PHYSICAL REVIEW D VOLUME 52, NUMBER 6 15 SEPTEMBER 1995

Can a particle interacting with a scalar field reach the speed of light?

Dan N. Vollick*
Department of Physics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, Wisconsin 53201
(Received 23 June 1994)

The motion of a particle interacting with a scalar field is examined. It is shown that the effective mass of the particle is a linear function of the scalar field and that the particle reaches the speed of light when its effective mass goes to zero if scalar field radiation is neglected. The equation of motion for the particle including radiation reaction has the same form as the Lorentz-Dirac equation. The radiation emitted diverges as the particle approaches the speed of light and prevents the particle from becoming luminal. The energy-momentum tensor for the particle and field is calculated and it is shown that there exists an interaction energy-momentum tensor which allows for violations of the weak energy condition.

PACS number(s): 03.30.+p, 03.20.+i, 03.50.−z

I. INTRODUCTION

It is well known that it is impossible to accelerate a particle of constant nonzero rest mass to the speed of light as it would require an infinite amount of energy [1]. If, on the other hand, the particle has a variable rest mass, it is of interest to see if the particle can reach the speed of light when its mass vanishes.

To examine this possibility I consider a collection of particles interacting with a scalar field. The equations of motion for the particles and field are derived from an action principle. The effective mass of a particle is given by $m(1+\alpha\phi)$ where m is the rest mass in the absence of the scalar field ϕ and α is a coupling constant. The particle equations of motion are solved in Minkowski space when $\phi=\phi(t)$ and when $\phi=\phi(\vec{x})$. In both cases the particle reaches the speed of light when its effective mass vanishes. These equations, however, do not include the backreaction produced by the scalar field radiation. The rate of scalar field four-momentum radiated by the particle is calculated and is shown to have the same form as the Larmor formula for the emission of electromagnetic radiation. The equations of motion for the particle will therefore have the same form as the Lorentz-Dirac equation. The radiation emitted by the particle diverges as the particle approaches the speed of light and prevents the particle from reaching the speed of light.

The energy-momentum tensor for the particles and field is calculated and it is shown that there exists an interaction energy-momentum tensor which depends on both the particles and the field. It is this interaction energy-momentum tensor which keeps the total energy of the system finite as the particle approaches the speed of light (neglecting radiation so the speed of light can be reached). The interaction energy-momentum tensor also allows for violations of the weak energy condition even if the particle and field energy-momentum tensors satisfy the weak energy condition.

Throughout this paper ϕ and its derivatives will be assumed to be finite, the metric will be taken to have the signature $(-,+,+,+)$, and the value of c will be set to 1.

II. FIELD AND PARTICLE EQUATIONS OF MOTION

Consider a collection of timelike particles interacting with a scalar field ϕ. The action will be taken to be

$$\begin{aligned}S=&-\sum_n m_n\int\sqrt{-g_{\mu\nu}U_n^\mu U_n^\nu}\,d\tau_n\\&+\tfrac{1}{2}\sum_n\int\lambda_n(\tau_n)[g_{\mu\nu}U_n^\mu U_n^\nu+1]d\tau_n\\&-\alpha\sum_n m_n\int\phi(x_n(\tau_n))d\tau_n-\tfrac{1}{2}\int\nabla^\mu\phi\nabla_\mu\phi\sqrt{g}\,d^4x\ ,\end{aligned}\tag{1}$$

where $x_n^\mu(\tau_n)$ and U_n^μ are the position and the velocity of the nth particle, τ_n is the proper time along its world line, m_n is its rest mass, $\lambda_n(\tau_n)$ are Lagrange multipliers, and α is a constant (see [2–4]) for a discussion of the dynamics of relativistic particles). The field equations are found by varying the action with respect to $\phi(x)$ and are given by

$$\Box\phi=\frac{\alpha}{\sqrt{g}}\sum_n m_n\int\delta^4(x^\mu-x_n^\mu(\tau_n))d\tau_n\ ,\tag{2}$$

where $\Box=\nabla^\mu\nabla_\mu$. The equations of motion for the particles are found by varying the action with respect to $x_n^\mu(\tau_n)$ and are given by

$$(m_n+\lambda_n)\left(\frac{dU_n^\mu}{d\tau_n}+\Gamma^\mu_{\alpha\beta}U_n^\alpha U_n^\beta\right)+\frac{d\lambda_n}{d\tau_n}U_n^\mu=-m_n\alpha\nabla^\mu\phi\ .\tag{3}$$

Contracting with U_μ^n gives

$$\frac{d\lambda_n}{d\tau_n}=m_n\alpha\frac{d\phi}{d\tau_n}\ .\tag{4}$$

*Present address: Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada, V8W 3P6.

Thus

$$\lambda_n = m_n \alpha\phi \ . \tag{5}$$

The equations of motion for timelike particles are then

$$\frac{d}{d\tau_n}[(1+\alpha\phi)U_n^\mu] + (1+\alpha\phi)\Gamma^\mu_{\alpha\beta}U_n^\alpha U_n^\beta = -\alpha\nabla^\mu\phi \ . \tag{6}$$

The "effective mass" of the nth particle is therefore given by $m_n(1+\alpha\phi)$. This mass has an infinite contribution from the ϕ field produced by the particle. This needs to be renormalized away, as does the usual infinite self-energy. Since luminal particles have zero rest mass it is possible that the particle reaches the speed of light when its effective mass vanishes. This possibility will be examined in the next section.

For null particles the action is

$$S = \tfrac{1}{2}\sum_n \int g_{\mu\nu}V_n^\mu V_n^\nu d\sigma_n + \tfrac{1}{2}\sum_n \int \lambda_n(\sigma_n) g_{\mu\nu}V_n^\mu V_n^\nu d\sigma_n$$
$$-\tilde{\alpha}\sum_n \int \phi(x(\sigma_n))d\sigma_n - \tfrac{1}{2}\int \nabla^\mu\phi\nabla_\mu\phi\sqrt{g}d^4x \ , \tag{7}$$

where σ_n are affine parameters and $V_n^\mu = \frac{dx_n^\mu}{d\sigma_n}$. The particle equations of motion are

$$(1+\lambda_n)\left(\frac{dV_n^\mu}{d\sigma_n} + \Gamma^\mu_{\alpha\beta}V^\alpha V^\beta\right) + \frac{d\lambda_n}{d\sigma_n}V^\mu = -\tilde{\alpha}\nabla^\mu\phi \ . \tag{8}$$

Contracting the V_μ^n gives

$$\tilde{\alpha}\frac{d\phi}{d\sigma_n} = 0 \ . \tag{9}$$

Thus $\tilde{\alpha} = 0$. Therefore null particles are completely decoupled from the ϕ field; they neither feel its effect nor act as its source. This behavior can be seen from the relativistic limit of (2) and (6). For simplicity consider (2) and (6) on a flat background space-time. Using

$$\int \delta^4(x^\mu - x_n^\mu(\tau_n))d\tau_n = \frac{1}{\gamma_n}\delta^3(\vec{x} - \vec{x}_n(t)) \tag{10}$$

gives

$$\Box\phi = \frac{\alpha}{\sqrt{g}}\sum_n \frac{m_n}{\gamma_n}\delta^3(\vec{x} - \vec{x}_n(t)) \ . \tag{11}$$

In the relativistic limit the right-hand side goes to zero [for more details see Eq. (23)]. The spacelike components of (6) can be written as (dropping the subscript n)

$$(1+\alpha\phi)\vec{v}\cdot\vec{a} = -\alpha\left(\gamma^{-4}\vec{v}\cdot\vec{\nabla}\phi + \gamma^{-2}v^2\frac{\partial\phi}{\partial t}\right) \tag{12}$$

and

$$(1+\alpha\phi)\hat{n}\cdot\vec{a} = -\alpha\gamma^{-2}\hat{n}\cdot\vec{\nabla}\phi \ , \tag{13}$$

where $\vec{v}$ and $\vec{a}$ are the three-velocity and the three-acceleration, respectively, and $\hat{n}$ is a unit vector normal to the velocity. Thus for ϕ not close to $-\frac{1}{\alpha}$ both $\vec{v}\cdot\vec{a}$ and $\hat{n}\cdot\hat{a}$ go to zero in the relativistic limit (the limit $\phi \to -1/\alpha$ will be discussed below).

III. MOTION IN A FLAT BACKGROUND SPACE-TIME

In order to gain some insight into the motion of particles interacting with the ϕ field consider a single particle moving in a given ϕ field in a flat space-time.

First consider the case when $\phi = \phi(t)$. The spatial components of (6) are

$$\frac{d}{d\tau}[(1+\alpha\phi)\vec{U}] = 0 \ , \tag{14}$$

where $\vec{U} = \gamma\vec{v}$. The solution (14) is

$$\vec{U} = \frac{\vec{U}_0}{1+\alpha\phi} \ , \tag{15}$$

where $\vec{U}_0$ is a constant vector. The magnitude of the three-velocity is

$$v = \frac{1}{\sqrt{1 + U_0^{-2}(1+\alpha\phi)^2}} \ . \tag{16}$$

Thus if the initial velocity is nonzero (i.e., $\vec{U}_0 \neq 0$) then $|\vec{U}| \to \infty$ as $\phi \to -\frac{1}{\alpha}$ and

$$\lim_{\phi\to-\frac{1}{\alpha}} v = 1 \ . \tag{17}$$

The three-acceleration of the particle is

$$\vec{a} = -\left(\frac{\alpha v^2}{U_0^2}\right)\frac{d\phi}{dt}(1+\alpha\phi)\vec{v} \tag{18}$$

so that $\vec{a} \to 0$ as the particle velocity approaches the speed of light. When the particle reaches the speed of light it decouples from the ϕ field and remains luminal.

Now consider the case when $\phi = \phi(\vec{x})$. the timelike component of (6) is

$$\frac{d}{d\tau}[(1+\alpha\phi)\gamma] = 0 \ . \tag{19}$$

Thus

$$v = \sqrt{1 - (1+\alpha\phi)/A^2} \ , \tag{20}$$

where A is an integration constant. Once again $v \to 1$ as $\phi \to -1/\alpha$. Note that this happens even if the initial velocity is zero.

The above calculations are idealized in the sense that radiation reaction has not been included. As the particle accelerates it radiates ϕ field radiation, gravitational radiation, and electromagnetic radiation if it is charged. One expects that radiation reaction will prevent the particle from reaching the speed of light. To calculate the ϕ field radiation produced by a particle I will follow Rohrlich's [5] derivation of the electromagnetic radiation produced by a point particle.

The field equation

$$\Box\phi = \alpha m \int \delta^4(x^\mu(\tau))d\tau \tag{21}$$

has the retarded solution

$$\phi = -\frac{\alpha m}{4\pi\rho} , \tag{22}$$

where ρ is the invariant $\rho = -U_\mu R^\mu$ and $R^\mu = x^\mu - x^\mu(\tau)$. Recall that $R^\mu R_\mu = 0$ at the retarded (and advanced) time. In noncovariant form (22) is

$$\phi = -\frac{\alpha m}{4\pi\gamma R(1-\vec{\beta}\cdot\hat{R})} . \tag{23}$$

Thus $\phi \to 0$ as $v \to 1$, as expected (except along the line $\vec{\beta}\cdot\hat{R} = 1$). Now define a spacelike u^μ such that $U^\mu u_\mu = 0$, $u^\mu u_\mu = 1$, and $R^\mu = \rho(u^\mu + U^\mu)$. The partial derivatives of ρ at the retarded time are given by

$$\partial^\mu \rho = u^\mu + \tilde{a} R^\mu , \tag{24}$$

where $\tilde{a} = a^\mu u_\mu$ and $a^\mu = dU^\mu/d\tau$. The energy-momentum tensor of the field is given by (see Sec. IV)

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}\eta^{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi . \tag{25}$$

Substituting (23) and (24) into (25) gives

$$T^{\mu\nu} = \frac{\alpha^2 m^2}{16\pi^2\rho^4}[u^\mu u^\nu + \tilde{a}(u^\mu R^\nu + u^\nu R^\mu) + \tilde{a}^2 R^\mu R^\nu - \tfrac{1}{2}\eta^{\mu\nu}(1+2\tilde{a}\rho)] . \tag{26}$$

The energy-momentum radiated by the particle in an interval $d\tau$ is given by (see Fig. 1)

$$dP^\mu_{\text{rad}} = \lim_{\rho\to\infty} \int_{d\sigma_1} T^{\mu\nu} d\sigma_\nu . \tag{27}$$

It can be shown [5] that (27) is surface independent and is therefore a four-vector. The only term which contributes to the radiation is the $R^\mu R^\nu$ term in (26). To calculate dP^μ_{rad} it is convenient to choose the surface $d\sigma_2$ shown in Fig. 2. The surface element is

$$d\sigma^\mu = u^\mu \rho^2 d\Omega c\, d\tau , \tag{28}$$

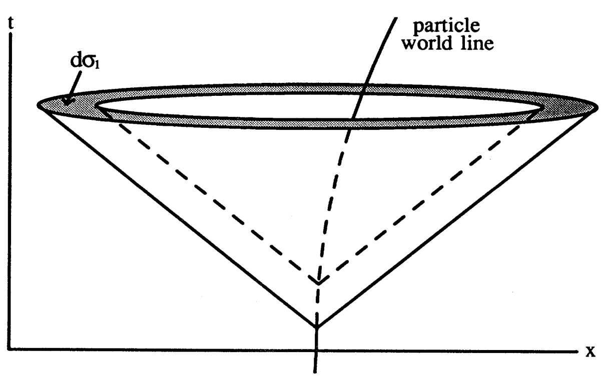

FIG. 1. The surface $d\sigma_1$ is the region of the plane t =const contained within the light cones produced at two infinitesimally close points on the particle world line.

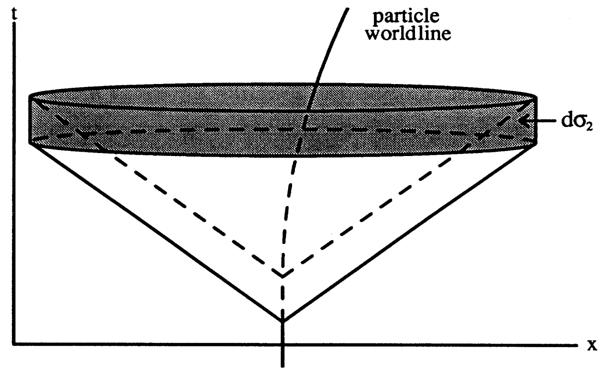

FIG. 2. The surface $d\sigma_2$ on which dP^μ_{rad} is evaluated.

where $d\Omega$ is an infinitesimal solid angle (this can be seen by going to the instantaneous rest frame of the particle). In the limit $\rho \to \infty$,

$$\frac{dP^\mu_{\text{rad}}}{d\tau} = \frac{\alpha^2 m^2}{16\pi^2} \int \tilde{a}^2 (u^\mu + U^\mu) d\Omega . \tag{29}$$

In the instantaneous rest frame of the particle,

$$\frac{dP^\mu_{\text{rad}}}{d\tau} = \frac{\alpha^2 m^2}{16\pi^2} \int (\vec{a}\cdot\hat{R})^2 (1;\hat{R}) d\Omega . \tag{30}$$

Integrating this gives

$$\frac{dP^\mu_{\text{rad}}}{d\tau} = \frac{\alpha^2 m^2}{12\pi} a^2 (1;\vec{0}) . \tag{31}$$

Since this is a four-vector the rate of radiation of four-momentum in an arbitrary inertial frame is

$$\frac{dP^\mu_{\text{rad}}}{d\tau} = \frac{\alpha^2 m^2}{12\pi} a^\lambda a_\lambda U^\mu . \tag{32}$$

This has the same form as the expression for electromagnetic radiation. In electrodynamics the constant $\alpha^2 m^2/12\pi$ is replaced by $2e^2/3$. The equation of motion of the particle will therefore have the same form as the Lorentz-Dirac equation [5,6] with $2e^2/3$ replaced by $\alpha^2 m^2/12\pi$. Thus

$$\frac{d}{d\tau}[(1+\alpha\phi)U^\mu] = -\alpha\partial^\mu\phi + \tau^*(\dot{a}^\mu - a_\lambda a^\lambda U^\mu), \tag{33}$$

where $\tau^* = \alpha^2 m/12\pi$, $\dot{a}^\mu = da^\mu/d\tau$, and $\Gamma^\mu \equiv \tau^*(\dot{a}^\mu - a^\lambda a_\lambda U^\mu)$ is the correction to the equations of motion. To eliminate possible runaway solutions the asymptotic condition

$$\lim_{\tau\pm\infty} a^\mu(\tau) = 0 \tag{34}$$

must be imposed (see Rohrlich [5] for a discussion of the properties of the Lorentz-Dirac equation).

It is interesting to note that the bound four-momentum of the particle (i.e., the bare momentum plus the bound field momentum) is given by

$$P^\mu_{\text{bound}} = m(1+\alpha\phi)U^\mu - m\tau^* a^\mu . \tag{35}$$

Thus, as in electrodynamics [7], the bound four-momentum is acceleration dependent. This means that the four-momentum radiated by the particle does not, in general, equal the change in $m(1+\alpha\phi)U^\mu$. However, if we impose the asymptotic conditions (34) the acceleration dependent term in (35) vanishes asymptotically and the total energy radiated equals the change in $m(1+\alpha\phi)U^\mu$.

Consider once again, the case where $\phi = \phi(t)$. To see when radiation reaction will be important $dP^\mu_{\rm rad}/d\tau$ and Γ^k can be estimated by using (14) and (15). The results are

$$\frac{dP^\mu_{\rm rad}}{d\tau} \approx \frac{\alpha^2 m\tau_* v^2 \dot{\phi}^2}{(1+\alpha\phi)^2} U^\mu \tag{36}$$

and

$$\Gamma^k \approx -\frac{\alpha\tau^* U_0^k}{(1+\alpha\phi)^2}\left(\ddot{\phi} - \frac{\alpha(2-v^2)\dot{\phi}^2}{(1+\alpha\phi)}\right) , \tag{37}$$

where $\dot{\phi} = d\phi/d\tau = \gamma d\phi/dt$. Therefore $dP^\mu_{\rm rad}/d\tau$ and Γ^k diverge as $\phi \to 1/\alpha$. Thus the radiation reaction will become important as the particle approaches the speed of light ($\dot{\phi}$ or $\ddot{\phi}$ nonzero).

To analyze the motion, consider one-dimensional motion and let $\gamma v = \sinh\xi$. Equation (33) becomes

$$(1+\alpha\phi)\dot{\xi} + \alpha\left(\dot{\phi}\tanh\xi + \frac{\partial\phi}{\partial x}\text{sech}\xi\right) = \tau^*\ddot{\xi} . \tag{38}$$

Consider motion with $\phi = \phi(t)$ near $\phi = -1/\alpha$ and with $\xi \gg 1$. The equation of motion then becomes

$$(1+\alpha\phi)\dot{\xi} + \alpha\dot{\phi} = \tau\ddot{\xi} . \tag{39}$$

If radiation reaction is neglected (i.e., $\tau^* = 0$) the solution is

$$\xi = \xi_0 - \ln(1+\alpha\phi) , \tag{40}$$

which diverges, as expected, when $\phi \to -1/\alpha$. For $\tau^* \neq 0$ Eq. (38) can be written as

$$\xi(\tau) = \alpha/\tau^* \int_0^\tau e^{\psi(\tau')} \int_0^{\tau'} e^{-\psi(\tau'')}\chi(\tau'')d\tau'' d\tau' + C_1 \int_0^\tau e^{\psi(\tau')}d\tau' + C_2 , \tag{41}$$

where C_1 and C_2 are constants,

$$\chi(\tau) = \dot{\phi}\tanh\xi + \frac{\partial\phi}{\partial x}\text{sech}\xi , \tag{42}$$

and

$$\psi(\tau) = \frac{1}{\tau^*}\int_0^\tau (1+\alpha\phi)d\tau'. \tag{43}$$

When $\phi = 0$ the term involving C_1 is a runaway term and is eliminated by the asymptotic conditions. When $\phi \neq 0$ the behavior as $\tau \to \pm\infty$ depends on ϕ. Since $\psi(\tau)$ and $\chi(\tau)d\tau$ are finite [note that $\dot{\phi}(\tau)d\tau = (d\phi/dt)dt$] for $|\tau| < \infty$ so is $\xi(\tau)$ and the particle does not reach the speed of light. It will be shown in Sec. IV that the energy of the particle (in the asymptotic regions) can be taken to be

$$E = m(1+\alpha\phi)\gamma . \tag{44}$$

Thus the energy of the particle will become negative when $1+\alpha\phi < 0$.

As an example consider $\xi \gg 1$ and the scalar field to be given by

$$1+\alpha\phi = -\tanh\left(\frac{\tau}{2\tau^*}\right) . \tag{45}$$

Note that in the absence of radiation the particle's acceleration vanishes as $\tau \to \pm\infty$. The solution to (39) is

$$\xi(s) = -\ln[(1+s)^{(1+s)}(1-s)^{(1-s)}] + C_1 s + C_2 , \tag{46}$$

where $s = -(1+\alpha\phi) = \tanh(\tau/2\tau^*)$ and C_1 and C_2 are constants. Thus $\xi(s)$ is finite for all $-1 \leq s \leq 1$. $\xi(s)$ has a local maximum at $s_m = \tanh(C_1/2)$, where it has the value

$$\xi(s_m) = -2\ln[1+\tanh(C_1/2)] + C_1 + C_2 . \tag{47}$$

It is also easy to show that $d^2\xi/ds^2 < 0$ for all $-1 < s < 1$ so that $\xi(s)$ is concave downwards. Thus for the condition $\xi \gg 1$ be satisfied it is necessary and sufficient that $\xi(\pm 1) \gg 1$. The asymptotic conditions are automatically satisfied, as can be seen from

$$\dot{\xi}(s) = -\frac{1}{2\tau^*}(1-s^2)\left[\ln\left(\frac{1+s}{1-s}\right) - C_1\right] , \tag{48}$$

where $\dot{\xi}(s) = d\xi/ds$. The change in energy $\Delta E = E(s=1) - E(s=-1) = 2m(2\ln 2 - C_2)$ is finite.

IV. THE INTERACTION ENERGY-MOMENTUM TENSOR

The energy-momentum tensor of the field and particles is given by

$$T^{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}}. \tag{49}$$

From (1) we have

$$T^{\mu\nu} = \sum_n \frac{m_n}{\sqrt{g}}\int (1+\alpha\phi)U_n^\mu U_n^\nu \delta^4(x - x_n(\tau_n))d\tau_n + \nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi . \tag{50}$$

There is therefore an interaction energy-momentum tensor given by

$$T^{\mu\nu}_{(I)} = \alpha\sum_n \frac{m_n}{\sqrt{g}}\int \phi(x_n)U_n^\mu U_n^\nu \delta^4(x - x_n(\tau_n))d\tau_n . \tag{51}$$

This energy-momentum tensor is necessary if

$$\nabla_\mu T^{\mu\nu} = 0 \tag{52}$$

is to give the correct equations of motion for the particles.

It is the interaction energy-momentum tensor which keeps the total energy of the system finite as the particle velocity approaches the speed of light (neglecting radiation so that the speed of light can be reached). The particle and interaction energy-momentum tensor can be written as

$$T^{\mu\nu}_{PI} = \sum_n (1+\alpha\phi)\gamma_n \frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\delta^3(\vec{x}-\vec{x}_n(t)). \quad (53)$$

From (14) and (19) it can be seen that $\gamma_n(1+\alpha\phi)$ is finite as the particle approaches the speed of light. Thus the total energy-momentum of the system remains finite as the particle approaches the speed of light. The quantity $m\gamma(1+\alpha\phi)$ can be called the energy of the particle.

The interaction energy-momentum tensor allows for violations of the weak energy condition even if the particle and field energy momentum tensors satisfy the weak energy condition individually. For example, if $\alpha\phi < -1$ and is constant then $T^{00} < 0$.

From (2) and (50) it can be seen that it is the trace of the particle energy-momentum tensor which acts as the source of the scalar field. In the continuum limit Eqs. (2) and (50) become

$$\Box\phi = -\alpha T_p \quad (54)$$

and

$$T^{\mu\nu} = (1+\alpha\phi)[(\rho+P)U^\mu U^\nu + Pg^{\mu\nu}] + [\nabla^\mu\phi\nabla^\nu\phi - \tfrac{1}{2}g^{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi] , \quad (55)$$

where T_p is the trace of the particle energy-momentum tensor, ρ is the rest mass density and P is the pressure.

V. CONCLUSION

The equations of motion for a collection of particles interacting with a scalar field were derived from an action principle. It was shown that the effective mass of a particle is given by $(1+\alpha\phi)m$ and that the velocity of the particle approaches the speed of light as the effective mass goes to zero if radiation reaction is neglected. Once the particle reaches the speed of light it decouples from the field and remains luminal.

The ϕ field four-momentum radiated by the particle was calculated and shown to have same form as the four-momentum radiated by an accelerating charged particle. The equation of motion of the particle including radiation reaction will therefore have the same form as the Lorentz-Dirac equation. This equation was then examined and it was found that radiation reaction prevents the particle from reaching the speed of light.

The energy-momentum tensor for the field and particles contains an interaction term which depends on both the field and particles. It is this interaction term which keeps the total energy and momentum of the system finite as the particle approaches the speed of light (neglecting radiation reaction so that the speed of light can be reached). The interaction energy-momentum tensor also allows for violations of the weak energy condition even if particle and field energy-momentum tensors satisfy the weak energy condition.

ACKNOWLEDGMENT

This work was supported in part by N.S.F. Grant No. PHY-9105935.

[1] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, New York, 1971).
[2] A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (MacMillan, New York, 1965).
[3] G. Kalman, Phys. Rev. **123**, 384 (1961).
[4] P. H. Lim, J. Math. Phys. **23**, 1641 (1982).
[5] F. Rohrlich, *Classical Charged Particles, Foundations of Their Theory* (Addison-Wesley, New York, 1965).
[6] P. A. M. Dirac, Proc. R. Soc. (London) **A167**, 148 (1938).
[7] C. Teitelboim, Phys. Rev. D **1**, 1572 (1970).