Strong coupling, unification, and recent data

Paul Langacker and Nir Polonsky*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 2 March 1995)

The prediction of strong coupling assuming (supersymmetric) coupling constant unification is reexamined. We find, using the new electroweak data, $\alpha_s(M_Z) \approx 0.129 \pm 0.010$. The implications of the large α_s value are discussed. The role played by the Z b quark width is stressed. It is also emphasized that high-energy (but not low-energy) corrections could significantly diminish the prediction. However, unless higher-dimension operators are assumed to be suppressed, at present one cannot place strong constraints on the superheavy spectrum. Nonleading electroweak threshold corrections are also discussed.

PACS number(s): 12.10.Dm, 11.10.Hi, 12.60.Jv

Assuming the minimal supersymmetric extension of the standard model (MSSM) [1] between the weak and some high scale, one finds [2] that the extrapolated electroweak and strong couplings approximately unify at a scale $M_G \sim 3 \times 10^{16}$ GeV (the grand unification scale). Alternatively, assuming coupling constant unification, one can use the precisely measured weak angle $s^2(M_Z)$ and fine-structure constant $\alpha(M_Z)$ to predict the Z-pole strong coupling $\alpha_s(M_Z)$. Model-dependent corrections are typically of order 10%, i.e., comparable to the experimental uncertainty in $\alpha_s(M_Z)$, and need to be included consistently [3]. Below, we update and extend our discussion of the $\alpha_s(M_Z)$ prediction [3–6]. We find that for the t-quark pole mass $m_t^{\rm pole} \gtrsim 160$ GeV, the positive corrections proportional to m_t^2 are sufficiently large that the sum of the (Yukawa, threshold, and operator) model-dependent corrections must cancel or be negative for unification to hold. Ignoring possible high-scale matching corrections, $\tan\beta \approx 1$ and heavy superpartners are preferred ($\tan\beta \equiv \langle H_2\rangle/\langle H_1\rangle$). However, large negative high-scale threshold and nonrenormalizable operator (NRO) corrections are possible. The former depend on the details of the grand-unified theory (GUT), while the latter [7] are gravitationally induced and are generic. Below, we review our formalism and discuss our results and their implications. We also comment on nonlogarithmic superpartner corrections, implications of the anomalous $Z \to b\bar{b}$ width, extended models, and on various aspects of the large QCD coupling. A comprehensive analysis is presented in Ref. [8].

The prediction for $\alpha_s(M_Z)$ reads[1]

$$\alpha_s(M_Z) = \alpha_s^{\rm OL}(M_Z) + 0.014 + H_{\alpha_s} + \frac{\alpha_s^2(M_Z)}{28\pi} +3.1 \times 10^{-7}\ {\rm GeV}^{-2}\left[(m_t^{\rm pole})^2 - (m_{t_0}^{\rm pole})^2\right] +\Delta_{\alpha_s}, \quad (1)$$

where

$$\alpha_s^{\rm OL}(M_Z) = \frac{7\alpha(M_Z)}{15 s_0^2(M_Z) - 3}, \quad (2)$$

is the lowest-order prediction, and[2]

$$s^2(M_Z) = s_0^2(M_Z) - 0.88 \times 10^{-7}\ {\rm GeV}^{-2} \times \left[(m_t^{\rm pole})^2 - (m_{t_0}^{\rm pole})^2\right], \quad (3)$$

where $s^2(M_Z)$ is the true [modified minimal subtraction scheme ($\overline{\rm MS}$)] weak angle and s_0^2 is the value it would have for $m_t^{\rm pole} = m_{t_0}^{\rm pole}$. The 0.014 correction is a (model-independent) two-loop gauge correction and the function H_{α_s} is a smaller (model-dependent) two-loop Yukawa correction. $\alpha_s^2/28\pi$ is a finite scheme-dependent term. The model-dependent function Δ_{α_s} sums threshold and NRO corrections at low and high scales. Substituting in (1) the ($\overline{\rm MS}$) input values [9–11] $\alpha(M_Z) = 1/(127.9 \pm 0.1)$ and

$$s_0^2(M_Z) = 0.2316 \pm 0.0003, \quad (4a)$$

$$m_{t_0}^{\rm pole} = 160^{+11}_{-12} + 13 \ln \frac{m_{h^0}}{M_Z} \quad (4b)$$

(m_{h^0} is the SM-like light Higgs boson mass[3]), one has (in the $\overline{\rm MS}$ scheme)

---

*Current address: Sektion Physik (LS Wess), University of Munich, 37 Thereisenstrasse, D-80333 Munich, Germany.

[1] Hypercharge is properly normalized, i.e., $s^2(M_G) = 3/8$.

[2] We do not explicitly treat smaller logarithmic dependences on $m_t^{\rm pole}$. They are included in the uncertainty. The 0.88 factor incorporates higher-order QCD corrections which were not included in [3].

[3] The authors of [11] perform a best fit to all W, Z, and neutral current data assuming $60 \le m_{h^0} \le 150$ GeV with a central value $m_{h^0} = M_Z$ for the SM-like light Higgs boson mass. (Other possible light particle corrections are discussed separately below.) In the (nonsupersymmetric) standard model one assumes a larger Higgs boson mass range $60 < m_{h^0} < 1000$ GeV with a central value of 300 GeV. This leads to the prediction $m_{t_0}^{\rm pole} = 175 \pm 11^{+17}_{-19}$ GeV, where the second uncertainty is from m_{h^0}.

$$\alpha_s(M_Z) - \Delta_{\alpha_s} = 0.129 \pm 0.001 + 3.1 \times 10^{-7}\ \text{GeV}^{-2} \times \left[(m_t^{\text{pole}})^2 - (160\ \text{GeV})^2\right] + H_{\alpha_s}. \quad (5)$$

The higher values of m_t^{pole} (e.g., compared to [3]) and lower value of the weak angle implied by recent data [11] increase the predicted α_s. An even higher central α_s value of 0.130 would be predicted for the value $m_t^{\text{pole}} = 174 \pm 16$ GeV suggested by the Collider Detector at Fermilab (CDF) t-quark candidate events [12]. Two-loop Yukawa corrections are negative but are typically negligible. They can be important if the Yukawa couplings of the t and/or b quark, h_t and h_b, respectively, are large, i.e., for $\tan\beta \approx 1$ or $\tan\beta \gtrsim 50$. We find [5]

$$-0.003 \lesssim H_{\alpha_s}(h_t, h_b) = H_{\alpha_s}(m_t^{\text{pole}}, \tan\beta) \lesssim 0. \quad (6)$$

For $h_t \sim \max[h_t(m_t)] \sim 1.1$ (and $h_b \sim 0$) one has [3] $H_{\alpha_s} \sim -0.1 \times \alpha_s^2 \times h_t^2 \sim -0.002$. In general, one can substitute a one-loop semianalytic expression for h_t^2 and integrate iteratively [13] (a similar procedure leads to our result for the gauge two-loop correction [3,8]).

The coupling constant unification is shown in detail in Fig. 1 for various values of $\alpha_s(M_Z) = 0.12 \pm 0.01$ and for $\Delta_{\alpha_s} = 0$ and $H_{\alpha_s} \sim -0.0005$. In the absence of threshold corrections, and for reasonable m_t^{pole}, coupling unification requires $\alpha_s(M_Z) \gtrsim 0.127$. Below, we show that typically $|\Delta_{\alpha_s}| \lesssim 0.01$. Thus, we obtain from coupling constant unification, assuming no conspiracies among different model-dependent corrections, $\alpha_s(M_Z) \gtrsim 0.12$. This is in a good agreement with Z-pole extractions of α_s, but is slightly higher than some extractions based on deep inelastic scattering (DIS) and quarkonium spectra. The prediction is compared with the data in Table I (from [14]). The α_s measurement and the possibility of light gluinos (that correct the α_s extrapolation between the quarkonium and weak scales by $\sim 10\%$) are further discussed in Ref. [15,16]. We note, in passing, that light colored scalars would correct the α_s extrapolation negligibly, i.e., a light scalar top quark would affect the extrapolation of α_s measured at low-energy to the Z pole by less than 1%.

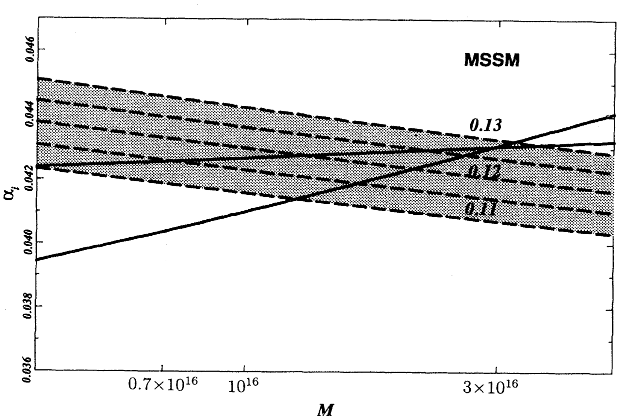

FIG. 1. MSSM evolution of $\alpha_{1,2}$ (solid lines) and of α_3 (dashed lines) in the vicinity of the $\alpha_{1,2}$ unification point (the scale M is in GeV). $\alpha_s(M_Z) = 0.110$, 0.115, 0.120, 0.125, 0.130; $m_t^{\text{pole}} = 160$ GeV; $\tan\beta = 4$; and $\Delta_{\alpha_s} = 0$.

TABLE I. Values of $\alpha_s(M_Z)$ extracted from different processes (and extrapolated to M_Z if relevant). The different values are ordered according to the energy scale of the relevant process.

Bjorken sum rules	$0.122^{+0.005}_{-0.009}$
$\tau \to$ hadrons (CLEO)	0.114 ± 0.003
$\tau \to$ hadrons (LEP)	0.122 ± 0.005
Deep inelastic scattering	0.112 ± 0.005
J/ψ (lattice)	0.110 ± 0.006
Υ (lattice)	0.115 ± 0.002
Υ, J/ψ (decays)	0.108 ± 0.010
$ep \to 2+1$ jet rate (DESY ep collider HERA)	0.121 ± 0.015
e^+e^- event shape (LEP)	0.123 ± 0.006
Z line shape (LEP)	0.126 ± 0.005
Prediction	0.13 ± 0.01

Models (in particular, NRO's) can be constructed with large ($\gtrsim 10-20\%$) and negative GUT scale contributions to Δ_{α_s}. Such models would violate our no-conspiracy assumption, but cannot be excluded. Hence, even if supersymmetry is characterized by experiment and the superpartner contribution to Δ_{α_s} (see below) is found to be positive, coupling constant unification will not be completely ruled out even for $\alpha_s(M_Z) \sim 0.11$. However, one will be able to sufficiently constrain GUT's only if the superpartner contribution is large and positive (i.e., if NRO's with perturbative coefficients are not sufficient to rectify the prediction).

The situation in the nonsupersymmetric extension is quite different since (a) supersymmetry doubles the GUT sector, (b) NRO's are typically suppressed in the nonsupersymmetric case by powers of $(M_G/M_{\text{Planck}}) \sim 10^{-5}$, and (c) the corrections $\propto \alpha_s^2(M_Z)$ are suppressed by a $\sim (0.07/0.13)^2$ factor in comparison to the MSSM [3,8]. One can rectify this situation by considering large logarithms and/or certain complicated chain-breaking scenarios with additional particles, i.e., intermediate scales (which, however, could be constructed to be $\sim 10^{16}$ GeV [17] or $\sim$1 TeV [18]). The predictive power of a desert theory is lost in such a case.

Next, we discuss in greater detail the possible model-dependent contributions to the $\sim 10\%$ correction function

$$\Delta_{\alpha_s} \approx \frac{-19\alpha_s^2}{28\pi} \ln\frac{M_{\text{SUSY}}}{M_Z} + \text{GUT threshold corrections} + \text{NRO corrections}. \quad (7)$$

The parameter M_{SUSY} [3] is a weighted sum of all superpartner and heavy Higgs boson mass logarithms which determines the (leading-logarithm) contribution to Δ_{α_s} [3] [$\Delta_{\alpha_s} \sim -0.003 \ln(M_{\text{SUSY}}/M_Z)$]. Specifically,

$$M_{\text{SUSY}} = \prod_i m_i^{-\frac{5}{38}\left[4b_1^i - \frac{96}{10}b_2^i + \frac{56}{10}b_3^i\right]}, \quad (8)$$

where the index i runs over all superpartner and heavy Higgs particles, and b_j^i is the contribution of particle i to the one-loop β function of the U(1), SU(2), and SU(3)

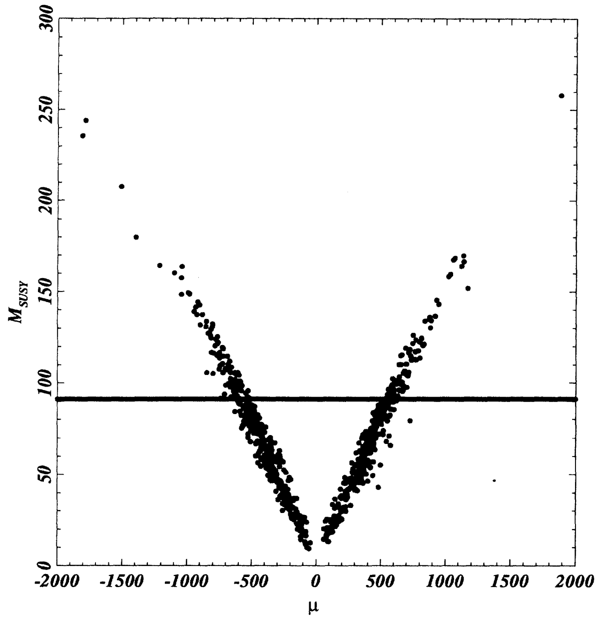

FIG. 2. $M_{\rm SUSY}$ as a function of the μ parameter. The different universal soft parameters and $\tan\beta$ are picked at random in the allowed parameter space (see text). $m_t^{\rm pole} = 160$ GeV. $M_{\rm SUSY} = M_Z$ is denoted for comparison. (All masses are in GeV.)

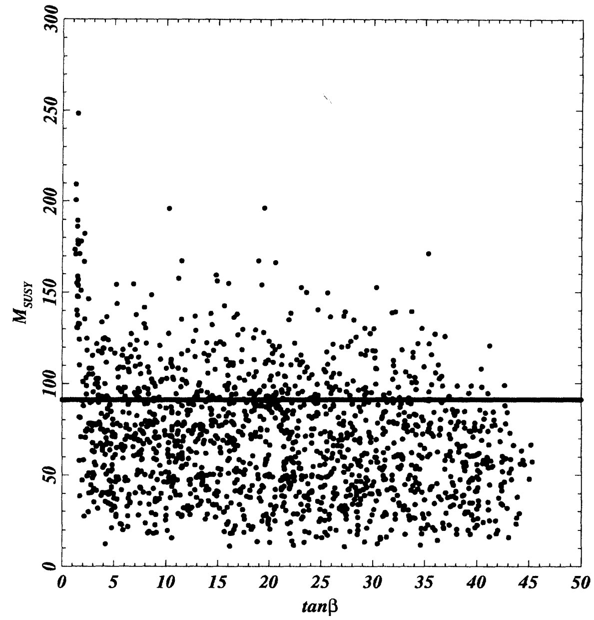

FIG. 3. Same as in Fig. 2 except a function of $\tan\beta$.

subgroup for $j = 1, 2, 3$, respectively [19]. Because of mass nondegeneracies between colored particles (whose masses are sensitive to the gluino mass), the Higgs and Higgsino particles (whose masses are sensitive to μ), and the scalar leptons (whose masses are sensitive to scalar mass boundary condition), and because of the different weights assigned to the different particles, $M_{\rm SUSY}$ is not simply the geometric mean of the m_i. In particular, the negative powers in (8) imply that $M_{\rm SUSY}$ can be (and generally is) much smaller than the actual masses of the superpartners. In Fig. 2 we calculate $M_{\rm SUSY}$ for more than 1000 arbitrary[4] MSSM's which are consistent with the electroweak symmetry breaking, a neutral lightest supersymmetric particle, and sparticle masses above experimental lower bounds and below ~ 2 TeV (see [20,21]). $M_{\rm SUSY}$ is proportional to the Higgsino mass parameter μ [22] and is indeed lower than the actual superpartner and Higgs boson masses. From Fig. 2 we have the approximate upper bound $M_{\rm SUSY} \lesssim 250 - 300$ GeV (or the lower bound $\Delta_{\alpha_s}^{\rm SUSY} \gtrsim -0.003$).

As mentioned above, H_{α_s} is large and negative for $\tan\beta \approx 1$. Also, $M_{\rm SUSY} \propto |\mu| \propto \sqrt{1/[\tan^2\beta - 1]}$ is maximized in that region of the parameter space ($M_{\rm SUSY}$ is shown as a function of $\tan\beta$ in Fig. 3). The proportionality factor depends on and grows with the superpartner masses. Thus, a heavy spectrum and $\tan\beta \sim 1$ are slightly preferred. This observation is consistent with $b-\tau$ Yukawa unification (which we do not require here), which is constrained by the interplay between the large predicted values of α_s and the Yukawa-unification preference of moderate α_s values [8]. (The large QCD radiative corrections to h_b constrain one to regions of the parameter space in which large Yukawa coupling can partially compensate for these corrections.[5]) In that region one has the spectacular constraint on the Higgs boson mass $m_{h^0} \lesssim 100$ (110) GeV for $m_t^{\rm pole} \lesssim 160$ (175) GeV at one loop (and a stronger bound applies at two loops) [24,20,8].

It was recently suggested that the Z-pole couplings should be extracted from the data assuming the full MSSM [25]. This is the case if the model contains some particles (aside from the SM-like Higgs boson) lighter than $\sim 100 - 150$ GeV. However, assuming the heavy MSSM limit, SU(2)-breaking mixing and other nonleading effects are negligible and our leading-logarithm formula, which is derived using renormalization-group techniques, is an excellent approximation. Otherwise, light particle (nonlogarithmic) effects can be accounted for in the same manner used to describe the quadratic m_t dependence [3,6,8], i.e., by the perturbative expansion[6]

$$s^2(M_Z) = s_0^2(M_Z) + \frac{s_0^2(1-s_0^2)}{1-2s_0^2}\left[\Delta r_Z^{\rm top} + \Delta r_Z^{\rm SUSY}\right], \quad (9)$$

[4]We assume universality of the soft parameters at M_G. However, similar results for $M_{\rm SUSY}$ hold in more general scenarios.

[5]Finite superpartner loops [23] modify only the allowed large $\tan\beta$ region.

[6]One could calculate the corrections to all fitted observables, or risk a minor inconsistency and calculate only (universal) corrections to the input parameter (M_Z in our case). The latter scheme, which we follow, is sufficient for our current purposes.

where Δr_Z [26] sums (universal) corrections to the Z-boson mass M_Z. The leading contribution to $\Delta r_Z^{\rm top}$ is given in Eq. (3) and $\Delta r_Z^{\rm SUSY}$ has been calculated in Ref. [27]. In fact, it is useful to subtract from $\Delta r_Z^{\rm SUSY}$ leading logarithms summed by $M_{\rm SUSY}$ and reserve $\Delta r_Z^{\rm SUSY}$ to denote only additional contributions of light superpartners. The correction function (7) is modified accordingly, $\Delta_{\alpha_s} \to \Delta_{\alpha_s} - 1.16\Delta r_Z^{\rm SUSY}$. The different contributions to $\Delta r_Z^{\rm SUSY}$ are correlated in a given model, and their interplay determines its magnitude and overall sign. We find [28] that nonlogarithmic corrections typically conspire with the m_t^2 term and increase the α_s prediction, in some cases, by a few percent. Thus, heavy superpartners are preferred beyond the leading order.

On a similar note, it has been observed that if supersymmetry significantly modifies the Z hadronic width (so that the $Z \to b\bar{b}$ anomaly is accounted for) then α_s extracted from the Z line shape is diminished significantly (e.g., $0.126 \to 0.112$) [11], and this effect was even promoted as a possible resolution of the discrepancy between low- and high-energy extractions of α_s [16]. Such a scenario would require either light Higgsinos and large Yukawa couplings or a very large $\tan\beta$ and a light pseudoscalar Higgs boson [29], i.e., $|\mu| \leq O(M_Z)$. However, a scheme with a small μ parameter is not favored in GUT models [21]. From our discussion above it is also clear that a solution involving light Higgsinos (or a light pseudo-scalar) is strongly disfavored by the α_s prediction: (1) The extracted α_s line-shape value would decrease (in agreement, however, with low-energy extractions); (2) the predicted α_s value would increase due to leading-logarithm $[\propto -\ln(|\mu|/M_Z)]$ and possibly nonlogarithmic threshold corrections; (3) the central value of the fitted $m_t^{\rm pole}$ [Eq. (4b)] would grow to 163 GeV, further increasing the α_s prediction by 0.0003. Thus, the $Z \to b\bar{b}$ anomaly, if not resolved, contains strong implications for supersymmetric models and could even rule out the simplest and most attractive unification scenarios.

Lastly, we consider possible high-scale contributions to the correction function. Unlike the MSSM case, in which the particles and their mass range are dictated by the model, the details of the high-scale corrections are ambiguous. In the minimal SU(5) model [30] negative threshold corrections in (7) due to super heavy color triplet Higgs supermultiplets are strongly constrained by the non-observation of proton decay [31], and the GUT-scale threshold correction contribution to Δ_{α_s} is typically positive. (This observation, however, need not hold in extended models.) Nevertheless, one cannot extract strong constraints on the GUT spectrum. Gravitationally induced operators (suppressed by $M_G/M_{\rm Planck} \sim 0.001$) split the M_G gauge couplings (in a correlated manner) and correct the α_s prediction in proportion to their effective strength η, which is a free parameter and can have either sign. One has[7] $\Delta_{\alpha_s}^{\rm NRO} \approx 0.005\eta$. Constraining the NRO corrections to stay perturbative so that the calculation is consistent (higher-order terms are negligible in this case) one has $|\eta| \lesssim 2$ ($|\eta| \sim 3$ is an extreme but still acceptable choice). Thus, NRO's with a non-negligible and negative η could smear light and heavy threshold corrections. Unless $\eta \gtrsim 0$ and/or $M_{\rm SUSY} \ll M_Z$ (which could also imply positive non-logarithmic corrections), no significant constraints can be placed on the superheavy spectrum at present. On the other hand, the minimal SU(5) model (where threshold corrections are strongly constrained) would require NRO's with $\eta < 0$ if $\alpha_s(M_Z) \lesssim 0.125$. (A similar observation was made recently in Ref. [32].) Thus, unification and quantum gravity may be inseparable.

Regarding the unification scale, corrections that increase the unification scale would typically also increase the prediction for $\alpha_s(M_Z)$ [33], and are thus difficult to construct [in particular, for $s^2(M_Z) \approx 0.2316$]. This is true for contributions to Δ_{α_s} as well as for an additional matter family $[\alpha_s(M_Z) \to 0.132]$ or additional pairs of Higgs doublets [which lead to nonperturbative values of $\alpha_s(M_Z)$]. This is easily understood if we write $\alpha_s^{\rm OL}$ as a function of the unification scale M and of $\alpha(M_Z)$ (see Fig. 4):

$$\alpha_s^{\rm OL}(M_Z) = \frac{8\alpha(M_Z)}{3 - 60\alpha(M_Z)t}, \tag{10}$$

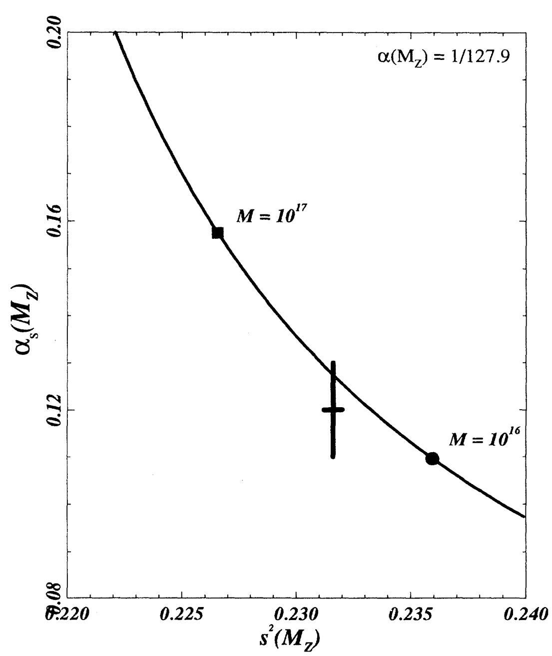

FIG. 4. The Z-pole weak angle and strong coupling are predicted as a function of the unification scale M. A given value of $s^2(M_Z)$ corresponds to a fixed choice for M, e.g., $s^2(M_Z) = 0.2359$ corresponds to $M = 10^{16}$ GeV. MSSM β functions are assumed. Two-loop Yukawa corrections are taken into account assuming $m_t^{\rm pole} = 160$ GeV and $\tan\beta = 4$. ($\Delta_{\alpha_s} = 0.$) $s^2(M_Z) = 0.2316 \pm 0.0003$ and $\alpha_s(M_Z) = 0.12 \pm 0.01$ are indicated for comparison.

[7]The proportionality factor is calculated here in the SU(5) theory [3], and its normalization is different by a factor of 4 from [3].

where $t = (1/2\pi)\ln(M/M_Z)$. Naively substituting, e.g., $M = M_{\rm string} \sim 5\times10^{17}$ GeV [34], one has $\alpha_s(M_Z) > 0.2$. By carefully adjusting operator and superheavy threshold correction contributions to Δ_{α_s}, one could increase M_G by an order of magnitude while maintaining an acceptable prediction for α_s [4,8]. However, in general, to rectify the string and unification scales (in level-one models) one has to compromise the predictive power of the unification scenario [35] so that the correlation between α_s and t is modified.

To conclude, we have shown that typically one expects a large QCD coupling in supersymmetric unified models (and even more so when considering a typical MSSM spectrum). This constitutes an interesting signature and has implications for, e.g., Yukawa unification, corrections to the unification scale, and the overall sign of the correction function Δ_{α_s}, and is in possible conflict with low-energy data. However, it does not yet allow a significant constraint on the superheavy spectrum because of possible gravitational corrections. We also pointed out the interesting role that the Z hadronic width might play in supersymmetric GUT's, and suggested a simple formula that extends our treatment of $m_t^{\rm pole}$-dependent electroweak corrections to the supersymmetric sector.

It is a pleasure to thank S. Dimopoulos for discussions and suggestions and J. Erler for discussions of precision data. This work was supported by U.S. Department of Energy Contract No. DE-AC02-76-ERO-3071.

[1] For a recent review, see, for example, H. P. Nilles, in *Testing The Standard Model*, edited by M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 633, and references therein.

[2] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); W. J. Marciano and G. Senjanovic, *ibid.* **25**, 3092 (1982); L. E. Ibanez and G. G. Ross, Phys. Lett. **105B**, 439 (1982); M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982); U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987); P. Langacker and M. Luo, *ibid.* **44**, 817 (1991); J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer, and H. Furstenau, *ibid.* **260**, 447 (1991); F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cimento **104A**, 1817 (1991).

[3] P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993).

[4] N. Polonsky, in *Proceedings of the International Workshop on Supersymmetry and Unification of Fundamental Interactions*, Boston, Massachusetts, 1993, edited by P. Nath (World Scientific, Singapore, 1993), p. 398.

[5] P. Langacker and N. Polonsky, Phys. Rev. D **49**, 1454 (1994).

[6] N. Polonsky, in *New Physics With New Experiments,* Proceedings of the XVI Kazimierz Meeting on Elementary Particle Physics, Kazimierz, Poland, 1993, edited by Z. Ajduk (World Scientific, Singapore, 1994), p. 322.

[7] C. T. Hill, Phys. Lett. **135B**, 47 (1984); Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984); J. McDonald and C. E. Vayonakis, Phys. Lett. **144B**, 199 (1984); Langacker and Polonsky [3]; L. J. Hall and U. Sarid, Phys. Rev. Lett. **70**, 2673 (1993); A. Vayonakis, Phys. Lett. B **307**, 318 (1993). Equivalently, nonminimal gauge kinetic functions were considered by J. Ellis *et al.*, Phys. Lett. **155B**, 381 (1985); M. Drees, *ibid.* **158**, 409 (1985). Large dimension-six operators (which are not consistent with a perturbative treatment) were considered by A. Datta, S. Pakrasa, and U. Sarkar, Phys. Rev. D (to be published). Implications for b-τ unification are studied in Ref. [5]. For recent discussions, see also Ref. [32].

[8] N. Polonsky, "*Unification and low-energy supersymmetry at one and two-loop orders*," a University of Pennsylvania dissertation, Report No. UPR-0641T, 1994 (unpublished), and references therein.

[9] We use the $\overline{\rm MS}$ scheme of W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); **68**, 898(E) (1992); S. Fanchiotti, B. Kniehl, and A. Sirlin, Phys. Rev. D **48**, 307 (1993), which decouples m_t and therefore differs by (small) $\ln(m_t/M_Z)$ threshold corrections from the alternative scheme of G. Degrassi, S. Fanchiotti, and A. Sirlin, Nucl. Phys. **B351**, 49 (1991), which was used in [3]. $\ln m_t/M_Z$ corrections to α were treated explicitly in [3], leading to the same numerical value of α used here. However, $s^2_{\rm new}(M_Z) - s^2_{\rm old}(M_Z) \approx -0.0002$, which is consistent with the 0.0003 residual uncertainty in our $s_0^2(M_Z)$.

[10] The value of (the $\overline{\rm MS}$) $\alpha(M_Z)^{-1} = 127.9 \pm 0.1$ used here employs the estimate of the hadronic correction $\Delta\alpha_{\rm had}$ in F. Jegerlehner, *Renormalizing the Standard Model*, in *Testing The Standard Model*, edited by M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 476. Recently, three new estimates have appeared, which, when translated to the $\overline{\rm MS}$ scheme, imply $\alpha(M_Z)^{-1} = 128.05 \pm 0.10$ [M. L. Swartz, SLAC Report No. PUB-6710) (unpublished)], $\alpha(M_Z)^{-1} = 127.96 \pm 0.06$ [A. D. Martin and D. Zeppenfeld, Phys. Lett. B **345**, 558 (1995)], and $\alpha(M_Z)^{-1} = 127.87 \pm 0.10$ [S. Eidelman and F. Jegerlehner, PSI Report No. PR-95-1 (unpublished)]. The most extreme deviation, suggested by Swartz, would imply a central value for $\alpha_s(M_Z)$ of 0.130 rather than 0.129. The change is mainly due to a lower central value of $s_0^2(M_Z) = 0.2314 \pm 0.0003$ (for $m_{t_0}^{\rm pole} = 150$ GeV), implying an increase of 0.0009 in $\alpha_s(M_Z)$. This is partially compensated by the direct effect (-0.0002) of the lower value of $\alpha(M_Z)$.

[11] J. Erler and P. Langacker, Phys. Rev. D **52**, 441 (1995).

[12] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **73**, 225 (1994).

[13] A. Kusenko (unpublished).

[14] Recent results, some of which are preliminary, are from Ref. [15]. Results from the Cern e^+e^- collider LEP are from S. Bethke, talk presented at *Radiative Corrections: Status and Outlook*, Gatlinburg, Tennessee, 1994; and from Ref. [11]. Lattice results are from A. X. El Khadra *et al.*, in *Lattice '93,* Proceedings of the International Symposium, Dallas, Texas, edited by T. Draper *et al.* [Nucl. Phys. B (Proc. Suppl.) **34**, (1994)]; C. T. H. Davies *et al.*, Phys. Lett. B **345**, 42 (1995). Other results are from I. Hinchliffe, in Particle Data Group, L. Montanet

et al., Phys. Rev. D **50**, 1173 (1994), p. 1297.

[15] B. R. Webber, in *Proceedings of the 27th International Conference on High Energy Physics*, Glasgow, Scotland, 1994, edited by P. J. Bussey and I. G. Knowles (IOP, London, in press), and references therein.

[16] The α_s discrepancy and its possible resolutions were discussed recently by M. Shifman, Minnesota Report No. UMN-TH-1323-94, 1994 (unpublished).

[17] L. Lavoura and L. Wolfenstein, Phys. Rev. D **48**, 264 (1993).

[18] U. Amaldi *et al.*, Phys. Lett. B **281**, 374 (1992).

[19] We defined the β-function coefficients $b_j^i \equiv a(S_i)N_i(j)t_i(j)$, where $a(S_i) = \frac{1}{3}, \frac{2}{3}, -\frac{11}{3}$ for a particle i of spin $S_i = 0, \frac{1}{2}, 1$, respectively, $N_i(j)$ is the appropriate multiplicity, and $t_i(j) = 0, \frac{1}{2}, 2, 3, \frac{3}{5}(\frac{Y}{2})^2$ for a singlet, a particle in the fundamental representation of SU(N), an SU(2) triplet, an SU(3) octet, and for $j = 1$ and a particle with hypercharge Y, respectively.

[20] P. Langacker and N. Polonsky, Phys. Rev. D **50**, 2199 (1994).

[21] N. Polonsky and A. Pomarol, Phys. Rev. Lett. **73**, 2292 (1994); Phys. Rev. D **51**, 6532 (1995).

[22] M. Carena, S. Pokorski, and C. E. M. Wagner, Nucl. Phys. **B406**, 59 (1993).

[23] L. J. Hall, R. Rattazzi, and U. Sarid, Phys. Rev. D **50**, 7048 (1994); M. Carena *et al.*, Nucl. Phys. **B426**, 269 (1994).

[24] V. Barger, M. S. Berger, P. Ohmann, and R. J. Phillips, Phys. Lett. B **314**, 351 (1993).

[25] A. Faraggi and B. Grinstein, Nucl. Phys. **B422**, 3 (1994); B. Lynn, Stanford Report No. SU-ITP-93-22, 1993 (unpublished); P. Chankowski, Z. Pluciennik, and S. Pokorski, Nucl. Phys. **B439**, 23 (1995); R. Barbieri, P. Ciafaloni, and A. Strumia, Pisa Report No. IFUP-TH-47-94, 1994 (unpublished). After completion of this work we received a paper by J. Bagger, K. Matchev, and D. Pierce, Phys. Lett. B **348**, 443 (1995), where similar issues are discussed and large positive corrections to $\alpha_s(M_Z)$ are also found.

[26] A. Sirlin, Phys. Lett. B **232**, 123 (1989).

[27] M. Drees, K. Hagiwara, and A. Yamada, Phys. Rev. D **45**, 1725 (1992).

[28] J. Erler, P. Langacker, and N. Polonsky (in progress).

[29] A. Djouadi *et al.*, Nucl. Phys. **B349**, 48 (1991); M. Boulware and D. Finnel, Phys. Rev. D **44**, 2054 (1991); G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. B **314**, 357 (1993); J. D. Wells, C. Kolda, and G. L. Kane, *ibid.* **338**, 219 (1994); M. Carena and C. E. M. Wagner, CERN Report No. TH-7393-94, 1994 (unpublished); J. E. Kim and G. T. Park, Phys. Rev. D **50**, 6686 (1994); **51**, 2444 (1995); D. Garcia, R. A. Jimenez, and J. Sola, Phys. Lett. B **347**, 309 (1995).

[30] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, *ibid.* **33**, 451 (1974); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[31] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69**, 725 (1992); J. Hisano *et al.*, Nucl. Phys. **B402**, 46 (1993); Tohoku Report No. TU-470, 1994 (unpublished).

[32] After completion of this work we received the papers by T. Dasgupta, P. Mamales, and P. Nath, Northeastern Report, 1995 and D. Ring, S. Urano, and R. Arnowitt, Texas A&M Report No. CTP-TAMU-01-95, 1995 (unpublished), where NRO's in the minimal SU(5) model are studied.

[33] We thank Savas Dimopoulos for stressing this point and for suggesting to plot Fig. 4.

[34] V. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988).

[35] See, for example, I. Antoniadis, J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **272**, 31 (1991); L. E. Ibanez, D. Lust, and G. G. Ross, *ibid.* **272**, 251 (1991); A. Faraggi, *ibid.* **302**, 202 (1993); L. E. Ibanez, *ibid.* **318**, 73 (1993); T. Kobayashi, D. Snematsu, and Y. Yamagishi, *ibid.* **329**, 27 (1994).