Isotropization of Bianchi-type cosmological solutions in Brans-Dicke theory

P. Chauvet*
Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa, P.O. Box. 55-534, C. P. 09340 México D. F., Mexico

J. L. Cervantes-Cota†
University of Konstanz, P.O. Box 5560, M 678, D-78434 Konstanz, Germany
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The cosmic, general analytic solutions of the Brans-Dicke theory for the flat Friedmann-Robertson-Walker (FRW) models containing perfect, barotropic, fluids are seen to belong to a wider class of solutions, which includes cosmological models with the open and the closed spaces of the FRW metric, as well as solutions for models with homogeneous but anisotropic spaces corresponding to the Bianchi-type metric classification, when all these solutions are expressed in terms of reduced variables. The existence of such a class lies in the fact that the scaled scalar field $\psi \equiv \phi a^{3(1-\beta)}$ (with $a^3 = a_1 a_2 a_3$ the "volume element" and β the barotropic index, $p = \beta\rho$) can be written as a quadratic function of the scaled time and this solution is independent of the metrics here employed. This reduction procedure permits one to analyze if explicitly given anisotropic cosmological solutions "isotropize" in the course of their time evolution. If this can happen, it could be claimed that there exists a subclass of solutions that is stable under anisotropic perturbations: This seems to be the case for the Bianchi type I, V, and IX.

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I. INTRODUCTION

The first authors to realize the possibility of giving a reason for the viability of the "cosmological principle" without the necessity of imposing highly special initial conditions before the "inflationary program" was developed were Hoyle and Narlikar [1]. However, their explanation came also before the cosmic microwave background (CMB) radiation was discovered, and nowadays most investigators believe that the steady state theory is untenable from the observations. Other investigators within Einstein's general relativity (GR) theory, notably Misner [2], tried to demonstrate, unsuccesfully, that the large-scale structure of the Universe, in particular its isotropy, could be attributed to the nature of the matter processes, such as dissipation, that took place at a very early stage of development of the Universe, independent of its initial conditions (chaotic cosmology), that is, that the Universe lost memory of any initially imposed anisotropy or inhomogeneity (Barrow and Matzner [3], Doroshkevich *et al.* [4], Misner [5], Rees [6], and Zel'dovich and Starobinsky [7]). More to the point, within theoretical cosmology in general and in the context of GR in particular, one is still looking for a satisfactory explanation to the following observational facts: the formation of galaxies and clusters of galaxies, which means that the Universe is not homogeneous locally, and on the other hand the CMB radiation that seems to be very nearly isotropic on account of its Planck spectrum and the lack of structure in its intensity, from which it has been concluded that the large-scale structure of the actual Universe must be homogeneous and isotropic. The standard big bang model, thought to give the most accurate description of the Universe, has the peculiarity that it appears to need a very special set of initial conditions to be viable. This state of affairs has produced several studies in different but related directions to obtain reasonable explanations to this conundrum, such as the "inflationary program" which nowadays is a popular approach, though not without some drawbacks, to solve also some other problems in cosmology (for a present review see Olive [8]). A most important reason why cosmological models that predict inflation in the early Universe are interesting is the hope they will explain the observed state of the Universe without appeal to highly special initial conditions. Even so, most inflationary cosmological models have assumed Friedmann-Robertson-Walker (FRW) symmetry from the outset. The horizon size in the FRW models suggests the possibility that physical interactions could have homogenized and isotropized the Universe, and therefore that its present state could have evolved from more general initial conditions.

In the context of general homogeneous cosmologies Barrow [9] has recently shown that by appealing to the Planck initial conditions for all stress energy densities, the Planck equipartition proposal (PEP), the "isotropy problem" is not present: A high level of CMB isotropy is predicted, in accordance with the bounds made by the Cosmic Background Explorer (COBE) satellite [10].

*Electronic address: pcha@xanum.uam.mx. Fax address: 52-5-724 4611.
†Electronic address: jorge@spock.physik.uni-konstanz.de. Fax address: 49-7531-88 3888.

Therefore, noninflationary theories of structure formation need no longer explain it as a separate issue.

Nevertheless, present observations are still far from elucidating all the properties that an actual model of the Universe should have. Even the degree of anisotropy of the primeval radiation or the counts of radio sources or galaxies in the various directions in the sky involve some uncertainties and agreement between the observed chemical composition and the predictions of the Friedmann models merely signifies that the time rate of change of the volume occupied by matter and the rate of expansion of a Friedmann model concide. On the other hand, astronomical observations seem to imply that isotropization must have occurred at a rather early epoch, maybe even for $z \gg 10^9$. It is feasible that for particular anisotropic models this could be achieved for the instantaneous values of the anisotropy parameters. But the case is that there are observational properties that depend on the degree of anisotropy over an extended period of time, again like the CMB isotropy. Therefore, it is interesting to find cosmological models which indeed tend to isotropize as the Universe evolves.

In GR, without the aid of a cosmological constant or inflation, Collins and Hawking [11] examined the question in terms of an "initial conditions" analysis. They obtained that the set of spatially homogeneous cosmological models approaching isotropy in the limit of infinite times is of measure zero in the space of all spatially homogeneous models, which in turn implies that the isotropy of the models is unstable to homogeneous and anisotropic perturbations. However, their definition of isotropization demands asymptotic stability of the isotropic solution. An asymptotic stability analysis of Bianchi models in GR [12] shows that, i.e., in the Bianchi type VII_h the anisotropy will not exactly vanish but can be bounded. In this sense, there the open FRW model may be stable.

Yet, other authors define the concept of isotropization in a different way (see Novikov [13], Mac Callum [14], and Zel'dovich and Novikov [15]). Therefore, in the literature concerned with the mathematical analysis of anisotropic models the term "isotropization" is often mentioned but its precise definition is author dependent. For Bianchi-type models it is claimed that a positive cosmological constant provides an effective mean of isotropizing homogeneous universes (Wald [16]). The "cosmic no-hair" conjecture has been proved for a number of models (see Refs. [17]), but in several cases it has assumed restrictive conditions (see Ref. [18]). In this context Barrow [19] has shown that contrary to previous expectations, perfect fluid cosmologies need not approach isotropy and homogeneity as $t \to \infty$.

Following the general line of thought on this subject put forward by Zel'dovich and Novikov, one declares that homogeneous cosmological models that isotropize are those that "approach a Friedmann model in the course of time as the Universe expands," which means that "its geometric and dynamic parameters as well as those concerning the distribution and motion of matter and radiation are nearly the corresponding quantities in a Friedmann model." Accordingly, one assumes that the idea conveyed by isotropization is the property that an arbitrary solution possesses to model our Universe which permits it to evolve from an initial, general state, into a state that is presently isotropic on a large scale and is, therefore, well described by a Friedmann solution. We reduce the general scope of the problem by assuming initial homogeneity, limiting ourselves to test the isotropization properties of certain specific noninflationary solutions in the Jordan-Brans-Dicke (JBD) cosmological theory. So, if at its outset the Universe was not in isotropic expansion, the above ideas imply that one can examine, in a first approximation, the properties of homogeneous but anisotropic models assumed to describe correctly the early stages of its expansion. Of these, only those Bianchi-type models whose group type comprises FRW models may isotropize: types I, V, VII_0, VII_h, and IX.

In this paper the above concept of isotropization is dealt with in a direct, but admittedly limited, way by qualifying and quantifying it through a "Raychaudhuri-type" equation common to all Bianchi-type models: Given an explicit solution, one can directly check if it may or may not approach a Friedmann regime in the course of its cosmological time evolution, specifically, if the different anisotropic scale factors of a Bianchi model in the various directions approach arbitrarily near to a Unique, single function of time. By this procedure one can then answer the question, at least for some representative spatially homogeneous models of the Bianchi type (I, V, IX), of whether, and if so, how in the JBD cosmological theory a present large-scale isotropy resulted from an initially anisotropic but homogeneous expanding universe.

II. FRW FIELD EQUATIONS

The JBD field equations for the FRW cosmology with a barotropic, perfect fluid, $p = \beta\rho$, $-1 < \beta < 1$ (the $\beta = \frac{1}{3}$, equation of state for incoherent radiation or ultrarelativistic matter is excluded) are

$$\rho a^{3(1+\beta)} = M_\beta, \qquad M_\beta = \text{const}, \tag{1}$$

$$3(1-\beta)\frac{a'}{a} = \left(\frac{\psi'}{\psi}\right) - \frac{(1-3\beta)m_\beta\eta + \eta_0}{\psi}, \qquad m_\beta = \frac{8\pi M_\beta}{3+2\omega}. \tag{2}$$

The dynamic equation is

$$\left(\frac{\psi''}{\psi}\right) - \frac{[2(2-3\beta)+3(1-\beta)^2\omega]m_\beta}{\psi} = \frac{-6(1-\beta)k}{a^{2(1-3\beta)}}, \qquad k = 0,\ \pm 1, \tag{3}$$

and the constraint equation is

$$\frac{3}{2(1-\beta)}\left(\frac{\psi''}{\psi}\right)-\frac{1}{(1-\beta)^2}\left(\frac{\psi'}{\psi}\right)^2-\frac{(1-3\beta)}{(1-\beta)^2}\left(\frac{(1-3\beta)m_\beta\eta+\eta_0}{\psi}\right)\left(\frac{\psi'}{\psi}\right)$$
$$+\frac{[2-3\beta+\frac{3}{2}(1-\beta)^2\omega]}{(1-\beta)^2}\left(\frac{(1-3\beta)m_\beta\eta+\eta_0}{\psi}\right)^2+\frac{3[2+\omega(1-\beta)(1+3\beta)]m_\beta}{2(1-\beta)\psi}=0, \quad (4)$$

where $\psi \equiv \phi a^{3(1-\beta)}$, ϕ is the JBD scalar field, a is the scale factor, ω the coupling parameter of the theory, η the "cosmic time parameter," η_0 an integration constant, and $()' = \partial_\eta$, where $dt = a^{3\beta}d\eta$ (for details see Chauvet and Pimentel [20] and references therein).

For $k = 0$, Eq. (3) is directly integrated. One gets

$$\psi = A\eta^2 + B\eta + C, \quad (5)$$

where A, B, and C are constants such that

$$A = \left[2 - 3\beta + \frac{3}{2}(1-\beta)^2\omega\right] m_\beta \quad . \quad (6)$$

Substitution of (5) and (6) in the constraint equation (4) hands out the following results. The constant B is undetermined and so, up to B, C also remains undetermined. Therefore, three different possible cosmic solutions to the FRW flat space ($k = 0$) exist distinguished by the sign of the determinant, $\Delta \equiv B^2 - 4AC$, which itself depends on the relation between the equation of state, through β, and the coupling parameter ω in a rather complex way. The behavior of the scalar field ϕ implies, for each type of determinant, $\Delta > 0$, $\Delta < 0$, and $\Delta = 0$, the possible existence of two branches: essentially, ones with ϕ an increasing function of time and the others with ϕ a decreasing function of time (the solutions are given explicitly and have been thoroughly discussed by Gurevich *et al.* [21], Ruban and Finkelstein [22], and Morganstern [23]; see also Chauvet [24] and Lorenz-Petzold [25] and also in other scalar-tensor gravity theories by Chauvet and Pimentel [20] and Barrow and Mimoso [26]).

ϕ is obtained by the straightforward integration of

$$\frac{\phi'}{\phi} = \frac{(1-3\beta)m_\beta\eta + \eta_0}{\psi} \quad (7)$$

and the scale factor a found from it through the definition of ψ:

$$a^{3(1-\beta)} = \frac{A\eta^2 + B\eta + C}{\phi} \quad . \quad (8)$$

Equation (4) does not involve the curvature constant k explicitly and so $\psi = A\eta^2 + B\eta + C$ is also a solution to actually both the open and closed space dynamics Eq. (3) provided that

$$\phi a^{(1+3\beta)} = \frac{2 + (1-\beta)(1+3\beta)\omega}{2(1+3\beta)k} m_\beta \ . \quad (9)$$

The same as in the flat space case A, B, and C are obtained from the constriction equation (4). For both $k = +1$ and -1, it is valid that

$$A = \frac{-(1-3\beta)^2 m_\beta}{(1+3\beta)} \ ,$$
$$B = -2\left(\frac{1-3\beta}{1+3\beta}\right)\eta_0 \ , \quad (10)$$
$$C = -\frac{\eta_0^2}{(1+3\beta)m_\beta} \quad .$$

Its determinant is then

$$\Delta = B^2 - 4AC = 0. \quad (11)$$

The explicit solutions for these two models are

$$\phi = \frac{-1}{(1+3\beta)m_\beta}\left(\frac{[2+(1-\beta)(1+3\beta)\omega]m_\beta{}^2}{-2k}\right)^{\frac{3(1-\beta)}{2(1-3\beta)}} \times[(1-3\beta)m_\beta\eta+\eta_0]^{-\frac{1+3\beta}{1-3\beta}} \quad (12)$$

and

$$a = \left(\frac{-2k}{[2+(1-\beta)(1+3\beta)\omega]m_\beta^2}\right)^{\frac{1}{2(1-3\beta)}} \times[(1-3\beta)m_\beta\eta+\eta_0]^{\frac{1}{1-3\beta}} . \quad (13)$$

Solutions for the JBD nonflat space were previously obtained by several authors (Morganstern [23] Lorenz-Petzold [27] and references therein). Recently, the vacuum and $\beta = 1/3$ solutions for all k in scalar-tensor theories with $\omega(\phi)$ (including JBD) were presented by Barrow [28] and extended for a stiff fluid by Mimoso and Wands [29], where use of the conformal invariance properties of these theories was made.

Next, we present the anisotropic Bianchi field equations in the above variables in order to analyze later their asymptotic solutions.

III. ANISOTROPIC FIELD EQUATIONS

Three extra equations, and simple modifications to the FRW equations (1)–(4) presented above, describe the Bianchi types I, V, and IX examined in this paper. Equations (1) and (2) remain formally the same, while Eq. (3) gets its "curvature" term modified and is then written

$$\left(\frac{\psi''}{\psi}\right) - \frac{[2(2-3\beta)+3(1-\beta)^2\omega]m_\beta}{\psi} = (1-\beta)a^{6\beta}\,{}^*R_j \ . \quad (14)$$

The constriction equation is a "Raychaudhuri type" so that the left hand side of Eq. (4) remains unaltered, but instead of being equal to zero as in the FRW cosmology, it is, in this case,

$$\frac{3}{2(1-\beta)}\left(\frac{\psi''}{\psi}\right)-\frac{1}{(1-\beta)^2}\left(\frac{\psi'}{\psi}\right)^2-\frac{(1-3\beta)}{(1-\beta)^2}\left(\frac{(1-3\beta)m_\beta\eta+\eta_0}{\psi}\right)\left(\frac{\psi'}{\psi}\right)$$
$$+\frac{[2-3\beta+\frac{3}{2}\omega(1-\beta)^2]}{(1-\beta)^2}\left(\frac{(1-3\beta)m_\beta\eta+\eta_0}{\psi}\right)^2+\frac{3[2+\omega(1-\beta)(1+3\beta)]m_\beta}{2(1-\beta)\psi}$$
$$=-(H_1-H_2)^2-(H_2-H_3)^2-(H_3-H_1)^2\equiv\ \sigma(\eta)\ . \quad (15)$$

σ is, for short, the "shear." $\sigma=0$ is a necessary condition to obtain a FRW cosmology since it implies $H_1=H_2=H_3$ (see Chauvet *et al.* [30]). If the sum of the squared differences of the Hubble expansion rates tends to zero, it would mean that anisotropic scale factors tend to a single function of time which is, presumably, the scale factor of a corresponding Friedmann model. However, in general not all Bianchi models contain a FRW space-time.

Three extra equations describe the dynamical evolution of the "anisotropic scale factors" a_1, a_2, and a_3:

$$(\psi H_i)'\ =\ [1+(1-\beta)\omega]m_\beta+\psi\ a^{6\beta}\ {}^*R_{ij},\quad i=1,2,3. \quad (16)$$

From Eqs. (14)–(16), and for the rest of this paper, we use the following notation and conventions: a is presently the mean scale factor, $a^3=a_1a_2a_3$, the H_i's $i=1,2,3$ are the Hubble expansion rates, $H_i=a_i'/a_i$, ${}^*R_j$ is the "spatial three-curvature" that belongs to a given Bianchi-type model, ${}^*R_j=\sum_{i=1}^{3}{}^*R_{ij}$ is a column sum, and the ${}^*R_{ij}$ are "partial curvature" terms pertaining to specific scale factor dynamic equations: in our case,

$$\begin{array}{cccc} & \mathrm{I} & \mathrm{V} & \mathrm{IX} \\ & 0, & 2/a_1^2, & [a_1^4-a_2^4-a_3^4+2a_2^2a_3^2]/(-2a^6), \\ {}^*R_{ij}\ = & 0, & 2/a_1^2, & [a_2^4-a_3^4-a_1^4+2a_1^2a_3^2]/(-2a^6), \\ & 0, & 2/a_1^2, & [a_3^4-a_1^4-a_2^4+2a_1^2a_2^2]/(-2a^6). \end{array} \quad (17)$$

In the next section it is first shown that $\psi=A\eta^2+B\eta+C$ includes solutions for the above, purposely chosen homogeneous but anisotropic, cosmological models. It will be shown that the obtained ψ class of solutions consists of two parts: the isotropic and the anisotropic one. Then, it will be clear that the latter approaches zero as the cosmic time parameter evolves, i.e., these solutions tend asymptotically to their corresponding isotropic group solutions, the FRW models.

IV. ANISOTROPIC SOLUTIONS AND THEIR ASYMPTOTIC BEHAVIOR

We show next that $\psi\ =\ A\eta^2+B\eta+C$ is a solution for the homogeneous but anisotropic, Bianchi-type cosmological models.

A. Bianchi type I

For this Bianchi-type model, ${}^*R_{\mathrm{I}}=0$. It is straightforward to see by substituting

$$\psi=A\eta^2+B\eta+C$$

into Eq. (14) that A has the same expression as the one given by Eq. (6) (from now on we attach a subindex to the A, B, and C to distinguish between the different Bianchi models):

$$A_{\mathrm{I}}\ =\ [2-3\beta+\tfrac{3}{2}(1-\beta)^2\omega]m_\beta\quad . \quad (18)$$

By direct substitution of the above results into Eq. (15) one finds that B_{I} remains undetermined and may be put equal to any convenient, but arbitrary value, and that

$$\begin{aligned}3(1-\beta)^2(3+2\omega)m_\beta\,C_{\mathrm{I}}&\\ =-\frac{(1-\beta)^2}{\omega^3}(h_1^2+h_2^2+h_3^2)-m_\beta\eta_0^2A_{\mathrm{I}}+B_{\mathrm{I}}^2&\\ +(1-3\beta)m_\beta\eta_0B_{\mathrm{I}}\ ,&\end{aligned} \quad (19)$$

where the h_i's are constants such that

$$H_i=\frac{1}{3}\frac{a^{3'}}{a^3}+\frac{h_i}{\psi}\quad . \quad (20)$$

The nonvanishing constants h_i's determine the anisotropic character of the solutions. They obey the condition

$$h_1+h_2+h_3=0. \quad (21)$$

By integration of Eq. (20), using Eq. (2) and Eq. (5), one finds explicitly $a\ =\ a(\eta)$ (first obtained by Ruban and Finkelstein [22]; see also Chauvet and Guzmán [31]).

For the Bianchi models in general, when Eq. (5) is substituted into the Raychaudhuri equation (15), one obtains the shear as a function of ψ and the h_i's (with $h_1 + h_2 + h_3 = 0$),

$$\sigma(\eta) = -\frac{3(h_1^2 + h_2^2 + h_3^2)}{\psi^2} \ . \tag{22}$$

This term permits, in addition to the only one allowed $\Delta = 0$ solution for the $k \neq 0$ FRW models, two other solutions with $\Delta \neq 0$ such that[1] $\sigma \to 0$ as $\eta \to \infty$ (or $t \to \infty$). It is in this sense that these solutions may evolve in the course of their time evolution to their isotropy solutions, depending on the value of the discriminant, discussed in Ref. [22] (note a mathematical characteristic of the anisotropic solutions shown by the above results, i.e., the relation between the exponents $h_{i's}$ and the B and C coefficients). Similar results have been recently reported in the context of more general, $\omega(\phi)$, scalar-tensor gravity theories; see Ref. [33].

B. Bianchi type V

Equation (5) is a solution for this Bianchi model, with A, B, and C equal to

$$\begin{aligned} A_{\rm v} &= -\frac{(1-3\beta)^2 m_\beta}{(1+3\beta)} \ , \\ B_{\rm v} &= -2\left(\frac{1-3\beta}{1+3\beta}\right)\eta_0 \ . \end{aligned} \tag{23}$$

$A_{\rm v}$ and $B_{\rm v}$ are equal to the ones obtained for the isotropic, $k = \pm 1$ cases, but

$$m_\beta\,(1+3\beta)\,C_{\rm v} = -\frac{(1+3\beta)^2(h_1^2+h_2^2+h_3^2)}{18\beta + \omega(1+3\beta)^2} - \eta_0^2 \ . \tag{24}$$

so that

$$\phi = \left[\frac{[2+(1-\beta)(1+3\beta)\omega]m_\beta}{-2(1+3\beta)}\right]^{\frac{3(1-\beta)}{2(1-3\beta)}} \times \left[A_{\rm v}\eta^2 + B_{\rm v}\eta + C_{\rm v}\right]^{-\frac{1+3\beta}{2(1-3\beta)}} \tag{25}$$

and

$$H_1 = \frac{1}{3}\frac{a^{3\prime}}{a^3} = \frac{a_1'}{a_1} = -\frac{1}{(1+3\beta)}\frac{(1-3\beta)m_\beta\eta + \eta_0}{(A_{\rm v}\eta^2 + B_{\rm v}\eta + C_{\rm v})} \ . \tag{26}$$

[1]In fact, one does not need to impose this condition, but just $\eta \gg \eta_*$ to warrant that the σ can be bounded from above. The value η_* should be such that it corresponds to times before nucleosynthesis takes place (or much earlier); otherwise, the standard nucleosynthesis can be altered (see Ref. [32]). Nevertheless, by assuming the PEP, this is even not necessary [9].

The scale factors are

$$a_1 = \left[\frac{-2(1+3\beta)}{[2+(1-\beta)(1+3\beta)\omega]m_\beta}\right]^{\frac{1}{2(1-3\beta)}} \times \left[A_{\rm v}\eta^2 + B_{\rm v}\eta + C_{\rm v}\right]^{\frac{1}{2(1-3\beta)}} \tag{27}$$

and

$$a_2 = a_1 \exp\left[\frac{-2h_2}{\sqrt{\Delta}}\,\mathrm{arctanh}\left(\frac{-2(1-3\beta)[(1-3\beta)m_\beta\eta + \eta_0]}{(1+3\beta)\sqrt{\Delta}}\right)\right] \ , \quad \Delta > 0, \tag{28}$$

or

$$a_2 = a_1 \exp\left[\frac{2h_2}{\sqrt{-\Delta}}\,\mathrm{arctan}\left(\frac{-2(1-3\beta)[(1-3\beta)m_\beta\eta + \eta_0]}{(1+3\beta)\sqrt{-\Delta}}\right)\right] \ , \quad \Delta < 0, \tag{29}$$

with

$$a_2 a_3 = a_1^2 \ . \tag{30}$$

These solutions are new (some other Bianchi type-V solutions in the JBD theory are also given in Ref. [34]). However, for $\Delta = 0$,

$$a_3 = a_2 = a_1 \tag{31}$$

is obtained. This last solution is clearly seen to be the one previously obtained for the isotropic, FRW model, with an open space ($k = -1$). Again, the $\Delta = 0$ is obtained only if $h_1 = h_2 = h_3 = 0$.

Independent of the value Δ might have, $h_1 + h_2 + h_3 = 0$ is always true. In the type-V models with $\Delta \neq 0$ one must have that $h_2 = -h_3$ with $h_1 = 0$. For the latter case, truly anisotropic solutions are obtained with

$$\Delta = B_{\rm v}^2 - 4A_{\rm v}C_{\rm v} = \frac{-8(1-3\beta)^2}{18\beta + (1+3\beta)^2\omega} h_2^2 \ . \tag{32}$$

$C_{\rm v}$, being proportional to the sum of the squares of the constants h_2 and h_3, carries information concerning the nature of the anisotropic character of this Bianchi-type model. For instance, by setting $B_{\rm v} = C_{\rm v} = 0$ from the beginning and by trying to solve the field equations one obtains just the isotropic part of the solutions; see Ref. [31].

Since Eq. (22) holds for all type-V models, the $\Delta \neq 0$ solutions could have had asymptotic behaviors to call them "nearly isotropic in appearance" if $\sigma \to 0$, when $\eta \to \infty$. In this regard one finds that for the $\Delta < 0$ solution a_2 tends to $a_1 \exp(\pi h_2/\sqrt{-\Delta})$ and a_3 tends to $a_1 \exp(-\pi h_2/\sqrt{-\Delta})$, and so when η reaches the value $(100\sqrt{-\Delta} - B_{\rm v})/2A_{\rm v}$ these two scale factors differ from each other by 1%, and this solution is then "99% near" the $\Delta = 0$ solution, corresponding to the FRW cosmology for $k = -1$ [see Eqs. (12) and (13)]; a similar behavior was also found for $\omega(\phi)$ theories; see Ref. [33]. On the other hand, for the $\Delta > 0$ model the scale factors can never approach to a same, single, function of η, because η is bounded; see Eq. (28). Nevertheless, it is significant that this last can be an inflationary solution without the need of an inflaton potential; however, the "no-hair conjecture" is not automatically satisfied, since the solutions are highly anisotropic. Then, the sign of the discriminant determines the type of solution. In GR the Bianchi type-V $\beta = 1/3$ (Ruban) solution tends to isotropize as the time goes to infinity. This is consistent with the asymptotical stability analysis for which the perturbated vacuum (Joseph) solution is asymptotically stable if $\beta > -1/3$; see Ref. [12] and references therein. Here, in the JBD Bianchi type-V model to find isotropizable solutions one must have $\Delta < 0$, implying that $18\beta + (1+3\beta)^2\omega > 0$, which for the experimentally required $\omega > 500$, is clearly satisfied. Indeed, not only the fluid type (β) but also ω plays a very important role in the solution's behavior.

C. Bianchi type IX

We analyze the solution for this model with the H_i's given by Eq. (20). However, in this case the h_i's cannot be constants. Instead, the h_i's$= h_i(\eta)$'s are now new, and unknown, functions of η.

With Eq. (20) substituted into Eq. (16), one must solve for

$$h_i' = a^{6\beta}\psi\ {}^*R_{i\,\mathrm{IX}} - \frac{\psi'' - [2(2-3\beta) + 3(1-\beta)^2\omega]m_\beta}{3(1-\beta)}, \qquad i = 1,2,3. \tag{33}$$

The sum of the above three equations is

$$a^{6\beta}\psi\ {}^*R_{\mathrm{IX}} = \frac{\psi'' - [2(2-3\beta) + 3(1-\beta)^2\omega]m_\beta}{(1-\beta)}\ , \tag{34}$$

given explicitly in terms of a_1, a_2, and a_3,

$$\frac{a_1^4 + a_2^4 + a_3^4 - 2(a_1^2a_2^2 + a_1^2a_3^2 + a_2^2a_3^2)}{2a^{6(1-\beta)}} = \frac{\psi'' - [2(2-3\beta) + 3(1-\beta)^2\omega]m_\beta}{(1-\beta)\psi}\ , \tag{35}$$

from which any chosen scale factor can be solved as function of the other two remaining ones.

Now, assuming the solution for ψ, given by Eq. (5), also to be valid, one has from Eq. (15) that

$$h_1^2 + h_2^2 + h_3^2 \equiv K^2 = -\frac{\omega^3}{2(1-\beta)^2}\left[P\eta^2 + Q\eta + S\right]\ , \tag{36}$$

where P, Q, and S are constants, given in terms of A_{IX}, B_{IX}, and C_{IX}, which stand for

$$\begin{aligned} P &= XA_{\mathrm{IX}} - [4A_{\mathrm{IX}} - Y](1-3\beta)^2 m_\beta\ , \\ Q &= XB_{\mathrm{IX}} - [4A_{\mathrm{IX}}\eta_0 - 2Ym_\beta\eta_0 + 2(1-3\beta)B_{\mathrm{IX}}](1-3\beta)m_\beta\ , \\ S &= XC_{\mathrm{IX}} - [2\Delta + 2(1-3\beta)m_\beta\eta_0 B_{\mathrm{IX}} - Ym_\beta^2\eta_0^2]\ , \end{aligned} \tag{37}$$

where

$$\begin{aligned} X &= 3(1+3\beta)(1-\beta)^2\omega m_\beta + 6(1-\beta)m_\beta - 2(1+3\beta)A_{\mathrm{IX}}\ , \\ Y &= 2(2-3\beta) + 3(1-\beta)^2\omega\ . \end{aligned} \tag{38}$$

The isotropic model solution that belongs to this Bianchi-type model is obtained when $\Delta = 0$, where one has that $h_1(\eta) = h_2(\eta) = h_3(\eta) = 0$ (see Sec. II).

Under any circumstance the functions h_i's, which must still obey the condition $h_1 + h_2 + h_3 = 0$, determine the anisotropic character of the solutions.

The h_i's can be given as

$$h_1 = -\left[\frac{\kappa^2 + 4\kappa + 1}{\sqrt{6}(\kappa^2 + \kappa + 1)}\right]K,$$

$$h_2 = \left[\frac{-\kappa^2 + 2\kappa + 2}{\sqrt{6}(\kappa^2 + \kappa + 1)}\right]K, \tag{39}$$

and

$$h_3 = \left[\frac{2\kappa^2 + 2\kappa - 1}{\sqrt{6}(\kappa^2 + \kappa + 1)}\right] K,$$

where now κ is another, new and yet unknown, function of η: Unfortunately, for $\Delta \neq 0$ we were not able to obtain the explicit functional dependence of $\kappa = \kappa(\eta)$. Even so, an asymptotic isotropic behavior, similar to other models, for the present solutions is also expected based on the strength of Eqs. (22) and (36).

V. DISCUSSION AND CONCLUSIONS

The JBD cosmological equations for barotropic perfect fluids are seen capable of being displayed, through the use of reduced variables, in a way which first permits one to obtain nontrivial, significant solutions with little effort and next, but more importantly, to express them in terms of the single function $\psi = A\eta^2 + B\eta + C$. The fact is that the aforementioned solutions belong to a class which embraces Bianchi-type models, some of which, in turn, comprise the FRW isotropy groups. Moreover, stated explicitly this class contains the general (analytic) matter solutions for the Bianchi type-I model as well as solutions for the other two Bianchi types examined in this paper, which in turn include special ones, a subclass, that tend asymptotically, as $\eta \gg \eta_*$, to the corresponding FRW solutions. The reason for the existence of this set is that the functional form of the product of the scalar field ϕ times a power of the mean scale factor, $a^3 = a_1 a_2 a_3$, as a function of the time parameter η is a solution to the equations used in this work independent of the metrics that give rise to any possible present anisotropy for Bianchi types-I, -V, and -IX models, and it has the FRW form. In other words, for a perfect fluid with a barotropic equation of state, we have shown that there exists a class of solutions for the Bianchi I, V, and IX types that contain their corresponding FRW models. The type-V solutions are new, as well as those for the type-IX, but in the latter case because of the complexity of the curvature terms it is only possible to give the explicit form, in terms of η, of the scale factors a_i's up to the single, unknown, function $\kappa = \kappa(\eta)$. Nevertheless, if $\eta \gg \eta_*$, an asymptotic isotropic behavior for a $\Delta \neq 0$ solution should be expected in view of Eqs. (22) and (36). Moreover, there are also other solutions obtained from ψ, through Eq. (5), which describe other Bianchi-type models; see Chauvet and Guzmán [31]. All of the above are salient and remarkable properties of the class of solutions that we have found. Even more, the ψ solutions also include the asymptotic solutions, the type-I models ($\Delta = 0, > 0$, and < 0), for all Bianchi types near the initial singularity, when spatial curvature terms can be neglected. They are in this sense, up to the matter terms, comparable to the Kasner vacuum solution of GR.

We want to note that in JBD theory the separate cosmological models are to be distinguished between themselves in a first instance, through the different values that the constants A, B, and C obtain. Also significant is the fact that B and C carry physical information on the nature of the presence, or even the absence, of the anisotropy that any given models may have. Remember that these constants also determine the value of the discriminant Δ in terms of the physical parameters β and ω. We stress the fact that solutions for the Bianchi models corresponding to $\Delta = 0$ that possess an isotropy group recover the FRW solutions: Type I goes into the corresponding flat FRW one, type V into the open FRW one, and the type IX into the closed FRW one. Meanwhile, for the anisotropic solutions with $\Delta \neq 0$ the models isotropize in the course of their time evolution by tending in the type-I case to its corresponding FRW model, while for types V and IX their evolution is toward the $\Delta = 0$ solution which is the only one available to describe a nonflat FRW cosmology in the JBD context.

The present observational evidence points to a high degree of isotropy of the CMB radiation, and it is a decisive argument in favor of the nowadays large-scale homogeneous and isotropic expansion of the Universe. Then, if the initial stages of the expansion had a homogeneous but anisotropic behavior, one could follow, within the JBD cosmological theory, how an actual nearly isotropic expansion can come about.

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