Nature of the decay $\varphi(1020) \to \rho(770)\pi$

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The arguments based on QCD suggest that direct transition could be equally a probable source of the decay $\varphi(1020) \to \rho(770)\pi \to \pi^+\pi^-\pi^0$ as conventional $\varphi\omega$ mixing. Modern data on the decays φ, $\omega \to e^+e^-$ are shown to point to an essential role of direct transition.

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At present, new investigations of the $\varphi(1020)$ meson decays with the detector CMD-2 [1] are under way at an upgraded e^+e^- facility VEPP-2M in Novosibirsk. Another detector, SND, aiming to study the same energy range should soon be put into operation at the same facility. The φ factory DAΦNE in Frascati will, probably, start operation in the near future. In connection with these developments, we would like to raise to a new level the old classic problem of the decay $\varphi(1020) \to \rho(770)\pi$.

The meson $\varphi(1020)$ composed of mainly an $s\bar{s}$ quark pair lies at the borderline between the domains of the light [$u\bar{u}$, $d\bar{d}$] and heavy [$c\bar{c}$, $b\bar{b}$] quarkonia. Hence, the mechanism of three-gluon annihilation, which qualitatively explains the Okubo-Zweig-Iizuka [OZI] rule [2–4] decays of heavy quarkonia J/ψ and $\Upsilon(1S)$ into the hadrons containing no heavy quarks, cannot be excluded for this state. In fact, let us apply to the φ meson the expression for the three-gluon width [5]:

$$\Gamma(\varphi \to 3g) = \frac{10}{9\pi}(\pi^2 - 9)\frac{\alpha_s^3(m_\varphi)}{\alpha^2}\Gamma(\varphi \to e^+e^-), \quad (1)$$

where $\alpha = 1/137$, s quarks are supposed to be heavy. Compare the result with the partial width of the OZI-violating decay $\varphi(1020) \to \rho(770)\pi$. Then one obtains that the branching ratio of the decay $\varphi(1020) \to \rho(770)\pi \to \pi^+\pi^-\pi^0$ corresponds to the QCD coupling constant $\alpha_s(m_\varphi) = 0.43$, which agrees with the QCD estimate, namely, $\alpha_s(m_\varphi) = 0.43$ at $\Lambda = 200$ MeV. [In the case of $\Lambda = 100$ MeV, the partial width of the three-gluon annihilation amounts to 1/3 of the $\varphi(1020) \to \rho(770)\pi \to 3\pi$ partial width.] Note that the $\rho\pi$ is the only final hadronic state not suppressed by the phase space volume, to which three gluons can convert, e.g., by means of a process shown in Fig. 1(a), exactly in the same manner as takes place in the decays of heavy quarkonia. The precise magnitude of the probability of this conversion cannot be evaluated at present. Nevertheless, it is hard to suggest the reason for its suppression, because the gluon conversion into the hadrons composed of light quarks occurs at large distances where the quark-gluon coupling constant is not small.

Evidently, such a diagram is not reduced to the $\varphi\omega$ mixing. In the mean time, the contribution of the three-gluon mechanism to the $\varphi\omega$ mixing, shown in Fig. 1(b), is negligible and has a wrong sign [6,7] in the sense that it predicts the interference minimum in the energy dependence of the cross section of the reaction $e^+e^- \to \omega, \varphi \to \pi^+\pi^-\pi^0$ [see Eq. (14) below] before the φ resonance but not after it, in contradiction with experiment [8].

Thus, there arises the conflict [9] between the explanation of the decay $\varphi(1020) \to \rho(770)\pi$ motivated by the description of heavy quarkonia and the conventional picture of the $\varphi\omega$ mixing of the ideally mixed states,

$$\omega^{(0)} = (u\bar{u} + d\bar{d})/\sqrt{2},$$
$$\varphi^{(0)} = s\bar{s}, \quad (2)$$

as the possible origin of this decay. In this picture, the decay $\varphi(1020) \to \rho(770)\pi \to \pi^+\pi^-\pi^0$ is caused by a small admixture of the nonstrange quarks in the wave function of the φ meson,

$$\varphi(1020) = s\bar{s} + \varepsilon_{\varphi\omega}(u\bar{u} + d\bar{d})/\sqrt{2}, \quad (3)$$

where $\varepsilon_{\varphi\omega}$ is the complex parameter of the $\varphi\omega$ mixing, $|\varepsilon_{\varphi\omega}| \ll 1$. It can be expressed through the nondiagonal polarization operator $\Pi_{\varphi\omega}$ according to the relation

$$\varepsilon_{\varphi\omega} = -\frac{\mathrm{Re}\Pi_{\varphi\omega} + i\mathrm{Im}\Pi_{\varphi\omega}}{m_\varphi^{(0)2} - m_\omega^{(0)2} - i\sqrt{s}(\Gamma_\varphi^{(0)} - \Gamma_\omega^{(0)})}. \quad (4)$$

Here $m_V^{(0)}$, $\Gamma_V^{(0)}$ are, respectively, the mass and width of the ideally mixed state $V = \omega^{(0)}, \varphi^{(0)}$. [See Eq. (2) and Ref. [10] for more details.] The magnitude of $\varepsilon_{\varphi\omega} \simeq 0.05$, which is necessary for the explanation of both the branching ratio $B(\varphi(1020) \to \rho(770)\pi) = 12.9 \pm 0.7\%$ [11,12] and the sign of the $\varphi\omega$ interference near the φ resonance, is considered to arise via the nonperturbative QCD effects of the type shown in Fig. 1(c) [13]:

$$\varepsilon_{\varphi\omega} \propto \alpha_s \frac{\langle 0|s\bar{s}|0\rangle\langle 0|u\bar{u} + d\bar{d}|0\rangle}{M^6}, \quad (5)$$

where M is the Borel mass entering into the QCD sum rule [13].

How well is the conventional picture of the $\varphi\omega$ mixing substantiated? To this end let us write the effective $\varphi\rho\pi$ coupling constant

$$g_{\varphi\rho\pi} = g_{\varphi\rho\pi}^{(0)} + \varepsilon_{\varphi\omega} g_{\omega\rho\pi}^{(0)}, \quad (6)$$

where the first term describes the direct transition

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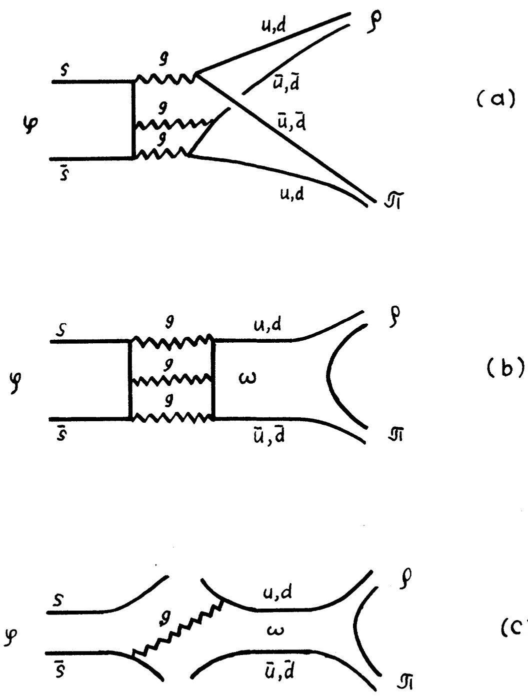

FIG. 1. Possible mechanisms of the decay $\varphi(1020) \to \rho(770)\pi$: (a) the three-gluon mechanism of the direct transition $\varphi \to \rho\pi$; (b) the $\varphi\omega$ mixing caused by the three-gluon mechanism; (c) the $\varphi\omega$ mixing due to the nonperturbative QCD effects. The gluon is denoted by g.

Fig. 1(a) while the second one corresponds to the $\varphi\omega$ mixing. The $\varphi\omega$ mixing would dominate if the mass difference of the ω and φ mesons were small, similar to the system of the neutral K mesons. In the present case, however, the difference $m_\varphi^{(0)2} - m_\omega^{(0)2} \simeq 0.4$ GeV^2 amounts to 40% of the characteristic scale 1 GeV^2, so that there are no reasons to expect that the mixing prevails over the direct transition. The value of the amplitude of the $\varphi\omega$ mixing is still unknown with full confidence. An old theoretical phenomenology [14–19] gave a broad spectrum of the possible values $-0.03 \le \mathrm{Re}\Pi_{\varphi\omega} \le +0.03$ GeV^2 [$-0.07 \le \mathrm{Re}\varepsilon_{\varphi\omega} \le +0.07$]. As for the QCD estimate, Eq. (5), the results of the papers [20,21] show that the magnitudes of the quark condensates $\langle 0|q\bar{q}|0\rangle$ are known only up to a factor ranging from 0.1 to 8.0. If, in addition, one takes into account the uncertainty of the Borel mass entering into the sum rule as M^{-6}, the lack of the firm ground for a sizable $\varphi\omega$ mixing in QCD should become evident.

The ratio of the leptonic widths [19]

$$R_{e^+e^-} \equiv \frac{\Gamma(\varphi \to e^+e^-, \sqrt{s} = m_\varphi)}{\Gamma(\omega \to e^+e^-, \sqrt{s} = m_\omega)} = \left|\frac{g_{\gamma\varphi}^{(0)} + \varepsilon_{\varphi\omega} g_{\gamma\omega}^{(0)}}{g_{\gamma\omega}^{(0)} - \varepsilon_{\varphi\omega} g_{\gamma\varphi}^{(0)}}\right|^2 \left(\frac{m_\omega}{m_\varphi}\right)^3 \quad (7)$$

is sensitive to the magnitude of the $\varphi\omega$ mixing. Here $g_{\gamma V}^{(0)} = e m_V^{(0)2}/f_V^{(0)}$ is the amplitude of the $\gamma V^{(0)}$ transition, m_ω and m_φ are the masses of the states with the $\varphi\omega$ mixing being taken into account. They are equal to, respectively, $m_\omega^{(0)}$ and $m_\varphi^{(0)}$, upon neglecting the second-order terms in the mixing. Assuming the ratio of the coupling constants implied by the nonrelativistic quark model,

$$f_\omega^{(0)}/f_\varphi^{(0)} = -\sqrt{2}, \quad (8)$$

one obtains that $R_{e^+e^-} = 2.0$ at $\mathrm{Re}\varepsilon_{\varphi\omega} \approx 0.05$ and $R_{e^+e^-} = 2.6$ at $\mathrm{Re}\varepsilon_{\varphi\omega} \approx 0$.

Until 1988, an experiment gave $R_{e^+e^-} = 1.9 \pm 0.4$ [22] and seemingly favored sizable $\varphi\omega$ mixing. Modern data [11] give $R_{e^+e^-} = 2.3 \pm 0.1$, pointing to a rather weak mixing. In fact, one gets from $R_{e^+e^-} = 2.3$ that $\mathrm{Re}\varepsilon_{\varphi\omega} \approx 0.02$, provided the ratio Eq. (8) is satisfied, resulting in an unacceptably low $B(\varphi(1020) \to \rho(770)\pi) = 2\%$. Hence, in view of Eq. (6), the coupling constant of the direct transition $\mathrm{Re}g_{\varphi\rho\pi}^{(0)} \simeq 0.5$ GeV$^{-1} \approx \frac{1}{29}\mathrm{Re}g_{\omega\rho\pi}^{(0)}$ is required to reconcile the data on the e^+e^- and $\rho\pi$ decay modes of the $\varphi(1020)$. Note that $\mathrm{Re}g_{\omega\rho\pi}^{(0)} = 14.3$ GeV^{-1}. [See, e.g., the results of the recent fits [8,23].]

Since the expression [24]

$$f_V^{(0)} = \frac{m_V^{3/2}}{2\sqrt{3}C_V|\psi(0, m_V)|} \quad (9)$$

holds, where C_V is expressed through the charge of the quark q as

$$C_\varphi = e_s = -1/3,$$
$$C_\omega = (e_u + e_d)/\sqrt{2} = 1/3\sqrt{2}, \quad (10)$$

the magnitude of $\varphi\omega$ mixing can be extracted from the e^+e^- data only in the case of the definite assumptions about the mass behavior of the wave function of the $q\bar{q}$ bound state $\psi(0, m_V)$ at the origin. Side by side with Eq. (8), which is valid in the case of the nonrelativistic behavior $|\psi(0, m_V)|^2 \propto m_V^3$ and corresponds to the Coulomb-like $q\bar{q}$ potential, currently popular is the behavior $|\psi(0, m_V)|^2 \propto m_V^2$, which is valid for heavy quarkonia J/ψ and $\Upsilon(1S)$. It gives

$$\Gamma(V \to e^+e^-, \sqrt{s} = m_V) \propto C_V^2 \quad (11)$$

in the absence of the mixing, independent of m_V. It follows from Eq. (11) that the ratios

$$\Gamma(\varphi \to e^+e^-) : \Gamma(J/\psi \to e^+e^-) : \Gamma(\Upsilon(1S) \to e^+e^-)$$
$$= e_s^2 : e_c^2 : e_b^2 = 1 : 4 : 1 \quad (12)$$

are satisfied, in perfect agreement with the data [11]. The corresponding $q\bar{q}$ potential varies like $r^{-1/2}$ [25], incorporating the property of the asymptotic freedom at the short distances. Extrapolating Eq. (11) to the $\omega\varphi$ mass range, one gets that

$$f_\omega^{(0)}/f_\varphi^{(0)} = -\sqrt{2m_\omega/m_\varphi} = -1.24, \quad (13)$$

giving $R_{e^+e^-} = 2.0$ in the absence of the $\varphi\omega$ mixing, $\mathrm{Re}\varepsilon_{\varphi\omega} \approx 0$, which coincides with the prediction of conventional $\varphi\omega$ mixing and the behavior $|\psi(0,m_V)|^2 \propto m_V^3$.

In order to explain the experimental magnitude of $R_{e^+e^-} = 2.3$, the extrapolation Eq. (13) demands $\mathrm{Re}\varepsilon_{\varphi\omega} \approx -0.026$, which leads to $B(\varphi \to \rho\pi)$ amounting to one fourth of the observed. Moreover, because of the minus sign, this sharply contradicts the measurements of the cross section of the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ [8],

$$\sigma = \frac{4\pi\alpha W(s)}{s^{3/2}}\left|\frac{(g^{(0)}_{\gamma\omega} - \varepsilon_{\varphi\omega}g^{(0)}_{\gamma\varphi})g^{(0)}_{\omega\rho\pi}}{m_\omega^2 - s - i\sqrt{s}\Gamma_\omega} + \frac{(g^{(0)}_{\gamma\varphi} + \varepsilon_{\varphi\omega}g^{(0)}_{\gamma\omega})g_{\varphi\rho\pi}}{m_\varphi^2 - s - i\sqrt{s}\Gamma_\varphi}\right|^2, \tag{14}$$

and results in the wrong sign of the interference pattern. Here s is the total center-of-mass energy squared, Γ_ω and Γ_φ are the total widths of the ω and φ mesons with the $\varphi\omega$ mixing being taken into account, and $W(s)$ is [23] the phase space factor for the 3π decay. To remove this contradiction, one should introduce, in view of Eq. (6), a sizable coupling constant of the direct transition $\mathrm{Re}g^{(0)}_{\varphi\rho\pi} \simeq 1.1\ \mathrm{GeV}^{-1} \approx \frac{1}{13}\mathrm{Re}g^{(0)}_{\omega\rho\pi}$.

From the other hand, to obtain the acceptable $\rho\pi$ branching ratio, the correct interference pattern in the cross section of the reaction $e^+e^- \to \omega, \varphi \to \pi^+\pi^-\pi^0$ and the experimental magnitude of $R_{e^+e^-} = 2.3$ in the case of conventional $\varphi\omega$ mixing, $\mathrm{Re}\varepsilon_{\varphi\omega} \approx 0.05$, one should change $f^{(0)}_\omega/f^{(0)}_\varphi$ to -1.53. The latter value corresponds to $|\psi(0,m_V)|^2 \propto m_V^{3.6}$ which, according to [25], follows from the $q\bar{q}$ potential $V(r) \propto r^{-1.2}$ and means an increase of effective QCD coupling at short distances, contrary to the property of the asymptotic freedom.

Thus, the existing data on the decays $\varphi(1020) \to e^+e^-$, $\rho(770)\pi$ and the interference pattern seem to point to a sizable, or even dominant, coupling constant $\mathrm{Re}g^{(0)}_{\varphi\rho\pi}$ of the direct transition as compared to the conventional $\varphi\omega$ mixing. The experimental elucidation of the situation is necessary. The improvement of the accuracy of the measurement of the ω and φ leptonic widths by a factor of 5 to 10 is urgent.

We, however, believe that the study of the nature of the decay $\varphi(1020) \to \rho(770)\pi$ is equally important. As shown in [10], to this end one should investigate the energy dependence of the cross section of the reaction $e^+e^- \to \omega, \varphi \to \pi^+\pi^-\pi^0$ in the vicinity of the $\varphi\omega$ interference minimum, $\sqrt{s} = 1.05$ GeV, where the cross section is very sensitive to a small difference between the $\varphi\rho\pi$ coupling constants,

$$\begin{aligned}\frac{\Delta g_{\varphi\rho\pi}}{g_{\varphi\rho\pi}} &\simeq i\sqrt{s}\varepsilon'\left.\frac{\Gamma_{\varphi K\bar{K}} + \Gamma_\varphi + \Gamma_{\omega 3\pi} - \Gamma_\omega}{\varepsilon'(m_\varphi^2 - m_\omega^2) + i\sqrt{s}\Gamma_{\varphi K\bar{K}}/\sqrt{2}}\right|_{\sqrt{s}=1.05}\\ &\simeq 0.02 + 0.04i,\\ \varepsilon' &= \mathrm{Re}g^{(0)}_{\varphi\rho\pi}/\mathrm{Re}g^{(0)}_{\omega\rho\pi},\end{aligned} \tag{15}$$

in the variants of the strong $[\mathrm{Re}\Pi_{\varphi\omega} \neq 0,\ \mathrm{Re}g^{(0)}_{\varphi\rho\pi} = 0]$ and weak $[\mathrm{Re}\Pi_{\varphi\omega} = 0,\ \mathrm{Re}g^{(0)}_{\varphi\rho\pi} \neq 0]$ $\varphi\omega$ mixing. Despite the apparent smallness, Eq. (15) results in a rather large relative difference of the cross sections $\Delta\sigma/\sigma \simeq 0.3$ caused by the destructive $\varphi\omega$ interference in a narrow energy range close to $\sqrt{s} = 1.05$ GeV. The difficulty of measuring small cross sections of about 0.1 nb at this energy is more than compensated by the expected high luminosity of φ factories. The real and serious difficulty is the necessity of making the cut by the energy of undetected photons in the final state with an accuracy better than 30 MeV, since the broad criteria of the event selection result, via the radiative corrections, in a smoothing of the difference of cross sections in the above $\varphi\omega$ mixing variants.

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