# Semi-infinite throat as the end-state geometry of two-dimensional black hole evaporation

Sukanta Bose,* Leonard Parker,† and Yoav Peleg‡
*Department of Physics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, Wisconsin 53201, USA*
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We study a modified two-dimensional dilaton gravity theory which is exactly solvable in the semiclassical approximation including back reaction. Infalling matter in an initially static radiationless spacetime forms a black hole if its energy is above a certain threshold. The black hole singularity is initially hidden behind a timelike apparent horizon. As the black hole evaporates by emitting Hawking radiation, the singularity meets the shrinking horizon in finite retarded time to become naked. A boundary condition exists at the naked singularity which preserves energy conservation, stability, and continuity of the metric and results in a unique end state for all evaporating black holes. The end-state geometry is static and asymptotically flat at its right spatial infinity, while its left spatial infinity is a semi-infinite throat extending into the strong coupling region. This end-state geometry is the ground state in our model.

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Hawking's discovery that black holes radiate thermally [1–3] gave rise to a long-standing question concerning the consequences of combining quantum theory and general relativity [4–8]. Does evolution from an initial pure state take place untarily to a final pure state or nonunitarily to a final mixed state? Intimately linked to this question is the final geometry resulting from black hole evaporation.

Here we present a specific two-dimensional (2D) dilaton gravity model in which a black hole evaporates leaving a static semi-infinite throat as the end-state or "remnant" geometry. Our model is a modification of the Callan-Giddings-Harvey-Strominger (CGHS) model [9]. We solve the semiclassical equations and get closed-form expressions for the metric and dilaton field.

The classical 2D CGHS action [9] is

$$S_{\rm cl} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left(e^{-2\phi}[R^{(2)} + 4(\nabla\phi)^2 + 4\lambda^2] -\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right) , \qquad (1)$$

where $\phi$ is the dilaton field, $R^{(2)}$ is the 2D Ricci scalar, $\lambda$ is a positive constant, $\nabla$ is the covariant derivative, and the $f_i$ are $N$ matter (massless scalar) fields. The action (1) describes a 2D effective theory in the throat region of a 4D almost extreme magnetically charged black hole [10,11]. It may also be regarded as a 2D arena in which some of the main questions about black hole evaporation can be studied. Among the classical solutions stemming from the action (1) are vacuum solutions, static black hole solutions, and dynamical solutions describing the formation of a black hole by collapsing matter fields. For a review see [12].

To study one-loop quantum corrections and back reaction one can use the trace anomaly for massless scalar fields in two dimensions, $\langle T^\mu_\mu\rangle = (\hbar/24)R^{(2)}$, and find the effective action $S_{\rm PL}$ for which $\langle T_{\mu\nu}\rangle = -\frac{2\pi}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}S_{\rm PL}$. This is the Polyakov-Liouville action [13]

$$S_{\rm PL} = -\frac{\hbar}{96\pi}\int d^2x\sqrt{-g(x)} \times\int d^2x'\sqrt{-g(x')}R^{(2)}(x)G(x,x')R^{(2)}(x') , \qquad (2)$$

where $G(x,x')$ is a Green's function for $\nabla^2$. Here we take the large-$N$ limit, in which $\hbar$ goes to zero while $N\hbar$ is held fixed. In that limit the quantum corrections for the gravitational and dilaton fields are negligible, and one needs to take into account only the quantum corrections for the matter (scalar) fields. The one-loop effective action is then $S_{(1)} = S_{\rm cl} + NS_{\rm PL}$. There are no known analytic solutions to this one-loop effective theory, though there are some numerical ones [14]. In order to find analytic solutions including semiclassical corrections, one can modify the action as in [15–17]. Our approach is similar in that we modify the original CGHS action (1) and find analytic solutions to the modified equations including back reaction. However, our analytic solutions yield *closed-form* expressions for the metric and dilaton field. This allows us to fully analyze the solutions.

We add to the classical action (1) a local covariant term of one-loop order:

$$S_{\rm corr} = \frac{N\hbar}{24\pi}\int d^2x\sqrt{-g}[(\nabla\phi)^2 - \phi R^{(2)}] . \qquad (3)$$

Now the total modified action including the one-loop Polyakov-Liouville term is

$$S_{\rm mod} = S_{\rm cl} + S_{\rm corr} + NS_{\rm PL} . \qquad (4)$$

Using null coordinates $z^\pm$ and conformal gauge $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}$ $(ds^2 = -e^{2\rho}dz^+dz^-)$, the action (4) can be written in the form

*Electronic address: bose@csd.uwm.edu
†Electronic address: leonard@cosmos.phys.uwm.edu
‡Electronic address: yoav@alpha2.csd.uwm.edu

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$$S_{\rm mod} = \frac{1}{\pi}\int dz^+dz^-\Bigg[\partial_-(\phi-\rho)\partial_+\left(e^{-2\phi}-\frac{\kappa}{2}(\phi-\rho)\right)$$
$$+\partial_+(\phi-\rho)\partial_-\left(e^{-2\phi}-\frac{\kappa}{2}(\phi-\rho)\right)+\lambda^2e^{2(\rho-\phi)}+\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i\Bigg] , \quad (5)$$

where $\kappa = N\hbar/12$. The kinetic action density is a bilinear symmetric form $(\partial_+\Theta)M(\phi)(\partial_-\Theta)$, where $\Theta$ is a vector comprised of the $(N+2)$ fields $\rho$, $\phi$, and the $N$ matter fields $f_i$ and $M(\phi)$ is an $(N+2)\times(N+2)$ symmetric matrix. One can verify that the determinant of $M$ is proportional to $e^{-4\phi}$, and unlike in other models of modified dilaton gravity [15–17], here this determinant is nonvanishing for all real values of $\phi$. Problems involving curvature singularities associated with the vanishing of this determinant are relevant only as $e^{-2\phi}\to 0$, which is a region of infinitely strong coupling. In the semiclassical theory, the strong coupling region is excluded by imposing suitable boundary conditions as discussed below [see the discussion of Eq. (17)]. [From the point of view of string theory, the action (5) with free fields $X \equiv e^{-2\phi}$ and $Y \equiv \phi-\rho$ (which are flat target space coordinates), describes a conformal field theory with tachyon and dilaton backgrounds $T = -4\lambda^2e^{-2Y}$ and $\Phi = -2X + 2\kappa Y$ [18], if conformal boundary conditions are imposed at $X = 0$ [19].] The action (5) is also invariant under the transformation[1] [15] $\delta\phi = \delta\rho = \epsilon e^{2\phi}$, with the conservation equation $\partial_\mu\partial^\mu(\phi-\rho) = 0$. We therefore can complete the gauge fixing by choosing the "Kruskal coordinates" $x^\pm(z^\pm)$ in which $\phi(x^+,x^-) = \rho(x^+,x^-)$. In this Kruskal gauge the equations of motion derived from the modified action (5) are the same as the classical ones

$$\partial_{x^+}\partial_{x^-}(e^{-2\rho(x^+,x^-)}) = \partial_{x^+}\partial_{x^-}(e^{-2\phi(x^+,x^-)}) = -\lambda^2 , \quad (6)$$

$$\partial_{x^+}\partial_{x^-}f_i(x^+,x^-) = 0 , \quad (7)$$

while the constraints get modified by nonlocal terms $t_\pm(x^\pm)$ arising from the Polyakov-Liouville action. In conformal gauge, one can use the trace anomaly of $N$ massless scalar fields $f_i$ to obtain $\langle T^f_{+-}\rangle = -\kappa\partial_+\partial_-\rho$ and integrate [20–22] the equation $\nabla^\mu\langle T^f_{\mu\nu}\rangle = 0$ to get the quantum corrections to the energy-momentum tensor of the $f_i$ matter fields:

$$\langle T^f_{\pm\pm}\rangle = \kappa[\partial^2_\pm\rho - (\partial_\pm\rho)^2 - t_\pm(z^\pm)] , \quad (8)$$

where $t_\pm(z^\pm)$ are integration functions determined by the specific quantum state $|\Psi\rangle$ corresponding to the expectation value $\langle\Psi|T^f_{\mu\nu}|\Psi\rangle \equiv \langle T^f_{\mu\nu}\rangle$. These functions can be determined by boundary conditions. Alternatively, Eq. (8) can be obtained by varying $NS_{\rm PL}$. Then the functions $t_\pm(z^\pm)$ arise from the homogeneous part of the Green's function in Eq. (2). Our modified constraints (in Kruskal gauge) are

$$\frac{\delta S_{\rm mod}}{\delta g^{\pm\pm}} = 0 \Rightarrow -\partial^2_{x^\pm}(e^{-2\phi(x^+,x^-)}) - (T^f_{\pm\pm})_{\rm cl} + \kappa t_\pm(x^\pm) = 0 , \quad (9)$$

where $(T^f_{\pm\pm})_{\rm cf} = \frac{1}{2}\sum_{i=1}^N(\partial_{x^\pm}f_i)^2$ is the classical (zero order in $\hbar$) contribution to the energy-momentum tensor of the $f_i$ matter fields. $\langle T^f_{\mu\nu}\rangle$ in (8) is the one-loop quantum correction of order $\hbar$, so the full energy-momentum tensor of the $f_i$ fields is $(T^f_{\mu\nu})_{\rm cl} + \langle T^f_{\mu\nu}\rangle + O(\hbar^2)$.

For a given classical matter distribution and a given $t_\pm(x^\pm)$ one finds the solution for the equations of motion (6) with the constraints (9)

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2x^+x^- - \int^{x^+}dx_2^+\int^{x_2^+}dx_1^+[(T^f_{++})_{\rm cl} - \kappa t_+(x_1^+)]$$
$$-\int^{x^-}dx_2^-\int^{x_2^-}dx_1^-[(T^f_{--})_{\rm cl} - \kappa t_-(x_1^-)] + a_+x^+ + a_-x^- + b , \quad (10)$$

where $a_\pm$ and $b$ are constants. First, let us consider the linear dilaton flat spacetime solution, $e^{-2\phi} = e^{-2\rho} = -\lambda^2x^+x^-$. It corresponds to the choice $(T^f_{\mu\nu})_{\rm cl} = 0$ and $t_\pm(x^\pm) = a_\pm = b = 0$. To determine the corresponding quantum state $|\Psi\rangle$ one must calculate $\langle T^f_{\pm\pm}\rangle$ in (8) using the given $t_\pm(x^\pm)$. In flat coordinates $\sigma^\pm$, which are related to the Kruskal coordinates $x^\pm$ by the conformal coordinate transformation $\pm\lambda x^\pm = e^{\pm\lambda\sigma^\pm}$, the expectation values (8) are $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = \kappa\lambda^2/4$. We see that unlike in the RST model, in our model $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle \neq 0$ for the linear dilaton solution. Because $\langle T^f_{\pm\pm}\rangle = \kappa\lambda^2/4$ and $\langle T^f_{+-}\rangle = 0$, the quantum state $|\Psi\rangle$ corresponding to the linear dilaton solution may describe a system in thermal equilibrium at temperature $T = \lambda/2\pi$.

In our model we also have *static* black hole solutions [23]. These correspond in Eq. (10) to the choice $(T^f_{\mu\nu})_{\rm cl} = t_\pm(x^\pm) = a_\pm = 0$ and $b = M/\lambda$. For these

[1]Unlike in the Russo-Susskind-Thorlacius (RST) model [15], in this model the transformation is exactly the same as in the classical case.

solutions at future and past null infinity ($\mathcal{I}^+$ and $\mathcal{I}^-$, respectively) one has $\langle T^f_{\pm\pm}\rangle = \kappa\lambda^2/4$; the solutions evidently describe a black hole in thermal equilibrium at temperature[2] $T_{\rm BH} = \lambda/2\pi$. This is as we would expect: A static black hole solution in a self-consistent semiclassical theory of Hawking radiation including back reaction is possible only if the black hole is in thermal equilibrium with incoming radiation.

In order to find the solution corresponding asymptotically to the Minkowski vacuum we can use (8) to find the solution for which $\langle T^f_{\pm\pm}(\sigma^\pm)\rangle = 0$. The functions $t_\pm(x^\pm)$ are determined by imposing appropriate boundary conditions on $\mathcal{I}^\pm$. We assume that on these boundaries the metric is flat, such that $\rho(\sigma^\pm)$ and its derivatives vanish in the asymptotically flat coordinates $\sigma^\pm$. Then the first two terms on the right hand side of (8) vanish on the boundary and we get

$$\langle T^f_{\pm\pm}(\sigma^\pm)\rangle|_{\rm boundary} = -\kappa t_\pm(\sigma^\pm)\ . \qquad (11)$$

We see from (11) that the Minkowski vacuum corresponds to $t_\pm(\sigma^\pm) = 0$. To find the corresponding $t_\pm(x^\pm)$ in "Kruskal coordinates" one can use the tensor transformation of $\langle T^f_{\pm\pm}\rangle$ in Eq. (8) (under a conformal coordinate transformation) and get

$$t_\pm(x^\pm) = \left(\frac{\partial\sigma^\pm}{\partial x^\pm}\right)^2 \{t_\pm(\sigma^\pm) - \tfrac{1}{2}D^S_{\sigma^\pm}[x^\pm]\} = \frac{1}{(2x^\pm)^2}\ , \qquad (12)$$

where $D^S_y[z]$ is the Schwarz operator $D^S_y[z] = \partial^3_y z/(\partial_y z) - \frac{3}{2}(\partial^2_y z/\partial_y z)^2$ and we use $t_\pm(\sigma^\pm) = 0$. Using (10), (12), and $(T^f_{\mu\nu})_{\rm cl} = 0$, we find that the general asymptotically Minkowski vacuum solution is

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\ln(-\lambda^2 x^+ x^-) + C\ , \quad (13)$$

where $C$ is a constant. In asymptotically flat coordinates $\sigma^\pm = t \pm \sigma$, we have

$$ds^2 = [1 - e^{-2\lambda\sigma}(\kappa\lambda\sigma/2 - C)]^{-1}(-dt^2 + d\sigma^2)\ ,$$
$$\phi(\sigma) = -\lambda\sigma - \frac{1}{2}\ln\left[1 - e^{-2\lambda\sigma}\left(\frac{\kappa\lambda}{2}\sigma - C\right)\right]\ . \qquad (14)$$

This solution is static, depending on the spatial coordinate $\sigma$ alone. On the boundaries $\mathcal{I}^\pm$, the solution approaches the linear dilaton flat spacetime solution, justifying our assumption. The reason this solution with no radiation at $\mathcal{I}^\pm$ and the earlier ones with radiation there all asymptotically approach the linear dilaton flat spacetime solution is that the coupling, $e^{2\phi}$, of the matter to the geometry vanishes exponentially fast at $\mathcal{I}^\pm$.

Before we turn to the question of the ground-state solution, let us consider the Arnowitt-Deser-Misner (ADM) masses of the various solutions we have found. Let us choose as reference solution one of the static radiationless solutions (14) with $C = C_0$. Then the ADM mass [24,25] of any other static solution (14) is $\lambda(C - C_0)$. On the other hand, the ADM mass of the linear dilaton solution as well as the static black hole solutions (relative to this ground state) is infinite. This is already clear from the fact that these solutions have nonvanishing radiation on $\mathcal{I}^\pm$ and can be checked explicitly by using the ADM mass definition [24]. Because the solutions (14) have lower ADM mass, it is plausible that the ground-state solution is one of these static radiationless solutions.

We next turn to the dynamical scenario in which the spacetime is initially described by one of the static solutions in (14) (not necessarily the reference solutions with $C = C_0$), and in which a black hole is formed by collapsing matter fields. First we consider the simple shock wave solution, but our results can be easily extended to general infalling matter configurations. The shock wave of infalling matter is described by $(T^f_{++})_{\rm cl} = (M/\lambda x_0^+)\delta(x^+ - x_0^+)$ and $(T^f_{--})_{\rm cl} = 0$ [9]. Unlike the RST model in which a shock wave always forms a black hole, here we take a general initial state geometry (14), and the shock wave forms a black hole only if $M$, the energy of the shock wave, is above a certain critical energy $M_{\rm cr}$. For $M < M_{\rm cr}$, the solution is stable and energy conserving [26,27]. Integrating $(T^f_{++})_{\rm cl}$ in (10) and using (12) and $a_\pm = 0$, we find the solution

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{\kappa}{4}\ln(-\lambda^2 x^+ x-)$$
$$-\frac{M}{\lambda x_0^+}(x^+ - x_0^+)\Theta(x^+ - x_0^+) + C\ , \quad (15)$$

where $\Theta(x)$ is the standard step function.

Before the shock wave, i.e., in the region $x^+ < x_0^+$, we have a static solution (14) which is not globally flat. Consider this static solution in the cases when $C < C^*$ and $C > C^*$, where

$$C^* = -\frac{\kappa}{4}[1 - \ln(\kappa/4)]\ . \qquad (16)$$

If $C < C^*$, then the scalar curvature $R^{(2)} = 8e^{-2\rho}\partial_+\partial_-\rho$ diverges on a timelike curve $\sigma = \sigma_s$, for which $e^{-2\phi(\sigma_s)} = 0$. Of course this is a region of strong coupling and one would expect to have higher-order quantum corrections there. On the other hand, if $C > C^*$, then the scalar curvature diverges on the null curves $x^+ = 0$ and $x^- = 0$. This null singularity is a finite affine-parameter distance away from any interior point in the spacetime. For $C > C^*$, the static solution has a region of strong coupling lying inside the spacetime near the curve $\sigma = \sigma_{\rm min} = (1/2\lambda)\ln(\kappa/4)$, where $e^{-2\phi}$ has its minimum. In both cases, $C < C^*$ and $C > C^*$, the region of strong coupling can be avoided in the semiclassical approximation by imposing boundary conditions on a suitable time-

[2]Since in 2D the Hawking temperature is mass independent, one may regard the linear dilaton solution as the zero mass limit of the static black hole solutions. This may explain the nonzero temperature of the linear dilaton solution in our model.

like hypersurface [28,15,19]. Such a hypersurface can be chosen to be the curve $e^{2\phi}$ = const. For static solutions this is equivalent to choosing this hypersurface to be the curve $\sigma$ =const. For the static solutions (14), $\langle T^f_{++}(\sigma^\pm)\rangle$ and $\langle T^f_{--}(\sigma^\pm)\rangle$ are constant on any timelike hypersurface $\sigma$ = const. Moreover,

$$\langle T^f_{++}(\sigma^\pm)\rangle|_{\sigma=\sigma_b} = \langle T^f_{--}(\sigma^\pm)\rangle|_{\sigma=\sigma_b} \quad \text{for any constant } \sigma_b . \quad (17)$$

This means that we can limit our model to a region in which the semiclassical approximation is valid by imposing reflecting boundary conditions (17) on any given timelike hypersurface $\sigma = \sigma_b$ that lies outside the region of strong coupling (these boundary conditions are also conformal [19]). The geometry before the shock wave is therefore a static geometry, defined in the region $\sigma > \sigma_s$ in the case $C < C^*$ (or $\sigma > \sigma_{\min}$ in the case $C > C^*$), with reflecting boundary conditions on $\sigma_b = \sigma_s + \delta$ (or on $\sigma_b = \sigma_{\min} + \delta$) where $\delta$ is a positive constant.

In the dynamical scenario, a shock wave moving along the null curve $x^+ = x_0^+$ hits the boundary curve $\sigma = \sigma_b$ when $x^- = x_0^-$, where $x_0^- = -(\lambda^2 x_0^+)^{-1}\exp(2\lambda\sigma_b)$. For all values of $M$ and $C$, the solution to the future of the shock wave ($x^+ > x_0^+$) is [see (15)]

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} + C , \quad (18)$$

where $\Delta = M/(\lambda^3 x_0^+)$. In the subcritical case ($M < M_{\rm cr}$) we find that no black hole forms and this solution is valid to the past of the null curve $x^- = x_0^-$. In this case we can choose boundary conditions in which the incoming shock wave is reflected, enabling us to continue the solution to the future of $x^- = x_0^-$. This subcritical solution is stable and unitary [26]. In the supercritical case ($M > M_{\rm cr}$), the solution (18) describes a black hole with a singularity at $e^{-2\phi} = 0$. In the remainder of this paper, we describe the properties of this dynamical black hole solution. The black hole singularity curve is

$$-\lambda^2 x_s^+(x_s^- + \Delta) - \frac{\kappa}{4}\ln(-\lambda^2 x_s^+ x_s^-) + \frac{M}{\lambda} + C = 0 . \quad (19)$$

Initially the singularity is behind an apparent horizon $\partial_+ e^{-2\phi} = 0$ [29], which is the curve

$$-\lambda^2 x_h^+(x_h^- + \Delta) = \frac{\kappa}{4} . \quad (20)$$

When the apparent horizon is formed, the black hole starts radiating. One can see this by calculating $\langle T^f_{\mu\nu}\rangle$ at future null infinity ($x^+ \to \infty$). From (18) we see that the asymptotically flat coordinates on $\mathcal{I}^+$ are $\hat\sigma^\pm$, related to $x^\pm$ by the conformal coordinate transformation, $\lambda\hat\sigma^+ = \ln(\lambda x^+)$ and $-\lambda\hat\sigma^- = \ln[-\lambda(x^- + \Delta)]$. Using (11) and (12) we get

$$\langle T^f_{--}(\hat\sigma^\pm)\rangle|_{\mathcal{I}^+} = \frac{\kappa\lambda^2}{4}\left(1 - \frac{1}{(1+\lambda\Delta e^{\lambda\hat\sigma^-})^2}\right) . \quad (21)$$

This is the "standard" Hawking radiation in 2D, where the Hawking temperature $T_H = \lambda/2\pi$ is a constant [9]. One can further verify that when the black hole evaporates over a long period of time, i.e., if $M \gg \kappa\lambda$, the spectrum of the Hawking radiation is indeed Planckian [2,30].

As the black hole evaporates by emitting Hawking radiation, the apparent horizon shrinks and eventually meets the singularity in a *finite* proper time. They intersect at (see Fig. 1)

$$x^+_{\rm int} = \frac{1}{\lambda^2\Delta}\left[\exp\left(\frac{4(M+\lambda C)}{\kappa\lambda} + 1\right) - \frac{\kappa}{4}\right]$$

and

$$x^-_{\rm int} = -\Delta\left\{1 - \frac{\kappa}{4}\exp\left[-\left(\frac{4(M+\lambda C)}{\kappa\lambda} + 1\right)\right]\right\}^{-1} . \quad (22)$$

At this point the singularity becomes naked. We show below that it is possible to impose a boundary condition in which a weak shock wave emanates from the intersection point, resulting in a solution that is stable (having non-negative ADM mass), conserves energy, and has a continuous metric.

Before considering the solution to the future of the null curve $x^- = x^-_{\rm int}$ (i.e., the end-state solution), we calculate the amount of energy $E_{\rm rad}$ radiated by the black hole up to the null curve $x^- = x^-_{\rm int}$. Integrating (21) over $\mathcal{I}^+$ (up to $x^-_{\rm int}$) gives the exact closed-form result

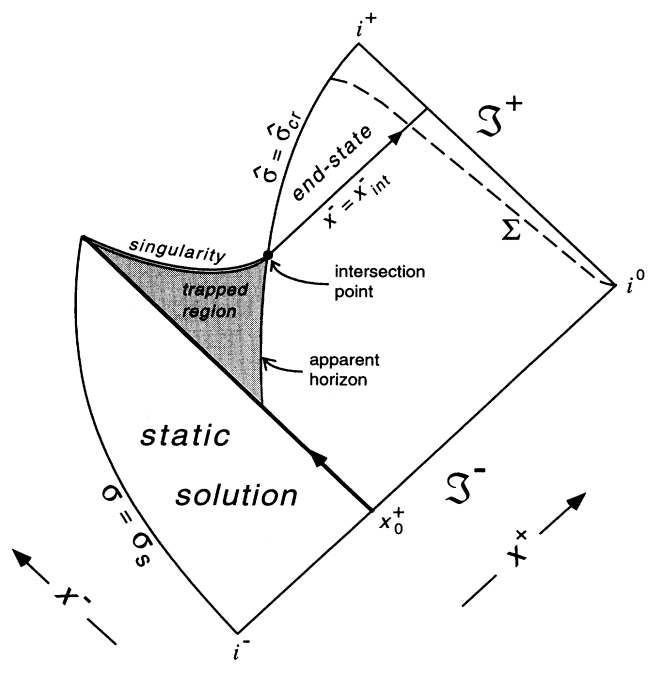


FIG. 1. Penrose diagram describing formation and subsequent evaporation of a black hole in our model.

$$E_{\rm rad} = \int_{-\infty}^{\hat\sigma^-_{\rm int}} \langle T^f_{--}(\hat\sigma^-)\rangle d\hat\sigma^- = M + \lambda C - \frac{\kappa\lambda}{4}[\ln(\kappa/4) - 1] - \frac{\kappa\lambda\Delta}{4x^-_{\rm int}} \,, \tag{23}$$

where $\hat\sigma^-_{\rm int} = \hat\sigma^-(x^-_{\rm int})$. The ADM mass [23,24] of the dynamical solution (15) (relative to the reference solution with $C = C_0$) is $M_{\rm ADM} = M + \lambda(C - C_0)$. We see that the black hole radiates almost all of its initial energy. The unradiated mass $\delta M$ remaining as $x^- \to x^-_{\rm int}$ (which is the Bondi mass) is

$$\delta M = M_{\rm ADM} - E_{\rm rad} = \frac{\kappa\lambda}{4}[\ln(\kappa/4) - 1] - \lambda C_0 + \frac{\kappa\lambda\Delta}{4x^-_{\rm int}} \,. \tag{24}$$

We now consider the solution to the future of the point of intersection $(x^+_{\rm int}, x^-_{\rm int})$. One candidate for such a solution would be the smooth continuation of solution (18). This solution is unstable because the total energy reaching $\mathcal{I}^+$ diverges as can be seen from Eq. (23) with $\hat\sigma^-_{\rm int}$ replaced by $\infty$. As was shown by RST in [15], one can avoid this instability by introducing a small outgoing shock wave originating at $(x^+_{\rm int}, x^-_{\rm int})$ such that this solution is matched to the stable ground-state solution to the future of the outgoing shock wave. In our case as well, the black hole solution can be matched to the stable ground state of our model by introducing a small outgoing shock wave. In this matching, the metric is continuous and energy is conserved. As we already discussed, the ground state in our model is one of the static solutions (14). Thus we try to find boundary conditions that match the solution (18) continuously to one of the static solutions (14). Remember that the asymptotically flat coordinates are $\hat\sigma^\pm$, so one should replace $\sigma$ in (14) with $\hat\sigma = \frac{1}{2}(\hat\sigma^+ - \hat\sigma^-)$. In the $x^\pm$ coordinates this static solution is [see (13)]

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\ln[-\lambda^2 x^+(x^- + \Delta)] + \hat C \,. \tag{25}$$

We would like to see if there exists a constant $\hat C$ such that on the null curve $x^- = x^-_{\rm int}$ the solutions (18) and (25) can be matched continuously. This is indeed the case and from (22), (18), and (25) we find that the unique solution has $\hat C$ equal to $C^*$ defined in (16). The end-state solution, or "remnant," is, therefore,

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+(x^- + \Delta) - \frac{\kappa}{4}\ln[-\lambda^2 x^+(x^- + \Delta)] - \frac{\kappa}{4}[1 - \ln(\kappa/4)] \,, \tag{26}$$

where $x^- > x^-_{\rm int}$. From the constraint equations (9) we find that

$$[T^f_{--}(\hat\sigma^-)]_{\rm cl} = \frac{1}{2}\sum_{i=1}^{N}(\partial_- f_i)^2 = \frac{\kappa\lambda\Delta}{4x^-_{\rm int}}\delta(\hat\sigma^- - \hat\sigma^-_{\rm int}) \,. \tag{27}$$

This describes a shock wave originating at the intersection point and carrying a small amount of negative energy, $\kappa\lambda\Delta/(4x^-_{\rm int})$, to null infinity. One may call it a "thunderpop" [15]. The solution (26) is one of the static solutions that is asymptotically flat (with no radiation) on $\mathcal{I}^+$. This means that there is no Hawking radiation after the thunderpop (27). Thus Eq. (23) gives all of the energy emitted in the form of Hawking radiation.

The mass remaining after the shock wave (27) has been emitted is $\delta M - \kappa\lambda\Delta/(4x^-_{\rm int})$. One readily verifies that this is equal to the mass of the "remnant" (relative to the reference solution with $C = C_0$) $M_{\rm rem} = \lambda(C^* - C_0)$. The fact that energy is exactly conserved, including terms of order $\hbar$, supports the self-consistency of our semiclassical theory. Notice that $C^*$ and therefore the "remnant" mass is independent of the mass $M$ of the infalling matter and of the constant $C$ describing the initial static geometry. Even more interesting is the fact that the end-state solution (26) with $\hat C = C^*$ is the critical solution separating the space of static solutions (25) into two classes: Those with a timelike singularity ($\hat C < C^*$) and those with a null singularity ($\hat C > C^*$).

Consider the late-time spacelike hypersurface $\Sigma$ shown in Fig. 1. Its right boundary ($\hat\sigma \to \infty$) is $i^0$, while its left boundary is the curve $\hat\sigma = \hat\sigma_{\rm cr}$, for which $e^{-2\phi} = 0$. For the critical solution we have $\partial_{x^+}(e^{-2\phi(\hat\sigma_{\rm cr})}) = e^{-2\phi(\hat\sigma_{\rm cr})} = 0$ and the curve $\hat\sigma = \hat\sigma_{\rm cr}$ is the analytical continuation to the region $x^- > x^-_{\rm int}$ of the curve that is an apparent horizon in the region $x^- > x^-_{\rm int}$. Note that in the region $x^- > x^-_{\rm int}$ the curve $\hat\sigma = \hat\sigma_{\rm cr}$ is not an apparent horizon. In fact the static solution (26) has no apparent horizon. To study the behavior of the spacetime near the curve $\hat\sigma = \hat\sigma_{\rm cr}$ let us define $\epsilon \equiv \hat\sigma - \hat\sigma_{\rm cr}$ and calculate the metric near $\epsilon = 0$. From (26) we get

$$ds^2 \to \frac{-d\hat t^2 + d\epsilon^2}{2\lambda^2\epsilon^2 + O(\epsilon^3)} \,, \tag{28}$$

where $\hat t = \frac{1}{2}(\hat\sigma^+ + \hat\sigma^-)$. The first nonvanishing term in the denominator of (28) is of order $\epsilon^2$, which means that the geometric structure near $\epsilon = 0$ is that of an *infinite throat.* Consider for example the distance along a $\hat t =$ const curve. The distance to $\hat\sigma = \hat\sigma_{\rm cr}$ diverges logarithmically, as it does in higher-dimensional extremal black holes. The end-state spacetime is geodescially complete. On the curve $\hat\sigma = \hat\sigma_{\rm cr}$, the Ricci scalar has the constant value $R^{(2)} = -4\lambda^2$ and the geometry is regular.

An appropriate choice of ground state is the static radiationless solution (14) with $C = C^*$. The solution is everywhere regular. Any solution (14) with smaller ADM mass ($C < C^*$) has a naked timelike singularity. Also if we choose the reference solution as the ground state (i.e., choose $C_0 = C^*$), then the mass remaining after the thunderpop (27) is zero. The end-state solution (26) resulting from black hole evaporation is the same as the static radiationless ground-state solution proposed here. Its geometrical structure is independent of the initial conditions. In our model it is a semi-infinite throat extending into the strong coupling region.

In this 2D semiclassical model when $M > M_{\rm cr}$ one does not recover all the information of the initial state from the end-state solution. For infalling matter described by a general $(T^f_{++})_{\rm cl}$ of compact support, the solution (10) will depend only on the first two moments of $(T^f_{++})_{\rm cl}$, $M = \lambda \int x^+ (T^f_{++})_{\rm cl} dx^+$, and $P_+ = \int (T^f_{++})_{\rm cl} dx^+$ [15]; the end-state solution will still be (26), but with $\Delta = \lambda^{-2} P_+$. The information encoded in this "remnant" (or more precisely, in its past null boundary $x^- = x^-_{\rm int}$) is only about $P_+$ and $M$. Thus in our model this end-state solution does not qualify as the "cornucopion" of [31]. However, the semi-infinite throat extends to a region of very strong coupling. There may be sufficient freedom in this strong coupling region to encode more information through strong quantum gravitational effects.

In this work we constructed an action in 2D dilaton gravity and showed that there exists a boundary condition preserving energy conservation, stability, and continuity of the metric, from which it follows that all evaporating black holes end in a unique ground-state geometry having a semi-infinite throat.

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