Applicability of perturbative QCD to $B \to D$ decays

Hsiang-nan Li
Department of Physics, National Chung-Cheng University, Chia-Yi, Taiwan, Republic of China
(Received 30 December 1994)

We examine the applicability of perturbative QCD to B meson decays into D mesons. We find that the perturbative QCD formalism, which includes Sudakov effects at intermediate energy scales, is applicable to the semileptonic decay $B \to Dl\nu$, when the D meson recoils fast. Following this conclusion, we analyze the two-body nonleptonic decays $B \to D\pi$ and $B \to DD_s$. By comparing our predictions with experimental data, we extract the matrix element $|V_{cb}| = 0.044$.

PACS number(s): 13.20.He, 12.15.Hh, 12.38.Bx

I. INTRODUCTION

Recently, perturbative QCD (PQCD) has been proposed to be an alternative theory for the study of B meson decays [1–3], which complements the approach based on the heavy-quark effective theory (HQET) [4] and the Bauer-Stech-Wirbel method [5]. The point is to include Sudakov effects [6], which arise from the all-order summation of large radiative corrections in the processes. It has been shown that these effects, suppressing contributions due to soft-gluon exchange, improve the applicability of PQCD to exclusive processes around the energy scale of few GeV [7]. The heavy b quark possesses a large mass scale located in the range of applicability [8]. The Sudakov factor for the heavy-to-light transition $B \to \pi l\nu$ has been derived in [2], and the perturbative evaluation of the associated differential decay rate is found to be reliable for the pion energy fraction above 0.3.

In this paper we shall investigate the applicability of the above PQCD formalism to heavy-to-heavy transitions, concentrating on the semileptonic decay $B \to Dl\nu$. Heavy-quark symmetry [9] has been employed in the analysis of this decay [10], whose amplitude is written as

$$A(P_1, P_2) = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\nu}\gamma_\mu(1-\gamma_5) l \langle D(P_2)|\bar{c}\gamma^\mu b|B(P_1)\rangle \ , \tag{1}$$

where $G_F = 10^{-5}$ GeV^{-2} is the Fermi coupling constant, and P_1 (P_2) is the B (D) meson momentum. The transition-matrix element $M^\mu = \langle D|\bar{c}\gamma_\mu b|B\rangle$ can be expressed in terms of a universal form factor ξ in the heavy-meson limit [9]:

$$M^\mu = \sqrt{m_B m_D}\,\xi(v_1 \cdot v_2)(v_1 + v_2)^\mu \tag{2}$$

with m_B (m_D) the B (D) meson mass. The velocities v_1 and v_2 are defined by the relations $P_1 = m_B v_1$ and $P_2 = m_D v_2$, respectively. The form factor ξ, called the Isgur-Wise (IW) function, depends only on the velocity transfer $v_1 \cdot v_2$ and is normalized to unity at zero recoil $v_1 \cdot v_2 = 1$ in the limits $m_B, m_D \to \infty$ [9].

The IW function has been usually regarded as sensitive to long-distance effects, and cannot be calculated in perturbation theory. For the behavior of ξ above zero recoil, there is only the model estimation from the overlap integrals of heavy-meson wave functions [11]. In this work we shall show that PQCD can give reliable predictions to ξ in the region with large $v_1 \cdot v_2$, where the heavy quark symmetry cannot provide any information of ξ. We argue that the IW function is dominated by a soft contribution in the slow D meson limit, at which the heavy-meson wave functions strongly overlap, and factorization theorems do not hold. However, when the D meson recoils fast, carrying energy much greater than m_D, the case is then similar to the $B \to \pi$ decays, and PQCD is expected to be applicable [12].

The above conclusion then indicates that two-body nonleptonic decays such as $B \to D\pi$ and $B \to DD_s$ can be analyzed reliably in the PQCD formalism. The $B \to D$ decays have been studied [12,13] based on the exclusive PQCD theory developed by Lepage and Brodsky [14]. However, these analyses lack quantitatively justification for the perturbative calculation and are highly sensitive to the variation of the heavy-meson wave functions. Our predictions for the branching ratios of these decay processes are comparable with those from the standard PQCD in [12,13] and lead to the value 0.044 for the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ by combining with experimental data [15]. On one hand, we derive the behavior of the IW function near the high end of $v_1 \cdot v_2$. On the other hand, the consistency of the extracted $|V_{cb}|$ with its currently accepted value justifies the application of our PQCD formalism to the semileptonic decays $B \to \pi l\nu$ [2] and $B \to \pi\pi$ [3].

A model-independent extraction of the matrix element $|V_{cb}|$ has been obtained from the semileptonic decay $B \to D^* l\nu$ in the framework of HQET [10]. The value of $|V_{cb}|$ was read off by extrapolating the experimental data to the zero-recoil point, at which the IW function is known to be equal to unity. In the present work, however, we must extract $|V_{cb}|$ by studying the behavior of the IW function at the opposite end of the velocity transfer, for which the PQCD analysis is reliable. Hence, the two-body decays are good candidates. Another possible method of extracting $|V_{cb}|$ has been proposed in [16], in which a sum rule for the relevant structure function of the inclusive semileptonic decay $b \to c$ was considered.

In Sec. II we derive the factorization formulas for the form factors involved in the $B \to Dl\nu$ decay, including the resummation of large radiative corrections to this transition. Numerical analysis is shown in Sec. III, along with the behavior of the IW function at large velocity transfer. The comparison of our predictions for the decays $B \to D\pi$ and $B \to DD_s$ with experimental data is also made. Section IV contains the conclusions.

II. FACTORIZATION

In this section we develop the factorization formula for the $B \to Dl\nu$ decay. The lowest-order factorization for the transition-matrix element M^μ is shown in Fig. 1, in which the bubbles represent the B and D mesons, and the symbol $\times$ represents the electroweak vertex where the lepton pair emerges. The b quark, denoted by a bold line, and its accompanying light quark carry the momenta $P_1 - k_1$ and k_1, respectively, which satisfy the on-shell conditions $(P_1 - k_1)^2 \approx m_b^2$ and $k_1^2 \approx 0$, m_b being the b quark mass. We shall work in the rest frame of the B meson such that the nonvanishing components of P_1 are $P_1^+ = P_1^- = m_B/\sqrt{2}$. k_1 contains a small number of transverse components $\mathbf{k}_{1T}$, and its minus component defines the momentum fraction $x_1 = k_1^-/P_1^-$. The assignment of the momenta for the D meson is similar, but with k_1, m_b, and x_1 replaced by k_2, m_c, and $x_2 = k_2^+/P_2^+$, respectively, m_c being the c quark mass.

The expressions for the components of P_2 are more complicated. At zero recoil the D meson sits at rest with the B meson, and we have $P_2 \propto P_1$. When the D meson takes the maximum energy, it moves fast, and P_2^+ is much greater than P_2^-. To show the relation between P_2^+ and P_2^-, it is most convenient to express them in terms of the velocity transfer $\eta = v_1 \cdot v_2$. Solving the equations $P_1 \cdot P_2 = \eta m_B m_D$ and $P_2^2 = m_D^2$, we obtain

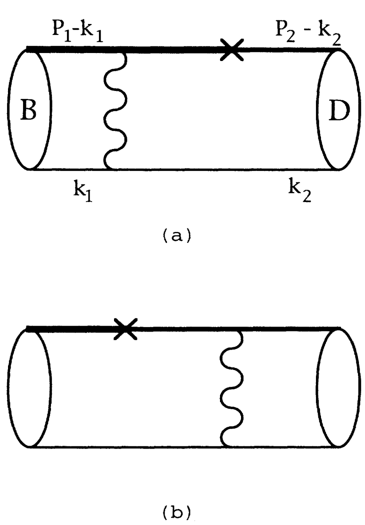

FIG. 1. Lowest-order diagrams of the decay $B \to Dl\nu$.

$$P_2^+ = \frac{\eta + \sqrt{\eta^2 - 1}}{\sqrt{2}} m_D \,,$$

$$P_2^- = \frac{\eta - \sqrt{\eta^2 - 1}}{\sqrt{2}} m_D \,. \tag{3}$$

The upper bound of η, corresponding to the maximum recoil of the D meson, is equal to $\eta_{\max} = (m_B/m_D + m_D/m_B)/2$. It is easy to check from Eq. (3) that $P_2^+ = P_2^- = m_D/\sqrt{2}$, as η takes the minimum value 1, and $P_2^+/P_2^- = m_B^2/m_D^2 \gg 1$ at $\eta = \eta_{\max}$.

We then consider higher-order corrections to the basic factorization picture. As analyzed before [2,6,7], these corrections produce large collinear logarithms, when the loop momentum is parallel to that of a light quark, or large soft logarithms, when the loop momentum is much smaller than the mass scale involved in the processes. The two types of large corrections may combine to give double logarithms. It has been shown that the double logarithms come from two-particle reducible diagrams in physical (axial) gauge, whose contributions are dominated by collinear enhancements for fast light mesons, and are dominated by soft enhancements for heavy mesons at rest [2]. Therefore, they can be absorbed into the corresponding wave functions, which involve similar dynamics. The all-order summation of the double logarithms in light-meson wave functions, such as a pion, has been performed in [7]. The resummation technique [17] has been extended to the case of heavy mesons in [2]. Combining the above results, we have derived the Sudakov factor for the heavy-to-light transition $B \to \pi l\nu$ [2].

The analysis of the Sudakov corrections to Fig. 1 is more complicated compared to that of the decay $B \to \pi l\nu$. Because of the dominance of soft contributions near the low end of η, we concentrate only on the large-η region. In this region radiative corrections on the D meson side involve three scales, $P_2^+ \gg m_D \gg k_{2T}$. Note that all the previous studies of resummation involve only two scales, for example, P^+ and k_T in the pion case and m_B and k_T in the B meson case [2]. The three scales produce various large logarithms of P_2^+/k_{2T}, P_2^+/m_D and m_D/k_{2T}, which complicate their organization. As a naive approximation, we keep only the largest one proportional to $\ln(P_2^+/k_{2T})$. The neglect of those logarithms containing m_D is equivalent to the neglect of $P_2^- \ll P_2^+$ in the analysis of radiative corrections to the D meson wave function. The D meson is then regarded as being light in the large η region, and the corresponding Sudakov factor for the decay $B \to Dl\nu$ can be approximated by that for the heavy-to-light transitions.

The factorization formula for M^μ in the transverse configuration space, with radiative corrections taken into account, is written as

$$M^\mu = \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b}_1}{(2\pi)^2}\frac{d^2\mathbf{b}_2}{(2\pi)^2} \mathcal{P}_D(x_2, \mathbf{b}_2, P_2, \mu) \times \tilde{H}^\mu(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, m_B, m_D, \mu) \mathcal{P}_B(x_1, \mathbf{b}_1, P_1, \mu), \tag{4}$$

in which both the B and D meson wave functions, $\mathcal{P}_B$ and $\mathcal{P}_D$, contain the evolution from the resummation of double logarithms performed in axial gauge. We have introduced the conjugate variable b_1 (b_2) to denote the separation between the two valence quarks of the B (D) meson. We shall employ the approximation $m_b \approx m_B = 5.28$ GeV and $m_c \approx m_D = 1.87$ GeV in Eq. (4). $\tilde{H}^\mu$ is the Fourier transform of the hard scattering amplitude H^μ to b space. μ is the renormalization and factorization scale.

Note that in the evaluation of H^μ we neglect those terms proportional to k_1^+ and k_2^- in the hard scattering amplitude following the kinematic ordering $k_1^+ \sim k_2^- \ll k_1^- \sim k_2^+$, which is valid in the large η region. For example, the gluon propagator in the lowest-order diagram is written as

$$\frac{1}{(k_1-k_2)^2+i\epsilon} \approx \frac{-1}{2k_1^- k_2^+ + (\mathbf{k}_{1T}-\mathbf{k}_{2T})^2}\,, \tag{5}$$

where $\mathbf{k}_T$ serves as the infrared cutoff of the Sudakov corrections. Once the approximation is made, the k_1^+ and k_2^- dependences, appearing only in the B and D meson wave functions, respectively, are integrated to give Eq. (4).

As stated above, near the high end of η the Sudakov factor e^{-S} for the decay $B \to Dl\nu$, which groups the large logarithmic corrections in $\mathcal{P}_B$, $\mathcal{P}_D$, and $\tilde{H}^\mu$, can be approximated by that for the heavy-to-light transition derived in [2], with the exponent S given by

$$\begin{aligned} S(x_i,b_i,m_B,m_D) &= s(x_1,b_1,P_1^-) \\ &\quad + \sum_{x=x_2,1-x_2} s(x,b_2,P_2^+) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \end{aligned} \tag{6}$$

where t is the largest mass scale associated with the hard gluon and will be specified later. The first term in Eq. (6) comes from the resummation of reducible corrections to the heavy-meson wave function [2]. The value of $\Lambda \equiv \Lambda_{\rm QCD}$ will be set to 100 MeV below. The complete expression for the factor $s(x,b,Q)$, including the leading and next-to-leading logarithms, is exhibited in the Appendix. It is observed that e^{-S} decreases quickly in the large b_i region and vanishes as $b_i > 1/\Lambda$. Therefore, long-distance contributions are suppressed, and the perturbative calculation becomes relatively reliable.

One may wonder whether the resummation of large radiative corrections can improve the applicability of PQCD near the low end of η. If we recognize that the D meson is regarded as a heavy meson in this region, and is dominated by similar dynamics to that of the B meson, the Sudakov factor for the decay $B \to Dl\nu$ can be taken as the combination of the expressions for heavy mesons [2] at two different mass scales, m_B and m_D. The Sudakov exponent S is then written as

$$\begin{aligned} S(x_i,b_i,m_B,m_D) &= s(x_1,b_1,P_1^-) + s(x_2,b_2,P_2^+) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right]. \end{aligned} \tag{7}$$

Obviously, it is not expected that our perturbative analysis with the above Sudakov suppression becomes self-consistent. The virtuality of the hard gluon in Fig. 1 diminishes as x_1 and x_2 are both small, leading to a large running coupling constant α_s. However, this nonperturbative region is not strongly suppressed by the Sudakov factor in Eq. (7). It is the extra exponent $s(1-x_2,b_2,P_2^+)$ in Eq. (6) that can provide the necessary suppression in the small x_2, or large $1-x_2$, region.

Having factored all the large logarithms into the Sudakov factor, we can then compute the hard scattering amplitude H^μ of the $B \to Dl\nu$ decay to the lowest order of α_s. From Fig. 1(a) we have

$$\begin{aligned} H^{(a)\mu} &= {\rm tr}\left[\gamma_\alpha \frac{\gamma_5(\not{P}_2+m_D)}{\sqrt{2N_c}}\gamma^\mu \frac{\not{P}_1-\not{k}_2+m_B}{(P_1-k_2)^2-m_B^2}\gamma^\alpha \frac{(\not{P}_1+m_B)\gamma_5}{\sqrt{2N_c}}\right] \frac{-g^2N_c\mathcal{C}_F}{(k_1-k_2)^2} \\ &= \frac{16\pi\alpha_s\mathcal{C}_F[m_Bm_D - x_2\zeta_1 m_D^2]}{[x_1x_2\zeta m_Bm_D+(\mathbf{k}_{1T}-\mathbf{k}_{2T})^2][x_2\zeta m_Bm_D+\mathbf{k}_{2T}^2]}P_1^\mu \\ &\quad + \frac{16\pi\alpha_s\mathcal{C}_F[m_B^2 + x_2\zeta_2 m_Bm_D]}{[x_1x_2\zeta m_Bm_D+(\mathbf{k}_{1T}-\mathbf{k}_{2T})^2][x_2\zeta m_Bm_D+\mathbf{k}_{2T}^2]}P_2^\mu\,, \end{aligned} \tag{8}$$

with

$$\begin{aligned} \zeta &= \eta + \sqrt{\eta^2-1}\,, \\ \zeta_1 &= \frac{1}{2} + \frac{\eta-2}{2\sqrt{\eta^2-1}}\,, \\ \zeta_2 &= \eta - 1 + \frac{2\eta^2-2\eta-1}{2\sqrt{\eta^2-1}}\,. \end{aligned} \tag{9}$$

The factors $(\not{P}_1+m_B)\gamma_5/\sqrt{2N_c}$ and $\gamma_5(\not{P}_2+m_D)/\sqrt{2N_c}$ come from the matrix structures of the B and D meson wave functions, respectively. $\mathcal{C}_F = 4/3$ is the color factor, and N_c the number of colors. Similarly, from Fig. 1(b) we get

$$\begin{aligned} H^{(b)\mu} &= \mathrm{tr}\left[\gamma_\alpha \frac{\gamma_5(\not{P}_2 + m_D)}{\sqrt{2N_c}}\gamma^\alpha \frac{\not{P}_2 - \not{k}_1 + m_D}{(P_2-k_1)^2 - m_D^2}\gamma^\mu \frac{(\not{P}_1 + m_B)\gamma_5}{\sqrt{2N_c}}\right]\frac{-g^2 N_c C_F}{(k_1-k_2)^2} \\ &= \frac{16\pi\alpha_s C_F[m_D^2 + x_1\zeta_2 m_B m_D]}{[x_1x_2\zeta m_B m_D + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2][x_1\zeta m_B m_D + \mathbf{k}_{1T}^2]}P_1^\mu \\ &\quad + \frac{16\pi\alpha_s C_F[m_B m_D - x_1\zeta_1 m_B^2]}{[x_1x_2\zeta m_B m_D + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2][x_1\zeta m_B m_D + \mathbf{k}_{1T}^2]}P_2^\mu . \end{aligned} \tag{10}$$

Note that $H^{(b)}$ can be obtained from $H^{(a)}$ by exchanging the variables associated with the B and D mesons. This permutation symmetry has been displayed manifestly in Fig. 1.

Performing the Fourier transform of Eqs. (8) and (10) to get $\tilde{H}^\mu$ and substituting them into (4), we obtain the factorization formula for $M^\mu = f_1 P_1^\mu + f_2 P_2^\mu$, where the form factors f_1 and f_2 are given by

$$\begin{aligned} f_1 &= 16\pi C_F \int_0^1 dx_1\, dx_2 \int_0^\infty b_1\, db_1 b_2\, db_2\, \phi_B(x_1,b_1)\phi_D(x_2,b_2) \\ &\quad \times[(m_B m_D - x_2\zeta_1 m_D^2)h(x_1,x_2,b_1,b_2) + (m_D^2 + x_1\zeta_2 m_B m_D)h(x_2,x_1,b_2,b_1)]\exp[-S(x_i,b_i,m_B,m_D)] \end{aligned} \tag{11}$$

and

$$\begin{aligned} f_2 &= 16\pi C_F \int_0^1 dx_1\, dx_2 \int_0^\infty b_1\, db_1 b_2\, db_2\, \phi_B(x_1,b_1)\phi_D(x_2,b_2) \\ &\quad \times[(m_B^2 + x_2\zeta_2 m_B m_D)h(x_1,x_2,b_1,b_2) + (m_B m_D - x_1\zeta_1 m_B^2)h(x_2,x_1,b_2,b_1)]\exp[-S(x_i,b_i,m_B,m_D)] , \end{aligned} \tag{12}$$

respectively, with

$$\begin{aligned} h(x_1,x_2,b_1,b_2) &= \alpha_s(t)K_0(\sqrt{x_1x_2\zeta m_B m_D}b_1)[\theta(b_1-b_2)K_0(\sqrt{x_2\zeta m_B m_D}b_1)I_0(\sqrt{x_2\zeta m_B m_D}b_2) \\ &\quad +\theta(b_2-b_1)K_0(\sqrt{x_2\zeta m_B m_D}b_2)I_0(\sqrt{x_2\zeta m_B m_D}b_1)] . \end{aligned} \tag{13}$$

The wave function ϕ_B (ϕ_D) comes from $\mathcal{P}_B$ ($\mathcal{P}_D$) in Eq. (4) with the evolution in P_1^- (P_2^+), which is the result of the resummation of reducible corrections, grouped into the Sudakov factor. The argument b in ϕ_B and ϕ_D denotes the intrinsic transverse momentum dependence of the wave functions [18], which is a nonperturbative object and cannot be handled in perturbation theory. K_0 and I_0 are the modified Bessel functions of order zero. We choose t as the largest scale associated with the hard gluon:

$$t = \max(\sqrt{x_1x_2\zeta m_B m_D}, 1/b_1, 1/b_2) . \tag{14}$$

III. NUMERICAL RESULTS

Before evaluating f_i, we compare our formulas with those derived in the framework of standard PQCD [12,13], where the $\mathbf{k}_T$ dependence in the hard scattering amplitude is neglected and the heavy meson wave functions, with $\mathbf{k}_T$ integrated, take the simple form of the δ function (the so-called peaking approximation):

$$\phi_B(x) = \frac{f_B}{2\sqrt{3}}\delta(x - x_B), \;\; \phi_D(x) = \frac{f_D}{2\sqrt{3}}\delta(x - x_D) . \tag{15}$$

Here $f_B = 0.12$ GeV and $f_D = 0.14$ GeV are the decay constants of the B and D mesons [19], respectively. Equations (11) and (12) are then reduced to the standard factorization formulas without b integrations, which lead to

$$\begin{aligned} f_1 &= \frac{4}{3}\pi C_F \alpha_s f_B f_D\left[\frac{m_B m_D - x_D\zeta_1 m_D^2}{x_B x_D^2 \zeta^2 m_B^2 m_D^2} + \frac{m_D^2 + x_B\zeta_2 m_B m_D}{x_B^2 x_D \zeta^2 m_B^2 m_D^2}\right] , \\ f_2 &= \frac{4}{3}\pi C_F \alpha_s f_B f_D\left[\frac{m_B^2 + x_D\zeta_2 m_B m_D}{x_B x_D^2 \zeta^2 m_B^2 m_D^2} + \frac{m_B m_D - x_B\zeta_1 m_B^2}{x_B^2 x_D \zeta^2 m_B^2 m_D^2}\right] . \end{aligned} \tag{16}$$

It is apparent that the above expressions are very sensitive to the values of x_B and x_D, and the coupling constant α_s must be regarded as a free parameter. We consider the nonleptonic decay $B \to D\pi$, which corresponds to the case of maximum recoil here with $\eta = \eta_{\max} = 1.59$. Setting $\alpha_s = 0.4$, $x_B = 0.07$, and $x_D = 0.2$ as in [13], we obtain $f_1 + f_2 = 1.3$, which gives a branching ratio comparable with experimental data [15]. However, if slightly different values such as $x_B = 0.07$ and $x_D = 0.15$ were inserted, the branching ratio becomes three times larger. On the other hand, simply setting α_s to a constant makes the justification of the perturbative calculation unavailable. Compared to the standard PQCD approach, our modified perturbative expressions are less sensitive to the profile change of the wave functions because of the inclusion of $\mathbf{k}_T$ in the hard scattering amplitude, which moderates the divergences from small x_B and x_D. Substituting Eq. (15) into (11) and (12), and performing the integrations over b_1 and b_2, we find that the latter set of x_B and x_D leads to a branching ratio only 50% larger than that from the former set.

We adopt the following model for the B meson wave function [20]:

$$\Phi_B(x,\mathbf{k}_T) = N_B\left[C_B + \frac{m_B^2}{1-x} + \frac{\mathbf{k}_T^2}{x(1-x)}\right]^{-2} . \quad (17)$$

The constants N_B and C_B are determined by the normalizations

$$\int_0^1 dx \int d^2\mathbf{k}_T \Phi_B(x,\mathbf{k}_T) = \frac{f_B}{2\sqrt{3}},$$
$$\int_0^1 dx \int d^2\mathbf{k}_T [\Phi_B(x,\mathbf{k}_T)]^2 = \tfrac{1}{2}, \quad (18)$$

which give $N_B = 0.923$ GeV^3 and $C_B = -27.877\,255$ GeV^2. ϕ_B is then defined by

$$\begin{aligned}\phi_B(x,b) &= \int d^2\mathbf{k}_T\, \Phi_B(x,\mathbf{k}_T) e^{i\mathbf{k}_T\cdot\mathbf{b}} \\ &= \frac{\pi N_B b x^2(1-x)^2}{\sqrt{m_B^2 x + C_B x(1-x)}} \\ &\quad \times K_1(\sqrt{m_B^2 x + C_B x(1-x)}\, b) . \end{aligned} \quad (19)$$

It is observed that ϕ_B peaks at $x \approx 0$ and decreases monotonically with x for fixed b, signifying the soft dynamics involved in the rest B meson.

If we assume a similar model for the D meson wave function, with m_B in Eq. (17) replaced by m_D, straightforwardly,

$$\begin{aligned}\phi_D(x,b) &= \frac{\pi N_D b x^2(1-x)^2}{\sqrt{m_D^2 x + C_D x(1-x)}} \\ &\quad \times K_1(\sqrt{m_D^2 x + C_D x(1-x)}\, b) , \end{aligned} \quad (20)$$

we obtain the constants $N_D = 0.136$ GeV^3 and $C_D = -3.495\,345$ GeV^2. The resulting wave function ϕ_D also peaks at small $x \approx 0.01$ for fixed b. However, the QCD sum-rule analysis in [21] has shown that the average momentum fraction of the light valence quark in a fast D meson is roughly 0.2. To be consistent with this observation, we employ Eq. (20) but with C_D determined by the requirement that ϕ_D takes the maximum value at $x \approx 0.2$ for $b \to 0$. We then have $C_D = -2.9$ GeV^2, along with $N_D = 0.240$ GeV^3 from the normalization $\int dx\, \phi_D(x,0) = f_D/(2\sqrt{3})$.

Results of f_1 and f_2 derived from Eqs. (11) and (12), respectively, with b_1 and b_2 integrated up to the same cutoff b_c are shown in Fig. 2. We find that at $\eta = 1.30$ approximately 55% of the contribution to f_i comes from the region with $\alpha_s(1/b_c) < 1$ or, equivalently, $b_c < 0.5/\Lambda$. The percentage of perturbative contribution increases with η, and for η above 1.39, more than 60% of the full contribution is accumulated in this region. It implies that our PQCD analysis of the decay $B \to Dl\nu$ in the range of $\eta \geq 1.39$ is relatively reliable. It is also found that the self-consistency of the perturbation theory becomes worse quickly for $\eta < 1.3$, as expected. Compared to a similar analysis of the decay $B \to \pi l\nu$ [2], in which about 80% of the whole contribution arises from the above perturbative region, PQCD does not work as well for the decay $B \to Dl\nu$ as for the $B \to \pi$ decay. The reason is that it is only a fair approximation to treat the D meson as being light because the ratio P_2^+/m_D is only equal to 2 even in the maximum recoil region. However, it is still sensible to compare our predictions with experimental data, since perturbative contribution indeed dominates [7].

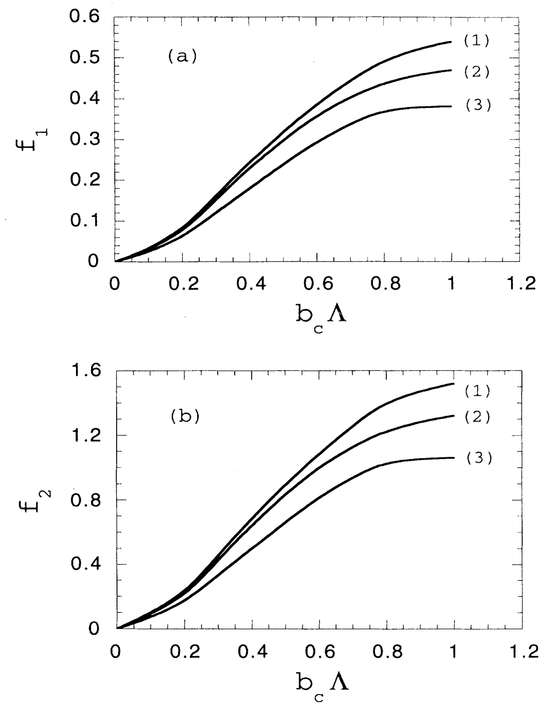

FIG. 2. Dependence of (a) f_1 and (b) f_2 on the cutoff b_c for (1) $\eta = 1.3$, (2) $\eta = 1.39$, and (3) $\eta = 1.59$.

Based on the above conclusion, we are led to consider the two-body nonleptonic decays such as $B \to D\pi$ and $B \to DD_s$, which can be best described by our PQCD formalism. The decay rate of the specific mode $\bar{B}^0 \to D^+\pi^-$ is given by

$$\Gamma = \frac{1}{64\pi} G_F^2 |V_{ud}|^2 |V_{cb}|^2 f_\pi^2 m_B^3 \left(1 - \frac{m_D^2}{m_B^2}\right)^3 |f_1 + f_2|^2 , \quad (21)$$

which is derived from the amplitude

$$A = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb} \langle \pi|\gamma_\mu(1-\gamma_5)|0\rangle\langle D|\bar{c}\gamma^\mu b|B\rangle \quad (22)$$

with the PCAC (partial conservation of axial vector current) relation $\langle \pi(P)|\gamma_\mu(1-\gamma_5)|0\rangle = i\sqrt{2} f_\pi P_\mu$ inserted, $f_\pi = 93$ MeV being the pion decay constant. Equation (22) is achieved following the conclusion in [22] that the nonfactorizable W-exchange contribution is negligible. The value of $f_1 + f_2$ in this case can easily be read of from the curves corresponding to $\eta = 1.59$ in Fig. 2, which is equal to 1.44. Substituting the matrix element $|V_{ud}| = 0.974$, we obtain $\Gamma = 8.4 \times 10^{-13}|V_{cb}|^2$ GeV, or equivalently, the branching ratio $B(\bar{B}^0 \to D^+\pi^-) = 1.65|V_{cb}|^2$ from the total width $(0.51\pm0.02)\times 10^{-9}$ MeV of the $\bar{B}^0$ meson [23]. Comparing with experimental data $B(\bar{B}^0 \to D^+\pi^-) = 3.2 \times 10^{-3}$, we extract the matrix element $|V_{cb}| = 0.044$, consistent with the currently accepted value [23]. Similarly, the decay rate for the mode $\bar{B}^0 \to D^+D_s^-$ is given by

$$\Gamma = \frac{1}{32\pi} G_F^2 |V_{cs}|^2 |V_{cb}|^2 f_{D_s}^2 \frac{m_D}{m_B^2} \sqrt{\eta'^2_{\max} - 1} \times |(m_B^2 - m_D^2 + m_{D_s}^2) f_1 + (m_B^2 - m_D^2 - m_{D_s}^2) f_2|^2 \tag{23}$$

with the matrix element $|V_{cs}| = 1.0$, the decay constant of the D_s meson $f_{D_s} = 0.16$ GeV [19], and the D_s meson mass $m_{D_s} = 1.97$ GeV. In this case we have the maximum velocity transfer $\eta'_{\max} = (m_B^2 + m_D^2 - m_{D_s}^2)/(2m_B m_D) = 1.39$, for which the corresponding values $f_1 = 0.47$ and $f_2 = 1.32$ are read off from Fig. 2. Equation (23) then gives the decay rate $\Gamma = 2.7 \times 10^{-12}|V_{cb}|^2$, or the branching ratio $B(\bar{B}^0 \to D^+D_s^-) = 5.3|V_{cb}|^2$. Experimental data show $B(\bar{B}^0 \to D^+D_s^-) = 9.9 \times 10^{-3}$, from which we extract $|V_{cb}| = 0.043$, close to that obtained from the decay $\bar{B}^0 \to D^+\pi^-$.

Because of the consistency of our predictions with experimental data, we can explore the behavior of the IW function near the high end of η reliably. For finite m_B and m_D, Eq. (2) is modified to

$$M^\mu = \sqrt{m_B m_D}[\xi_+(v_1\cdot v_2)(v_1+v_2)^\mu + \xi_-(v_1\cdot v_2)(v_1-v_2)^\mu] , \tag{24}$$

where $\xi_+ \to \xi$ and $\xi_- \to 0$ in the heavy-meson limit. A simple manipulation gives the relations

$$\xi_\pm = \tfrac{1}{2}\left(\sqrt{m_B/m_D} f_1 \pm \sqrt{m_D/m_B} f_2\right) . \tag{25}$$

The dependence of ξ_+ and ξ_- on η is shown in Fig. 3, which exhibits a falloff and an increase with η, respectively. The magnitude of ξ_- indeed diminishes as stated above.

Note that in the analysis based on the heavy-quark symmetry only the single form factor ξ is involved, as shown in Eq. (2). HQET and PQCD are basically two different approaches to $B \to D$ decays, and which one is more appropriate depends on the region we are considering. From Fig. 3 it is observed that the form factor ξ_- becomes smaller, and thus only ξ_+ is important in the low-η region, as required by heavy-quark symmetry. However, ξ_- increases with η, indicating that this symmetry breaks down gradually when the D meson moves fast, and then PQCD may serve as an alternative tool. In fact, the heavy-quark symmetry also breaks down, when $1/m$ corrections are included in HQET [11]. These corrections usually increase with η [11], consistent with the observation made here. This is the reason the matrix element $|V_{cb}|$ is extracted from the behavior of the IW function at zero recoil, $\xi(1) = 1$, in HQET [10] and at maximum recoil in PQCD. Therefore, HQET and PQCD are complementary to each other in the study of B meson decays as stated in the Introduction.

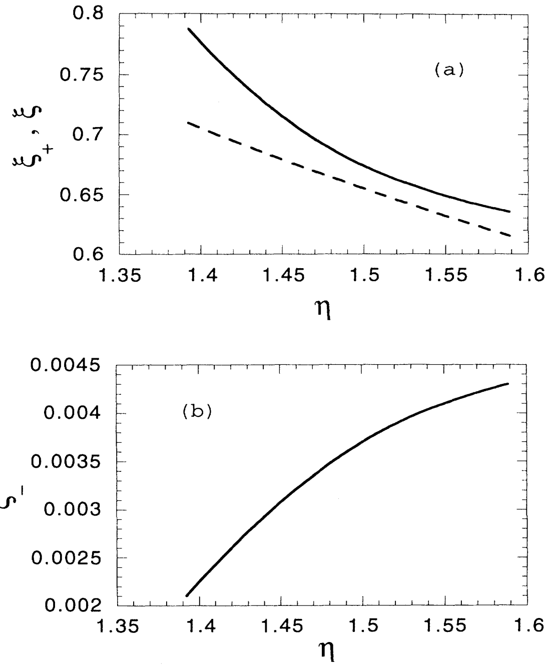

FIG. 3. Dependence of (a) ξ_+ and (b) ξ_- on η derived from our PQCD formalism. The dependence of ξ on η from the model calculation in [11] (dashed line) is also shown.

A model calculation of the IW function in terms of the overlap integrals of the heavy-meson wave functions has been performed [11], which leads to

$$\xi(\eta) = \frac{2}{\eta+1} \exp\left[-(2\rho^2-1)\frac{\eta-1}{\eta+1}\right] \tag{26}$$

with the parameter $\rho \approx 1$. The behavior of ξ is also shown in Fig. 3. It is observed that our predictions for ξ_+ are close to ξ at large η and begin to deviate from ξ as $\eta < 1.39$. The match confirms the applicability of PQCD to heavy meson decays in the large recoil region.

Finally, the differential decay rate for the specific mode $\bar{B}^0 \to D^+ l^- \bar{\nu}$ with vanishing lepton masses is given by

$$\frac{d\Gamma}{d\eta} \equiv |V_{cb}|^2 R(\eta) = |V_{cb}|^2 \frac{1}{48\pi^3} G_F^2 m_B m_D^4 (\eta^2-1)^{3/2} |f_1+f_2|^2 . \tag{27}$$

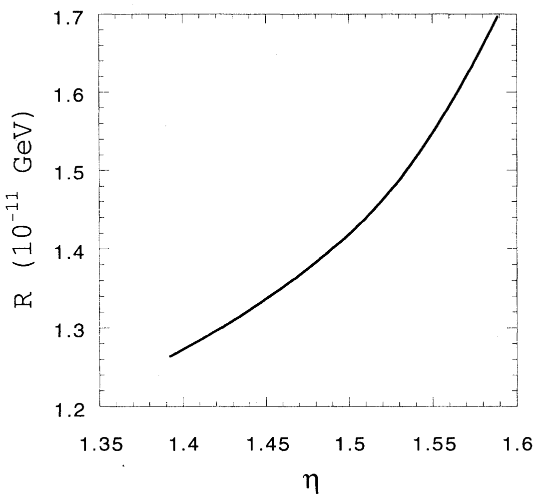

FIG. 4. Dependence of $R(\eta)$ on η derived from the PQCD analysis.

Substituting the results of f_i into Eq. (27), we derive the behavior of $R(\eta)$ for $\eta \geq 1.39$ as in Fig. 4, which shows an increase with η. Once experimental data for the spectrum of the decay $\bar{B}^0 \to D^+ l^- \bar{\nu}$ are available, the matrix element $|V_{cb}|$ can also be extracted from the curve in Fig. 4.

IV. CONCLUSIONS

In this paper we have applied the PQCD formalism to the semileptonic decay $B \to Dl\nu$ and found that the perturbative calculation is reliable for the velocity transfer above 1.4. The point is to include the resummation of large radiative corrections in the process, which improves the applicability of PQCD. The intrinsic transverse momentum dependence also plays an essential role in the calculation. We emphasize that our analysis does not involve any phenomenological parameter and is insensitive to the profile change of the wave functions. The perturbative calculation is shown to be self-consistent by considering the magnitude of the running coupling constant, which defines the region where perturbation theory is reliable.

Our predictions are satisfying in the sense that they match the model estimation of the IW function at the high end of the velocity transfer, and the values 0.044 and 0.043 for the matrix element $|V_{cb}|$ are extracted from the decays $B \to D\pi$ and $B \to DD_s$, respectively. The agreement of perturbative predictions with experimental data justifies the approximation of regarding the D meson as a light meson, which has been employed in this work. We then confirm the perturbative analysis of the decay $B \to \pi l\nu$ in [2], which is important for the extraction of $|V_{ub}|$. On the other hand, it is also worthwhile to apply the same formalism to $B \to D^*, D^{**}$ decays [24], in which spin effects introduce more form factors. To explore the relations among these form factors will provide further justification for the PQCD analysis presented here.

ACKNOWLEDGMENTS

I thank T. Huang and J. Bolz for pointing out the errors in the coefficients of the Sudakov exponent, and F. T. Cheng, G. Sterman, and H. L. Yu for helpful discussions. This work was supported by the National Science Council of R.O.C. under Grant No. NSC84-2112-M194-006.

APPENDIX

In this appendix we show the derivation of the exponent $s(x, b, Q)$ in Eq. (6). We start with Eq. (5.42) in Ref. [6]:

$$s(x,b,Q) = \int_{1/b}^{xQ} \frac{d\mu}{\mu} \left[\ln\left(\frac{xQ}{\mu}\right) A(g(\mu)) + B(g(\mu)) \right], \tag{A1}$$

in which the factors $A(g)$ and $B(g)$ are expanded as

$$\begin{aligned} A(g) &= A^{(1)} \frac{\alpha_s}{\pi} + A^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 \\ B(g) &= \frac{2}{3} \frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma - 1}}{2}\right), \end{aligned} \tag{A2}$$

in order to take into account the next-to-leading logarithms. The running coupling constant α_s is written as

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_1 \ln(\mu^2/\Lambda^2)} - \frac{\beta_2}{\beta_1^3} \frac{\ln\ln(\mu^2/\Lambda^2)}{\ln^2(\mu^2/\Lambda^2)}. \tag{A3}$$

The above coefficients β_i and $A^{(i)}$ are

$$\begin{aligned} &\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \\ &A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln\left(\frac{e^{\gamma}}{2}\right), \end{aligned} \tag{A4}$$

where $n_f = 4$ is the number of quark flavors and γ is the Euler constant. Performing the integration in Eq. (A1), we obtain s, which is given in terms of the variables

$$\hat{q} \equiv \ln(xQ/\Lambda), \quad \hat{b} \equiv \ln(1/b\Lambda) \tag{A5}$$

by [7]

$$
\begin{aligned}
s = & \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b}) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) \\
& + \frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q}) - \ln^2(2\hat{b})] \\
& + \frac{A^{(1)}\beta_2}{8\beta_1^3}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] \\
& - \frac{A^{(1)}\beta_2}{16\beta_1^4}\left[\frac{2\ln(2\hat{q})+3}{\hat{q}} - \frac{2\ln(2\hat{b})+3}{\hat{b}}\right] - \frac{A^{(1)}\beta_2}{16\beta_1^4}\frac{\hat{q}-\hat{b}}{\hat{b}^2}[2\ln(2\hat{b})+1] \\
& + \frac{A^{(2)}\beta_2^2}{1728\beta_1^6}\left[\frac{18\ln^2(2\hat{q})+30\ln(2\hat{q})+19}{\hat{q}^2} - \frac{18\ln^2(2\hat{b})+30\ln(2\hat{b})+19}{\hat{b}^2}\right] \\
& + \frac{A^{(2)}\beta_2^2}{432\beta_1^6}\frac{\hat{q}-\hat{b}}{\hat{b}^3}[9\ln^2(2\hat{b})+6\ln(2\hat{b})+2]\,. \qquad \text{(A6)}
\end{aligned}
$$

The previous studies involving the Sudakov logarithms pick up only the first six terms in Eq. (A6), which are more important than the remaining ones in the large-Q region. Note that the coefficients of the fifth and sixth terms are different from those in Refs. [6,7]. It can be easily checked that with these corrections the results for the pion form factor in [7] are reduced only by a few percent. An explicit examination on the form factors f_i in $B \to D$ decays shows that the partial expression, including only the first six terms, gives predictions smaller than those from the full expression by less than 5%. Hence, for simplicity, this partial expression is substituted into (11) and (12). Note that s is defined for $\hat{q} \geq \hat{b}$ and is set to zero for $\hat{q} < \hat{b}$. As a similar treatment, the complete Sudakov factor $\exp(-S)$ is set to unity, if $\exp(-S) > 1$, during the numerical analysis.

[1] R. Akhoury, G. Sterman, and Y.-P. Yao, Phys. Rev. D **50**, 358 (1994).
[2] H.-n. Li and H. L. Yu, Report No. CCUTH-94-04 (unpublished); Phys. Rev. Lett. **74**, 4388 (1995).
[3] H.-n. Li, Phys. Lett. B **348**, 597 (1995).
[4] H. Georgi, Phys. Lett. B **240**, 447 (1990).
[5] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); **34**, 103 (1987).
[6] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
[7] H.-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); H.-n. Li, Phys. Rev. D **48**, 4243 (1993).
[8] A. Szczepaniak, E. M. Henley, and S. J. Brodsky, Phys. Lett. B **243**, 287 (1990); S. J. Brodsky, in *Particle Physics—The Factory Era*, Proceedings of the Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell *et al.* (World Scientific, Singapore, 1991), p. 1; in *QCD—20 Years Later*, Proceedings of the Workshop, Aachen, Germany, 1992, edited by P. M. Zerwas and H. A. Kastrup (World Scientific, Singapore, 1993), p. 520.
[9] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[10] M. Neubert, Phys. Lett. B **264**, 455 (1991).
[11] M. Neubert and V. Rieckert, Nucl. Phys. **B382**, 97 (1992).
[12] J. G. Körner and P. Kroll, Phys. Lett. B **293**, 201 (1992).
[13] C. E. Carlson and J. Milana, Phys. Lett. B **301**, 237 (1993).
[14] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[15] K. Berkelman and S. L. Stone, Annu. Rev. Nucl. Part. Sci. **41**, 1 (1991).
[16] J. D. Bjorken, I. Dunietz, and J. Taron, Nucl. Phys. **B371**, 111 (1992); T. D. Cohen and J. Milana, Phys. Rev. D **50**, 50 (1994).
[17] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
[18] R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993); J. Bolz, R. Jakob, P. Kroll, M. Bergmann, and N. G. Stefanis, Z. Phys. C **66**, 267 (1995).
[19] C. Bernard, J. Labrenz, and A. Soni, Phys. Rev. D **49**, 2536 (1994); C. Bernard, talk presented at the CTEQ summer school on QCD analysis and phenomenology, Missouri, 1994 (unpublished).
[20] F. Schlumpf, Report No. SLAC-PUB-6335 (unpublished).
[21] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[22] A. N. Kamal and T. N. Pham, Phys. Rev. D **50**, 395 (1994).
[23] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[24] H.-n. Li, Report No. CCUTH-95-05 (unpublished).