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Improved perturbative QCD analysis of the pion-photon transition form factor

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We reexamine the transition form factor in π^0 coupling to $\gamma^*\gamma^*$ in the framework of a perturbative QCD approach based on the modified factorization formula. Sudakov suppression is less important here than in other exclusive channels and can be used to check the hard-scattering approach of Brodsky and Lepage in this process.

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I. INTRODUCTION

The study of exclusive processes provides an interesting check of the hard-scattering approach in the framework of perturbative QCD [1–3] based on the factorization formula. The validity of this approach in the energy range of a few GeV has been questioned by some authors [4,5] who claim that the main contribution comes from the end-point regions of x (the fractional momentum of the valence quark in the parton model) where the running coupling constant α_s becomes large; thus, the perturbative expansion is illegal.

However, other authors [6,7] have shown that the perturbative calculation remains valid if the Sudakov correction, for soft-gluon exchange, is taken into account. They have calculated explicitly the Sudakov effects and proposed a modified factorization formula. The gluonic radiative corrections have to be summed up into an exponential factor $\exp(-S)$. This Sudakov form factor selects components of the hadronic wave function with small values of $\mathbf{b}$ (the transverse separation between valence quarks in configuration space) and makes the perturbative calculation more self-consistent.

Previous calculations of exclusive processes may be reexamined with this new approach. Many interesting channels have indeed been investigated recently [8–14]. With Sudakov suppression, the predictions of the factorization formula are somewhat smaller than the available experimental data [15–18]. An important, though delicate, application of Sudakov effects is the prediction of the nucleon's magnetic form factor [9,13].

One of the successful applications of the hard-scattering approach concerns the determination of the meson-photon transition form factor. Brodsky and Lepage (BL) [2] have proposed a simple interpolation formula for $F_{\pi\gamma}(Q^2)$ between its asymptotic expression ($Q^2 \to \infty$) and the current-algebra prediction ($Q^2 \to 0$). Notice that the asymptotic behavior was predicted by various authors [19].

Recently, it was shown [20] that pseudoscalar-meson production by two off-shell photons can be used to check this standard QCD approach versus the vector-dominance model.

The meson-photon transition form factor (with only one photon off shell) has been investigated within the modified hard-scattering approach including both transverse momentum effects and Sudakov corrections [21]. The experimental data [17,18] seem to favor the asymptotic wave function; the Chernyak-Zhitnitsky (CZ) one gives a too large result in that case.

In this paper we follow the approach of the authors of Ref. [21] in order to study the transition form factor in π^0 coupling to $\gamma^*\gamma^*$. In Sec. II, we shall discuss in detail the Sudakov effects in the π-γ^* transition form factor; in Sec. III, we shall summarize our results and present our conclusion.

II. SUDAKOV EFFECTS IN THE PION-PHOTON TRANSITION FORM FACTOR

The transition form factor, in reactions where the meson is produced by one on-shell and one off-shell photon, reads [2]

$$F_{\pi\gamma}(Q^2) = \frac{2}{\sqrt{3}Q^2}\int_0^1 dx \frac{\phi_\pi^*(x)}{x(1-x)}[1 + O(\alpha_s, m^2/Q^2)] \ . \tag{1}$$

Let us notice that (i) this form factor has no dependence on $\alpha_s(Q^2)$ in leading order, and (ii) the mass corrections $O(m^2/Q^2)$ are important only at low Q^2.

Using the asymptotic distribution amplitude

$$\phi_\pi = \sqrt{3} f_\pi x(1-x) \ , \tag{2}$$

one gets

$$F_{\pi\gamma}(Q^2) = \frac{2f_\pi}{Q^2} \ \text{ as } Q^2 \to \infty \ . \tag{3}$$

Interpolating between this expression and the current-algebra result at $Q^2 = 0$, Brodsky and Lepage predict

$$F_{\pi\gamma}(Q^2) \simeq 0.27 \left(1 + \frac{Q^2}{\Lambda_\pi^2}\right)^{-1} \text{GeV}^{-1} \ , \tag{4}$$

where $\Lambda_\pi^2 = 8\pi^2 f_\pi^2$; with the value $f_\pi \simeq 93$ MeV of the pion decay constant, one gets, for the mass scale parameter, $\Lambda_\pi \simeq 0.83$ GeV.

The transition form factor for the $\pi^0\gamma^*\gamma^*$ vertex is obtained, in leading order, as

$$F_{\pi\gamma^*}(Q^2,Q'^2)=\frac{4}{\sqrt{3}}\int_0^1 dx\frac{\phi_\pi^*(x)}{xQ^2+(1-x)Q'^2}\ , \qquad (5)$$

assuming Q^2 and/or $Q'^2 \gg m_\pi^2$, and taking account of the symmetry (due to invariance under charge conjugation) of $\phi_\pi(x)$ in x, $1-x$.

Using again the asymptotic distribution amplitude, one here obtains [20]

$$F_{\pi\gamma^*}(Q^2,Q'^2)=2f_\pi\frac{Q^4-Q'^4-2Q^2Q'^2\ln(Q^2/Q'^2)}{(Q^2-Q'^2)^3}$$
$$\text{as } Q^2 \text{ and/or } Q'^2\to\infty\ . \qquad (6)$$

Applying again the Brodsky-Lepage interpolation procedure, one is led to the expression [which appears as a generalization of formula (4)]

$$F_{\pi\gamma^*}(Q^2,Q'^2)=0.27\left(1+\frac{X^2}{\Lambda_\pi^2}\right)^{-1}\ \text{GeV}^{-1}\ , \qquad (7)$$

with

$$X^2=\frac{(Q^2-Q'^2)^3}{Q^4-Q'^4-2Q^2Q'^2\ln(Q^2/Q'^2)}\ . \qquad (8)$$

Now, using the modified factorization formula [6,7], we reexpress Eq. (5) in transverse configuration space:

$$F_{\pi\gamma^*}(Q^2,Q'^2)=\int dx\int\frac{d^2b}{4\pi}\hat\Psi(x,\mathbf{b})\hat T_H(x,\mathbf{b},Q,Q')\exp[-S(x,b,Q_0)]\ , \qquad (9)$$

where the two-dimensional vector $\mathbf{b}$ (of modulus b) represents the transverse separation between quark and antiquark in configuration space. It is the Fourier conjugate of $\mathbf{k}_T$, the relative transverse momentum of the quarks. In momentum space the hard-scattering amplitude reads

$$T_H(x,\mathbf{k}_T,Q,Q')=2\sqrt{6}C_\pi\left(\frac{1}{(1-x)Q^2+xQ'^2+\mathbf{k}_T^2}+\frac{1}{xQ^2+(1-x)Q'^2+\mathbf{k}_T^2}\right)\ , \qquad (10)$$

where $C_\pi=(e_u^2-e_d^2)/\sqrt{2}$ is the charge factor of the quarks; the Fourier transform of T_H is given by

$$\hat T_H(x,\mathbf{b},Q,Q')=\frac{2\sqrt{6}}{\pi}C_\pi K_0\big(\sqrt{(1-x)Q^2+xQ'^2}\,b\big)\ . \qquad (11)$$

K_0 is the modified Bessel function of order zero.

The Sudakov exponent $S(x,b,Q_0)$ in (9) is expressed as

$$S(x,b,Q_0)=s(x,b,Q_0)+s(1-x,b,Q_0)-\frac{4}{\beta_0}\ln\frac{\ln(t/\Lambda_{\rm QCD})}{\ln(1/b\Lambda_{\rm QCD})}\ , \qquad (12)$$

where $\beta_0 = 11-2n_f/3$ ($n_f=3$ is the number of quark flavors); we assume $\Lambda_{\rm QCD}=200$ MeV, while $t=\max\big(\sqrt{(1-x)Q^2+xQ'^2},1/b\big)$ is the largest mass scale in $\hat T_H$.

The expression of $s(\xi,b,Q_0)$ with $\xi=x,1-x$ is given in detail in [7,9]. The parameter Q_0^2 is chosen to be

$$2Q_0^2=-(q-q')^2\ , \qquad (13)$$

where q and q' are the four-momenta of the two off-shell photons. In the perturbative (PQCD) approach, the pion is assumed to be on shell; thus,

$$Q_0^2=Q^2+Q'^2\ . \qquad (14)$$

Following [7], we set $\exp(-S)$ to unity in the small-b region; when b increases, $\exp(-S)$ decreases and vanishes as $b\to 1/\Lambda_{\rm QCD}$.

The authors of Refs. [11,21] have proposed to include both the Sudakov corrections and the intrinsic transverse-momentum dependence of the hadronic wave function in this new approach. Following this assumption, we write the wave function as

$$\hat\Psi(x,\mathbf{b})=\frac{f_\pi}{2\sqrt{3}}\varphi(x)\Sigma\big(\sqrt{x(1-x)}\,b\big)\ , \qquad (15)$$

with the normalization condition $\int_0^1\varphi(x)dx=1$.

The Fourier transform Σ of the $\mathbf{k}_T$-dependent part of the wave function is assumed to be a simple Gaussian; using $\Sigma(0)=4\pi$, it reads

$$\Sigma\big(\sqrt{x(1-x)}\,b\big)=4\pi\ \exp[-x(1-x)b^2/(4a^2)]\ . \qquad (16)$$

We also assume that the transverse size parameter a takes the value of 0.861 GeV^{-1} for the asymptotic wave function $\varphi_{(\rm as)}=6x(1-x)$ and of 0.673 GeV^{-1} for the CZ one, $\varphi_{\rm CZ}=30x(1-x)(2x-1)^2$.

III. NUMERICAL RESULTS AND CONCLUSION

We display in Fig. 1, for various values of Q'^2, the variation of the transition form factor vs Q^2, using the

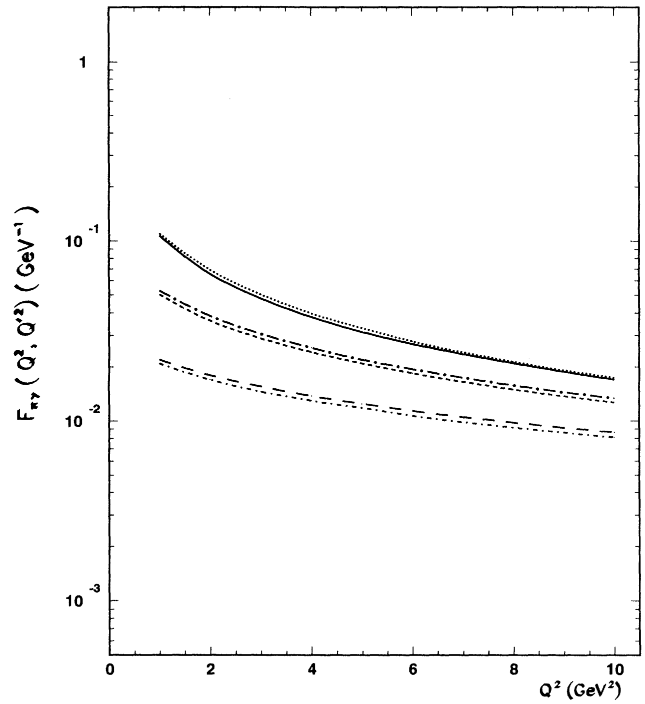

FIG. 1. Pion-photon transition form factor vs Q^2. With Sudakov corrections and transverse momentum effects, $Q'^2 = 0$ (solid line), $Q'^2 = 1$ GeV^2 (long-dash–dotted line), and $Q'^2 = 5$ GeV^2 (long-dashed line). The dotted, short-dashed, and short–dash-dotted lines are the corresponding curves obtained by applying the interpolation formula. In both procedures the asymptotic wave function is used.

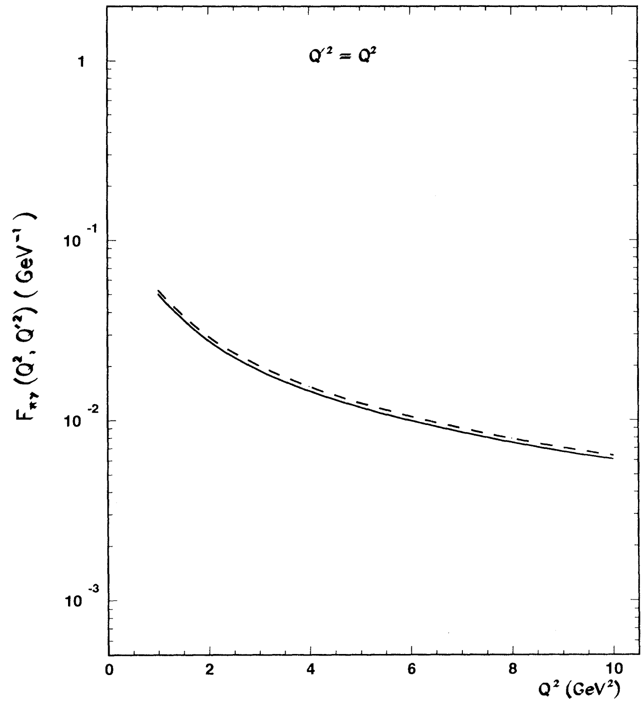

FIG. 2. Pion-photon transition form factor vs Q^2 in the configuration where $Q^2 = Q'^2$, with Sudakov corrections and transverse momentum effects (dashed line) and using the interpolation formula (solid line).

asymptotic wave function and taking account of Sudakov corrections and transverse momentum effects. For $Q'^2 = 0$, our results are in agreement with those obtained by the authors of Ref. [21]; like them, we check that the CZ wave function should be discarded since it does not fit the existing experimental data.

A comparison between the interpolation formula and the modified hard-scattering approach is displayed in Fig. 2 for the configuration where $Q^2 = Q'^2$. Let us notice that the modified (dashed) curve remains practically independent of the particular distribution amplitude considered.

While the interpolation procedure is, of course, not unique, the inclusion of Sudakov corrections leads to an unambiguous procedure; in addition, there is no need of scales other than $\Lambda_{\rm QCD}$.

As we show in Fig. 1 the transition form factor including Sudakov effects is extremely similar to that obtained by applying the interpolation procedure in the manner of Brodsky and Lepage, even in the case of two off-shell photons [formula (7)]. It is difficult to find a precise explanation for that similarity. At least the fact that the Sudakov suppression is less brutal than in other processes can be explained by the fact that the form factor has no dependence on α_s in leading order and that the propagator in the hard-scattering amplitude T_H is always off shell in the end-point regions for finite values of Q^2 and Q'^2.

An interesting kinematic configuration is the symmetric one, when $Q^2 = Q'^2$. With Sudakov corrections and transverse momentum effects, the independence of the form factor with respect to the distribution amplitude chosen, which was already noticed in Ref. [20], remains basically preserved, as already noticed above. It results in the type of experiment using this configuration being really a test of the hard-scattering approach rather than of a particular distribution amplitude.

In order to check the feasibility of measurements of the form factors considered, using e^+e^- colliding beams, we have computed the cross sections of the reaction $e^+e^- \to e^+e^-\pi^0$ with a c.m. energy of 10 GeV (chosen to be that of a $B\bar{B}$ factory) and an integrated luminosity $\int L\,dt = 10^{40}$ cm^{-2}. For $Q^2_{\rm min} = 1$ GeV^2 (defining $Q^2_{\rm min}$ as the minimal value of both Q^2 and Q'^2), one expects about 310 events with these assumptions.

Our study can easily be extended to other pseudoscalar mesons, i.e., η or η'. Pseudoscalar-meson production in $\gamma^*\gamma^*$ collisions, using e^+e^- colliding beams, is an interesting channel for checking the improved hard-scattering approach of exclusive processes, based on the modified factorization formula.

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