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New asymptotic expansion method for the Wheeler-DeWitt equation

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A new asymptotic expansion method is developed to separate the Wheeler-DeWitt equation into the time-dependent Schrödinger equation for a matter field and the Einstein-Hamilton-Jacobi equation for the gravitational field including the quantum back reaction of the matter field. In particular, the nonadiabatic basis of the generalized invariant for the matter field Hamiltonian separates the Wheeler-DeWitt equation completely in the asymptotic limit of m_P^2 approaching infinity. The higher order quantum corrections of the gravity to the matter field are found. The new asymptotic expansion method is valid throughout all regions of superspace compared with other expansion methods with a certain limited region of validity. We apply the new asymptotic expansion method to the minimal FRW universe.

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I. INTRODUCTION

Recently quantum field theory in a curved spacetime for matter fields has been studied in the context of quantum cosmology. In quantum cosmology one derives, from the Wheeler-DeWitt (WDW) equation, a tentative quantum gravity theory, the quantum field theory for the matter fields in the curved space which is the time-dependent Schrödinger equation, a Tomonaga-Schwinger functional equation. The time-dependent Schrödinger equation for matter fields has an advantage over the canonical quantization on the solution space of wave equation for the matter fields in that one can take into account the higher order quantum corrections of the gravity to the matter fields and the quantum back reaction of the matter fields to the gravity. Banks [1] showed that the semiclassical limit (WKB approximation) of the WDW equation for pure gravity was the Einstein-Hamilton-Jacobi (EHJ) equation for gravity, which turned out equivalent to the classical Einstein equation [2]. In Ref. [3] the induced gauge potential due to some cosmological mode was considered in semiclassical gravity. Semiclassical gravity was extended to quantum cosmological models for gravity coupled to matter fields by including the expectation value, a back reaction, of the matter fields to the EHJ equation [4–8]. In a quantum cosmological model for gravity coupled to matter fields whose mass scale is much smaller than the Planck mass, there is not only the back reaction of the matter fields to the EHJ equation but also the geometric phases to the quantum states of the matter fields [9–13].

In this paper we develop a new asymptotic expansion method which separates the WDW equation for gravity coupled to a matter field into the time-dependent Schrödinger equation for the matter field and the EHJ equation for gravity including the quantum back reaction of the matter field. When one expands with respect to some basis of quantum states for the matter field and introduces a cosmological time the WDW equation is equivalent to a matrix equation which involves cosmological time-dependent elements. In particular it is shown in the nonadiabatic basis of the eigenstates of the generalized invariant for the matter field Hamiltonian that the WDW equation is equivalent to the matrix equation which consists of dominant diagonal elements and nondiagonal perturbation elements proportional to the asymptotic parameter $1/m_P^2$. In other words we are able to separate the WDW equation into the time-dependent matrix equation for the matter field and the EHJ equation with the back reaction of the matter field. We obtain the higher order quantum corrections of gravity to the matter field as a power series of $1/m_P^2$. We also compare the new asymptotic expansion method with other related works. Finally, we apply the new asymptotic expansion method to the minimal FRW universe.

The organization of this paper is as follows. In Sec. II we introduce the new asymptotic expansion method for the WDW equation. In Sec. III the higher order quantum corrections of gravity to the matter field are obtained. In Sec. IV we compare the new asymptotic expansion method with other methods used to derive the time-dependent Schrödinger equation for the matter field. Finally in Sec. V we apply the new asymptotic expansion method to the Friedmann-Robertson-Walker universe minimally coupled to a free massive scalar field. The quantum back reaction of the matter field and the EHJ equation are found explicitly using the generalized invariant for a well-known time-dependent harmonic oscillator.

II. NEW ASYMPTOTIC EXPANSION METHOD

With the fundamental constants such as c the speed of light, $\hbar$ the Planck constant, and G the gravitational constant, inserted explicitly, the WDW equation takes the form

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$$\left[-\frac{\hbar^2}{2m_P^2}G_{ab}\frac{\delta^2}{\delta h_a \delta h_b} - 2m_P^2 c^2 \sqrt{h}\,{}^{(3)}R(h_a) + \hat{H}_m(\pi_\phi, \phi, h_a)\right]\Psi(h_a, \phi) = 0, \quad (2.1)$$

where $m_P^2 = c^2/32\pi G$ is the Planck mass squared and G_{ab} is the symmetrized supermetric on the superspace of the three-geometry. There are two asymptotic parameters $1/\hbar \to \infty$ and $m_P^2 \to \infty$ for the WDW equation, which can be compared with just one asymptotic parameter $1/\hbar \to \infty$ for the Schrödinger equation in quantum mechanics. The asymptotic limit $m_P^2 \to \infty$ can be considered as the limiting case either of $G \to 0$ or $c^2 \to \infty$ or both of them. We shall consider here only the weak gravitational coupling limit $G \to 0$ without any loss of generality.

The method in this paper is neither to evolve the wave functions of the WDW equation from some Cauchy initial data [14] nor to separate the matrix effective gravitational field Hamiltonian equation by expanding the wave functions with respect to some basis for the matter field Hamiltonian [12]. The key point of our method is to separate asymptotically the Schrödinger equation for the matter field by introducing a cosmological time via the gravitational action which in turn includes the quantum back reaction of the matter field. This can be achieved by setting the wave function in the form

$$\Psi(h_a, \phi) = \exp\left[\frac{i}{\hbar}S(h_a)\right]\Phi(\phi, h_a). \quad (2.2)$$

Once we use the asymptotic parameter $1/\hbar \to \infty$ in the WDW equation just as for the WKB expansion in quantum mechanics, then we obtain the intermediate equation

$$\left[\frac{1}{2m_P^2}G_{ab}\left(2i\hbar\frac{\delta S}{\delta h_{(a}}\frac{\delta}{\delta h_{b)}} - \frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b} + \hbar^2\frac{\delta^2}{\delta h_a \delta h_b} - i\hbar\frac{\delta^2 S}{\delta h_a \delta h_b}\right) + 2m_P^2 c^2\sqrt{h}\,{}^{(3)}R(h_a) - \hat{H}_m(\pi_\phi, \phi, h_a)\right]\Phi(\phi, h_a) = 0, \quad (2.3)$$

where S is the gravitational action to be determined consistently later on and (a, b) denotes symmetrization with respect to the indices a and b. Assuming that the prefactor of the wave function in Eq. (2.2) oscillates and is peaked around a classical trajectory, we may first introduce the cosmological time along the classical trajectory in this oscillatory region of the superspace by

$$\frac{\delta}{\delta\tau} \equiv \frac{1}{m_P^2}G_{ab}\frac{\delta S}{\delta h_{(a}}\frac{\delta}{\delta h_{b)}}, \quad (2.4)$$

and then rewrite Eq. (2.3) as

$$\left[i\hbar\frac{\delta}{\delta\tau} - \hat{H}_m(\pi_\phi, \phi, h_a) + 2m_P^2 c^2\sqrt{h}\,{}^{(3)}R(h_a) - \frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b} - i\frac{\hbar}{2m_P^2}G_{ab}\frac{\delta^2 S}{\delta h_a \delta h_b} + \frac{\hbar^2}{2m_P^2}G_{ab}\frac{\delta^2}{\delta h_a \delta h_b}\right]\Phi(\phi, h_a) = 0. \quad (2.5)$$

It should be remarked that Eq. (2.5) can be interpreted as the exact time-dependent Schrödinger equation for the matter field including the gravitational quantum back reaction, since if we neglect the last two terms in Eq. (2.5), which is valid in the limit $1/m_P^2 \to 0$, it is nothing but the conventional time-dependent Schrödinger equation [1,4,15]

$$i\hbar\frac{\delta}{\delta\tau}\Phi(\phi, h_a) = \hat{H}_m(\pi_\phi, \phi, h_a)\Phi(\phi, h_a) \quad (2.6)$$

provided that the gravitational action satisfies the conventional EHJ equation

$$\frac{1}{2m_P^2}G_{ab}\frac{\delta S^{(0)}}{\delta h_a}\frac{\delta S^{(0)}}{\delta h_b} - 2m_P^2 c^2\sqrt{h}\,{}^{(3)}R(h_a) = 0\,. \quad (2.7)$$

The second to the last term $-i(\hbar/2m_P^2)G_{ab}(\delta^2 S/\delta h_a \delta h_b)$ not only is asymptotically small as a power of $1/m_P^2$ but also violates the unitarity of matter field equation, so we shall not consider this term in this paper.

Our method to be developed below differs from others in that instead of taking the conventional EHJ equation in Eq. (2.7) to obtain the conventional time-dependent Schrödinger equation in Eq. (2.6) we treat the exact time-dependent Schrödinger equation in Eq. (2.5), expand the exact quantum states of the matter field by some basis, and finally solve the matrix equation equivalent to Eq. (2.5). For the sake of simplicity, we shall use the ket-bra vector notation for the quantum states of the matter field, and act the bra vector on the ket vector to denote the inner product. The exact quantum state in Eq. (2.5) for the matter field can be expanded by some basis $|\Phi_k(\phi, h_a)\rangle$ which constitutes a complete set of orthonormal vectors as

$$\Phi(\phi, h_a) = \sum_k c_k(h_a)\,|\Phi_k(\phi, h_a)\rangle\,, \quad (2.8)$$

where $c_k(h_a)$ are coefficient functions of the gravitational field only. Both $c_k(h_a)$ and $|\Phi_k(\phi, h_a)\rangle$ are still unknowns which will be determined systematically later on. Then Eq. (2.5) is equivalent to the matrix equation

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a) + \Omega^{(0)}(h_a)c_k(h_a) + \sum_n \left[-\Omega^{(1)}_{kn}(h_a) + \Omega^{(2)}_{kn}(h_a) + \frac{\hbar^2}{2m_P^2}\Omega^{(3)}_{kn}(h_a)\right] \times c_n(h_a) = 0, \quad (2.9)$$

where

$$\Omega^{(0)}(h_a) = -\frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b} + 2m_P^2c^2\sqrt{h}^{(3)}R(h_a),$$
$$\Omega^{(1)}_{kn}(h_a) = \langle\Phi_k(\phi, h_a)|\,\hat{H}_m(\pi_\phi, \phi, h_a)\,|\Phi_n(\phi, h_a)\rangle,$$
$$\Omega^{(2)}_{kn}(h_a) = i\hbar\Big\langle\Phi_k(\phi, h_a)\Big|\frac{\delta}{\delta\tau}\Big|\Phi_n(\phi, h_a)\Big\rangle,$$
$$\Omega^{(3)}_{kn}(h_a) = G_{ab}\Big\langle\Phi_k(\phi, h_a)\Big|\frac{\delta^2}{\delta h_a\delta h_b}\Big|\Phi_n(\phi, h_a)\Big\rangle + G_{ab}\frac{\delta^2}{\delta h_a\delta h_b}\delta_{kn} - 2i(A_{(a})_{kn}\frac{\delta}{\delta h_{b)}}, \quad (2.10)$$

where

$$(A_a)_{kn} = i\Big\langle\Phi_k(\phi, h_a)\Big|\frac{\delta}{\delta h_a}\Big|\Phi_n(\phi, h_a)\Big\rangle. \quad (2.11)$$

The task of solving the WDW equation (2.1) is now reduced to that of solving the matrix equation (2.9). However, in the asymptotic limit of $1/m_P^2 \to 0$, we may neglect the last term of summation in Eq. (2.9) and get the approximate equation

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a) + \Omega^{(0)}c_k(h_a) + \sum_n \left[-\Omega^{(1)}_{kn}(h_a) + \Omega^{(2)}_{kn}(h_a)\right] \times c_n(h_a) = 0. \quad (2.12)$$

On the other hand, if one introduces a generalized invariant obeying the invariant equation

$$\frac{d}{d\tau}\hat{I}(h_a) = \frac{\partial}{\partial\tau}\hat{I}(h_a) - \frac{i}{\hbar}\left[\hat{I}(h_a), \hat{H}_m(h_a)\right] = 0 \quad (2.13)$$

for the matter field Hamiltonian, then there is a well-known decoupling theorem [16] for the generalized invariant such that

$$\Omega^{(1)}_{kn}(h_a) = \Omega^{(2)}_{kn}(h_a) \quad (2.14)$$

for different eigenstates, $k \neq n$, of the generalized invariant

$$\hat{I}(h_a)\,|\Phi_k(\phi, h_a)\rangle = \lambda_k\,|\Phi_k(\phi, h_a)\rangle. \quad (2.15)$$

Since all of the off-diagonal equations in Eq. (2.12) vanish, we have only the diagonal equations left

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a) + \left[\Omega^{(0)}(h_a) - \Omega^{(1)}_{kk}(h_a) + \Omega^{(2)}_{kk}(h_a)\right]c_k(h_a) = 0. \quad (2.16)$$

Recollecting that the gravitational action S is still undetermined, we may take a simple ansatz of the form

$$\Omega^{(0)}(h_a) - \Omega^{(1)}_{kk}(h_a) = 0, \quad (2.17)$$

and

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a) + \Omega^{(2)}_{kk}(h_a)c_k(h_a) = 0. \quad (2.18)$$

Equation (2.17) is nothing but the familiar EHJ equation with the quantum back reaction of the matter field included

$$\frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b} - 2m_P^2c^2\sqrt{h}\;^{(3)}R(h_a) + \langle\Phi_k(\phi, h_a)|\,\hat{H}_m(\pi_\phi, \phi, h_a)\,|\Phi_k(\phi, h_a)\rangle = 0. \quad (2.19)$$

It should be noted that the EHJ equation (2.19) is an implicitly coupled nonlinear equation in which the kth eigenstate is defined by the generalized invariant (2.13), which in turn is defined using the cosmological time (2.4) through the gravitational action S. This means that the gravitational action does depend on the mode number k, i.e., $S_{(k)}$, and so does the cosmological time $\tau_{(k)}$. The physical implication is that there are an infinite number of gravitational actions $S_{(k)}$, where k runs over all quantum numbers of the generalized invariant which satisfies Eq. (2.19). The cosmological time $\tau_{(k)}$ is defined along each gravitational action $S_{(k)}$. It was shown in Ref. [14] that there is a spectrum of infinite number of wave functions that depend on the modes. The gravitational action $S_{(k)}$ corresponds to the wave function $\Psi_{(k)}$ and the cosmological time is defined along the classical trajectory around the peak of wave function. Now, the wave function (2.2) becomes

$$\Psi_{(k)}(h_a, \phi) = \exp\left[\frac{i}{\hbar}S_{(k)}(h_a)\right]\Phi_{(k)}(\phi, h_a) \quad (2.20)$$

and so does the quantum state of the matter field

$$\Phi_{(k)}(\phi, h_a) = \sum_n c_n(h_a)\,|\Phi_{(k)n}(\phi, h_a)\rangle. \quad (2.21)$$

We find the solution to Eq. (2.18),

$$c^{(0)}_k(h_a) = d_k \exp\left[\frac{i}{\hbar}\int\Omega^{(2)}_{kk}(h_a)d\tau_{(k)}\right], \quad (2.22)$$

where d_k is a constant. It was implicitly assumed that all the Eqs. (2.9) through (2.19) should depend on the mode number (k); we shall, however, work with the (k) mode only and drop the mode number hereafter throughout this paper.

III. HIGHER ORDER QUANTUM CORRECTIONS

We now turn to the time-dependent Schrödinger equation (2.5) for the matter field with the gravitational action, S and thereby the cosmological time τ determined by Eq. (2.19) and Eq. (2.4), respectively. The time-dependent Schrödinger equation, the gravitational action, and the cosmological time are all defined along the (k)th mode, whose mode number will be omitted. Its matrix equivalent in Eq. (2.9) now satisfies

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a) + \Omega^{(2)}_{kk}(h_a)c_k(h_a) + \frac{\hbar^2}{2m_P^2}\sum_n \Omega^{(3)}_{kn}(h_a)c_n(h_a) = 0. \quad (3.1)$$

By substituting

$$c_k(h_a) = \exp\left[\frac{i}{\hbar}\int \Omega^{(2)}_{kk}(h_a)d\tau\right]\tilde{c}_k(h_a) \quad (3.2)$$

into Eq. (3.1) we obtain the equation

$$i\hbar\frac{\delta}{\delta\tau}\tilde{c}_k(h_a) + \frac{\hbar^2}{2m_P^2}\sum_n \tilde{\Omega}^{(3)}_{kn}(h_a)\tilde{c}_n(h_a) = 0, \quad (3.3)$$

where

$$\tilde{\Omega}^{(3)}_{kn}(h_a) = \exp\left[-\frac{i}{\hbar}\int \Omega^{(2)}_{kk}(h_a)d\tau\right]\Omega^{(3)}_{kn}(h_a) \times \exp\left[\frac{i}{\hbar}\int \Omega^{(2)}_{nn}(h_a)d\tau\right]. \quad (3.4)$$

The simple ansatz (2.18) was a solution to the asymptotic approximate diagonal equation (2.16) and, therefore, should be regarded as the zeroth order solution

$$\tilde{c}^{(0)}_k(h_a) = d_k, \quad (3.5)$$

to Eq. (3.1) with the EHJ equation (2.19) still satisfied. We may find the solution for Eq. (3.3) perturbatively in a power series of $\hbar/2m_P^2$:

$$\tilde{c}_k = \sum_n \left(\frac{\hbar}{2m_P^2}\right)^n \tilde{c}^{(n)}_k, \quad (3.6)$$

where they satisfy the recursion equation

$$i\frac{\delta}{\delta\tau}\tilde{c}^{(l)}_k(h_a) + \sum_n \tilde{\Omega}^{(3)}_{kn}(h_a)\tilde{c}^{(l-1)}_n(h_a) = 0. \quad (3.7)$$

The solution to the recursion equation is the ordered lth multiple integral

$$\tilde{c}^{(l)}_k = i^l \sum_{n_i, i=1,\dots,l} \int \tilde{\Omega}^{(3)}_{kn_1}(h_a)\int \tilde{\Omega}^{(3)}_{n_1n_2}(h_a) \times\cdots\int \tilde{\Omega}^{(3)}_{n_{l-1}n_l}(h_a)\tilde{c}^{(0)}_{n_l}. \quad (3.8)$$

For an instance, when an initial data $\tilde{c}^{(0)}_n = d_k\delta_{kn}$, d_k = const, is imposed, the quantum state of the matter field including the gravitational correction up to the first order is

$$\Phi(\phi, h_a) \sim d_k \exp\left[\frac{i}{\hbar}\int \Omega^{(2)}_{kk}(h_a)d\tau\right]\left[|\Phi_k(\phi, h_a)\rangle + i\frac{\hbar}{2m_P^2}\sum_n \int \tilde{\Omega}^{(3)}_{kn}(h_a)d\tau\,|\Phi_n(\phi, h_a)\rangle\right]. \quad (3.9)$$

Thus the first order transition to the other states from the initially prepared state is suppressed by a factor of $1/m_P^2$ because there comes a factor of $\hbar$ from the integration and similarly the nth order transition is suppressed by a factor of $(1/m_P^2)^n$. It is to be noted that the procedure employed in this paper is quite similar to the perturbation method for a quantum system with a small perturbation term; here the parameter for the smallness of the perturbation term is $\hbar/2m_P^2$ and Eq. (3.8) is the perturbative series for the given choice of the basis of eigenstates of the generalized invariant, which is also the solution for the unperturbed equation (2.6).

IV. COMPARISON WITH OTHER RELATED WORKS

In this section, we shall compare the result of this paper with other related works.

First, we shall answer in part the question whether the conventional EHJ equation (2.7) or the EHJ equation (2.19) is right through the investigation of a quantum cosmological model. Suppose a quantum cosmological model with the gravitational super-Hamiltonian

$$\hat{H}_g(h_a) = \frac{1}{2m_P^2}G_{ab}\hat{\pi}^a\hat{\pi}^b - 2m_P^2c^2\sqrt{h}\ {}^{(3)}R(h_a). \quad (4.1)$$

When coupled to the matter field, the gravitational super-Hamiltonian (4.1) leads to the WDW equation (2.1). Quantizing the gravitational Hamiltonian Eq. (4.1) by substituting $\hat{\pi}^a = (\hbar/i)(\delta/\delta h_a)$, taking the wave function of the form $\Psi(h_a) \sim \exp[iS(h_a)/\hbar]$, and keeping dominant terms only, we just obtain the conventional EHJ equation

$$\frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b} - 2m_P^2c^2\sqrt{h}\ {}^{(3)}R(h_a) = 0, \quad (4.2)$$

which is the same as Eq. (2.7). Finally, putting the conventional EHJ equation (4.2) and neglecting the last two terms in Eq. (2.5), we obtain the time-dependent Schrödinger equation

$$i\hbar\frac{\delta}{\delta\tau}\Phi(\phi, h_a) = \hat{H}_m(\pi_\phi, \phi, h_a)\Phi(\phi, h_a). \quad (4.3)$$

We found the exact quantum states in terms of the eigenstate of the generalized invariant [12]:

$$\Phi(\phi, h_a) = \sum_k d_k \exp\left[\frac{i}{\hbar}\int \Omega^{(2)}_{kk}(h_a) d\tau\right] |\Phi_k(\phi, h_a)\rangle. \quad (4.4)$$

It is to be noted that the conventional EHJ equation (4.2) is equal to the approximation $\Omega^{(0)}(h_a) = 0$, and the gravitational field-dependent coefficients in Eq. (4.4) can also be determined by directly integrating Eq. (2.16) with both $\Omega^{(0)}(h_a) = 0$ and $\Omega^{(1)}(h_a) = 0$ substituted. However, the lowest order solution (2.18) of the coefficient function does not depend on the gravitational field, i.e., is a constant and the higher order solution (3.8) depends on the gravitational field. The significant difference on the coefficient functions comes from the fact that in this paper we have used the EHJ equation (2.19) with the quantum back reaction of the matter field rather than the conventional EHJ equation (4.2). Below we put forth a criterion on whether Eq. (2.19) or Eq. (4.2) should be used for the correct gravitational action.

In order to show the relation between the EHJ equation (2.19) and the conventional EHJ equation (4.2), we expand the action (2.19) perturbatively in the inverse power of the Planck mass:

$$S_k = \sum_n m_P^{2(1-n)} S_k^{(n)}, \quad (4.5)$$

whose lowest order action obeys

$$O(m_P^2) : \frac{1}{2} G_{ab} \frac{\delta S_k^{(0)}}{\delta h_a} \frac{\delta S_k^{(0)}}{\delta h_b} - 2c^2 \sqrt{h}\; {}^{(3)}R(h_a) = 0, \quad (4.6)$$

and the first two higher order actions obey

$$O(m_P^0) : G_{ab} \frac{\delta S_k^{(0)}}{\delta h_{(a}} \frac{\delta S_k^{(1)}}{\delta h_{b)}} - \Omega^{(1)}_{kk} + \Omega^{(2)}_{kk} = 0,$$

$$O(m_P^{-2}) : G_{ab} \left(\frac{\delta S_k^{(0)}}{\delta h_{(a}} \frac{\delta S_k^{(2)}}{\delta h_{b)}} + \frac{1}{2} \frac{\delta S_k^{(1)}}{\delta h_a} \frac{\delta S_k^{(1)}}{\delta h_b} \right) = 0. \quad (4.7)$$

It is worthy to note that the lowest order contribution (4.6) gives nothing but the conventional EHJ equation (4.2). So the above question whether Eq. (2.19) or Eq. (4.2) should be used is closely related to the question whether the asymptotic expansion of the gravitational action (4.5) gives the correct gravitational action.

However, contrary to the belief widely accepted that the conventional EHJ equation (4.2) gives the correct gravitational action, we give a counterexample showing that the asymptotic expansion (4.5) leads to a wrong gravitational action. In the case of the Friedmann-Robertson-Walker universe minimally coupled to a scalar field with a power-law potential, the WDW equation takes the form

$$\left[\frac{\hbar^2}{2m_P^2} \frac{1}{a} \frac{\partial^2}{\partial a^2} - 2m_P^2 c^2 a - \hbar^2 \frac{1}{a^3} \frac{\partial^2}{\partial \phi^2} + 2a^3 U(\phi) \right] \Psi(a, \phi) = 0, \quad (4.8)$$

from which it follows that $G_{aa} = -1/a$ and $\hat{H}_m(\phi, a) = -\hbar^2 \partial^2 / a^3 \partial \phi^2 + 2a^3 U(\phi)$. The EHJ equation (4.2) becomes

$$\frac{1}{2m_P^2} \frac{1}{a} \left(\frac{\partial S}{\partial a} \right)^2 + 2m_P^2 c^2 a = 0. \quad (4.9)$$

Direct integration by quadrature yields the gravitational action $S(a) = \pm i m_P^2 c a^2$ and the wave function

$$\Psi(a, \phi) = \exp\left(\pm \frac{1}{\hbar} m_P^2 c a^2 \right) \Phi(\phi, a). \quad (4.10)$$

Likewise, the lowest order gravitational action (4.6) also has the same value $S^{(0)} = \pm i c a^2$. Therefore, the cosmological time Eq. (2.4) leads to an imaginary one $\tau = \pm i \ln(a/2c)$. Both the conventional EHJ equation (4.2) and the dominant term of the asymptotic expansion of the gravitational action (4.5) always lead to the wave functions with an exponential behavior due to the curvature term, the second term in Eq. (4.8). However, both in the adiabatic basis method [17] which expands the wave functions by the gravitational field-dependent eigenfunctions of the matter field Hamiltonian and in the superadiabatic expansion method [14] in which transitions among different eigenstates are taken into account during the evolution of the Universe, the resulted wave functions show not only the exponential behavior for a large three-geometry but also the oscillatory behavior for an intermediate three-geometry depending on the quantum number of the matter field Hamiltonian. The oscillatory behavior of wave functions is inevitable to the classical Lorentzian universe such as the present Friedmann-Robertson-Walker universe. Therefore, in the case of the minimally coupled Friedmann-Robertson-Walker universe it can be inferred that the conventional EHJ equation (4.2) has a certain limited region of the large three-geometry for the validity, whereas the EHJ equation (2.19) holds not only for the large three-geometry prevailing with the curvature term but also for the intermediate three-geometry prevailing with the quantum back reaction of the matter field. The EHJ equation (2.19) should be used in order to give the correct gravitational action valid for all the regions of superspace.

Second, we shall compare the new asymptotic expansion method with the adiabatic expansion method. Quite similarly as in the new asymptotic expansion method in which one expands the quantum state of matter field by the nonadiabatic basis of the eigenstates of the generalized invariant, in the adiabatic expansion method one expands the quantum state of the matter field by the adiabatic basis of the instantaneous eigenstates of the matter field Hamiltonian itself and includes the quantum back reaction of the matter field with respect to the adiabatic basis. By defining the instantaneous eigenstates

$$\hat{H}_m(h_a) |\Phi_k(\phi, h_a)\rangle_{ad} = H_k(h_a) |\Phi_k(\phi, h_a)\rangle_{ad}, \quad (4.11)$$

and expanding the quantum state

$$\Phi(\phi, h_a) = \sum_k c_{ad,k}(h_a) |\Phi_k(\phi, h_a)\rangle_{ad}, \quad (4.12)$$

one obtains the matrix equation

$$i\hbar\frac{\delta}{\delta\tau}c_{ad,k}(h_a)+\Omega^{(0)}(h_a)c_{ad,k}(h_a)+\sum_n\left[-\Omega^{(1)}_{ad,kn}(h_a)+\Omega^{(2)}_{ad,kn}(h_a)+\frac{\hbar^2}{2m_P^2}\Omega^{(3)}_{ad,kn}(h_a)\right]c_{ad,n}(h_a)=0, \quad (4.13)$$

where

$$\Omega^{(0)}(h_a)=-\frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b}+2m_P^2c^2\sqrt{h}\ {}^{(3)}R(h_a),$$

$$\Omega^{(1)}_{ad,kn}(h_a)={}_{ad}\left\langle\Phi_k(\phi,h_a)\right|\hat{H}_m(\pi_\phi,\phi,h_a)\left|\Phi_n(\phi,h_a)\right\rangle_{ad},$$

$$\Omega^{(2)}_{ad,kn}(h_a)=i\hbar\ {}_{ad}\left\langle\Phi_k(\phi,h_a)\middle|\frac{\delta}{\delta\tau}\middle|\Phi_n(\phi,h_a)\right\rangle_{ad},$$

$$\Omega^{(3)}_{ad,kn}(h_a)=G_{ab}\ {}_{ad}\left\langle\Phi_k(\phi,h_a)\middle|\frac{\delta^2}{\delta h_a\delta h_b}\middle|\Phi_n(\phi,h_a)\right\rangle_{ad}+G_{ab}\frac{\delta^2}{\delta h_a\delta h_b}\delta_{kn}-2i(A_{(ad,a})_{kn}\frac{\delta}{\delta h_{b)}}, \quad (4.14)$$

where

$$(A_{ad,a})_{kn}=i_{ad}\left\langle\Phi_k(\phi,h_a)\middle|\frac{\delta}{\delta h_a}\middle|\Phi_n(\phi,h_a)\right\rangle_{ad}. \quad (4.15)$$

Again, in the asymptotic limit of $1/m_P^2\to 0$, we may neglect the last term in Eq. (4.13) and get the approximate equation

$$i\hbar\frac{\delta}{\delta\tau}c_{ad,k}(h_a)+\Omega^{(0)}(h_a)c_{ad,k}(h_a)+\sum_n\left[-\Omega^{(1)}_{ad,kn}(h_a)+\Omega^{(2)}_{ad,kn}(h_a)\right]c_n(h_a)=0. \quad (4.16)$$

It should, however, be remarked that in the adiabatic expansion method the off-diagonal elements of the coupling matrix do not vanish, $\Omega^{(1)}_{kn}\neq\Omega^{(2)}_{kn}$ for $k\neq n$, in strong contrast with those in the nonadiabatic expansion method. Therefore, one should solve the whole adiabatic approximate matrix equation (4.16) with the elements of the coupling matrix accounting for transition between different eigenstates instead of the frequently used adiabatic diagonal equation in Eq. (4.16):

$$i\hbar\frac{\delta}{\delta\tau}c_k(h_a)+\left(\Omega^{(0)}(h_a)-\Omega^{(1)}_{kk}(h_a)+\Omega^{(2)}_{kk}(h_a)\right)c_k(h_a)=0. \quad (4.17)$$

Because $\Omega^{(1)}_{ad,kn}$ for $k\neq n$, have an order of magnitude comparable to $\Omega^{(1)}_{ad,kk}$, it is not justified to use the adiabatic diagonal equation (4.17), which is frequently used in the literature under the assumption that the off-diagonal elements be neglected. From Eq. (4.17) one also obtains the frequently used EHJ equation with the adiabatic quantum back reaction of the matter field:

$$\frac{1}{2m_P^2}G_{ab}\frac{\delta S}{\delta h_a}\frac{\delta S}{\delta h_b}-2m_P^2c^2\sqrt{h}\ {}^{(3)}R(h_a)+{}_{ad}\left\langle\Phi_k(\phi,h_a)\right|\hat{H}_m(\pi_\phi,\phi,h_a)\left|\Phi_k(\phi,h_a)\right\rangle_{ad}=0. \quad (4.18)$$

Here we have the same subtlety as in Sec. II that the EHJ equation with the adiabatic quantum back reaction depends on the mode number k. So we have the gravitational action $S_{(k)}$ and the cosmological time $\tau_{(k)}$. The difference between the EHJ equation (4.18) and the EHJ equation (2.19) is that in the former the quantum back reaction of the matter field is explicitly given, whereas in the latter it is implicitly determined via the cosmological time which is defined by the gravitational action.

Third, there has been a recent study of geometric phases as a mechanism for the asymmetry of the cosmological time [12]. In particular, in the basis of eigenstates of the generalized invariant one may define a gauge potential (Berry connection)

$$A_a(h_a)=i\vec{U}^*(\phi,h_a)\frac{\delta}{\delta h_a}\vec{U}^T(\phi,h_a), \quad (4.19)$$

where

$$\vec{U}(\phi,h_a)=\begin{pmatrix}|\Phi_0(\phi,h_a)\rangle\\|\Phi_1(\phi,h_a)\rangle\\\vdots\\|\Phi_n(\phi,h_a)\rangle\\\vdots\end{pmatrix} \quad (4.20)$$

is a column vector, and the asterisk * and the superscript ^{T} denote dual and transpose operations, respectively. One can show that

$$\frac{\delta}{\delta\tau}\vec{U}(\phi,h_a)=-i\frac{1}{m_P^2}G_{ab}\frac{\delta S}{\delta h_{(a}}A^T_{b)}\vec{U}(\phi,h_a), \quad (4.21)$$

and

$$G_{ab}\frac{\delta}{\delta h_a}\frac{\delta}{\delta h_b}\vec{U}(\phi,h_a)=G_{ab}\left[\frac{\delta A_b^T}{\delta h_a}+(A_aA_b)^T\right]\vec{U}(\phi,h_a). \quad (4.22)$$

Then the matrix equivalent (2.9) to the WDW equation is entirely determined by the gravitational action and the gauge potential. It should be remarked again that the matter field Hamiltonian gives not only a back reaction to the EHJ equation (2.19) but also the geometric phase term

$$\langle\Phi_k(\phi,h_a)|\,i\hbar\frac{\delta}{\delta\tau}\,|\Phi_k(\phi,h_a)\rangle=\frac{\hbar}{m_P^2}G_{ab}\frac{\delta S}{\delta h_{(a}}\left(A_{b)}\right)_{kk} \quad (4.23)$$

to the quantum state of the matter field. It should also be noted that the gauge potential defined in terms of the eigenstates of the generalized invariant does always give nontrivial diagonal elements in strong contrast with the gauge potential defined in terms of instantaneous eigenstates of the Hamiltonian whose diagonal elements vanish for real eigenstates. On the other hand, when the gravitational field Hamiltonian (4.1) acts on the wave function expanded by the eigenstates of the generalized invariant as

$$\Psi(h_a,\phi)=\vec{U}^T(\phi,h_a)\cdot\vec{\Psi}(h_a), \quad (4.24)$$

it becomes the matrix nonadiabatic gravitational Hamiltonian equation and acquires the induced gauge potential (4.19) [12]:

$$\hat{H}_g \vec{\Psi}(h_a) = \left[\frac{1}{2m_P^2} G_{ab}(\hat{\pi}^a - A_a)(\hat{\pi}^b - A_b) - 2m_P^2 c^2 \sqrt{h}\, {}^{(3)}R(h_a)\right] \vec{\Psi}(h_a). \tag{4.25}$$

It was shown that there was a remarkable decoupling theorem as mentioned earlier canceling the off-diagonal gauge potential and the expectation value of the matter field Hamiltonian at the classical level. After the matter field Hamiltonian is included, the total Hamiltonian $\hat{H}_g + \hat{H}_m$ acting on the kth wave function

$$\Psi_k(h_a, \phi) = \exp\left(\frac{i}{\hbar} S_k(h_a)\right) |\Phi_k(\phi, h_a)\rangle \tag{4.26}$$

leads to

$$\frac{1}{2m_P^2} G_{ab} \frac{\delta S_k}{\delta h_a} \frac{\delta S_k}{\delta h_b} - \frac{1}{m_P^2} G_{ab} \frac{\delta S_k}{\delta h_{(a}} (A_{b)})_{kk} + \frac{1}{2m_P^2} G_{ab} (A_a)_{kk} (A_b)_{kk}$$

$$- 2m_P^2 c^2 \sqrt{h}\, {}^{(3)}R(h_a) + \langle \Phi_k(\phi, h_a) | \hat{H}_m(\pi_\phi, \phi, h_a) | \Phi_k(\phi, h_a)\rangle = 0. \tag{4.27}$$

This equation can be rewritten as

$$\frac{1}{2m_P^2} G_{ab} \left(\frac{\delta S_k}{\delta h_a} - A_a\right) \left(\frac{\delta S_k}{\delta h_b} - A_b\right) - 2m_P^2 c^2 \sqrt{h} + \langle \Phi_k(\phi, h_a) | \hat{H}_m(\pi_\phi, \phi, h_a) | \Phi_k(\phi, h_a)\rangle = 0. \tag{4.28}$$

Equation (4.27) is the diagonal equation of Eq. (2.9) with the wave function (4.26) and the zeroth order solution c_k = const. In our new asymptotic expansion of the WDW equation what corresponds to the gauge potential is resigned to the matrix equation (2.9) for the matter field.

V. MINIMAL FRW UNIVERSE

We consider in detail the Friedmann-Robertson-Walker universe minimally coupled to a free massive scalar field, whose WDW equation is

$$\left[\frac{\hbar^2}{2m_P^2} \frac{1}{a} \frac{\partial^2}{\partial a^2} - 2m_P^2 c^2 a - \hbar^2 \frac{1}{a^3} \frac{\partial^2}{\partial \phi^2} + m^2 a^3 \phi^2\right] \Psi(a, \phi) = 0. \tag{5.1}$$

Here m is the mass of the scalar field and the matter field Hamiltonian is given by

$$\hat{H}_m = \frac{1}{a^3} \hat{\pi}_\phi^2 + m^2 a^3 \phi^2 . \tag{5.2}$$

In order to find the generalized invariant, the first thing to do is to find the classical equation of motion for Eq. (5.2):

$$\ddot{\phi}(\tau) + 3\frac{\dot{a}(\tau)}{a(\tau)} \dot{\phi}(\tau) + 4m^2 \phi(\tau) = 0, \tag{5.3}$$

where the cosmological time will be determined later on through Eq. (2.4). The cosmological scale factor a and the scalar field ϕ depend implicitly on the cosmological time. Under the assumption that the classical solutions $\phi_1(\tau)$ and $\phi_2(\tau)$ to Eq. (5.3) are given explicitly, it is known that the generalized invariant is given by [18]

$$\hat{I}(\tau) = g_-(\tau) \frac{\hat{\pi}_\phi^2}{2} + g_0(\tau) \frac{\hat{\pi}_\phi \hat{\phi} + \hat{\phi} \hat{\pi}_\phi}{2} + g_+(\tau) \frac{\hat{\phi}^2}{2}, \tag{5.4}$$

where

$$g_-(\tau) = c_1 \phi_1^2(\tau) + c_2 \phi_1(\tau) \phi_2(\tau) + c_3 \phi_2^2(\tau),$$

$$g_0(\tau) = -\frac{a^3(\tau)}{2} \left\{ c_1 \phi_1(\tau) \dot{\phi}_1(\tau) + \frac{c_2}{2} \left[\dot{\phi}_1(\tau) \phi_2(\tau) + \phi_1(\tau) \dot{\phi}_2(\tau)\right] + c_3 \phi_2(\tau) \dot{\phi}_2(\tau) \right\},$$

$$g_+(\tau) = \left[\frac{a^3(\tau)}{2}\right]^2 \left[c_1 \dot{\phi}_1^2(\tau) + c_2 \dot{\phi}_1(\tau) \dot{\phi}_2(\tau) + c_3 \dot{\phi}_2^2(\tau)\right]. \tag{5.5}$$

We may introduce the cosmological time-dependent creation and annihilation operators of the generalized invariant

$$\hat{b}^\dagger(\tau) = \left[\sqrt{\frac{\omega_0}{2g_-(\tau)}} - i\sqrt{\frac{1}{2\omega_0 g_-(\tau)}}g_0(\tau)\right]\hat{\phi}$$
$$-i\sqrt{\frac{g_-(\tau)}{2\omega_0}}\hat{\pi}_\phi,$$
$$\hat{b}(\tau) = \left[\sqrt{\frac{\omega_0}{2g_-(\tau)}} + i\sqrt{\frac{1}{2\omega_0 g_-(\tau)}}g_0(\tau)\right]\hat{\phi}$$
$$+i\sqrt{\frac{g_-(\tau)}{2\omega_0}}\hat{\pi}_\phi, \qquad (5.6)$$

where

$$\omega_0 = \sqrt{g_+(\tau)g_-(\tau) - g_0^2(\tau)} \qquad (5.7)$$

is a constant of motion. The generalized invariant can be written as

$$\hat{I}(\tau) = \omega_0\left[\hat{b}^\dagger(\tau)\hat{b}(\tau) + \frac{1}{2}\right]. \qquad (5.8)$$

The eigenstates are the number states

$$|n,\tau\rangle = \frac{1}{\sqrt{n!}}\hat{b}^{\dagger n}(\tau)\,|0,\tau\rangle\,, \qquad (5.9)$$

where the ground state is annihilated by the cosmological time-dependent annihilation operator

$$\hat{b}(\tau)\,|0,\tau\rangle = 0. \qquad (5.10)$$

In the number state representation the gauge potential (4.19) reads that

$$A(\tau) = i\alpha(\tau)\left[\hat{b}^\dagger(\tau)\hat{b}(\tau) + \frac{I}{2}\right]$$
$$+\frac{i}{2}\left[\beta(\tau)\hat{b}^2(\tau) - \beta^*(\tau)\hat{b}^{\dagger 2}\right], \qquad (5.11)$$

where

$$\alpha(\tau) = \frac{1}{2i}\frac{g_-(\tau)}{\omega_0}\frac{\partial}{\partial\tau}\left[\frac{g_0(\tau)}{g_-(\tau)}\right],$$
$$\beta(\tau) = -\frac{1}{2}\frac{1}{g_-(\tau)}\frac{\partial g_-(\tau)}{\partial\tau} + \alpha(\tau). \qquad (5.12)$$

The gauge potential (5.11) has the matrix notation

$$A_{kn}(\tau) = i\alpha(\tau)\left(n + \frac{1}{2}\right)\delta_{k,n}$$
$$+\frac{i}{2}\beta(\tau)\sqrt{n(n-1)}\delta_{k,n-2}$$
$$-\frac{i}{2}\beta^*(\tau)\sqrt{(n+1)(n+2)}\delta_{k,n+2}, \qquad (5.13)$$

and the gauge potential squared

$$A^2_{kn}(\tau) = \left[\left(-\alpha^2 + \frac{1}{2}\beta\beta^*\right)(n^2+n) + \frac{1}{2}\beta\beta^* - \frac{1}{4}\alpha^2\right]\delta_{k,n}$$
$$-\frac{1}{2}\alpha\beta(2n-1)\sqrt{n(n-1)}\delta_{k,n-2} + \frac{1}{2}\alpha\beta^*(2n+3)\sqrt{(n+1)(n+2)}\delta_{k,n+2}$$
$$-\frac{1}{4}\beta^2\sqrt{n(n-1)(n-2)(n-3)}\delta_{k,n-4} + \frac{1}{4}\beta^{*2}\sqrt{(n+1)(n+2)(n+3)(n+4)}\delta_{k,n+4}. \qquad (5.14)$$

One can show that

$$\Omega^{(1)}_{kk}(\tau) = \hbar\frac{2\left[\omega_0^2 + g_0^2(\tau)\right] + 2m^2a^6g_-^2(\tau)}{2\omega_0 a^3 g(\tau)}\left(k + \frac{1}{2}\right), \qquad (5.15)$$

$$\Omega^{(2)}_{kk}(\tau) = -\frac{\hbar}{2}\frac{g_-(\tau)}{\omega_0}\frac{\partial}{\partial\tau}\left[\frac{g_0(\tau)}{g_-(\tau)}\right]\left(k + \frac{1}{2}\right), \qquad (5.16)$$

$$\Omega^{(3)}_{kn}(\tau) = \frac{a}{\left(\frac{\partial S}{\partial a}\right)^2}\left(i\frac{\partial A_{kn}}{\partial\tau} - A^2_{kn}\right) - i\frac{\partial}{\partial\tau}\left(\frac{a}{\frac{\partial S}{\partial a}}\right)A_{kn} - \frac{1}{a}\frac{\partial^2}{\partial a^2}\delta_{kn} - 2iA_{kn}\frac{\partial}{\partial a}. \qquad (5.17)$$

Ωs are functions of the cosmological time (2.4) through the dependence of the classical solutions (5.3) and thereby the generalized invariant (5.5) on $a(\tau)$. Therefore Eq. (2.19) which now reads

$$-\frac{1}{2m_P^2}\frac{1}{a}\left(\frac{\partial S}{\partial a}\right)^2 - 2m_P^2c^2a + \Omega^{(1)}_{kk}(\tau) = 0, \qquad (5.18)$$

together with the definition of the cosmological time

$$\frac{\partial}{\partial\tau} = -\frac{1}{a}\frac{\partial S}{\partial a}\frac{\partial}{\partial a}, \qquad (5.19)$$

determines the action S as a function of $a(\tau)$.

VI. SUMMARY AND DISCUSSION

In summary, we have developed a new asymptotic expansion method according to which the Wheeler-DeWitt equation was separated into the Einstein-Hamilton-Jacobi equation with the quantum back reaction of the matter field included and the time-dependent Schrödinger equation for the matter field. In the new asymptotic expansion the Wheeler-DeWitt equation was equivalent to two coupled nonlinear functional equations consisting of the cosmological time (2.4) and of the time-dependent Schrödinger equation (2.5) for the matter field or the matrix representation (2.9) of it. The time-dependent Schrödinger equation or the matrix equation for the matter field included quantum gravitational corrections. In particular we have found the exact quantum state (2.8) in the nonadiabatic basis of eigenstates of the generalized invariant for the matter field Hamiltonian by solving the matrix equation whose solution consists of Eq. (2.17) and Eqs. (3.6) and (3.8). The zeroth order quantum state in Eq. (2.8) was the nonadiabatic basis itself, which motivated the use of the generalized invariant to solve the time-dependent Schrödinger equation in Ref. [12]. It was found that the quantum corrections of gravity gave rise to transition of quantum states of the matter field whose first order transition rate (3.9) was suppressed by the factor of $1/m_P^2$, and the nth order transition rate by the factor of $(1/m_P^2)^n$. Moreover, Eq. (2.17) was nothing but the Einstein-Hamilton-Jacobi equation (2.19) with the quantum back reaction of the matter field included. The Einstein-Hamilton-Jacobi equation is an implicitly coupled nonlinear equation for the gravitational action by which the cosmological time (2.4) used to define the generalized invariant is defined. Since the quantum back reaction of the matter field depends on the mode number, the cosmological time as well as the gravitational action do depend on the mode number. The physical meaning of the mode-dependent gravitational actions is that the time-dependent Schrödinger equation (2.5) or its matrix equation (2.9) should be defined along the corresponding gravitational action. In fact there are infinite number of wave functions which are peaked along the gravitational action [14]. The higher order quantum corrections of the gravity to the matter field in Secs. III and V are calculated along a specific mode-number gravitational action.

It has been shown through an investigation of the minimally coupled Friedmann-Robertson-Walker universe that the Einstein-Hamilton-Jacobi equation (2.19) gives indeed the correct gravitational action rather than the conventional Einstein-Hamilton-Jacobi equation (4.2) providing an oscillatory regime necessary for the emergence of the Lorentzian universe. The new asymptotic expansion method based on the generalized invariant has the advantage that one has already cancelled the off-diagonal terms between two different eigenstates due to the remarkable decoupling theorem of the generalized invariant, whereas one has to take care of the off-diagonal terms between two different instantaneous eigenstates of the matter field Hamiltonian in the conventional adiabatic expansion method. Furthermore, by introducing the gauge potential (Berry connection) (4.19) we were able to express explicitly the gravitational correction (4.22) as well as the back reaction of the matter field.

Finally, we have applied the new asymptotic expansion method to the Friedmann-Robertson-Walker universe with a minimal scalar field. The generalized invariant was found in terms of the classical solutions of the matter Hamiltonian, from which the back reaction (5.15), the geometric phase (5.16), and the coupling matrix (5.17) of the gravitational corrections are derived explicitly.

Considering an analogy, if any, between a quantum cosmological model of gravitational field and matter fields and a quantum system of heavy particles and light particles, the gravitational field and heavy particles behave as slow variables, and the matter fields and light particles behave as fast variables [12]. In the quantum jargon of the fast and slow variables the new asymptotic expansion method provides us with a very systematic method to separate one equation for the fast variables and the other equation for the slow variables. The fast variables obey a parameter-dependent quantum mechanical equation whose parameter is determined by the slow variables and the slow variables satisfy a classical Einstein-Hamilton-Jacobi equation with the quantum back reaction of the fast variables. In quantum cosmological models the gravitational field with the Planck mass scale behaves as a heavy particle obeying classical Einstein-Hamilton-Jacobi equation with the quantum back reaction of the matter fields which consists of the gauge potential as well as the expectation value of the matter field Hamiltonian. The time-dependent Schrödinger equation for the matter field can also be solved using the generalized invariant. The free massive scalar fields in quantum cosmological models have been solved in Ref. [12] and have provided a mechanism for the cosmological entropy production during an expansion and recollapse of the Universe. It is a new feature of the new asymptotic expansion of the Wheeler-DeWitt equation that the geometric phases are an inevitable consequence to the quantum states of the matter field as well as the expectation value of the matter field Hamiltonian to the Einstein-Hamilton-Jacobi equation which is the counterpart of the classical Einstein-Hamilton-Jacobi equation. The gauge potential in the nonadiabatic basis cannot be gauged away even for the real eigenstates. However, the gauge potential can always be gauged away for the adiabatic basis of real eigenstates of the matter field Hamiltonian in the conventional expansion method. Thus the argument [12] that the cosmological time asymmetry may have origin in the geometric phase still survives.

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