Stringy evidence for D=11 structure in a strongly coupled type-IIA superstring

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Witten proposed that the low energy physics of a strongly coupled D=10 type-IIA superstring may be described by D=11 supergravity. To explore the stringy aspects of the underlying theory we examine the stringy massive states. We propose a systematic formula for identifying nonperturbative states in D=10 type-IIA superstring theory, such that, together with the elementary excited string states, they form D=11 supersymmetric multiplets, in SO(10) representations. This provides hints for the construction of a conjectured weakly coupled D=11 theory that is dual to the strongly coupled D=10 type-IIA superstring.

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I. INTRODUCTION

Recently several proposals have been made about duality connections between various string theories in several dimensions. These seem to provide a handle on the behavior of strongly interacting string theories in various dimensions. In this paper we will concentrate on the type-IIA D=10 superstring in the strong coupling limit. In a recent paper Witten [1] made the remarkable suggestion that 11-dimensional supergravity, including all the states that come from compactifying the 11th dimension, represents the *low energy sector* of the *strongly coupled* type-IIA D=10 superstring. He suggested that nonperturbative black hole or monopole-type states that are alleged to arise in the strongly coupled D=10 string theory are precisely the massive Kaluza-Klein states of D=11 supergravity. Also, as was known for a long time, the original massless string states are the massless D=11 supergravity states. He based his reasoning on U duality and hints provided in related previous work [2].

I first give a short summary of Witten's idea in a form that will suggest generalizations. The argument relies on extending the D=10, N=2 super Poincaré algebra with a central extension. There is mounting evidence that the D=10 type-IIA superalgebra develops a central extension Z nonperturbatively,

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = p^\mu\left(C\gamma_\mu\right)_{\alpha\beta} + \varepsilon^{ij}C_{\alpha\beta}\,Z, \qquad (1.1)$$

and that there are states that carry nontrivial values of Z. Witten argued that the values of Z should be given by $Z = cW/\lambda$ where c is a pure constant, W is quantized in terms of a unit, $W = nW_0$, and λ is the coupling constant of the interacting string. It is useful for our arguments later to regard the central extension Z as the 11th component of momentum for an 11-dimensional superalgebra:

$$\{Q_A, Q_B\} = P\,(C\Gamma)_{AB}\ , \qquad (1.2)$$

where $A, B = 1, 2, \ldots, 32$, and P is a momentum in 11 dimensions $P = (p^\mu, Z)$. Then (1.1) and (1.2) are the same superalgebra. It is useful to rewrite the algebra in the rest frame $P \to P' = (M_{11}, \vec{0}, 0)$ where $M_{11}^2 = M^2 - Z^2$. If M_{11} vanishes, it is not possible to go to the rest frame. It is known that when the mass M_{11} vanishes the supersymmetry representations are "short" and contain $\mathbf{2^7}$ bosons plus $\mathbf{2^7}$ fermions. These states have $M = |Z| = c\,|W|/\lambda$ and they are called the "BPS (Bogomolnyi, Prasad, Sommerfield) saturated states" in [1]. When $M_{11} \neq 0$ the supersymmetry representations are "long," and contain the seeds of an SO(10) little group, as will be discussed below.

For zero charge $W = 0$ the "BPS saturated states" are precisely the massless states of the string theory, and their quantum numbers are determined by the $\mathbf{2}^7_B + \mathbf{2}^7_F$ short multiplet. Since the little group for massless states in D=11 is SO(9), it is natural to expect that the massless string states should be classified by SO(9). So, even though in D=10 the little group for massless states is only SO(8), this argument demands that the SO(8) states have just the property to fit into SO(9) multiplets. Indeed, the massless states which come from the Ramond-Ramond vacuum sector $|\text{vac}\rangle_L|\text{vac}\rangle_R$, are classified under SO(8) as $(8_v + 8_-)_L \times (8_v + 8_+)_R$, where $8_v, 8_\pm$ are the vector and spinor representations of SO(8). This product yields the following representations for the massless bosons and fermions:

$$\text{SO}(8): \quad (1 + 8 + 28 + 35 + 56)_B + [8_+ + 8_- + 56_+ + 56_-]_F\ , \qquad (1.3)$$

and, as is well known, they do form the SO(9) multiplets

$$\text{SO}(9): \quad \mathbf{2}^7_B = (44 + 84)_B,\ \ \mathbf{2}^7_F = \mathbf{128}_F\ , \qquad (1.4)$$
$$\phi_{(IJ)} = 44,\ \ \phi_{[IJK]} = 84,\ \ \psi_{\alpha I} = 128$$

which are interpreted as the graviton, antisymmetric tensor, and gravitino of D=11 supergravity.

For nonzero charge $W \neq 0$ the BPS saturated states are massive $M = c\,|W|/\lambda$, but as argued above, they are classified in the same short multiplets. Witten interpreted them as the Kaluza-Klein excitations of the graviton, antisymmetric tensor, and the gravitino, coming from compactifying the 11th dimension of D=11 supergravity. Furthermore he suggested that their interactions at low energies are given precisely by the full theory of D=11 supergravity.

These arguments suggest that there may be some richer, stringy, D=11-like theory that is dual to D=10 type-IIA superstring, whose spectrum and interactions may be studied in either the language of the D=11 theory (if one could guess it) or directly in the original D=10 theory. If this is true we should be able to see more evidence of D=11 by studying the type-IIA theory. In particular it should be possible to see that the stringy spectrum of the D=10 string theory exhibits some D=11 property. Indeed we have known for quite some time that the first stringy massive level of the type-IIA theory does exhibit a D=11 dimensional structure [3]. Namely the states fill the long multiplet $\mathbf{2}_B^{15}+\mathbf{2}_F^{15}$ of D=11 supersymmetry, and they are classified in SO(10) representations, i.e., with the little group of massive states (the rotation group) in 1+10 dimensions. This was discovered in the process of analyzing D=11 supermembrane theory, but the same arguments apply directly to type-IIA string as is evident in [3]. It will also be repeated below.

In this paper we will make a proposal that extends this observation to include the higher stringy levels, but only after including nonperturbative states. We will give a systematic prescription for the masses and SO(8) or SO(9) quantum numbers of the nonperturbative stringy states, and will carry out an explicit analysis of the D=11 structure successfully up to string level 5. The perturbative states combined with the nonperturbative ones will form complete supermultiplets at each string level. In the weak coupling limit the nonperturbative states become infinitely heavy or decouple, and the spectrum reduces to the usual type-IIA perturbative string states. In the infinite coupling limit there is a Kaluza-Klein tower of degenerate massive states forming D=11 supermultiplets at each stringy level.

II. PERTURBATIVE STRING SPECTRUM

In 10 dimensions (1-time + 9-space) the rotation group is SO(9). A massive state at rest must come in degenerate multiplets of SO(9), where the multiplet represents the spin components. When one actually constructs the states of the type-IIA superstring in the lightcone gauge there is only a manifest SO(8) symmetry (see the Appendix). This is because manifest Lorentz invariance is broken by the choice of gauge. However, since the theory is actually Lorentz invariant one finds that the SO(8) representations can be reassembled into SO(9) representations (see, e.g., [4]).

Furthermore, there is separate supersymmetry for the left movers and right movers (type IIA). For left movers, or right movers, there are 16 supercharges. For a massive state at rest $p^\mu = (M,\vec{0})$ the supercharges form a 16-dimensional Clifford algebra for left movers $\left\{S^L_\alpha, S^L_\beta\right\} = M\delta_{\alpha\beta}$, and a similar one for right movers. The automorphism group of this algebra is SO(16). It is useful to embed SO(9) in this SO(16) by mapping the vector of SO(16) into the 16-dimensional spinor of SO(9). The 16 supercharges may be rearranged into 8 fermionic creation operators and 8 annihilation operators. At the most 8 powers of the creation operators can be applied on a given state. The repeated action of the supercharges can be organized into the two spinor representations $\mathbf{2}_B^7+\mathbf{2}_F^7$ of SO(16), where the subscripts B, F imply that they are bosonic (even powers) or fermionic (odd powers) operators respectively. The SO(9) content is obtained by decomposing these SO(16) representations into the SO(9) representations

$$\mathbf{2}_B^7 = \mathbf{44}+\mathbf{84}, \quad \mathbf{2}_F^7 = \mathbf{128}\ . \tag{2.1}$$

Therefore, for either the left movers or right movers, the type-IIA superstring states must be arranged into massive supermultiplets of the form

$$r \times \{(44+84)_B + 128_F\}\ , \tag{2.2}$$

where r is a representation of SO(9) that may be considered as the lowest state in the supermultiplet (which may be bosonic or fermionic) and the factor $\{(44+84)_B + 128_F\}$ represents the action of the supercharges.

The closed string state is obtained by taking the direct product of left and right movers at the same excitation level l, with $L_0 = -M^2 + l$, $\tilde{L}_0 = -M^2 + \tilde{l}$ and $L_0 = \tilde{L}_0 = 0$. The combined left- and right-moving states at level l take the form

$$\left(\sum_i r_i^{(l)}\right)_L \times \left(\sum_i r_i^{(l)}\right)_R \times \begin{bmatrix} \{(44+84)_B + 128_F\}_L \\ \times\{(44+84)_B + 128_F\}_R \end{bmatrix}\ , \tag{2.3}$$

where an identical collection of representations $\left(\sum_i r_i^{(l)}\right)$ occur for left or right moving states at each level l. These represent the lowest states in the supermultiplet.

Thus, to chracterize the states of the type-IIA superstring it is sufficient to give the collection of states $\sum_i r_i^{(l)}$ that occur at each level l. Up to level 5 these are computed in the Appendix, and are given in the following table in terms of SO(9) representations:

$$\begin{array}{ll} \underline{\text{Level}} & \underline{\text{SO(9) representations } \sum_i r_i^{(l)}} \\ l=1: & \mathbf{1}_B \\ l=2: & \mathbf{9}_B \\ l=3: & \mathbf{44}_B + \mathbf{16}_F \\ l=4: & \begin{cases} (9+36+156)_B \\ +\mathbf{128}_F \end{cases} \\ l=5: & \begin{cases} (1+36+44+84+231+450)_B \\ +[16+128+576]_F\,. \end{cases} \end{array} \tag{2.4}$$

Levels $l = 0,1,2$ were previously given [4] while the results for levels $l = 4,5$ are new. The SO(9) irreducible tensor structure of these representations are given in (1.4) and below

$$\phi_{(IJKL)} = 450, \quad \phi_{(IJ,K)} = 231, \quad \phi_{(IJK)} = 156, \quad \phi_{[IJ]} = 36, \quad \psi_{\alpha(IJ)} = 576. \tag{2.5}$$

Indices in square brackets $\phi_{[IJ]}$ are antisymmetrized, in round parantheses $\phi_{(IJK)}$, etc., are completely symmetrized and traces projected out, $\phi_{(IJ,K)}$ corresponds to a Young tableaux with (2,1) boxes and traces projected out, while $\psi_{\alpha(IJ)}$ is a mixed spinor tensor with a projection involving a γ matrix $[16 \times 45 - 16 \times 9 = 576]$.

III. D=11 SUPERMULTIPLET STRUCTURE

A. Perturbative states

Type-IIA superstring in D=10 has two supercharges of opposite chirality. Each supercharge has 16 components and all states can be classified as SO(9) supermultiplets as seen above. However, there is a higher supermultiplet structure. To understand this, first notice that, at rest, the two supercharges combined form a 32-dimensional Clifford algebra, which may be divided into 16 creation operators and 16 annihilation operators. The isomorphism group of this algebra is SO(32). The 16 creation operators form the 16-dimensional representation of the rotation group SO(9). However, it is useful to regard this SO(9) as being embedded in SO(32) as

$$\mathrm{SO}(32) \supset \mathrm{SU}(16) \supset \mathrm{SO}(10) \supset \mathrm{SO}(9) , \tag{3.1}$$

where the embedding is done by classifying the 16 creation (annihilation) operators in the $\mathbf{16}$ ($\overline{\mathbf{16}}$) of SU(16), which is the $\mathbf{16}$ ($\overline{\mathbf{16}}$) of SO(10) and the $\mathbf{16}$ ($\mathbf{16}$) of SO(9). This embedding exhibits an intermediate SO(10) which will play an essential role below. This is the embedding that we used some time ago [3].

To take advantage of this higher structure, let us reorganize the perturbative levels as follows. A massive supermultiplet in the type-IIA superstring is obtained by starting from any representation of SO(9) (which may represent either bosons or fermions) and applying the 16 fermionic creation supercharges on it repeatedly. At the most 16 powers can be applied. The SO(9) content of the nth power of the generator is obtained by antisymmetrizing the $\mathbf{16}$ of SO(9) n-times, i.e., $[16^n]$. The SO(9) content of this antisymmetrization is better understood by embedding it in SO(32), since the same procedure forms the two spinor representations of SO(32). Thus, the even powers form the spinor $\mathbf{2}_B^{15}$ and the odd powers form $\mathbf{2}_F^{15}$ where the subscripts B, F stand for bosons or fermions. Each spinor may be decomposed into representations of SU(16). The $\mathbf{2}_B^{15}$ contains the completely antisymmetric SU(16) tensors with $0,2,4,6,8,10,12,14,16$ indices, and likewise the $\mathbf{2}_F^{15}$ contains the completely antisymmetric SU(16) tensors with $1,3,5,7,9,11,13,15$ indices. Therefore, under SU(16) we have the representations

$$\mathbf{2}_B^{15} = \begin{cases} \mathbf{1} + \mathbf{120} + \mathbf{1820} + \mathbf{8008} + \mathbf{12870} \\ +\overline{\mathbf{1}} + \overline{\mathbf{120}} + \overline{\mathbf{1820}} + \overline{\mathbf{8008}} , \end{cases}$$
$$\mathbf{2}_F^{15} = \begin{cases} \mathbf{16} + \mathbf{560} + \mathbf{4368} + \mathbf{11440} \\ +\overline{\mathbf{16}} + \overline{\mathbf{560}} + \overline{\mathbf{4368}} + \overline{\mathbf{11440}} . \end{cases} \tag{3.2}$$

These are further decomposed under SO(10) and then under SO(9) [3]. Thus, any massive supermultiplet must have the structure of the long D=11 supermultiplet

$$R \times \left\{ \mathbf{2}_B^{15} + \mathbf{2}_F^{15} \right\} , \tag{3.3}$$

where R is some collection of SO(9) representations [rather than SO(10) at this stage] representing either a boson or fermion, and the structure $\left\{ \mathbf{2}_B^{15} + \mathbf{2}_F^{15} \right\}$ comes from applying the supercharges on it.

By comparing to the perturbative spectrum in Eq. (2.3) we can determine that

$$\left\{ \mathbf{2}_B^{15} + \mathbf{2}_F^{15} \right\} = \begin{cases} \{(44+84)_B + 128_F\}_L \\ \times\{(44+84)_B + 128_F\}_R , \end{cases}$$
$$\text{perturbative}: \quad R = R_P^{(l)} \equiv \left(\textstyle\sum_i r_i^{(l)}\right)_L \times \left(\textstyle\sum_i r_i^{(l)}\right)_R . \tag{3.4}$$

The point is that the action of the supercharges represented by $\left\{ \mathbf{2}_B^{15} + \mathbf{2}_F^{15} \right\}$ has a higher symmetry structure. In particular we focus on its SO(10) subgroup since it is the rotation group in 10 spacelike dimensions. Indeed, from the D=11 the superalgebra point of view the SO(10) has the correct interpretation to be associated with one additional spacelike dimension.

The question is whether the factor R also has the SO(10) symmetry? The answer is affirmative [3] for the first excited level $l = 1$ since, according to (2.4) we have only SO(9) singlets

$$l = 1: \quad R_P^{(1)} = 1_B \times 1_B = 1,$$

which are also SO(10) singlets. Thus, in addition to the arguments given by Witten about the existence of hidden D=11 structure at the massless level, we have a first hint that D=11 evidence may show at the massive stringy levels as well. The argument above relies only on the supersymmetry structure and is completely general as far as the first level is concerned.

At levels $l > 1$ we need to analyze R_P in more detail. For example at level 2 we have, from (3.4) and (2.4),

$$l = 2: \quad R_P^{(2)} = 9_B \times 9_B , \tag{3.5}$$

which clearly cannot be complete SO(10) multiplets. So, there is no way that the perturbative spectrum has D=11 structure by itself.

B. Nonperturbative states

We now postulate that the D=10 type-IIA superstring has additional nonperturbative states that emerge just like monopoles or black holes in nonlinear theories. According to the supersymmetry algebra their mass should satisfy $M \geq \frac{c}{\lambda} |W|$. In the weak coupling limit of the type-IIA superstring these states are presumably infinitely heavy, or infinitely weakly coupled (i.e., nonperturbative $e^{-c/\lambda}$) , and therefore do not appear in the perturbative theory. In the strong coupling limit, if there is

D=11 structure, the extra nonperturbative states should become degenerate with the perturbative states just in such a way as to give complete SO(10) supermultiplets. In this case, since the D=11 momentum of the state has the form $P = (p^\mu, cW/\lambda)$, we conjecture a stringy mass relation of the type

$$P^2 = M_{11}^2 = M^2 - c^2W^2/\lambda^2 = n, \tag{3.6}$$

where the stringy $n = 0, 1, 2, \ldots$ is a positive integer.

The $n = 0$ case is the sector that saturates the BPS bound, and therefore the states form the short multiplets of D=11 supersymmetry with $\mathbf{2}_B^7 + \mathbf{2}_F^7 = (44+84)_B + \mathbf{128}_F$, just like the massless states, since for all of them the D=11 mass vanishes, $P^2 = M_{11}^2 = 0$. The elementary massless states of the string have $W = 0$, but the nonperturbative states have $W \neq 0$. The $n = 0$ case is the sector discussed by Witten, and interpreted as Kaluza-Klein modes of D=11 supergravity, with charges $W = kW_0$, and $M = \frac{c}{\lambda}|W|$.

In our case we investigate the conjectured stringy nonperturbative states, with $n \geq 1$. The masses will be $M = \sqrt{n + c^2W^2/\lambda^2}$. Just like the $n = 0$ case, it is natural to expect one state with $W = 0$, and an infinite number of charged states with $W = kW_0$. At infinite coupling $c^2W^2/\lambda^2 \to 0$, all the new nonperturbative states of level n have the same mass $M^2 = n$, which is similar to the spectrum of the elementary massive string states. At weak coupling the $W \neq 0$ states become infinitely heavy and disappear from the spectrum. But how about the $W = 0$, $M^2 = n$ nonperturbative states which remain at finite mass at weak coupling. Assuming that the $W = 0$ states do not exist at all is one option, but it seems unnatural, since $W = 0$ is interpreted as just a zero value for the 11th component of the momentum. Instead I will assume that its coupling to the elementary string states is nonperturbative, e.g., $g \sim \exp(-c/\lambda)$ rather than a power of λ. This would explain why it would not be seen in weakly coupled perturbation expansion of the type-IIA superstring.

Our aim here is to answer the question "what is the massive stringy spectrum that exhibits D=11 structure and SO(10) symmetry at strong coupling?" The implication is that the extra nonperturbative states together with the perturbative states form complete SO(10) supermultiplets. However, can we provide a concrete formula for identifying all such states at all massive levels? Here we make a proposal for all levels, and show that it works at least up to level 5.

As a first hint consider how the $l = 2$ states in (3.5) can be completed to SO(10) multiplets. We need to add singlets for both left movers and right movers at the same mass. By examining (2.4) we see that the only place where singlets appear are at the previous level ($l = 1$). This suggests the presence of new nonperturbative states at level l (with $l = 2$ in the present example), whose quantum numbers are those of the perturbative states of the previous levels. Indeed this hint leads to the following systematic formula.

In the infinitely strong coupling limit (or for the $W = 0$ state) we suggest that the mass formula for left and right movers that incorporates all perturbative and nonperturbative states follows from putting the non-perturbative string on shell as follows:

$$L_0 = -M^2 + l' + l = 0, \qquad \tilde{L}_0 = -M^2 + \tilde{l}' + \tilde{l} = 0, \tag{3.7}$$

where l' is a positive integer and l represents the level of excitation with the elementary superstring oscillators as defined before. Then integer l determines the SO(9) content of the state as given in (2.4). The factors $(-M^2 + l')$, $(-M^2 + \tilde{l}')$ play the role of D=11 squared masses $(-M_{11}^2)$ for left and right movers, respectively. Thus, at a fixed 10-dimensional (10D) mass level $M^2 = n = l' + l$ the integers $(l', l), (\tilde{l}', \tilde{l})$ take the values

$$(l', l) = (0, n) + (1, n-1) + \cdots + (n-1, 1), \qquad (\tilde{l}', \tilde{l}) = (0, n) + (1, n-1) + \cdots + (n-1, 1). \tag{3.8}$$

The states with the highest level ($l' = 0, l = n$) are isomorphic to the perturbative level $l = n$ states that were discussed in the previous section. The others are the nonperturbative states with $l' \neq 0$. The SO(9) quantum number of the state (l', l) is identical to those listed in Eq. (2.4), as determined only by l, but the new states listed in (3.8) are distinguished from those in (2.4) by their quantum number l', as well as the W charge, if any.

Note that we have not included the state ($l' = n, l = 0$) even though the mass formula $M^2 = l' + l = n$ allows it. The reason is that the D=11 left or right masses vanish when $l = \tilde{l} = 0$, i.e., $M_{11}^2 = (M^2 - l') = (M^2 - \tilde{l}') = 0$ and then the supermultiplet is the short one (the BPS saturated states) $\mathbf{2}_B^7 + \mathbf{2}_F^7 = (44+84)_B + \mathbf{128}_F$. These states are stringy partners of the Kaluza-Klein excitations of the D=11 supergravity multiplet. They may not exist at all, or they may be interpreted as point-like states (from the D=10 point of view) that would fit into the type of discussion given by Witten. The stringy states must have $l \neq 0$ and must form the long multiplets $\{\mathbf{2}_B^{15} + \mathbf{2}_F^{15}\} \times R$ as described in (3.3) and (3.4). Therefore, since ($l' = n, l = 0$) are not such states they are excluded from the stringy list in (3.8).

In this section we simply want to use this mass formula irrespective of its origin and show that it works. However, in addition to the mass formula we need a scheme for the *multiplicity* of each state listed in (3.8). We will explore two schemes for the multiplicities. These will differ from each other by how many times it is possible to obtain the same value of l'. Each scheme works at least up to level 5, but each one provides a different view on the origin of l' and the dynamics of the weakly coupled dual theory. The simplest scheme is to take a *single copy* of each state listed in (3.8). We now show how the SO(9) representations reassemble to give SO(10) representations at each mass level $M^2 = n \geq 1$.

At mass level $M^2 = 1$ there are only the states (0,1) that correspond to the $\mathbf{2}_B^{15} + \mathbf{2}_F^{15}$ perturbative states already discussed above. They form the D=11 supermultiplet

$$M^2 = 1 : 1 \times \left\{\mathbf{2}_B^{15} + \mathbf{2}_F^{15}\right\}. \tag{3.9}$$

This is SO(10) covariant.

At mass level $M^2 = 2$ we have, for left movers,

$$\{(0,2)+(1,1)\}_L : \quad R = \{9_B + 1_B\}_L \tag{3.10}$$

and the same set of representations for the right movers. These SO(9) representations are read off directly from Eq. (2.4) through their l values. Therefore, the full set of perturbative and nonperturbative states form the D=11 supermultiplet

$$M^2 = 2 : (10 \times 10)\left\{\mathbf{2}_B^{15} + \mathbf{2}_F^{15}\right\}. \tag{3.11}$$

At mass level $M^2 = 3$ we read off from Eq. (2.4) the SO(9) content of $(0,3)+(1,2)+(2,1)$. For either left or right movers this is $R = (44+9+1)_B + 16_F$. These reassembe into SO(10) representations so that the D=11 supermultiplet is

$$M^2 = 3 : \{54_B + 16_F\} \times \left\{54_B + \overline{16}_F\right\} \times \left\{\mathbf{2}_B^{15} + \mathbf{2}_F^{15}\right\}, \tag{3.12}$$

where $\phi_{(XY)} = 54$ is the symmetric traceless tensor for SO(10) and $\mathbf{16}, \overline{\mathbf{16}}$ are the two spinor representations.

At mass level $M^2 = 4$ the SO(9) content of $(0,4)+(1,3)+(2,2)+(3,1)$ is obtained from (2.4). For left movers the SO(9) representations reassemble into SO(10) as

$$\mathrm{SO}(9) = \left\{ \begin{array}{l} (1 + 2\times 9 + 36 + 156 + 44)_B \\ +\,(16+128)_F\ , \end{array} \right. \tag{3.13}$$

$$\mathrm{SO}(10) = (45+210)_B + \mathbf{144}_F. \tag{3.14}$$

Therefore the full set of perturbative and nonperturbative states included in the D=11 supermultiplet is

$$M^2 = 4 : \left\{ \begin{array}{c} \{(45+210)_B + 144_F\} \\ \times\left\{(45+210)_B + \overline{144}_F\right\} \\ \times\left\{2_B^{15} + 2_F^{15}\right\}\ . \end{array} \right. \tag{3.15}$$

Here the SO(10) tensors are

$$\begin{array}{c} \phi_{(XYZ)} = 210, \quad \phi_{[XY]} = 45, \\ \\ \psi_{\alpha X} = 144, \quad \psi_{\bar\alpha X} = \overline{144}, \end{array} \tag{3.16}$$

and their SO(10)$\to$SO(9) decomposition is

$$\begin{array}{l} \mathbf{210} \to \mathbf{156} + \mathbf{44} + \mathbf{9} + \mathbf{1}, \\ \mathbf{45} \to \mathbf{36} + \mathbf{9}, \\ \mathbf{144} \to \mathbf{128} + \mathbf{16}. \end{array} \tag{3.17}$$

Finally, at mass level $M^2 = 5$ the SO(9) content of $(0,5)+(1,4)+(2,3)+(3,2)+(4,1)$ for left movers, as obtained from (2.4), reassemble into SO(10) representations as follows;

$$\mathrm{SO}(9) = \left\{ \begin{array}{l} \left(\begin{array}{c} 2\times(1+9+36+44) \\ +84+156+231+450 \end{array} \right)_B \\ +\,[2\times(16+128)+576]_F\,, \end{array} \right. \tag{3.18}$$

$$\mathrm{SO}(10) = \left\{ \begin{array}{c} (1+120+320+660)_B \\ +\,[144+720]_F\,. \end{array} \right. \tag{3.19}$$

Therefore the full set of perturbative and nonperturbative states forming the D=11 supermultiplet is

$$M^2 = 5 : \left\{ \begin{array}{c} \left\{ \begin{array}{c} (1+120+320+660)_B \\ +\,[144+720]_F \end{array} \right\} \\ \times \left\{ \begin{array}{c} (1+120+320+660)_B \\ +\,\left[\overline{144}+\overline{720}\right]_F \end{array} \right\} \\ \times\left\{2_B^{15} + 2_F^{15}\right\}. \end{array} \right. \tag{3.20}$$

The SO(10) tensors are

$$\begin{array}{l} \phi_{(UXYZ)} = 660, \quad \phi_{(XY,Z)} = 320, \quad \phi_{[XYZ]} = 120, \\ \psi_{\alpha(XY)} = 720, \quad \psi_{\alpha X} = 144, \\ \psi_{\bar\alpha(XY)} = \overline{720}, \quad \psi_{\bar\alpha X} = \overline{144}. \end{array} \tag{3.21}$$

Their SO(10)$\to$SO(9) decomposition is

$$\begin{array}{l} \mathbf{660} \to \mathbf{450} + \mathbf{156} + \mathbf{54} + \mathbf{9} + \mathbf{1}, \\ \mathbf{320} \to \mathbf{231} + \mathbf{44} + \mathbf{36} + \mathbf{9}, \\ \mathbf{120} \to \mathbf{84} + \mathbf{36}, \\ \mathbf{720} \to \mathbf{576} + \mathbf{128} + \mathbf{16}, \\ \mathbf{144} \to \mathbf{128} + \mathbf{16}. \end{array} \tag{3.22}$$

C. Higher multiplicity scheme

The simplest scheme for the multiplicity of the states that was used above may be effectively reformulated as follows. Introduce a new left-moving oscillator β_{-1} that has a single mode at level 1. A similar one $\tilde\beta_{-1}$ is introduced also for right movers. Then we may identify the number operators $l' = \beta_{-1}\beta_1$ and $\tilde l' = \beta_{-1}\beta_1$ while the states $(l',l)\,,(\tilde l',\tilde l)$ can be built as

$$\begin{array}{c} (l',l) \longleftrightarrow (\beta_{-1})^{l'}\,|l\rangle_L, \\ \\ (\tilde l',\tilde l) \longleftrightarrow \left(\tilde\beta_{-1}\right)^{\tilde l'}\,|\tilde l\rangle_R, \end{array} \tag{3.23}$$

where $|l\rangle_L, |\tilde l\rangle_R$ are the original string oscillator states given in (2.4). This construction should be regarded as a mnemonic to keep track of the states, and while it is not excluded, it need not represent necessarily an additional oscillator in the dual theory. This minimal approach clearly gives a single copy of each state $(l',l)\,,(\tilde l',\tilde l)$.

It is tempting to explore the idea of extra oscillators, leading to higher multiplicities. The most attractive case would be to boldly introduce all modes for an extra dimension. This would account naturally for all stringy Kaluza-Klein partners in the conjectured D=11 dual theory. Let the modes be β_n with the usual Heisenberg algebra $[\beta_n,\beta_m] = n\delta_{n+m}$. Similarly introduce also right movers $\tilde\beta_n$. The zero mode is the Kaluza-Klein charge that appears as the 11th momentum in the superalgebra $\beta_0 = \tilde\beta_0 \sim W$. We may now construct many more states with the same value of $l', \tilde l'$:

$$l' = \sum_n \beta_{-n}\beta_n, \quad \tilde l' = \sum_n \tilde\beta_{-n}\tilde\beta_n\ . \tag{3.24}$$

At level l' the states (l',l) are

$$\beta_{-l'}|l\rangle_L \oplus \beta_{-l'+1}\beta_{-1}|l\rangle_L \oplus \cdots \oplus (\beta_{-1})^{l'}\,|l\rangle_L\ . \quad (3.25)$$

The multiplicity is equal to the number of partitions $p(l')$ of the integer l'. Thus, in this scheme we would have the following states up to mass level $M^2 = 5$

$M^2 = n$	States $\sum p(l') \times (l', l)$
1	$(0,1)$
2	$(0,2) + (1,1)$
3	$(0,3) + (1,2) + 2(2,1)$
4	$(0,4) + (1,3) + 2(2,2) + 3(3,1)$
5	$(0,5) + (1,4) + 2(2,3) + 3(3,2) + 5(4,1)$.

(3.26)

The remarkable fact is that these states also can be reorganized into complete SO(10) supermultiplets. This is done as follows.

There is nothing new at mass levels $M^2 = 1, 2$. At mass level $M^2 = 3$, first use one factor of each state to obtain the result $\{54_B + 16_F\}$ as before. The remaining $(2,1)$ is just a singlet of SO(10); that is, it has the same SO(10) content as mass level $n = 1$. Therefore, the full collection of SO(10) states is $\{(54 + 1)_B + 16_F\}$ for left movers, and the same one for the right movers. These multiply the overall factor $\mathbf{2}_B^{15} + \mathbf{2}_F^{15}$ as before.

Similarly, at mass level $M^2 = 4$, after reproducing the previous collection of SO(10) states in (3.14) by using one factor of each state, there remains $(2,2) + 2(3,1)$. This may be regarded as two sets $(2,2) + (3,1)$ and $(3,1)$ each having multiplicities one, and furthermore having the same SO(9) or SO(10) content as the collection of all the states in mass levels $n = 1, 2$ as listed in (3.26). Using the known result for those levels, we see that the extra states correspond to the SO(10) multiplets $(10 + 1)_B$. Therefore the full collection of left-moving SO(10) states is $(1 + 10 + 45 + 210)_B + [16 + 144]_F$. Similarly, for right movers.

Finally at mass level $M^2 = 5$, the same procedure indicates that in addition to the states in (3.19) there are those coming from the levels $(2,3) + 2(3,2) + 4(4,1)$. Again, these may be regarded as several sets, each containing single multiplicities:

$$\begin{aligned} &(2,3) + (3,2) + (4,1)\ , \\ &(3,2) + (4,1)\ , \\ &(4,1)\ , \\ &(4,1)\ . \end{aligned} \quad (3.27)$$

These have the same SO(9) or SO(10) quantum numbers computed for the single multiplicity scheme at the lower mass levels $n = 1, 2, 3$. Therefore the total SO(10) multiplets are

$$(5 \times 1 + 2 \times 10 + 54 + 120 + 320 + 660)_B + [16 + 144 + 720]_F\ . \quad (3.28)$$

The pattern is clear. If we have already established that the single multiplicity scheme gives SO(10) multiplets, then the expanded scheme also gives it since the additional states at mass level $M^2 = n$ have the same SO(10) quantum numbers as the states in the mass levels $M^2 = (n-2), (n-3), \ldots, 1$. We have already shown this up to level 5, and an iterative proof can be given for all mass levels.

Recall that in addition to these stringy Kaluza-Klein partners, the dual theory presumably has also an infinite tower of pointlike Kaluza-Klein states for all possible values of the 11th momentum W (all of which become degenerate at infinite coupling).

IV. COMMENTS

I have made a proposal for identifying the conjectured nonperturbative stringy states that uncover a hidden $D{=}11$ structure. This involves stringy structures that are not included in Witten's discussion, but which must be there if his proposal is more than an accident at low energies. In order to tighten the arguments one should look for a possible role of discrete symmetries, similar to, or beyond the conjectured SL(2,Z) symmetry and the associated U duality [2]. There may be a symmetry that commutes with the SO(8) or even SO(9), but not with SO(10). Combining such a symmetry with SO(10) may generate a much more restrictive symmetry of the string theory that could be sufficient to elucidate the conjectured 11-dimensional structure and the strong coupling behavior of the theory. Evidence for such a symmetry would begin with finding repetitions of the same SO(8) or SO(9) representations at the same mass levels, such that they would form multiplets of the extra symmetry. We indeed find such repetitions at various stages of our analysis as is evident in the Appendix and the text. However, more is needed to understand if this is due to a symmetry.

It may be that restoring the light-cone oscillators would make it easier to investigate the presence of symmetries in a covariant quantization. In this connection I suspect that a new construction of an SL(2,R) current algebra that uses the light-cone oscillators [5] would be useful. Note that the arbitrary $c = 0$ stress tensor T' used in this construction may be taken as the stress tensor of the type-IIA theory by including ghost fields. Extra effective dimensions may arise through such a mechanism.

One approach for searching for the nonperturbative states is to investigate a string field theory type formulation. In this connection recent proposals for the field theoretic formulation of the superstring by Berkovits and Vafa seem promising [6].

The extra oscillators $\beta_n, \tilde{\beta}_n$ are intriguing. Is it possible to include them directly in the discussion of the $D{=}10$ type-IIA superstring, and not only in the dual theory? In this connection, perhaps it is useful to recall that the covariant quantization of the Green-Schwarz string has never been fully understood. Could there be a possibility of a Liouville-like mode that decouples perturbatively, but which is present nonperturbatively? If so, it would account naturally for the extra dimension.

The dual $D{=}11$ theory that we are seeking, especially when discussed in terms of the oscillators $\beta_n, \tilde{\beta}_n$, is beginning to look like a membrane theory, as it shares some similar features to the D=11 supermembrane theory that we studied some time ago, although not quite the same. The D=11 theory that is dual to type-IIA superstring may be a new membranelike theory. In any case, in view of the approach we have pursued here, it may be useful to revise also the previous work [3,7,8], using the new hints as a guide for the construction of a consistent "supermembrane theory" in 11 dimensions.

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APPENDIX

In this appendix we construct the SO(9) content of the elementary string states for D=10 type-IIA superstring, up to level 5. The results were previously known for levels 0,1,2,3 [4], but they are new for levels 4 and 5.

1. Notation

We work directly in the light-cone gauge. Therefore, we start with manifest SO(8) symmetry. Vector indices $i = 1, 2, \ldots, 8$ denote the vector 8_v of SO(8), while the spinor indices $a, \dot{a} = 1, 2, \ldots, 8$, denote the spinor representations $8_+, 8_-$, respectively. The string oscillators are classified as $\alpha^i_{-n} = 8_v$, $S^a_{-n} = 8_+$ for left movers and $\tilde{\alpha}^i_{-n} = 8_v$, $\tilde{S}^{\dot{a}}_{-n} = 8_-$ for right movers. The zero modes $S^a_0 = 8_+$, $\tilde{S}^{\dot{a}}_0 = 8_-$ give the Ramond vacuum $|8_v + 8_-\rangle_L$ for left movers and $|8_v + 8_+\rangle_R$ for right movers.

SO(9) vector indices are denoted by $I = 1, 2, \ldots, 9$ and spinor indices by $\alpha = 1, 2, \ldots, 16$. SO(10) vector indices are denoted $X = 1, 2, \ldots, 10$ and spinor indices by $\alpha, \bar{\alpha} = 1, 2, \ldots, 16$ for the spinor representations $16, \overline{16}$ of SO(10), respectively. The group or subgroup decomposition of these representations is

$$\begin{array}{lll} \mathrm{SO}(10) & \mathrm{SO}(9) & \mathrm{SO}(8) \\ \phi_X = 10 & \left\{ \begin{array}{l} \phi_I = 9 \\ \phi = 1 \end{array} \right. & \left\{ \begin{array}{l} (\phi_i = 8_v) + (\phi' = 1) \\ + (\phi = 1) \end{array} \right. \\ \psi_\alpha = 16 & \psi_\alpha = 16 & (\psi_a = 8_+) + (\bar{\psi}_{\dot{a}} = 8_-) \\ \bar{\psi}_{\bar{\alpha}} = \overline{16} & \psi'_\alpha = 16 & (\psi'_a = 8_+) + (\bar{\psi}'_{\dot{a}} = 8_-) \end{array} \tag{A1}$$

Thus, the decomposision is obtained by specializing the index $X \to I \oplus 10 \to i \oplus 9 \oplus 10$ and $\alpha, \bar{\alpha} \to \alpha \to a \oplus \dot{a}$.

For SO(n) all antisymmetric tensors are irreducible multiplets, but tensors with symmetric vector indices or mixed spinor-vector indices are reducible. Irreducible multiplets are obtained provided these tensors give zero when contracted with the Krönecker δ function or with a γ matrix. Thus, for SO(8), SO(9), or SO(10) writing irreducible representations in the form $\phi_{(XY)}$, or $\psi_{\alpha X}$, etc., implies that these tensors are constrained as follows:

$$\phi_{(XY)}\, \delta^{XY} = 0, \quad \left(\gamma^X\right)_\alpha^{\ \beta}\, \psi_{\beta X} = 0. \tag{A2}$$

Combining these rules with the decomposition of indices in the above table, one can figure out the group or subgroup decomposition of higher representations. For example,

$$\begin{array}{lll} \mathrm{SO}(10) & \mathrm{SO}(9) & \mathrm{SO}(8) \\ \phi_{(XY)} & \left\{ \begin{array}{l} \phi_{(IJ)} + \\ \phi_{I\,or\,J} + \phi \end{array} \right. & \left\{ \begin{array}{l} (\phi_{(ij)} + \phi_{i\,or\,j} + \phi') \\ + (\phi_{i\,or\,j} + \phi'') + \phi \end{array} \right. \\ = 54 & = 44 + 9 + 1 & \left\{ \begin{array}{l} = (35_v + 8_v + 1) \\ + (8_v + 1) + 1 \end{array} \right. \\ \psi_{\alpha X} & \psi_{\alpha I} + \psi'_\alpha & \left\{ \begin{array}{l} \psi_{ai} + \psi_{ai} \\ + \psi_{\dot{a}i} + \psi_{\dot{a}i} \\ + (\psi_a + \psi_{\dot{a}}) \\ + (\psi'_a + \psi'_{\dot{a}}) \end{array} \right. \\ = 144 & 128 + 16 & \left\{ \begin{array}{l} = \left(\begin{array}{l} 56_+ + 56_- \\ + 8_+ + 8_- \end{array} \right) \\ \quad + (8_+ + 8_-). \end{array} \right. \end{array} \tag{A3}$$

2. Massless sector

The Ramond-Ramond vacuum ($l = 0$) has the following SO(8) classification of bosons and fermions:

$$\begin{aligned} &|8_v + 8_-\rangle_L \times |8_v + 8_+\rangle_R \\ &\quad = (8_v \times 8_v + 8_- \times 8_+)_B + (8_v \times 8_+ + 8_v \times 8_-)_F \\ &\quad = (1 + 35_v + 28 + 56_v + 8_v)_B \\ &\qquad + (56_+ + 8_- + 56_- + 8_+)_F \end{aligned} \tag{A4}$$

$$\mathrm{SO}(9): \quad = (44 + 84)_B + 128_F \ .$$

So, SO(9) emerges only after combining left with right.

3. Level 1

At massive levels, the left movers and right movers separately must exhibit the SO(9) structure since each sector behaves like the open string. Another reason, based on the representations of supersymmetry, was given in the text. Therefore, we will first reorganize the left-moving SO(8) states into SO(9) long supermultiplets $(\mathbf{2}_B^7 + \mathbf{2}_F^7) \times r$. The right moving states have the identical SO(9) structure. The SO(10) structure will become apparent only when left and right are put together, as done in the text.

At level 1, there are left and right moving oscillators applied on the left and right vacuum. For left movers the SO(9) structure can be seen by writing out the SO(8) content of the oscillators and vacuum

$$\begin{aligned}\text{Left}: \quad & (\alpha^i_{-1} \oplus S^a_{-1})|\text{vac}\rangle_L \\ & = (8_v + 8_+) \times (8_v + 8_-) , \\ \text{SO}(9) & = [(44+84)_B + 128_F]_L .\end{aligned} \tag{A5}$$

4. Level 2

The oscillators applied on the left vacuum are classified under SO(8) as

$$\begin{aligned}\text{Left}: \quad & \{(\alpha^i_{-2} \oplus S^a_{-2}) \oplus [(\alpha^i_{-1} \oplus S^a_{-1})^2]_{\text{SUSY}}\}\,|\text{vac}\rangle_L \\ & = \{(8_v + 8_+) + [(8_v + 8_+)^2]_{\text{SUSY}}\} \times (8_v + 8_-) ,\end{aligned} \tag{A6}$$

where the subscript "SUSY" means symmetrization of identical bosons and antisymmetrization of identical fermions. It can be rewritten as

$$\begin{aligned}[(8_v + 8_+)^2]_{\text{SUSY}} & = \begin{pmatrix} (8_v \times 8_v)_S + (8_+ \times 8_+)_A \\ + (8_v \times 8_+) \end{pmatrix} \\ & = (1 + 35_v + 28)_B + (8_- + 56_+)_F \\ & = 8_v \times (8_v + 8_+) ,\end{aligned} \tag{A7}$$

where the subscripts S, A mean symmetrization and antisymmetrization, respectively. The important last step is the rewriting in terms of an overall factor $(8_v + 8_+)$. This allows rewriting all the SO(8) states for the left movers in (A6) in the form of SO(9) multiplets

$$\begin{aligned}\text{Left}: \quad & (1 + 8_v) \times (8_v + 8_+) \times (8_v + 8_-) \\ & = 9 \times [(44+84)_B + 128_F] .\end{aligned} \tag{A8}$$

5. Level 3

The oscillators that are applied on the vacuum have the SO(8) structure

$$\begin{aligned}\text{Left}: \quad & \begin{bmatrix} (\alpha^i_{-3} \oplus S^a_{-3}) \\ \oplus(\alpha^i_{-2} \oplus S^a_{-2})(\alpha^i_{-1} \oplus S^a_{-1}) \\ \oplus [(\alpha^i_{-1} \oplus S^a_{-1})^3]_{\text{SUSY}} \end{bmatrix} |\text{vac}\rangle_L \\ & = \begin{bmatrix} (8_v + 8_+) + (8_v + 8_+)^2 \\ + [(8_v + 8_+)^3]_{\text{SUSY}} \end{bmatrix} \times (8_v + 8_-) .\end{aligned} \tag{A9}$$

By an analysis similar to (A7) we can rewrite the cubic supersymmetrized product by factoring out $(8_v + 8_+)$ as

$$\left((8_v + 8_+)^3\right)_{\text{SUSY}} = (35_v + 8_-) \times (8_v + 8_+) . \tag{A10}$$

Then the left-moving states may be rewritten as SO(9) multiplets as follows:

$$\begin{aligned}\text{Left}: \quad & \left\{ \begin{matrix} [1 + (8_v + 8_+) + (35_v + 8_-)] \\ \times (8_v + 8_+) \times (8_v + 8_-) \end{matrix} \right\} , \\ \text{SO}(9) & = [44_B + 16_F] \times [(44+84)_B + 128_F] .\end{aligned} \tag{A11}$$

The SO(9) $\to$ SO(8) decomposition is

$$44 \to 1 + 8_v + 35_v . \tag{A12}$$

6. Level 4

The oscillators that are applied on the vacuum have the SO(8) structure

$$\begin{aligned}\text{Left}: \quad & \begin{bmatrix} (\alpha^i_{-4} \oplus S^a_{-4}) \\ \oplus(\alpha^i_{-3} \oplus S^a_{-3})(\alpha^i_{-1} \oplus S^a_{-1}) \\ \oplus [(\alpha^i_{-2} \oplus S^a_{-2})^2]_{\text{SUSY}} \\ \oplus(\alpha^i_{-2} \oplus S^a_{-2})\,[(\alpha^i_{-1} \oplus S^a_{-1})^2]_{\text{SUSY}} \\ \oplus [(\alpha^i_{-1} \oplus S^a_{-1})^4]_{\text{SUSY}} \end{bmatrix} |\text{vac}\rangle_L \\ & = \begin{bmatrix} (8_v + 8_+) + (8_v + 8_+)^2 \\ + [(8_v + 8_+)^2]_{\text{SUSY}} \\ (8_v + 8_+) \times [(8_v + 8_+)^2]_{\text{SUSY}} \\ + [(8_v + 8_+)^4]_{\text{SUSY}} \end{bmatrix} \times (8_v + 8_-) .\end{aligned} \tag{A13}$$

First we work on rewriting the supersymmetrized quartic by pulling out a factor of $(8_v + 8_+)$ as

$$[(8_v + 8_+)^4]_{\text{SUSY}} = [(8_v + 112) + 56_-] \times (8_v + 8_+) , \tag{A14}$$

where $\phi_{(ijk)} = 112$ is the completely symmetric traceless SO(8) tensor in three indices. Combining this result with (A7) and inserting them in (A13) we may pull out the overall $(8_v + 8_+)$ factor to exhibit the SO(9) classification as

$$\begin{aligned}\text{Left}: \quad & \left\{ \begin{matrix} 1 + (8_v + 8_+) + 8_v \\ +8_v \times (8_v + 8_+) \\ + (8_v + 112) + 56_- \end{matrix} \right\} \\ & \times (8_v + 8_+) \times (8_v + 8_-) \\ & = \left\{ \begin{matrix} \begin{pmatrix} 2 \times 1 + 3 \times 8_v \\ +28 + 35_v + 112 \end{pmatrix}_B \\ + (8_+ + 8_- + 56_+ + 56_-)_F \end{matrix} \right\} \\ & \times (8_v + 8_+) \times (8_v + 8_-) , \\ \text{SO}(9) & = \{(9 + 36 + 156)_B + 128_F\} \\ & \times [(44+84)_B + 128_F] ,\end{aligned} \tag{A15}$$

where $\phi_{(IJK)} = 156$ and $\phi_{[IJ]} = 36$ are the three-index traceless symmetric and the two-index antisymmetric SO(9) tensors, respectively. The SO(9) $\rightarrow$ SO(8) decomposition is

$$\begin{aligned} \mathbf{36} &\rightarrow \mathbf{8}_v + \mathbf{28}\ , \\ \mathbf{156} &\rightarrow \mathbf{1} + \mathbf{8}_v + \mathbf{35}_v + \mathbf{112}\ , \\ \mathbf{128} &\rightarrow \mathbf{8}_+ + \mathbf{8}_- + \mathbf{56}_+ + \mathbf{56}_-\ . \end{aligned} \tag{A16}$$

7. Level 5

The oscillators that are applied on the vacuum have the SO(8) structure

$$\text{Left}: \left[\begin{array}{c} (\alpha^i_{-5} \oplus S^a_{-5}) \\ \oplus(\alpha^i_{-4} \oplus S^a_{-4})(\alpha^i_{-1} \oplus S^a_{-1}) \\ \oplus(\alpha^i_{-3} \oplus S^a_{-3})(\alpha^i_{-2} \oplus S^a_{-2}) \\ \oplus(\alpha^i_{-3} \oplus S^a_{-3}) \left[(\alpha^i_{-1} \oplus S^a_{-1})^2\right]_{\text{SUSY}} \\ \oplus(\alpha^i_{-1} \oplus S^a_{-1}) \left[(\alpha^i_{-2} \oplus S^a_{-2})^2\right]_{\text{SUSY}} \\ \oplus(\alpha^i_{-2} \oplus S^a_{-2}) \left[(\alpha^i_{-1} \oplus S^a_{-1})^3\right]_{\text{SUSY}} \\ \oplus \left[(\alpha^i_{-1} \oplus S^a_{-1})^5\right]_{\text{SUSY}} \end{array}\right] |\text{vac}\rangle_L$$

$$= \left[\begin{array}{c} (8_v + 8_+) + 2 \times (8_v + 8_+)^2 \\ +2 \times (8_v + 8_+) \times \left[(8_v + 8_+)^2\right]_{\text{SUSY}} \\ +(8_v + 8_+) \times \left[(8_v + 8_+)^3\right]_{\text{SUSY}} \\ + \left[(8_v + 8_+)^5\right]_{\text{SUSY}} \end{array}\right] \times (8_v + 8_-)\ . \tag{A17}$$

First we work on rewriting the supersymmetrized quintic by pulling out a factor of $(8_v + 8_+)$ as follows:

$$\left[(8_v + 8_+)^5\right]_{\text{SUSY}} = \left[\begin{array}{c} (28 + 35 + 294)_B \\ + (224_- + 8_-)_F \end{array}\right] \times (8_v + 8_+)\ , \tag{A18}$$

where

$$\phi_{(ijkl)} = 294, \quad \psi_{\dot{a}(ij)} = 224_- \tag{A19}$$

are the SO(8) representations. Combining this result with (A7) and (A10) and inserting them in (A17) we may pull out the overall $(8_v + 8_+)$ factor and then exhibit the SO(9) classification as follows:

$$\text{Left}: \left\{\begin{array}{c} 1 + 2 \times (8_v{+}8_+) + 2 \times 8_v{\times}(8_v{+}8_+) \\ +(35_v{+}8_-) \times (8_v{+}8_+) \\ +(28 + 35{+}294){+}\,(224_- + 8_-) \end{array}\right\} \times \{(8_v + 8_+) \times (8_v + 8_-)\}$$

$$= \left\{ \left(\begin{array}{c} 3{\times}1 + 4 \times 8 + 3 \times 28 + 3 \times 35 \\ +56 + 112 + 160 + 294 \end{array}\right)_B + \left[\begin{array}{c} 3 \times 8_+ + 3 \times 8_- + 2 \times 56_+ \\ +2 \times 56_- + 224_+ + 224_- \end{array}\right]_F \right\} \times \{(8_v + 8_+) \times (8_v + 8_-)\}\ . \tag{A20}$$

These form the SO(9) multiplets

$$\text{SO}(9) = \left\{\begin{array}{c} (450 + 231 + 84 + 44 + 36 + 1)_B \\ + [576 + 128 + 16]_F \end{array}\right\} \times \{(44 + 84)_B + 128_F\}\ , \tag{A21}$$

where $\phi_{(IJKL)} = 450$, $\phi_{(IJ,K)} = 231$, $\phi_{[IJK]} = 84$, $\psi_{\alpha(IJ)} = 576$ are the SO(9) irreducible tensors. Their SO(9) $\rightarrow$ SO(8) decomposition is

$$\begin{aligned} \mathbf{84} &\rightarrow \mathbf{28} + \mathbf{56} \\ \mathbf{231} &\rightarrow \mathbf{8} + \mathbf{28} + \mathbf{35} + \mathbf{160}\ , \\ \mathbf{450} &\rightarrow \mathbf{1} + \mathbf{8} + \mathbf{35} + \mathbf{112} + \mathbf{294}\ , \\ \mathbf{576} &\rightarrow \mathbf{8}_+ + \mathbf{8}_- + \mathbf{56}_+ + \mathbf{56}_- + \mathbf{224}_+ + \mathbf{224}_-\ . \end{aligned} \tag{A22}$$

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