Background thermal contributions in testing the Unruh effect

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(Received 14 October 1994)

We consider inertial and accelerated Unruh-DeWitt detectors moving in a background thermal bath and calculate their excitation rate. It is shown that for fast moving detectors such a thermal bath does not affect substantially the excitation probability. Our results are discussed in connection with a possible proposal of testing the Unruh effect in high energy particle accelerators.

PACS number(s): 04.60.−m, 03.70.+k

I. INTRODUCTION

It is already two decades since Hawking discovered the striking result that quantum mechanics may induce black holes to evaporate [1]. Many different questions related with such an effect have been clarified since that time, and a number of other ones are presently under investigation [2]. The so-called Unruh effect [3,4] has played an outstanding role in emphasizing some of the exquisite features present in the Hawking effect. According to the Unruh effect, a detector uniformly accelerated through the *inertial* vacuum responds as being in a thermal bath characterized by a temperature proportional to its proper acceleration. We call *inertial* vacuum the no-particle state as described by a family of observers following an *inertial* timelike Killing field ∂_t in Minkowski space. The Unruh effect is a direct consequence of the fact that the particle content of a field theory is very frame dependent [3,5]. Later, Bell and Leinaas [6] raised the very interesting possibility of interpreting the observed depolarization of electrons in particle accelerators in terms of the Unruh effect. They argued that electrons could be used as sensitive thermometers because of the fact that the coupling between the spin and the magnetic field induces a splitting between the "spin up" and "spin down" levels. Thus, the observed depolarization of electron beams might be interpreted in the electron's rest frame because of the thermal bath predicted by Unruh. Since electrons in *linear accelerators* do not have time to reach equilibrium in the polarization distribution, Bell and Leinaas decided to consider electrons in *storage rings* [6]. In this case, ultrarelativistic electrons following uniform circular motion experience in their rest frame an "effective" thermal bath characterized by a temperature which differs from the original Unruh temperature by a numerical factor $\pi/\sqrt{3}$ [7,8]. Although other effects [9] may play an important role in this phenomenon, Bell and Leinaas' results can be used to analyze the depolarization phenomenon apart from Thomas precession contributions. At conditions reached at the CERN e^+e^- collider LEP, electrons are typically accelerated at $a = 2.9 \times 10^{23} m/s^2$, which corresponds to an Unruh temperature of $\hbar a/2\pi ck = 1200$ K. This is only 4 times larger than typical laboratory temperatures. In this vein, it would be desirable to consider a detector accelerated in a background thermal bath rather than in the inertial vacuum in order to investigate in what extent finite-temperature corrections should be taken into account when testing the Unruh effect under real laboratory conditions. The detector excitation will represent the electron depolarization, since both ones share the common feature of being two-level systems [6]. We show that in real accelerator conditions the major contribution to the detector's response comes from the inertial vacuum rendering the contribution because of the presence of the background thermal bath unimportant. It corroborates the usual assumption, when testing the Unruh effect in storage rings, of considering the electrons as being accelerated in the Minkowski vacuum [6,10].

The paper is organized as follows: In Sec. II we study *inertial* detectors evolving in a background thermal bath and show that because of time dilatation the faster the detector moves, the less it interacts with the thermal bath. In Sec. III we replace the inertial detectors by *uniformly accelerated* ones, and calculate finite-temperature corrections in the detector's excitation rate because of the external thermal bath. In Sec. IV we consider detectors moving *circularly with constant speed,* and discuss our results in connection with the proposal of testing the Unruh effect in storage rings. Final conclusions are summarized in Sec. V. Natural units will be used ($\hbar = c = k = 1$) unless stated otherwise, and the signature adopted is $(+ - - -)$.

II. INERTIAL DETECTORS IN A BACKGROUND THERMAL BATH

We will show in this section that the faster a detector moves in a background thermal bath the less the detector interacts with the bath. This is so because time dilatation induces a fast moving detector to interact preferentially with low frequency modes. Although a thermal bath is rich of low frequency modes, the phase space volume element ($\propto \omega^2 d\omega$) suppresses infrared contributions.

Let us begin considering an Unruh-DeWitt detector [3,11]. It is basically a two-level device which may be either in the ground state $|E_0\rangle$ or in the excited state $|E\rangle$. The detector will be described by a monopole $\hat{m}(\tau)$ coupled to a massless scalar field $\hat{\phi}(x^\mu)$ through the interaction action

$$S_I = \int_{-\infty}^{+\infty} d\tau \; c(\tau)\hat{m}(\tau)\hat{\phi}\left[x^\mu(\tau)\right], \qquad (2.1)$$

where $x^\mu(\tau)$ is the detector's world line and τ is its proper time. Here $c(\tau)$ is a switching function through which the detector is turned on or off, and plays the role of a small coupling parameter. In this section it will be enough to consider a permanently switched on detector, i.e., $c(\tau) = c_0 = \text{const}$. In the Heisenberg picture the monopole operator is time evolved as

$$\hat{m}(\tau) = e^{i\hat{H}_0\tau}\hat{m}(0)e^{-i\hat{H}_0\tau}, \qquad (2.2)$$

where $\hat{H}_0|E\rangle = E|E\rangle$ for any detector's energy eigenstate $|E\rangle$.

The amplitude for the detector to be excited and simultaneously absorbing a particle $|\mathbf{k}\rangle$ is

$$\mathcal{A}^{\rm exc}_{\rm abs} = \langle 0| \otimes \langle E|S_I|E_0\rangle \otimes |\mathbf{k}\rangle. \qquad (2.3)$$

Using the expansion of the scalar field in plane waves (see, e.g., [12]), and assuming that our detector follows an inertial world line $x = y = 0$; $z = vt$; $t = \tau/\sqrt{1-v^2}$ (where $v = |\mathbf{v}|$ is the detector's speed with respect to the background thermal bath), we obtain

$$\mathcal{A}^{\rm exc}_{\rm abs} = \frac{c_0}{\sqrt{4\pi\omega}}\delta\left[\Delta E - \frac{\omega - k_z v}{\sqrt{1-v^2}}\right], \qquad (2.4)$$

where $\Delta E = E - E_0$, and $\omega = |\mathbf{k}|$. We will assume the selectivity $\langle E|\hat{m}(0)|E_0\rangle \equiv 1$ since it only depends on the internal details of the detector, and it can be always factored out. The amplitude for the detector to be excited and simultaneously emitting a particle into the vacuum vanishes because of energy conservation. Thus, at the tree level, the *total* excitation rate per total proper time $T^{\rm tot}$ of the detector will be

$$\frac{\mathcal{P}^{\rm exc}}{T^{\rm tot}} = \frac{1}{T^{\rm tot}} \int d^3\mathbf{k} |\mathcal{A}^{\rm exc}_{\rm abs}|^2 \left[\frac{1}{e^{\beta\omega} - 1}\right], \qquad (2.5)$$

where $T^{\rm tot} = 2\pi\delta(0)$ [12], and we have added into brackets the usual absorption weight associated with a thermal bath at a temperature β^{-1}.

As a consequence of (2.4), fast moving detectors will only interact with low frequency modes. The very behavior of the detector will be determined in (2.5) by the competition between the thermal bath, which is rich of low frequency modes, and the phase space volume element ($\propto \omega^2 d\omega$) which tends to suppress infrared contributions. Substituting (2.4) in (2.5), and performing the integrations, we obtain

$$\frac{\mathcal{P}^{\rm exc}}{T^{\rm tot}} = \frac{c_0^2\beta^{-1}\sqrt{1-v^2}}{4\pi v} \ln\left[\frac{1 - e^{-\beta\Delta E\sqrt{1+v}/\sqrt{1-v}}}{1 - e^{-\beta\Delta E\sqrt{1-v}/\sqrt{1+v}}}\right]. \qquad (2.6)$$

In the limit $v \to 0$ the detector responds with a Planckian spectrum

$$\frac{\mathcal{P}^{\rm exc}}{T^{\rm tot}} = \frac{c_0^2}{2\pi}\frac{\Delta E}{e^{\beta\Delta E} - 1}, \qquad (2.7)$$

as expected, while as $v \to 1$ the excitation rate per total proper time vanishes (see Fig. 1). This suggests that in testing the Unruh effect in ultrarelativistic regimes as in particle accelerators [6,10], background thermal contributions should be small. This issue will be investigated in detail in Sec. IV.

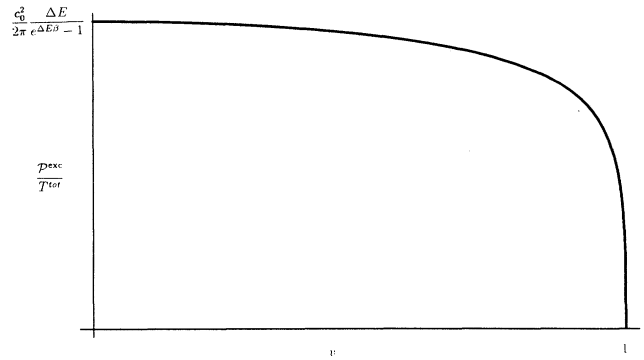

FIG. 1. The excitation probability of an inertial detector moving with speed v in a thermal bath characterized by a temperature β^{-1} is plotted. Faster the detector moves less it interacts with the thermal bath. Here we have chosen arbitrarily $\beta = 2.5 \times 10^{-14}$ s, and $\Delta E = 9.7 \times 10^{14}$ s^{-1}.

III. UNIFORMLY ACCELERATED DETECTORS IN A BACKGROUND THERMAL BATH

Before analyzing the most interesting case of circularly moving detectors, let us investigate the response of a uniformly accelerated detector in a background thermal bath. By uniformly accelerated detectors we mean *linearly* accelerated detectors with constant proper acceleration. The total excitation probability for a detector evolving in a background thermal bath characterized by a temperature β^{-1} can be written as [13]

$$\mathcal{P}^{\rm exc} = \int_{-\infty}^{+\infty} d\tau c(\tau) \int_{-\infty}^{+\infty} d\tau' c(\tau')\, e^{-i\Delta E(\tau-\tau')}$$
$$\times G^+_\beta[x^\mu(\tau), x^\mu(\tau')], \qquad (3.1)$$

where

$$G^+_\beta[x^\mu(\tau), x^\mu(\tau')]$$
$$= -\sum_{n=-\infty}^{+\infty} \frac{(4\pi^2)^{-1}}{(t - t' - i\beta n - i\varepsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2} \qquad (3.2)$$

is the Wightman function, $x^\mu(\tau)$ is the world line of the accelerated detector, and τ is its proper time. The world line of a detector moving in the z axis with constant proper acceleration a is

$$t = \frac{1}{a}\sinh a\tau, \quad z = \frac{1}{a}\cosh a\tau, \quad x = y = 0. \tag{3.3}$$

Substituting (3.3) in the Wightman function (3.2) we obtain

$$G^+_\beta(\tau,\tau') = -\frac{a^2}{16\pi^2}\sum_{n=-\infty}^{+\infty}\frac{1}{[\sinh a\Delta\tau/2 + i(n\beta a - \epsilon)e^{a\xi}/2]\,[\sinh a\Delta\tau/2 + i(n\beta a - \epsilon)e^{-a\xi}/2]}, \tag{3.4}$$

where $\Delta\tau \equiv \tau - \tau'$, and $\xi \equiv (\tau+\tau')/2$. Using the identity (the prime indicates that $n=0$ is excluded from the sum)

$$\sum_{n=-\infty}^{+\infty}{}' \frac{1}{(A+iBn)(A+iCn)} = \frac{2}{C(C-B)}\sum_{n=1}^{+\infty}\frac{1}{(A^2/C^2+n^2)} - \frac{2}{B(C-B)}\sum_{n=1}^{+\infty}\frac{1}{(A^2/B^2+n^2)}, \tag{3.5}$$

in conjunction with [14]

$$\sum_{n=1}^{+\infty}\frac{1}{x^2+n^2} = \frac{\pi}{2x}\coth \pi x - \frac{1}{2x^2}, \tag{3.6}$$

we can cast (3.4) in the form

$$G^+_\beta(\tau,\tau') = G^+_{\text{vac}}(\Delta\tau) + G^+_{\text{ther}}(\Delta\tau,\xi). \tag{3.7}$$

The first term in (3.7) corresponds to the pure vacuum contribution [13]:

$$G^+_{\text{vac}}(\Delta\tau) = -\frac{a^2}{16\pi^2}\sinh^{-2}(a\Delta\tau/2 - i\varepsilon), \tag{3.8}$$

while the second term corresponds to the background thermal bath contribution

$$G^+_{\text{ther}}(\Delta\tau,\xi) = \frac{a^2}{16\pi^2}\sinh^{-2}(a\Delta\tau/2) + \frac{a}{16\pi\beta\sinh a\xi\,\sinh(a\Delta\tau/2)}\left[\coth\frac{2\pi\sinh(a\Delta\tau/2)}{a\beta e^{-a\xi}} - \coth\frac{2\pi\sinh(a\Delta\tau/2)}{a\beta e^{a\xi}}\right]. \tag{3.9}$$

The fact that G^+_{ther} depends on ξ reflects the fact that this is a nonstationary situation. Notice that $G^+_{\text{ther}}(\Delta\tau,\xi)$ does not diverge at any point. In particular $G^+_{\text{ther}}(\Delta\tau=0,\xi) = 1/12\beta^2$. Asymptotically $G^+_{\text{ther}}(\Delta\tau,\xi)$ behaves as (see Fig. 2)

$$G^+_{\text{ther}}[|\Delta\tau| \gg (a^{-1},\beta),\ \xi] \sim e^{-a|\Delta\tau|}, \quad G^+_{\text{ther}}[\Delta\tau\neq 0,\ |\xi| \gg (a^{-1},\beta)] \sim e^{-a|\xi|}. \tag{3.10}$$

Clearly, G^+_{ther} vanishes in the limit $\beta \to +\infty$, and thus $G^+_{\beta\to+\infty}(\tau,\tau') = G^+_{\text{vac}}(\Delta\tau)$.

Now, we are ready to investigate the total excitation rate, $\mathcal{P}^{\text{exc}} = \mathcal{P}^{\text{exc}}_{\text{vac}} + \mathcal{P}^{\text{exc}}_{\text{ther}}$, of a detector uniformly accelerated in a background thermal bath. Here, we shall consider the detector as being switched on only during a finite period of proper time $|\tau| < T_0/2$, where $T_0 = \text{const} \in \mathbf{R}_+$. Concerning the pure vacuum contribution $\mathcal{P}^{\text{exc}}_{\text{vac}}$ it could be calculated for some continuous $c(\tau)$ by letting (3.8) into (3.1). Notwithstanding, we will use directly the results of Ref. [15] where the calculations were performed in the detector's rest frame. The excitation probability for a detector uniformly accelerated in the *inertial* vacuum, and kept switched on for long enough, $T_0 \gg a^{-1}, \Delta E^{-1}$, is

$$\mathcal{P}^{\text{exc}}_{\text{vac}} \approx \frac{c_0^2}{2\pi}\frac{\Delta E}{e^{2\pi\Delta E/a} - 1}T_0. \tag{3.11}$$

Notice the linear dependence with T_0 (see Fig. 3), which is exactly what one should expect due to the Unruh effect [13]. Here $c_0 = \text{const}$ is the coupling constant between the field and the monopole while the detector is kept switched on. In the regime considered above, the detailed form of $c(\tau)$ is not important. The only restriction is that $c(\tau) \in C^0$, since discontinuities in $c(\tau)$ would result in ultraviolet divergences [15].

The thermal correction $\mathcal{P}^{\text{exc}}_{\text{ther}}$ on the pure vacuum term (3.11) will be obtained by introducing (3.9) in (3.1),

$$\mathcal{P}^{\text{exc}}_{\text{ther}} = c_0^2\int_{-T_0/2}^{+T_0/2} d\tau \int_{-T_0/2}^{+T_0/2} d\tau'\, e^{-i\Delta E\Delta\tau} G^+_{\text{ther}}(\Delta\tau,\xi), \tag{3.12}$$

where we have already considered the fact that the detector is kept switched on only during a finite amount of proper time T_0. The integrals above were solved numerically for the arbitrary values $\Delta E = 9.7\times 10^{14}\ \text{s}^{-1}$, $a = 9.7\times 10^{14}\ \text{s}^{-1}$, $\beta = 2.5\times 10^{-14}$ s (see discussion in the

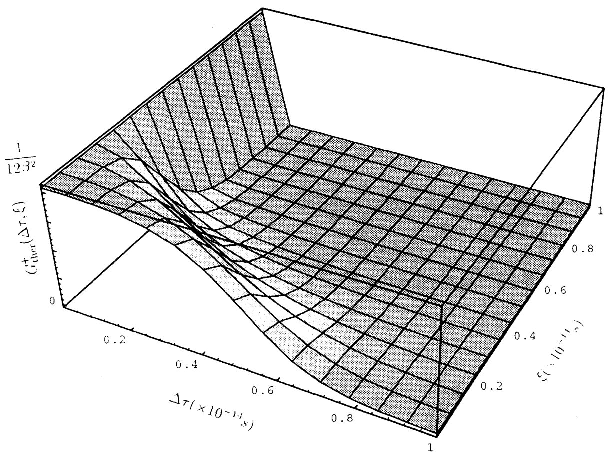

FIG. 2. $G^{+}_{\rm ther}(\Delta\tau,\xi)$ is finite everywhere, and asymptotically it decreases exponentially except along the ξ axis on which it is constant. Notice that $G^{+}_{\rm ther}(\Delta\tau,\xi)$ is completely symmetric in the other quadrants. Here we have chosen arbitrarily $\beta = 2.5\times10^{-14}$ s, and $a = 9.7\times10^{14}$ s^{-1}.

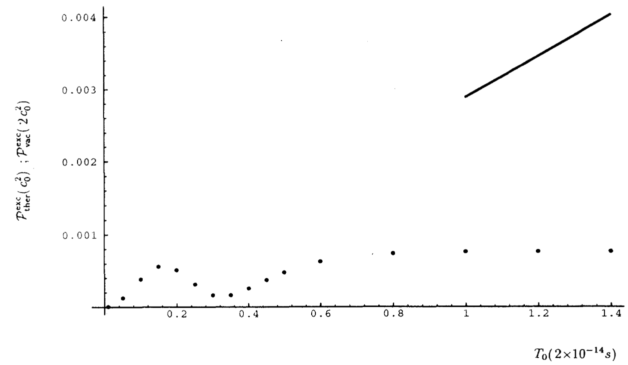

FIG. 3. $\mathcal{P}^{\rm exc}_{\rm vac}$ and $\mathcal{P}^{\rm exc}_{\rm ther}$ are plotted as a function of T_0. $\mathcal{P}^{\rm exc}_{\rm vac}$, which is represented with a full line, increases linearly for large T_0, while $\mathcal{P}^{\rm exc}_{\rm ther}$, which is represented with a dotted line, increases much slower. This is a consequence of the fact that $G^{+}_{\rm ther}$ decreases exponentially for large T_0, except along the "monorail" $\Delta\tau = 0$. Eventually, it reflects the fact that faster the detector moves with respect to the background thermal bath, less the detector interacts with it. Here we have chosen $\beta = 2.5\times10^{-14}$ s, $a = 9.7\times10^{14}$ s^{-1}, and $\Delta E = 9.7\times10^{14}$ s^{-1}.

next paragraph), and plotted as a function of T_0 in Fig. 3. It is clear from this figure that the pure vacuum contribution $\mathcal{P}^{\rm exc}_{\rm vac}$ increases with T_0 much faster than the background thermal contribution $\mathcal{P}^{\rm exc}_{\rm ther}$. Actually, the background thermal bath is only important in a transient initial period when the velocity of the detector is small. This is a consequence of the fact that $G^{+}_{\rm ther}$ decreases exponentially for large T_0 (see Fig. 2), except along the ξ axis. The situation above is clearly nonstationary. Eventually, Fig. 3 just reflects the fact that faster the detector moves less it interacts with the background thermal bath as discussed in Sec. II. In order to obtain a closed form to the background thermal contribution $\mathcal{P}^{\rm exc}_{\rm ther}$ when β^{-1} is small, we can expand $G^{+}_{\rm ther}(\Delta\tau,\xi)$ in terms of β factors. For this purpose, we have used Eq. (1.411.8) of [14] in (3.9) obtaining

$$G^{+}_{\rm ther}(\Delta\tau,\xi) = \frac{1}{12\beta^2} - \sum_{n=2}^{+\infty} \frac{(4\pi)^{2n-2}B_{2n}\sinh^{2n-2}(a\Delta\tau/2)\sinh[(2n-1)a\xi]}{(2n)!a^{2n-2}\beta^{2n}\sinh a\xi}, \tag{3.13}$$

where B_n are the Bernoulli numbers, and $\beta > 2a^{-1}|e^{a\xi}\sinh(a\Delta\tau/2)|$. Substituting (3.13) in (3.12) gives, up to first order,

$$\mathcal{P}^{\rm exc}_{\rm ther} = c_0^2\frac{\sin^2 \Delta E T_0/2}{3\beta^2\Delta E^2} + O(\beta^{-4}). \tag{3.14}$$

The values of ΔE, a, and β used above were *arbitrarily* chosen in this section just to illustrate the role played by the background thermal bath in linearly accelerated detectors. Nevertheless, as explained below, they will have a clear physical motivation in the next section, where we analyze *detectors moving circularly.* Electrons in particle accelerators have their spin coupled to the magnetic field. It induces a splitting of the "spin up," and "spin down" levels. The energy gap associated with such a splitting is $\Delta E = 2|\mu||\mathbf{B}|$, where $\mu \approx e/2m_e$ is the electron's magnetic moment (it is assumed the gyromagnetic factor to be $g = 2$), and $\mathbf{B}$ is the magnetic field. Following [6] we consider the depolarization of an accelerated electron as representing the excitation of the detector, since both ones share the common feature of being two-level systems. Apart from Thomas precession contributions more detailed calculations are not supposed to change the order of magnitude of the results obtained, and consequently our conclusion that the depolarization of accelerated electrons is usually a vacuum effect rather than a background thermal bath effect. At LEP conditions an electron has a typical Lorentz factor of $\gamma \equiv (1-v^2)^{-1/2} = 10^5$, and proper acceleration of $a = 2.9\times10^{23} m/$ s^2, which corresponds to $a = 9.7\times10^{14}$ s^{-1} in natural units. The energy gap between the two spin levels is $\Delta E \approx a = 9.7\times10^{14}$ s^{-1} [6]. The lab time for building up the polarization is about 2 h, which corresponds to 7.2×10^{-2} s in the electron's proper time. Finally, the background thermal bath has a temperature corresponding to $\beta = (300\ {\rm K})^{-1} \approx 2.5\times10^{-14}$ s. It is also interesting to give an approximate numerical value for the coupling constant c_0. Although c_0 does not affect the relative contributions of the background and Unruh thermal baths, it does determine the overall excitation rate. In order to obtain a physical estimate for c_0, we compare the depolarization of an accelerated electron at zero temperature coupled with an electromagnetic field as obtained in Eq. (24) of [6] with our Eq. (3.11). This leads us immediately to

$$c_0 \approx \frac{\sqrt{8}e\Delta E}{m_e} \approx 10^{-6}, \tag{3.15}$$

where e, m_e is the electron's charge and mass, respectively. The important point to notice here is that $c_0 \ll 1$, which corroborates our tree-level calculations.

Before finalizing this section, we recall the importance that the Unruh effect may have in hadronic physics. It may be possible that the large acceleration present in some heavy-ion processes induce the appearance of a relevant Unruh thermal bath in the rest frame of the involved particles producing observable effects. Barshay and Troost [16], for instance, suggested that the large transverse acceleration of projectile and target which occurs in high-energy hadronic collisions is connected with the thermal emission of particles at Unruh temperature of about 100 MeV $\approx 10^{12}$ K. Other recent papers suggesting the possible relevance of the Unruh effect for quarks can be also found [17]. Quarks in bag models can be seen as having a high acceleration corresponding to an Unruh temperature of about 140 MeV. Our results suggest that the background thermal bath may be neglected in these cases. This is so because in the typical regions where heavy ions (or quarks) are free, they have small acceleration, and high velocity. Thus, in this regime both the background and Unruh thermal baths are not important. In the region where the heavy ions (or quarks) interact, the acceleration is very high corresponding to an Unruh temperature of 10^{12} K. Since in the "moment" of the interaction the typical velocities are small, we can compare the Unruh temperature above, 10^{12} K, directly with the background temperature of 300 K. This corroborates the usual procedure of neglecting the background thermal bath in these cases. Although some points like whether the heavy ions (and quarks) have time to thermalize in the Unruh thermal bath deserve a more careful investigation, the results obtained so far are stimulating.

IV. CIRCULARLY MOVING DETECTORS IN A BACKGROUND THERMAL BATH

Now, let us analyze the most favorable case to identify observable effects due to the existence of the Unruh thermal bath: circularly moving detectors. The world line of a detector describing a circular motion with radius R and constant speed v is

$$t = \gamma\tau\ ,\quad x = R\cos\omega\gamma\tau\ ,\quad y = R\sin\omega\gamma\tau\ ,\quad z = 0, \tag{4.1}$$

where $\omega = v/R$. The proper acceleration of the detector is $a \equiv \sqrt{a_\mu a^\mu} = v^2\gamma^2/R$, where $a^\mu = u^\nu\nabla_\nu u^\mu$.

Substituting (4.1) in (3.2), and using (3.5) and (3.6) we decompose the relevant Green function again in a pure vacuum part and in a background thermal part as

$$D^+_\beta(\Delta\tau) = D^+_{\rm vac}(\Delta\tau) + D^+_{\rm ther}(\Delta\tau), \tag{4.2}$$

where

$$D^+_{\rm vac}(\Delta\tau) = (4\pi^2)^{-1}\left[-\gamma^2(\Delta\tau - i\epsilon)^2 + 4v^4\gamma^4\sin^2(a\Delta\tau/2v\gamma)/a^2\right]^{-1}, \tag{4.3}$$

and

$$D^+_{\rm ther}(\Delta\tau) = -\frac{1}{4\pi^2\gamma^2\Delta\tau^2}\left(\left[\frac{4v^4\sin^2(A\Delta\tau/2v)}{A^2\Delta\tau^2} - 1\right]^{-1} + \frac{\pi A\Theta\Delta\tau^2}{4v^2\sin(A\Delta\tau/2v)}\left\{\coth\left[\pi\Theta\Delta\tau \times\left(1 - \frac{2v^2}{A\Delta\tau}\sin\frac{A\Delta\tau}{2v}\right)\right] - (v \to -v)\right\}\right). \tag{4.4}$$

Here $A \equiv a/\gamma$, and $\Theta \equiv \gamma/\beta$ play the role of an effective proper acceleration, and an effective background temperature, respectively. $D^+_{\rm ther}(\Delta\tau)$ is finite everywhere. In particular, $\lim_{\Delta\tau\to 0} D^+_{\rm ther}(\Delta\tau) = 1/12\beta^2$. Asymptotically, $D^+_{\rm ther}(\Delta\tau \gg 1) \sim (4\pi^2\gamma^2\Delta\tau^2)^{-1}$. This guarantees that the detector's response per unit time is finite. The fact that D^+_β does not depend on ξ reflects the fact that this situation is stationary.

In order to calculate the average vacuum excitation rate, $d\mathcal{P}^{\rm exc}_{\rm vac}/dT$, for ultrarelativistic detectors it is convenient to express (4.3) as

$$D^+_{\rm vac}(\Delta\tau) = (4\pi^2)^{-1}[-(\Delta\tau - i\epsilon)^2 - a^2\Delta\tau^4/12 + O(\gamma^{-1})]. \tag{4.5}$$

Substituting (4.5) in (3.1), we obtain, for $\gamma \gg 1$,

$$\frac{d\mathcal{P}^{\rm exc}_{\rm vac}}{dT} \approx \frac{c_0^2 a e^{-\sqrt{12}\Delta E/a}}{4\pi\sqrt{12}}, \tag{4.6}$$

which was first obtained in [7]. Thus at LEP we expect $d\mathcal{P}^{\rm exc}_{\rm vac}/dT \approx 7.0 \times 10^{11} c_0^2$. We can also obtain an approximate physical value for the coupling constant c_0 by comparing (4.6) with Eq. (10) obtained by Jackson [18] for electrons circulating in the vacuum. This leads us to the value $c_0 \approx 10^{-6}$ as in Sec. III.

In order to compare the vacuum contribution (4.6) with the average thermal contribution, $d\mathcal{P}^{\rm exc}_{\rm ther}/dT$, we substitute (4.4) in (3.1) obtaining

$$\frac{d\mathcal{P}^{\rm exc}_{\rm ther}}{dT} = c_0^2\int_{-\infty}^{+\infty} d(\Delta\tau)\, e^{-i\Delta E\Delta\tau} D^+_{\rm ther}(\Delta\tau). \tag{4.7}$$

Evaluating numerically this expression with LEP values we obtain $d\mathcal{P}^{\rm exc}_{\rm ther}/dT \approx 3 \times 10^8 c_0^2$. Thus, after the equilibrium in the polarization distribution is reached the pure vacuum contribution is expected to be about 3 orders of magnitude larger than the background thermal contribution.

Before concluding, we notice that for *"quasi-inertial"* detectors, i.e. $A \ll \Theta, \Delta E$, the background thermal contribution for *circularly* moving detectors (4.7) can be approximated by the background thermal contribution for *inertial* detectors (2.6). (Notice that these two situations are stationary in contrast with the linearly accelerated one.) In particular, in the limit $A \to 0$, (4.7) turns out to be exactly (2.6). This is a consistency check for the results obtained in Secs. II and IV. This is so because a detector moving with some finite velocity v, but with $A \to 0$, means that it is circulating in a ring with arbitrarily large radius. In this case, we must expect the detector to behave as an *inertial* detector moving with the same velocity v, as we have indeed obtained. At LEP conditions we have $A = 9.7 \times 10^9$ s^{-1}, $\Theta = 4.0 \times 10^{18}$ s^{-1}, and $\Delta E = 9.7 \times 10^{14}$ s^{-1}. Since $A \ll \Theta, \Delta E$, as required to consider the detector as "quasiinertial," one could have used directly (2.6) to estimate the average thermal contribution obtaining $d\mathcal{P}^{\rm exc}_{\rm ther}/dT \approx 2.9 \times 10^8 c_0^2$.

V. CONCLUSION

We have derived the response of inertial and accelerated detectors in a background thermal bath. Faster the detector moves, less important will be the background thermal bath. This is so because time dilatation induces the detector to interact only with the low frequency modes present in the bath. Although the thermal bath is rich of low frequency modes, the phase space volume element suppresses infrared contributions in the excitation probability. Bell and Leinaas suggested that the depolarization of electrons in storage rings could be explained through the Unruh effect, i.e., because of the appearance of a thermal bath in the electron's rest frame of about 1200 K. In their analysis it is assumed that the electrons are accelerated in the *inertial vacuum.* We have estimated whether considering the fact that the electrons are actually accelerated in a *background thermal bath* of about 300 K would add or not any substantial contribution in the depolarization rate. We obtain under LEP conditions the interesting result that although the Unruh thermal bath is only about 4 times hotter than the background thermal bath, the term $\mathcal{P}^{\rm exc}_{\rm ther}$ because of the external bath is various orders of magnitude smaller than the pure vacuum contribution $\mathcal{P}^{\rm exc}_{\rm vac}$. Concerning the proposal of testing the Unruh effect in storage rings, it corroborates the usual assumption of considering the electrons as being accelerated in the inertial vacuum [6,10]. According to our results background thermal corrections will be only relevant in nonultrarelativistic situations with moderate acceleration.

ACKNOWLEDGMENTS

We would like to acknowledge discussions with Dr. F. Kokubun in the early stages of this work, and our gratitude to Dr. A. Higuchi for his suggestions in the final manuscript. This article was supported by Coordenadoria de Aperfeiçoamento de Pessoal de Nível Superior (S.C.), and by Conselho Nacional de Desenvolvimento Científico e Tecnológico (G.M.).

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