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Light-cone view on the applicability of perturbative QCD for exclusive processes

Chueng-Ryong Ji, Alex Pang, and Adam Szczepaniak
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695
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It is pointed out in the example of the pion form factor that the usual factorized hard scattering amplitude T_H in perturbative QCD is derived from the light-cone time-ordered perturbative expansion. In the light-cone perturbative expansion, the natural variable to make a separation of perturbative contributions from contributions intrinsic to the bound-state wave function itself is the light-cone energy, rather than the gluon virtuality of T_H. We find that the "legal" PQCD contribution defined by the light-cone energy cut saturates to the full PQCD prediction without any cut in the smaller Q^2 region as compared to that defined by the gluon four-momentum square cut. This is due to the contribution from highly off-energy-shell gluons in the end-point regions of the phase space.

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It has long been discussed whether perturbative QCD (PQCD) is applicable to exclusive processes at currently available experimental energies [1–7]. In view of the current trend of upgrading the electron beam energy at the Continuous Electron Beam Accelerator Facility (CEBAF) [8], this issue needs to be further clarified. The explicit criticism of the PQCD applicability can be found in the example of pion and proton form factors [2]. For the main critique, it has been argued that in order for perturbation to be consistent the four-momentum squared of the exchanged gluon, k_g^2, has to be larger than the typical hadronic scale $\mu^2 \lesssim 1$ GeV^2. Thus the "legal" PQCD contribution was defined by requiring $k_g^2 > \mu^2$. If μ^2 was taken to be 1 GeV^2, then the "legal" PQCD contribution was too small to compare with the currently available experimental data.

However, there is not yet a consensus on what value of μ^2 should be taken to define the "legal" PQCD. Furthermore, the ambiguity of the scale $\tilde{Q}^2$ for the argument of the QCD running coupling constant $\alpha_s(\tilde{Q}^2)$ and the renormalization scheme dependence in the PQCD expansion add more uncertainty to the criticism of PQCD [9]. These issues are quite delicate because the hard scattering amplitudes in the leading order PQCD are very sensitive to the values of μ^2 and $\tilde{Q}^2$. An important question for evaluating exclusive amplitudes in the transition region between hard and soft QCD processes is how to analytically separate perturbative contributions from contributions intrinsic to the bound-state wave function itself. In this respect, we note that the factorization of the covariant hard scattering amplitude from the nonperturbative quark distribution amplitude is originated from the light-cone quantization method of the QCD Fock state expansion [1]. In the renormalization group approach on the light cone [10], the variable that makes this separation is the light-cone energy, rather than gluon virtuality of the hard scattering amplitude. In this paper, we investigate this point in the explicit example of the pion form factor calculation using the light-cone perturbation theory. We first explicitly show that the sum of six light-cone time-ordered diagrams shown in Fig. 1 is equivalent to the usual PQCD hard scattering amplitude T_H and illustrate our point in leading twist level (see Fig. 2 and the corresponding discussions). However, we will attempt to include at least the higher twist effects which arise within the lowest Fock component of the hadron. More recently, Li and Sterman [6] have shown that by including the Sudakov effect the PQCD calculation can be made self-consistent at much lower Q^2 values. An essential step in their work was to bring the intrinsic transverse momentum dependence back in the hard scattering amplitude. The transverse momentum was neglected in the leading twist approximation but could become important in the end-point region. Thus we consider higher twist effects such as the intrinsic transverse momentum and the mass

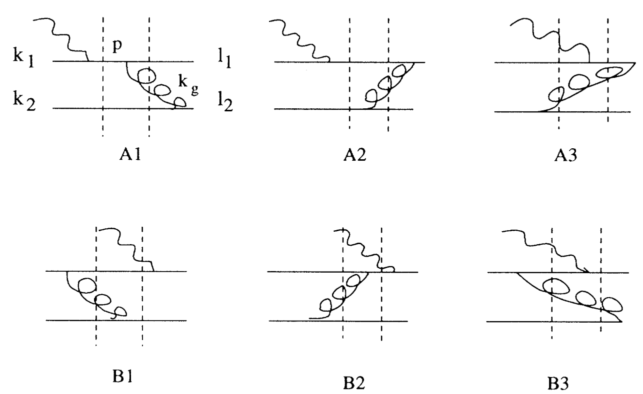

FIG. 1. Leading order light-cone time-ordered diagrams for the pion form factor, where $k_1 = (x_1P^+, k_\perp)$, $k_2 = (x_2P^+, -k_\perp)$, $l_1 = (y_1P'^+, y_1q_\perp + l_\perp)$, $l_2 = (y_2P'^+, y_2q_\perp - l_\perp)$, and $q = (0, q_\perp^2, q_\perp)$ in $(+,\perp)$ components with the $-$ component being determined by on-mass-shell conditions. Here q, P, and P' are the momentum of the photon, the pion in the initial state, and the pion in the final state, respectively. In each diagram, the instantaneous diagrams for the intermediate quark and gluon are implicitly included by using the technique shown in Ref. [1].

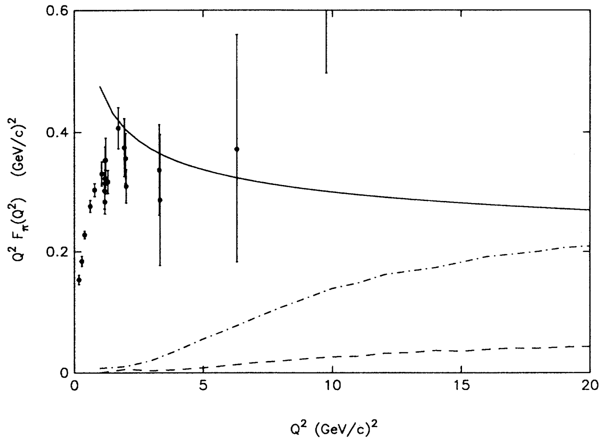

FIG. 2. Leading twist PQCD results using Dziembowski's wave function with $\beta = 0.46$ GeV. Each curve is described in the text.

of the constituents in examining the self-consistency of PQCD. Jacob and Kroll [7] had also considered the role of transverse momentum by repeating the Li-Sterman analysis but with the transverse momentum effect in the wave function as well. However, we note that our main point in this paper is not the inclusion of the higher twist effect but the observation that the natural variable to separate the hard and soft contributions is the light-cone energy rather than the gluon virtuality of T_H. This observation is independent of whether or not we include the higher twist effect. While the present discussion focuses on the pion form factor, the point addressed here is applicable to any other exclusive processes. Some works advocating a cutoff in invariant mass as a light-cone Hamiltonian regulator and natural variable in light-cone wave functions can be found in Refs. [11,12] which also discuss the advantages and consistency of this procedure.

The PQCD calculation of the pion form factor in the leading order can be given by [11]

$$F_\pi(Q^2) = \int dx\, dy\, d^2\vec{k}_\perp d^2\vec{l}_\perp \psi(x,\vec{k}_\perp) \times T(x,y,\vec{k}_\perp,\vec{l}_\perp,\vec{q}_\perp)\psi(y,\vec{l}_\perp) , \quad (1)$$

where $\psi(x,\vec{k}_\perp)$ is the light-cone wave function of the two-body Fock state and $T(x,y,\vec{k}_\perp,\vec{q}_\perp)$ is obtained by the two-body irreducible diagrams. In order to show that the sum of the light-cone time-ordered diagrams is equivalent to the covariant Feynman diagrams, the reducible one-gluon-exchange diagrams which can be obtained by the light-cone two-body bound-state equation of $\psi(x,\vec{k}_\perp)$ are also needed in the leading order calculation [13]. Thus, in the leading order of the light-cone PQCD, the six light-cone time-ordered diagrams shown in Fig. 1 were calculated in order to establish the equivalence to the covariant hard scattering amplitude. Also, the results are then gauge invariant. In each diagram, the instantaneous diagrams for the intermediate quark and gluon are included using the technique shown in Ref. [1].

The main structure of each diagram is given by $A_i = N_i/D_{1i}D_{2i}$, where N_i is the numerator expressing the spinor and γ-matrix algebra of the light-cone perturbation theory and D_{1i} and D_{2i} are the energy denominators designed by the dashed lines in Fig. 1. Even though the detailed expressions of all D_{1i} and D_{2i} depend on the time ordering, the general features are the same. For example, for A_1 we obtain

$$D_1 = \frac{1}{x_1x_2}\{(x_2\vec{q}_\perp + \vec{k}_\perp)^2 + m^2 - x_1x_2M_\pi^2\} , \quad (2)$$

$$D_2 = \frac{1}{y_1x_2(y_2-x_2)}\left\{ y_1y_2\left(x_2\vec{q}_\perp + \vec{k}_\perp - \frac{x_2}{y_2}\vec{l}_\perp\right)^2 + \frac{x_2}{y_2}(y_2-x_2)\vec{l}_\perp^{\,2}\right\} + \left(\frac{m^2}{y_1} + \frac{m^2}{x_2} - M_\pi^2\right) . \quad (3)$$

Here $x_i = k_i^+/P^+$, $y_i = l_i^+/P'^+$ are the momentum fractions of the ith constituent for the plus component in the initial and final states, respectively, with $x_1 + x_2 = 1$ and $y_1 + y_2 = 1$. When transverse momentum and masses are neglected, one can verify that, by multiplying the plus components of the intermediate quark and gluon momenta, D_1 and D_2 give the quark and gluon propagators, respectively, i.e., x_2Q^2 and $x_2y_2Q^2$, which appear in the covariant leading twist calculation. Applying the condition $M_\pi < 2m$ of the bound-state pion, D_1 and D_2 do not develop any singularity and they are positive in the entire phase space. Notice that $x(=x_1) > y(=y_1)$ for A_1 and, therefore,

$$\frac{m^2}{y} + \frac{m^2}{1-x} - M_\pi^2 > \frac{m^2}{x(1-x)} - M_\pi^2 > 0 . \quad (4)$$

We verified that the same properties hold for all energy denominators D_{1i} and D_{2i}. To leading order and leading twist in the light-cone gauge $A^+ = 0$, the explicit results for the six diagrams in Fig. 1 are given by

$$A_1 = \frac{8\theta(y_2-x_2)}{x_2y_2q_\perp^2} + \frac{8\theta(y_2-x_2)}{(y_2-x_2)x_2y_2q_\perp^2} , \quad (5)$$

$$A_2 = \frac{8x_1\theta(x_2-y_2)}{x_2^2y_1q_\perp^2} + \frac{8x_1\theta(x_2-y_2)}{(x_2-y_2)x_2^2y_1q_\perp^2} , \quad (6)$$

$$A_3 = \frac{8(x_2-y_2)\theta(x_2-y_2)}{x_2^2y_1y_2q_\perp^2} - \frac{8(x_2y_1+x_1y_2)\theta(x_2-y_2)}{(x_2-y_2)x_2^2y_1y_2q_\perp^2} , \quad (7)$$

$$B_1 = \frac{8\theta(x_2-y_2)}{x_2y_2q_\perp^2} + \frac{8\theta(x_2-y_2)}{(x_2-y_2)x_2y_2q_\perp^2} , \quad (8)$$

$$B_2 = \frac{8y_1\theta(y_2-x_2)}{x_1y_2^2q_\perp^2} + \frac{8y_1\theta(y_2-x_2)}{(y_2-x_2)x_1y_2^2q_\perp^2} , \quad (9)$$

$$B_3 = \frac{8(y_2-x_2)\theta(y_2-x_2)}{x_1x_2y_2^2q_\perp^2} - \frac{8(x_1y_2+x_2y_1)\theta(y_2-x_2)}{(y_2-x_2)x_1x_2y_2^2q_\perp^2} . \quad (10)$$

If we sum Eqs. (5)–(10) and include the common color factor and QCD running coupling constant, we obtain the usual hard scattering amplitude:

$$T_H = \frac{64\pi\alpha_s}{3Q^2}\left(\frac{e_1}{x_2 y_2} + \frac{e_2}{x_1 y_1}\right) . \tag{11}$$

Thus we showed that the results of calculating the six diagrams are equivalent to the covariant calculation of the two Feynman diagrams. Here our main point is that the larger are the light-cone energy denominators, the smaller is the light-cone time uncertainty of the parton system. Therefore the natural variable to separate perturbative contributions on the light cone from contributions intrinsic to the bound-state wave function itself is the light-cone energy [10]. Then, our criterion is that the boost-invariant quantity given by the multiplication of the plus component of the meson momentum (P^+) with the light-cone energy deficit Δk^-, that is, the light-cone energy denominator D_i, is greater than some hadronic scale μ^2;

$$P^+ D_i > \mu^2 . \tag{12}$$

In the rest of the text, we will take $P^+ = 1$ for convenience. This should be contrasted to the previous criteria based on the gluon virtuality of T_H. If one includes the intrinsic transverse momenta $\vec{k}_\perp$ and $\vec{l}_\perp$ and the mass m of the quarks, the light-cone gauge part proportional to $1/k_g^+$ leads to a singularity even though the Feynman gauge part $g_{\mu\nu}$ gives the regular amplitude. This is due to the gauge-invariant structure of the amplitudes. The covariant derivative $D_\mu = \partial_\mu + igA_\mu$ makes both the intrinsic transverse momenta $\vec{k}_\perp, \vec{l}_\perp$ and the transverse gauge degree of freedom $g\vec{A}_\perp$ be of the same order, indicating the need of the higher Fock state contributions to ensure the gauge invariance. However, we can show that the $1/k_g^+$ terms of $A_1 + B_2 + B_3$ and $B_1 + A_2 + A_3$ are proportional to the light-cone energy differences given by $\Delta_x = M_\pi^2 - (m^2 + \vec{k}^2)/x(1-x)$ and $\Delta_y = M_\pi^2 - (m^2 + \vec{l}_\perp^{\,2})/y(1-y)$. Thus we calculate the higher twist effects in the approximation of $\Delta_x = \Delta_y = 0$ to avoid the involvement of the higher Fock state contributions. Since our main argument is based on consideration of the energy denominators, the small changes of the numerator and the choice of the wave functions, etc., do not alter our conclusions.

For the numerical calculation, we used the light-cone wave function suggested by Dziembowski [14] which has the Gaussian parameter β. Note that the quark distribution amplitudes obtained by $\beta = 0.32$ GeV and $\beta = 0.46$ GeV are very similar to the asymptotic quark distribution amplitude and the double-hump-shaped quark distribution amplitude obtained by Chernyak and Zhitnitsky [15], respectively. Figure 2 shows the leading twist case (i.e., $k_\perp$ and masses are neglected) for the Chernyak-Zhitnitsky-type distribution amplitude. We compared the results obtained by different definitions of "legal" PQCD (i.e., different ways of cutting the integral). The solid curve corresponds to the full answer; the dashed liner is obtained by requiring $|k_g^2| > \mu^2 = 1$ GeV^2 and the dash-dotted curve for both $|D_1|$ and $|D_2| > \mu^2 = 1$ GeV^2. As we have discussed earlier, it is not yet clear what the μ value should be. However, we took the more conservative value $\mu = 1$ GeV for the illustration. As one can see, the curve with the light-cone energy cut saturates to the curve without any cut in a much smaller Q^2 region compared to the curve with the four-momentum square cut. A very similar pattern can also be obtained for the asymptotic quark distribution amplitude. We also note that the difference between the dashed curve and the dash-dotted curve would be even larger if one required the quark to be far off mass shell in the covariant calculation as well. The large difference between the dashed and the dash-dotted curves comes from the contribution in the region of $1-\mu/Q < x \sim y < 1$, i.e., the shaded area shown by Fig. 3. While this region is certainly near to the end points of the quark distribution amplitudes, the gluons in this region are highly off energy shell. Thus, from the light-cone quantization point of view, this region should be included in the "legal" PQCD. When transverse momentum and masses are included as shown in Fig. 4, the dash-dotted curve ($|D_1|, |D_2| > 1$ GeV^2) saturates to the solid curve (full answer) even faster, indicating that the PQCD calcu-

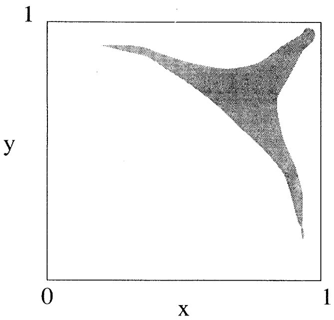

FIG. 3. Phase space of the momentum fractions. The shaded area indicates the region where $1 - \mu/Q < x \sim y < 1$.

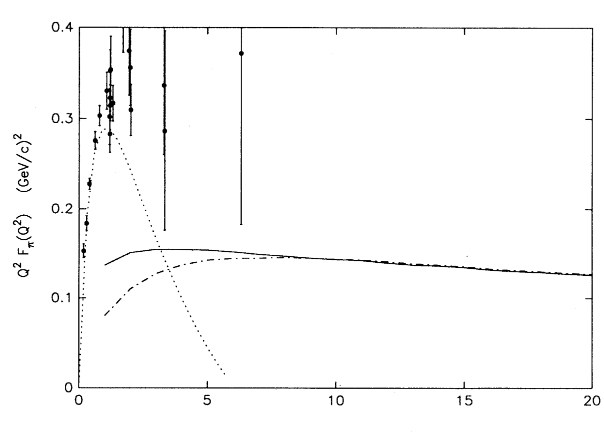

FIG. 4. PQCD results including the higher twist effects using Dziembowski's wave function with $\beta = 0.32$ GeV. Each curve is described in the text.

lation becomes self-consistent at even lower Q^2. Here we use $\beta = 0.32$ GeV to compare with the soft contribution (dotted curve) calculated in Ref. [14]. However, we should stress that our main point based on energy denominators is independent of the choice of the wave functions. The reason for the faster saturation is because the light-cone PQCD result is dominated by the contribution from the region where $D_1, D_2 > \mu^2$ when higher twist effects are included. This can be seen from Eqs. (2)–(4). Because of Eq. (4), one can prevent D_1 and D_2 from being zero. Furthermore, the gluon light-cone energy becomes nearly on shell at the kinematic point: $x_2 = y_2$ and $l_\perp = k_\perp + y_2 q_\perp$. For x_2, y_2 not nearly zero, i.e., away from the end-point region, $l_\perp \sim q_\perp$, and there is a suppression coming from the wave function evaluated at large $l_\perp$. Near the end-point region, $l_\perp \sim k_\perp$, and $l_\perp$ is not necessarily large. However, there is again a suppression factor $\exp(-\beta^2 m^2/x_1 x_2)$ evaluated at small x values. Therefore the wave function naturally suppresses the soft contribution.

One may consider the definition of a "legal" region for PQCD as the one in which higher order corrections are not large. The absence of large logarithms due to radiative corrections is the basic requirement for such a region. A further investigation along this line is needed.

In conclusion, we point out that the gluon four-momentum square cut may have cut too much to define the "legal" PQCD. As we observed from the numerical computation, the "legal" PQCD contribution defined by the light-cone energy cut saturates to the full PQCD prediction in a much smaller Q^2 region when it is compared with that defined by the gluon four-momentum square cut. We find that if $x \sim y$ the gluons are highly off energy shell even if they are in the end-point regions $x, y > 1 - \mu/Q$. We also observe that the PQCD calculation becomes more self-consistent when higher twist terms ($k_\perp$ and m) are taken into account. This observation is model independent since our consideration is based on the structure of the energy denominators.

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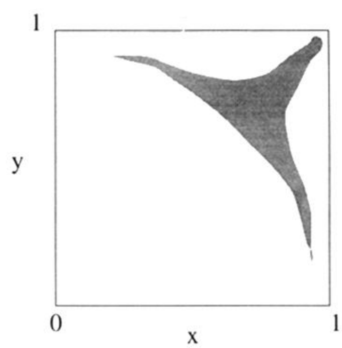

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