_Q \bar{Q}^{\alpha L(-p)}(\vec{k}')\gamma^m Q^{\alpha L(p)}(\vec{k}) . \tag{2.7}$$

Now, using the explicit decuplet baryon states in the relativistic framework and Eq. (2.7), the matrix element appearing on the right-hand side of Eq. (2.6) can be written in general, with cyclic indices i, j, k, as

$$\langle B_{3/2}(-\vec{p})|J^m(\vec{0})|B_{3/2}(\vec{p})\rangle = \frac{1}{(2\pi)^3}\left(\frac{3R_B^2}{2\pi}\right)^{3/2}\int d\vec{k}\sum_{i=1}^{3}\exp\left[-\frac{3R_B^2}{2}\left\{\vec{k}-(\lambda_j+\lambda_k)\left(\frac{M_B}{p^0}\right)\vec{p}\right\}^2\right]$$
$$\times\exp[-2R_B^2(\lambda_j^2+\lambda_k^2+\lambda_k^2+\lambda_j\lambda_k)(M_B/p^0)^2\vec{p}^{\,2}]e_{Qi}\bar{u}_{1/2}^{L(-p)}(\vec{k}')\gamma^m u_{1/2}^{L(p)}(\vec{k}) \ , \quad (2.8)$$

where e_{Qi} is the charge of the constituent quark Q_i and $\vec{k}' = \vec{k} - 2(\lambda_j + \lambda_k)\vec{p}$ with λ_i's as fractions of energy carried by the quarks inside the hadron which are in fact in the present model taken to be proportional to their respective constituent quark masses [9].

However, in the static limit, for the ground states of decuplet baryons as considered here, neglecting the contributions of $O(|\vec{p}|^2/m^2)$ and higher from the expansion of (2.8) and comparing with Eq. (2.6), one obtains a general relativistic expression for the magnetic moment of a decuplet baryon B as

$$\mu_{B'} = \sum_{i=1}^{3} e_{Qi}\left\{\frac{1}{2M_B'}\left(1-\frac{4g_{Qi}^2}{3R_B^2}\right)+g_{Qi}(\lambda_j+\lambda_k)\right\} . \quad (2.9)$$

B. Charge radii of decuplet baryons

In this subsection we deduce the expressions for charge radii of decuplet baryons in the present field-theoretic quark model. To do so we define, in the Breit frame, the electric form factor of a decuplet baryon, with the momentum transfer variable $t = -4\vec{p}^{\,2}$, as [9]

$$G_E^B(t) = (2\pi)^3(p^0/M_B)\langle B_{3/2}(-\vec{p})|J_{\rm em}^0(\vec{0})|B_{3/2}(\vec{p})\rangle \ , \quad (2.10)$$

where the charge density operator in terms of the four component quark field operator $\psi_Q(x)$ of the model is written as [9]

$$J_{\rm em}^0(\vec{0}) = \sum_Q e_Q\bar{\psi}_Q(\vec{0})\gamma^0\psi_Q(\vec{0}) \ , \quad (2.11)$$

which in momentum space becomes

$$J_{\rm em}^0(\vec{0}) = (2\pi)^{-3}(p^0/M_B)\sum_{Q,\alpha}e_Q\int d\vec{k}'d\vec{k}Q_{Ir}^{\alpha\dagger}(L(-p)k')u_r^\dagger(\vec{k}')u_s(\vec{k})Q_{Is}^{\alpha}(L(p)k) \ . \quad (2.12)$$

Thus substituting Eq. (2.12) in (2.10) one obtains the electric form factor of a decuplet baryon B as

$$G_E^B(t) = (p^0/M_B)^2\left\langle B_{3/2}(-\vec{p})\left|\sum_Q e_Q\int d\vec{k}'\,d\vec{k}\,Q_{Ir}^{\alpha\dagger}(L(-p)k')u_r^\dagger(\vec{k}')u_s(\vec{k})Q_{Is}^{\alpha}(L(p)k)\right|B_{3/2}(\vec{p})\right\rangle \quad (2.13)$$

which with the explicit decuplet baryon states and equal time algebra yields

$$G_E^B(\tau) = \sum_{i=1} e_{Qi}\left(1+\frac{g_{Qi}^2}{2}(\lambda_j+\lambda_k)^2\tau\right)\exp\left[-\frac{R_B^2}{2}(\lambda_j^2+\lambda_k^2+\lambda_j\lambda_k)\tau\right] \ , \quad (2.14)$$

with i,j,k as the cyclic indices and $\tau = t/[1-(t/4M_B^2)] = -(M_B/p^0)^2 4\vec{p}^{\,2}$. Now, using Eq. (2.14), one obtains a general expression for the charge radius of a decuplet baryon B as

$$\langle R_{\rm ch}^2\rangle_B^{1/2} = \left[6\sum_{i=1}^{3}e_{Qi}\left\{\frac{R_B^2}{2}(\lambda_j^2+\lambda_k^2+\lambda_j\lambda_k)+\frac{g_{Qi}^2}{2}(\lambda_j+\lambda_k)^2\right\}\right] . \quad (2.15)$$

Thus we will be using in the following section the general expressions for magnetic moment and charge radii as described in Eqs. (2.5), (2.9), and (2.15) to estimate them for different decuplet baryons.

III. RESULTS AND DISCUSSION

We now estimate the magnetic moments and charge radii of the decuplet baryons using their expressions as obtained in the earlier section. Our calculations primarily are based on the model parameters such as constituent quark masses, the harmonic oscillator radii of decuplet baryons in addition to the experimentally measured baryon masses [18]. We may note here that the constituent quark mass m_Q enters here through the g_Q's appearing in the quark spinors and λ_i's, the energy fractions carried by the constituent quarks as [9,16]

$$g_{Qi} = \frac{1}{2m_{Qi}} \quad \text{and} \quad \lambda_i = \frac{m_{Qi}}{\sum_{i=1}^{3} m_{Qi}} \ . \quad (3.1)$$

Such a parametrization of g_{Qi}'s and λ_i's of the present model were in fact reasonably successful in explaining varieties of hadronic phenomena [8–17]. Thus we take the constituent quark masses for u, d, and s quarks as [16]

$$m_u = 0.308 \text{ GeV}, \quad m_d = 0.295 \text{ GeV}, \quad \text{and} \quad m_s = 0.485 \text{ GeV} , \tag{3.2}$$

from the earlier applications [8–17] of the model.

However, we take the harmonic oscillator radius for all the decuplet baryons to be the same and also the same as that of the octet baryons: i.e.,

$$R_B^2 = 14.5 \text{ GeV}^{-2} . \tag{3.3}$$

Such a harmonic oscillator radius when used to estimate the $\Delta^+ p$ transition amplitudes in the present model estimates it as

$$A_{3/2} = \sqrt{3} A_{1/2} = \left\langle \Delta^{++}_{3/2}(\vec{0}) \middle| \sum_Q \vec{J}_Q \cdot \vec{A} \middle| p_{1/2}(-\vec{p})\gamma(\vec{p}) \right\rangle$$
$$= -\left(\frac{2|\vec{p}|}{3}\right)^{1/2} \left(\frac{4R_p^2 R_{\Delta^+}^2}{(R_p^2 + R_{\Delta^+}^2)^2}\right)^{3/2} \exp\left(-\frac{R_p^2 R_{\Delta^+}^2}{3(R_p^2 + R_{\Delta^+}^2)} \vec{p}^{\,2}\right) (e_u g_u - e_d g_d) , \tag{3.4}$$

which in fact with the above mentioned model parameters becomes $-0.176\,22$ GeV$^{-1/2}$ in contrast with its experimentally measured value of (-0.258 ± 0.011) GeV$^{-1/2}$ [18]. However, a smaller harmonic oscillator radius for Δ^+ yields a smaller amplitude. We may note here that though the present estimation of the $\Delta^+ p$ transition amplitude is different from its measured value it still is in agreement with other model estimations such as the MIT bag model [19], and the same with pionic cloud effect [20] estimations. This in fact has been a mystery at present with all the quark models, except those dealing with chiral bag models [21], to explain the $\Delta^+ p$ transition amplitudes. Further, in the present estimation of $A_{3/2}$ the effects due to pionic and gluonic currents have not been considered whose contributions we believe can enhance the same.

Thus, with the model parameters reported above and the expressions (2.5) and (2.7) of the earlier section, we also estimate the magnetic moments for all the decuplet baryons both in the nonrelativistic and relativistic frames and report them in Table I where we also observe a reasonable agreement with the experiment, wherever available [1,2]. The ratios of magnetic moments such as $\mu_{\Delta^{++}}/\mu_p$ and $\mu_{\Omega^-}/\mu_\Lambda$ have also been experimentally observed [1,2]. Here, we also estimate these ratios with the μ_p and μ_Λ of the present model as was estimated earlier [8,9,16] and have reported them in Table I, along with their experimental measurements [1,2].

Now, we make a comparative analysis of the presently estimated magnetic moments with other theoretical investigations. In the simple nonrelativistic quark model (NQM) [18] the magnetic moment of a decuplet baryon is estimated by taking it as the sum of the magnetic moments of the individual constituent quarks and, when we compare our estimations with theirs, we find they are very close to our nonrelativistic estimations. In another analysis [4] in the Skyrme model the magnetic moments of decuplet baryons have been estimated and we find these are not much different from ours. Also, in the cloudy bag model (CBM) [5] the magnetic moments of decuplet baryons such as Δ^{++} and Ω^- have been estimated and these estimations differ from ours as well as from the experiment. In a recent lattice calculation, using the lattice simulation of quenched QCD [6], mag-

TABLE I. Estimated magnetic moments (in nm) of decouplet baryons in the present investigation compared with the experiment and other theoretical estimations.

μ_B	Present estimation Nonrel.	Present estimation Rel.	Ref. [3]	Ref. [4]	Ref. [5]	Ref. [6]	Ref. [18]	Expt. [1,2]
$\mu_{\Delta^{++}}$	5.55	5.23	4.76	4.53	6.54	6.09	5.56	4.52±0.50
μ_{Δ^+}	2.74	2.58	2.38	2.09		3.05	2.73	
μ_{Δ^0}	−0.06	−0.078	0.00	−0.36		0.00	−0.09	
μ_{Δ^-}	−2.87	−2.68	−2.38	−2.80		−3.05	−2.92	
$\mu_{\Sigma^{*+}}$	3.08	3.05	1.82	2.55		3.16	3.09	
$\mu_{\Sigma^{*0}}$	0.27	0.289	−0.27	−0.02		0.33	0.27	
$\mu_{\Sigma^{*-}}$	−2.53	−2.43	−2.36	−2.60		−2.50	−2.56	
$\mu_{\Xi^{*0}}$	0.60	0.68	−0.60	0.40		0.58	0.63	
$\mu_{\Xi^{*-}}$	−2.20	−2.13	−2.41	−2.31		−2.08	−2.20	
μ_{Ω^-}	−1.86	−1.80	−2.35	−1.98	−2.52	−1.73	−1.84	−1.94±0.17
$\mu_{\Delta^{++}}/\mu_p$	1.98	1.87	1.69	1.98	2.34	2.18	2.00	1.62±0.18
$\mu_{\Omega^-}/\mu_\Lambda$	2.98	2.93	3.41	3.73	4.13	3.6	3.00	3.16±0.28

netic moments of decuplet baryons have been estimated with which, when we compare the results of the present investigation, we observe a reasonable agreement. In a most recent analysis [3] in light front relativistic quark model [22], the magnetic moments of decuplet baryons have been estimated and though their results are close to ours still a subtle observation points out that for Δ^{++} theirs is closer to experiment [1] than ours, whereas for Ω^- ours is closer to the experiment [2] than theirs. However, the datum quoted in Table I for the $\mu_{\Delta^{++}}$, being an indirect measurement, is extracted from an analysis of the pion bremsstrahlung data and thus it becomes a model-dependent quantity. Therefore, it also acquires large errors associated with this model dependence and so we believe one may not take very seriously the discrepancy with respect to this. In addition we may note that all the decuplet baryons need not have the same harmonic oscillator radius. In fact the higher mass ones are expected to have a smaller radius (especially for Ω^-) and when we do so we obtain a smaller magnetic moment for Ω^-. To be more specific, a change of $R^2_{\Omega^-}$ from 14.5 to 9 GeV^{-2} yields a change in the magnetic moment of Ω^- from -1.86 to -1.82 nm and -1.80 to -1.77 nm in the nonrelativistic and relativistic frames respectively. Thus one may note here a slower variation of the magnetic moment of Ω^- with respect to the variation in its harmonic oscillator radius.

The ratios of magnetic moments $\mu_{\Delta^{++}}/\mu_p$ and $\mu_{\Omega^-}/\mu_\Lambda$ have been calculated in the NQM [18] as 2 and 3, respectively, which when we compare with ours, we find that they are not far from ours. However, the estimations due to the present investigation are in better agreement with the experimental measurements of the E756 Collaboration [2]. In fact, in Table I, we have reported the estimations of these ratios due to the NQM [18], Skyrme model [4], CBM [5], lattice simulation [6], and light front relativistic quark model [3] along with ours and experiment [1,2].

Further, because of the consistency of the present model investigation in the Δ^+p transition amplitude estimation with other model investigations [19,20] we also make here an attempt to estimate all other transition amplitudes utilizing the equations in parallel with Eq. (3.4), where in fact the baryons and the corresponding quarks involved in the interaction are to be replaced. Such estimations we have also tabulated in Table II with those estimated in the quark model [23], quenched lattice model [24], along with the available experimental measurements [18]. We may note here that, in Eq. (3.4), for the sake of simplicity we have reported the nonrelativistic calculation only [25]. In addition to the transition amplitudes, we also estimate here the decuplet (B) to octet (B') transition magnetic moments in the nonrelativistic and relativistic frames as

$$\mu_{BB'} = \frac{4}{3\sqrt{2}}\left(\frac{4R_B^2R_{B'}^2}{(R_B^2+R_{B'}^2)^2}\right)^{3/2}\left[e_{Q_1}g_{Q_1}\left(1-\frac{g_{Q_1}^2}{(R_B^2+R_{B'}^2)}\right)-e_{Q_3}g_{Q_3}\left(1-\frac{g_{Q_3}^2}{(R_B^2+R_{B'}^2)}\right)\right] \tag{3.5}$$

and

$$\mu_{BB'} = \frac{4}{3\sqrt{2}}\left(\frac{4R_B^2R_{B'}^2}{(R_B^2+R_{B'}^2)^2}\right)^{3/2}\left[e_{Q_1}\left\{\left(\frac{M_{B'}+M_B}{4M_{B'}M_B}\right)\left(1-\frac{8g_{Q_1}^2}{3(R_B^2+R_{B'}^2)}\right)+g_{Q_1}(\lambda_2+\lambda_3)\right\}\right.$$
$$\left.-e_{Q_3}\left\{\left(\frac{M_{B'}+M_B}{4M_{B'}M_B}\right)\left(1-\frac{8g_{Q_2}^2}{3(R_B^2+R_{B'}^2)}\right)+g_{Q_3}(\lambda_1+\lambda_2)\right\}\right]. \tag{3.6}$$

With the earlier mentioned parameters we estimate them and report them in Table III along with an estimation in an independent particle potential model [26]. At present there are no direct experimental data in this regard and thus we believe such measurements in the future can give a further understanding regarding the electromagnetic structure of baryons.

Next with the same set of parameters as taken above we also estimate charge radii of all the decuplet baryons in the framework of the present model, using the expres-

TABLE II. Estimated helicity amplitudes (in 10^{-3} GeV$^{1/2}$) of decuplet-octet transitions of the present investigation compared with other theoretical investigations.

	Present invest.		Ref. [23]		Ref. [24]		Ref. [18]	
Transitions	$A_{3/2}$	$A_{1/2}$	$A_{3/2}$	$A_{1/2}$	$A_{3/2}$	$A_{1/2}$	$A_{3/2}$	$A_{1/2}$
$p\gamma \leftrightarrow \Delta^+$	−176.22	−101.74	−175	−101	−195	−125	−258±11	−141±5
$n\gamma \leftrightarrow \Delta^0$	178.11	101.67	175	101	195	125		
$\Sigma^+\gamma \leftrightarrow \Sigma^{*+}$	−138	−79	−131	−75	−125	−86		
$\Sigma\gamma \leftrightarrow \Sigma^{*0}$	−58	−33	−55	−32	−52	−34		
$\Sigma^-\gamma \leftrightarrow \Sigma^{*-}$	21	12	20	12	21	17		
$\Xi^0\gamma \leftrightarrow \Xi^{*0}$	143	82	137	79	130	82		
$\Xi^0\gamma \leftrightarrow \Xi^{*-}$	−22	−12	−21	−12	−21	−16		

TABLE III. Estimated transition magnetic moments (in nm) in the present investigation compared with other theoretical investigations.

$\mu_{BB'}$	Present investigation Nonrel.	Rel.	Ref. [26]
$\mu_{\Delta^+ p}$	2.64	2.57	2.448
$\mu_{\Delta^0 n}$	2.64	2.55	2.415
$\mu_{\Sigma^{*+}\Sigma^+}$	2.33	2.28	2.205
$\mu_{\Sigma^{*0}\Sigma^0}$	1.00	0.95	0.983
$\mu_{\Sigma^{*-}\Sigma^-}$	−0.31	−0.35	−0.239
$\mu_{\Xi^{*0}\Xi^0}$	2.33	2.40	2.163
$\mu_{\Xi^{*-}\Xi^-}$	−0.31	−0.35	−0.233

TABLE IV. Estimated charge radii $\langle R^2_{\text{ch}}\rangle^{1/2}_B$ (in fm) of decuplet baryons in the present investigation compared with other theoretical investigations.

Baryons	Present investigation	Ref. [27]	Ref. [28]	Ref. [26]
Δ^{++}	1.18	1.112	1.337	1.02
Δ^+	0.82			0.723
Δ^0	0.16			0.00
Δ^-	0.84			0.723
Σ^{*+}	0.97			0.788
Σ^{*0}	0.34			0.276
Σ^{*-}	0.84			0.689
Ξ^{*0}	0.49			0.381
Ξ^{*-}	0.82			0.651
Ω^-	0.78	0.670	0.387	0.608

sion (2.15) of the earlier section and have reported them in Table IV. Because of the lack of experimental observations in this regard the estimations here stand as model predictions of the present model and may be verified in future experiments. However, there are theoretical estimations such as MIT bag model [27], soliton model [28], and independent quark potential model [26] estimations which, when compared with the present investigation, show a reasonable agreement, as reported in Table IV. We may also note here that in addition to the presently considered electromagnetic current operator there can be other contributions such as gluonic and pionic ones due to gluonic and pionic exchange current operators [29], respectively. However, the present investigation for the baryon decuplet charge radii and magnetic moments has not considered them and to this effect it lacks gauge invariance [29].

Like its earlier success, the presently applied field-theoretic quark model [8,9] has reproduced the observed magnetic moments of decuplet baryons reasonably along with a prediction for the unobserved ones and the charge radii in a much simplified and tractable manner which again could be possible due to the harmonic oscillator ansatz for the baryon wave function. One also observes here that different theoretical models are constrained while consistently getting the observed magnetic moments for Δ^{++} and Ω^-, and we believe that they will be further constrained once the final results come from the E800 collaboration, a succeeding experiment in this line, which are in fact expected to be here in the very near future [30].

ACKNOWLEDGMENTS

The authors would like to thank S. P. Misra and N. Barik for valuable suggestions and encouragement and they also thank K. C. Roy and P. Das for many discussions.

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