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Top quark and Higgs boson masses in dynamical symmetry breaking

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A model for composite electroweak bosons is reexamined to establish approximate ranges for the initial predictions of the top quark and Higgs boson masses. Higher order corrections to this four-fermion theory at high mass scale, where the theory is matched to the standard model, have little effect, as do wide variations in this scale. However, including all one loop evolution and defining the masses self-consistently, at their respective poles, shifts the top quark and Higgs boson masses somewhat from the earlier calculated positions. These masses exhibit a moderate dependence on the measured strong coupling: for example, with $\alpha_S(m_W) = 0.115(0.125)$, one finds $m_t \sim 180(185)$ GeV and $m_H \sim 130(135)$ GeV.

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I. INTRODUCTION

In this paper we refine predictions for the top quark and Higgs boson masses made in an earlier work on dynamical symmetry breaking [1]. The specific four-fermion model of dynamical symmetry breaking presented in Ref. [1] [see the Lagrangian in Eq. (1) below] may perhaps ultimately be viewed as the low-mass limit of a gauge theory at some very high scale Λ, with primordial boson masses $M_B = O(\Lambda)$. This scale then acts as an effective cutoff for the four-fermion theory. Certainly, no explanation is presented here for the number and character of elementary fermions included in the model or for the large disparity in mass scales, i.e., $m_f \ll M_B$. Rather, a central point of our calculation is that new, composite, bosons with masses near $2m_f$ arise naturally in the theory. These are just fermion-antifermion bound states produced by the four-fermion interaction. This phenomenon is well described in the papers of Nambu and Jona-Lasinio (NJL) on the four-fermion theories [2], and has been exploited by many authors [3–8]. Since the scale Λ at which any new physics enters is so high, the theory is in fact a weak coupling, albeit constrained, version of the standard model for scales well below Λ.

Previously [1] we abstracted simple, asymptotic relationships between the masses of composite standard model bosons and those of the model fermions, principally the top quark, from the four-fermion theory and used these as boundary conditions on the standard model renormalization group (RG) equations. This was done at a matching scale $\mu \sim M_{\mathrm{GUT's}}$, where the electroweak (EW) sector can be treated as approximately independent of QCD $[\mathrm{SU}(3)_c]$. Values for the top quark and Higgs boson masses then followed from downward evolution of the top-quark–Higgs-boson and Higgs self-couplings to scales near m_W, assuming no intervening structure. This scale μ has no fundamental significance, but is introduced as a mathematical convenience to tie the four-fermion theory to the standard model. It is a scale, however, at which all couplings should be small and might be defined as a scale below which the strong interactions begin to affect the weak.

In the present work we show that modifications in these asymptotic relationships, due to higher order corrections in the four-fermion theory at the upper scale μ, are considerably reduced in magnitude when evolved down to the much lower scale of m_t or m_H. Also, large changes in μ, even several orders of magnitude, affect the top quark and Higgs boson masses remarkably little. However, a more consistent handling of the RG evolution moves the central predictions for m_t and m_H from approximately 165 GeV and 140 GeV in Ref. [1] to nearer 180 GeV and 130 GeV, respectively. The actual values for these masses depend somewhat on the measured value of the $\mathrm{SU}(3)_c$ coupling at the W mass (see Fig. 3).

II. THE FOUR-FERMION THEORY

In Ref. [1], we indicated that a four-fermion interaction including vector terms led to rather low, well-determined masses for the top quark and Higgs boson. The model is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi} i(\gamma\cdot\partial)\psi - \tfrac{1}{2}[(\bar{\psi}G_S\psi)^2 - (\bar{\psi}G_S\tau\gamma_5\psi)^2] - \tfrac{1}{2}G_B^2(\bar{\psi}\gamma_\mu Y\psi)^2 - \tfrac{1}{2}G_W^2(\bar{\psi}\gamma_\mu\tau P_L\psi)^2, \qquad (1)$$

in which very specific vector interactions have been added to the usual scalar and pseudoscalar terms of NJL. The form of the vector terms is uniquely dictated by the standard model. The field operator is $\psi = \{f_i\}$, and the index i runs over all fermions, $i = \{(t,b,\tau,\nu_\tau),(c,s,\ldots),\ldots\}$.

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The scalar-coupling matrix G_s is taken diagonal and the dimensionful couplings are adjusted to produce the known fermion masses dynamically; in practice only the top acquires an appreciable mass. The model admits bound states corresponding to the Higgs bosons as well as the gauge bosons of the standard electroweak theory, and is essentially equivalent to the standard model below some high mass scale μ. It is the vector terms in Eq. (1) which ensure the existence of the Z and W as composites with masses of the order of m_t, thus naturally explaining why these standard model bosons and the top quark appear to have about the same mass. The usual NJL treatment with the scalar interaction produces an analagous result for the Higgs boson.

In reexamining the predictions for the top quark and Higgs boson we do not presume to seek overly precise values for their masses, but rather attempt to determine the latitude in masses present in the modeling. Such a study is especially timely[1] in light of the search for the top now being carried out at Fermilab [9]. The apparent paucity of top events in this data suggests a high mass for the top, certainly it now seems m_t is greater than 130 GeV and possibly considerably higher. Present analyses of data from the CERN e^+e^- [10] with respect to EW corrections suggest[1] $m_t = 166 \pm 30$ GeV.

As usual in NJL a necessary fine-tuning of the scalar coupling is accomplished by solving the scalar gap equation, whence diagonalization of the scalar action yields the Higgs boson mass formula

$$m_H(\mu) = 2m_t(\mu)[1 + O(g_t^2)] , \quad (2)$$

where g_t is the dimensionless Yukawa top-quark–Higgs-boson coupling defined below [see Eq. (6)]. This fine-tuning relates the dimensionful scalar coupling to the cutoff Λ:

$$G_t^2 = \frac{1}{\Lambda^2 - m_t^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right)} . \quad (3)$$

Bound states also exist in the vector channels defined by Eq. (1), corresponding to the W, the Z, and the photon. Kinetic and mass terms for each composite boson are generated in the effective action [1]. A similar fine-tuning of the vector coupling is required, but here with the added physical interpretation that the photon mass should vanish [1]. This latter constraint leads, at lowest order in the electroweak and Yukawa couplings, to the mass relationship

$$m_W^2(\mu) = \tfrac{3}{8} m_t^2(\mu) . \quad (4)$$

To the same order in couplings, the required diagonalization of the neutral vector boson action results in

$$\sin^2(\theta_W) = \left(\sum_i Q_i^2\right)^{-1} = \tfrac{3}{8} , \quad (5)$$

with the denominator on the right-hand side of Eq. (5) being summed over the charges Q_i in any one generation.

The dimensionful couplings of the four-fermion theory are replaced, after fine-tuning and wave function renormalization, by the dimensionless couplings of the standard model [11,1], and the gradient expansion of the effective action is in fact an expansion in these dimensionless electroweak couplings. One has, for the scalars,

$$g_S = G_S Z_S^{-1/2},$$
$$Z_S = \frac{1}{2}\mathrm{Tr}\left[G_S^2 \frac{1}{(\partial^2 + M^2)^2}\right] , \quad (6)$$

where the fermion-scalar coupling matrix is for the present taken diagonal:

$$(G_S)_{ij} = G_i \delta_{ij} . \quad (7)$$

Similarly, for the vector couplings one has

$$\frac{g_2}{2} = \frac{G_W}{\sqrt{Z_W}} \quad (8)$$

and

$$\frac{g'}{2} = \frac{G_B}{\sqrt{Z_B}} \quad (9)$$

as well as the usual relationship between g_2 and g':

$$g_2 \sin(\theta_W) = g' \cos(\theta_W) . \quad (10)$$

From Eqs. (2), (4), and (5), valid presumably at a scale μ where any mixing between the EW and strong sectors is small, but still well below the cutoff Λ, we derive values for the top quark and Higgs boson masses at a scale near m_W. The theory leading to these equations is equivalent to the electroweak sector of the standard model below μ, and the framework for connecting the scales μ and m_W is provided by the standard model RG. Thus $SU(3)_c$ influences on the top quark and Higgs boson masses are included through the renormalization group, below the matching scale μ. We demonstrate below that indeed all couplings are small near μ, so that cross couplings between EW and strong is small there, a mathematical necessity for our proposed use of Eqs. (2), (4), and (5) as boundary conditions to the RG evolution. We note that all triviality points are well above this scale, and indeed above the Planck scale, the latter a likely place for our cutoff and for potentially new physics.

III. RENORMALIZATION GROUP EVOLUTION

We turn now to the calculation of smaller effects, neglected in the initial work due to corrections in the four-fermion theory of higher order in the electroweak couplings, and to a more consistent treatment of the evolution downward to experimental mass scales. Our basic

[1] We note that this manuscript was submitted before the first successful results of the Collider Detector at Fermilab (CDF) [20] and D0 [21] discovering the top quark. Present values for m_t from CDF and D0 are $178 \pm 8 \pm 12$ GeV and $199 \pm 20 \pm 22$ GeV, respectively. The central LEP [22] value for m_t from EW loop corrections has by now moved to $177 \pm 11 \pm 19$ GeV.

equations are (1) the boundary condition relationships between the Higgs boson, top quark, and W masses including dependence on electroweak couplings and quark masses, and (2) the RG evolution equations for the top-quark–Higgs-boson and Higgs-self-couplings g_t and λ. Defining (see [12,13])

$$\kappa_t = \frac{g_t^2}{2\pi} , \quad (11)$$

one has

$$\frac{d\kappa_t}{dt} = \frac{9}{4\pi}\kappa_t^2 - \frac{4}{\pi}\kappa_t\alpha_S - \frac{9}{8\pi}\kappa_t\alpha_W - \frac{17}{4\pi}\kappa_t\alpha_1 , \quad (12)$$

with $\alpha_S, \alpha_W, \alpha_1$ taken equal to the $\alpha_3, \alpha_2, \alpha_1$ of Refs. [12,13], while $t = \ln(\frac{q}{m})$. With these choices one finds

$$m_t = g_t v , \qquad m_W = \frac{g_W}{2} v , \quad (13)$$

where v is the standard EW vacuum expectation value (VEV).

Also taking $m_H^2 = 2\lambda v^2$, the evolution equation for the Higgs boson self-coupling is, to the same (one-loop) order [14]:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left\{12\lambda^2 + 6\lambda g_t^2 - 3g_t^4 - \frac{3}{2}\lambda\left(3g_W^2 + g'^2\right) + \frac{3}{16}[2g_W^4 + (g_W^2 + g'^2)^2]\right\} . \quad (14)$$

Redefining the standard choice of couplings [12],

$$\alpha_1 = \frac{5}{3}\alpha' \text{ with } \alpha_1 = \frac{g_1^2}{4\pi},\ \alpha' = \frac{g'^2}{4\pi} , \quad (15)$$

and setting

$$\sigma = \frac{\lambda}{4\pi} , \quad (16)$$

one obtains

$$\frac{d\sigma}{dt} = \frac{1}{2\pi}\left\{12\sigma^2 + 6\sigma\kappa_t - 3\kappa_t^2 - \frac{9}{2}\sigma\left(\alpha_W = \frac{1}{5}\alpha_1\right) + \frac{3}{16}\left[2\alpha_W^2 + \left(\alpha_W + \frac{3}{5}\alpha_1\right)\right]\right\} . \quad (17)$$

Equations (2) and (4) impose boundary conditions on Eqs. (12) and (17) at the scale μ. These are, to lowest order,

$$m_t^2 = \frac{8}{3}m_W^2$$

and

$$m_H^2 = 4m_t^2 = \frac{32}{3}m_W^2 , \quad (18)$$

which can be restated to include higher orders:

$$\frac{\kappa_t}{\alpha_2}(\mu) = \frac{4}{3} + O(g_i^2) \text{ and } \frac{\sigma}{\alpha_2}(\mu) = \frac{4}{3} + O(g_i^2) . \quad (19)$$

Corrections to Eq. (19) come from two sources: higher order $1/N$, multiloop contributions to the effective action, and more trivial $1/\ln(\Lambda)$ terms within the lowest order. The latter arise, for example, from the proper generalized form of Eq. (4):

$$m_W^2 = \frac{1}{2}\frac{\sum_i m_i^2\left[\ln\left(\frac{\Lambda^2}{m_i^2}+1\right) - 1\right]}{\sum_i \frac{r_i}{6}\left[\ln\left(\frac{\Lambda^2}{m_i^2}+1\right) - \frac{11}{6}\right]} , \quad (20)$$

where the sum is over all fermions and

$$r_i = \cos^{-2}(\theta_W)[\sin^4(\theta_W)(y_{Li}^2 + y_{Ri}^2) - 2\sin^2(\theta_W)\cos^2(\theta_W)y_{Li}\tau_i^z + \cos^4(\theta_W)\tau_i^z\tau_i^z] . \quad (21)$$

y_{Li}, y_{Ri}, and τ_i^z are the fermion hypercharges and weak isospins. Equation (4) is obtained from Eq. (20) by keeping only the top quark mass and ignoring terms of order $[\ln(\Lambda)]^{-1}$. These terms are of higher order in the electroweak couplings: for example the Higgs-boson–top quark Yukawa coupling is, from Eq. (6), proportional to $[\ln(\Lambda)]^{-1}$.

We note parenthetically that the basic SU(5) symmetry evident in Eqs. (4) and (5) results from the $\bar{\mathbf{5}}$ + **10** generational structure $[(u_L, d_L), u_R, d_R, (e_L, \nu_L), e_R]$ built into the present model, and follows from Eq. (20) in the limit of large Λ. We find that the several percent change implied in Eq. (20) relative to Eq. (4), at the scale μ, produces a considerably smaller change in m_t at m_W, less than 1%. Thus, to the accuracy meaningful here, we can perhaps ignore these corrections as well as other higher order $1/N$ effects arising from discarded, incoherent, summations over fermions.

IV. SOLUTION OF THE RG EQUATIONS

It is possible to obtain an explicit solution to Eq. (12), and a perturbative solution for Eq. (17). For the top quark evolution one has, making a simple transformation of Eq. (12),

$$\frac{d}{dt}\frac{1}{\kappa_t} = -\frac{9}{4\pi} + \frac{1}{\kappa_t}\left(\frac{4}{\pi}\alpha_S + \frac{9}{8\pi}\alpha_W + \frac{17}{40\pi}\alpha_1\right), \quad (22)$$

which has the one-parameter family of solutions

$$\frac{1}{\kappa_t} = \frac{(1+\alpha_{S0}b_S t)^{8/7}(1+\alpha_{W0}b_W t)^{27/38}}{(1-\alpha_{10}b_1 t)^{17/82}}\left[D - \frac{9}{4\pi}\int_0^t dt'\frac{(1-\alpha_{10}b_1 t')^{17/82}}{(1+\alpha_{S0}b_S t')^{8/7}(1+\alpha_{W0}b_W t')^{27/38}}\right] . \quad (23)$$

Here α_{S0}, α_{W0}, and α_{10} are the couplings at $t = \ln\frac{m_W}{m_W} = 0$, and the constants $b_S = \frac{7}{2\pi}$, $b_W = \frac{19}{12\pi}$, and $b_1 = \frac{41}{20\pi}$ determine the evolution of the SU(3), SU(2), and U(1) couplings, respectively. The constant D in Eq. (23) is given by

$$D = \frac{1}{\kappa_t(0)} , \tag{24}$$

and directly yields the running top mass at the scale m_W from

$$m_t^2(m_W) = \frac{2\kappa_t(0)}{\alpha_W(0)} m_W^2(m_W) . \tag{25}$$

To self-consistently determine the physical top quark mass as a pole in the top quark propagator, one must run $m_t(m_W)$ back up to get $m_t(m_t)$.

The cross coupling in Eq. (17) complicates its solution. The pure scalar self-coupling result

$$\sigma_0(t) = \frac{\sigma_0(0)}{1 - \frac{6}{\pi}\sigma_0(0)t} , \tag{26}$$

may be improved perturbatively:

$$\sigma(t) = \sigma_0(t) + \sigma_1(t) . \tag{27}$$

Linearizing in the small correction $\sigma_1(t)$ produces

$$\sigma_1(t) = e^{-v(t)} \int_{t_\mu}^{t} dt' g(t') e^{v(t')} , \tag{28}$$

with

$$v(t) = -\int_{t_\mu}^{t} dt' f(t') , \tag{29a}$$

$$f(t) = \frac{12}{\pi}\sigma_0(t) + \frac{3\kappa_t(t)}{\pi} - \frac{9}{4\pi}\left[\alpha_2(t) + \frac{1}{5}\alpha_1(t)\right] , \tag{29b}$$

and

$$g(t) = \frac{3}{\pi}\sigma_0(t)\kappa_t(t) - \frac{3\kappa_t^2}{2\pi} + \frac{3}{32\pi}\left[2\alpha_2^2(t) + \left(\alpha_2(t) + \frac{3}{5}\alpha_1(t)\right)^2\right] . \tag{29c}$$

Boundary conditions are introduced at $t_\mu = \ln\frac{\mu}{m_W}$ through

TABLE I. The SU(3), SU(2), and U(1) couplings, as well as the Higgs top-, and the Higgs self-coupling are shown at both the scales $m_W = 80.1$ GeV and $\mu = 7.5 \times 10^{12}$ GeV. At the upper scale, all these couplings are comparable, and may be considered small.

q	α_S	α_W	α_1	κ_t	σ
m_W	0.107	0.0344	0.0169	0.0880	0.0111
μ	0.0267	0.0239	0.0234	0.0319	0.0319

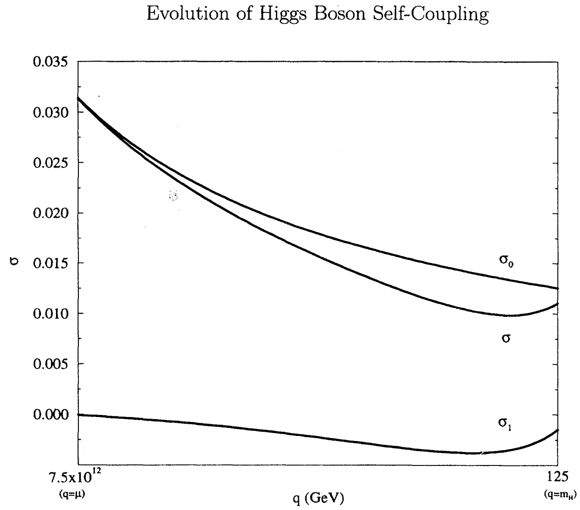

FIG. 1. Evoluton of the reduced Higgs self-coupling $\sigma = \sigma_0 + \sigma_1$ over the range from m_W to $\mu = 10^{14}$. The perturbation σ_1 remains small.

$$\sigma_1(t_\mu) = 0, \quad \sigma_0(t_\mu) = \kappa_t(t_\mu) = \tfrac{4}{3}\alpha_2(t_\mu) + O(\alpha_i^2) . \tag{30}$$

Since $\sigma_1(t)$ is small over the range m_H to μ (see Fig. 1) there is no need to include higher orders.

Results from numerical integration of Eqs. (23), (28), and (29) are displayed in Table I, and Figs. 1–4. We have varied the inputs to these calculations, the strong and electroweak couplings $\alpha_{i0} (i = \{1, W, S\})$, over a reasonable range, somewhat wider than the flexibility allowed by present experiments. The W mass is fixed at

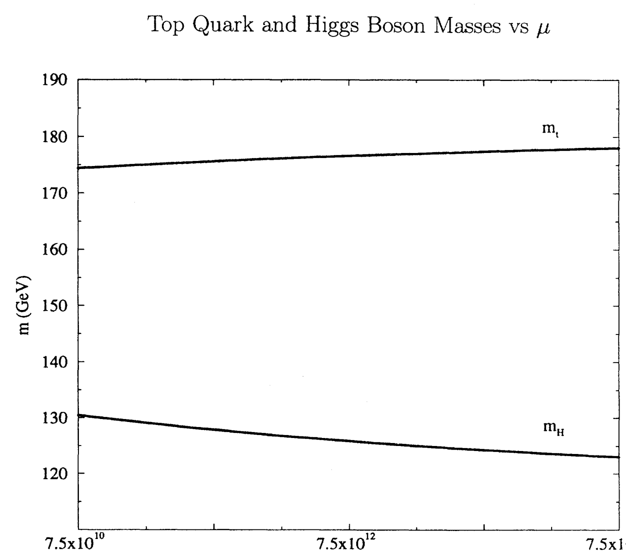

FIG. 2. Variation of the top quark and Higgs boson masses with the matching scale μ over a range from 10^{10} to 10^{14} GeV. The scale $\mu = 7.5 \times 10^{12}$, for which $\sin^2[\theta(\mu)] = \frac{3}{8}$, is defined as a central value.

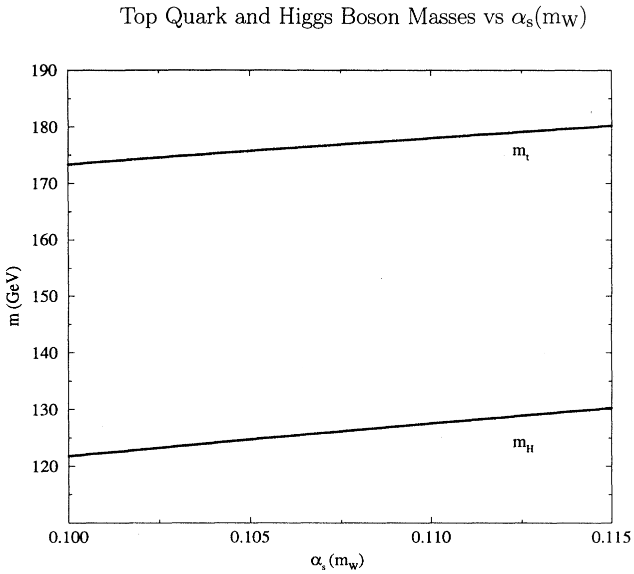

FIG. 3. Variation of m_t and m_H with the strong coupling. Present LEP data suggest a higher value of this coupling than previously, so perhaps one must consider α_{S0} between 0.107 and at least 0.120, as possible.

80.1 GeV, and this alone sets the magnitudes of the m_t and m_H predictions; there are no free parameters in the theory, the couplings and m_W being determined from experiment. The cutoff Λ, which is surely well above μ, has essentially no effect on m_t and m_H. Any dependence other than logarithmic on Λ has been eliminated by fine-tuning, while residual $\ln(\Lambda)$ presence is transmuted into dependence on the experimentally measured dimensionless couplings.

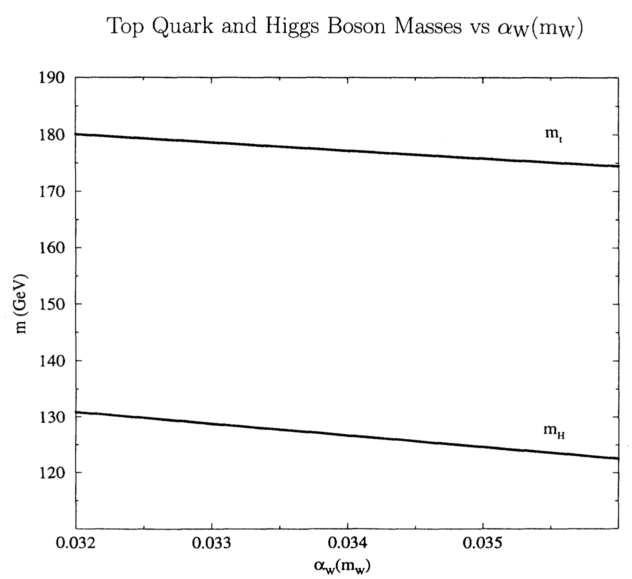

FIG. 4. Variation of m_t and m_H with the weak coupling; $\alpha_W = 0.0344$ is the central value.

The effect of imposing boundary conditions sharply at a scale μ remains to be examined. As we noted above, a minimum value for μ is that point, evolving downward in mass, at which the g_i become interdependent. For example, the top quark evolution is strongly influenced by $SU(3)_c$ from $\mu \sim 10^{14}$ downward, and even the running of α_W is significant. Varying μ over 4 orders of magnitude from $\mu = 10^{10}$ GeV to $\mu = 10^{14}$ GeV has practically no effect on m_t, and only a small effect on m_H. This remarkable result, demonstrated in Fig. 2, lends credence to our use of a sharp boundary condition.

The one physical parameter sensitive to μ is the weak mixing angle θ_W. We indicated [1] that, for one loop evolution, $\sin^2(\theta_W)$ achieves its experimental value ~ 0.23 (at a scale m_W) for the choice $\mu \sim 10^{13}$ GeV. Unlike grand unified theories (GUT's), the present theory need not have a single scale at which the gauge couplings are equal, however, and one need only have $\sin^2(\theta_W)$ near $\frac{3}{8}$ at "some" scale. A degree of unification does exist in the model, and this simply implies that the standard model should evolve smoothly into the effective four-fermion theory where the couplings become weak. At such a point the couplings are found to obey simple relations. There is, of course, no explicit baryon decay present in the model described by the Lagrangian [Eq. (1)]. Table I displays the couplings at scale μ; the α_i are the experimental values at m_W evolved upward to μ, while $\kappa_t(\mu)$ is obtained from the boundary condition $\frac{\kappa_t}{\alpha_2} = \frac{4}{3}$. It is clear that the couplings are indeed all small at μ, again justifying the placing of the boundary conditions there.

Figures 3 and 4 show the variations of m_t and m_H with the strong and electroweak couplings, respectively. The strong coupling is, perhaps, less well known. Using as a representative low value $\alpha_{S0} = 0.108$, as well as $\alpha_{W0} = 0.0344$, and $\alpha_{10} = 0.0169$ [10,16], we get $m_t \simeq 175$ GeV and $m_H \simeq 125$ GeV. Included in the top mass is a 6-GeV reduction from evolving the top self-consistently to its proper mass at $q = m_t$; for the Higgs boson this effect is smaller. Clearly, m_t and m_H are most influenced by the strong coupling, rising to close to 180 and 130 GeV, respectively, for a coupling $\alpha_{S0} = 0.115$ (and even higher for the present LEP claim for α_{S0}; see footnote 1). The major ambiguity in our predictions arises then from the present lack of precision in the measurement of the strong coupling at a scale near m_W or m_Z. Over the past few years one has had global searches which yield, for example: $\alpha_S(m_Z) = 0.118 \pm 0.008$ [18], or using just electroweak data $\alpha_S(m_Z) = 0.120 \pm 0.006$ [19,22]. Figure 3 covers this range and indicates to what extent m_t and m_H increase with increasing $\alpha_S(m_W)$.

Further small contributions to Eq. (19), from nonleading log terms in defining the top pole and from running the W mass, are ignored here. It is clear from the figures that m_H is somewhat more sensitive to all these changes, and the remaining uncertainty in its predicted mass larger. This uncertainty, nevertheless, may be usefully bounded by noting [1] that a rather large arbitrary variation in the boundary condition ratio m_H/m_t from 2 to $\sqrt{8}$ produces less than a 15-GeV change in m_H. One must also keep in mind that the top is confined and its mass therefore subject to some ambiguity in definition.

V. CONCLUSIONS AND COMMENTS

In summary, it is clear the model leads, in a parameter-free fashion, to remarkably stable predictions for the top quark and Higgs boson masses. The only inputs were the experimentally known couplings and the W mass. A characteristic prediction of this type of theory is $m_H \leq m_t$, so that the Higgs bosons, which is almost purely a $t\bar{t}$ bound state, is deeply bound.

In view of the present dearth of events from the Fermilab experiments with D0 and CDF, the prediction for the top quark in Fig. 3 (near 180 GeV) may not be wholly wild (see footnote 1). The somewhat low prediction for the Higgs boson mass (near 130 GeV) may take considerably longer to test.

One point certainly worthy of more comment is the apparent composite nature of the photon in the above. At first sight this would seem to be a weakness of this approach. This point has been discussed by many authors, first by Bjorken [3], and by some [5,6] who use the photon mass-vanishing condition to convert the global symmetry which led to U(1) current conservation, into a local gauge invariance. We do not with to pursue this argument here but note simply that the vector gap equation of Ref. [1] implies that the vacuum polarization contribution to the photon inverse propagator is in fact purely transverse, as in QED, consistent with current conservation. The inverse photon propagator for the composite photon may be written [1,3]

$$\Delta_{\rho\sigma}^{-1}(q) = \delta_{\rho\sigma} + \Delta_{0\rho\sigma}^{-1}(q) + E^2\Pi_{\rho\sigma}(q) \ , \tag{31}$$

where $E(\mu) = G_W(\mu)\cos(\Theta_W)$ is the dimensionful electromagnetic coupling in the four-fermion theory. A zero-order term consistent with the free Lagrangian generated for the composite photon, and vanishing for on-shell photons, is included. The vector gap equation, which in this context may be viewed as a renormalization condition, is obtained by setting

$$\Delta_{\rho\sigma}^{-1}(0) = \delta_{\rho\sigma} + E^2\Pi_{\rho\sigma}(0) = 0 \ . \tag{32}$$

With the proper definition of $\Delta_{0\rho\sigma}^{-1}(q)$, and because of current conservation, one can introduce a transverse vacuum polarization satisfying

$$q^\rho\Pi_{\rho\sigma}^{\perp}(q) = q^\rho[\Pi_{\rho\sigma}(q) - \Pi_{\rho\sigma}(0)]0 \ . \tag{33}$$

Finally, after a further, dimension-altering, wave function renormalization [see Eq. (9)] for the vector bosons, one can define a renormalized propagator in complete accord with the standard model:

$$\Delta_{\rho\sigma}^{-1^R} = \Delta_{0\rho\sigma}^{-1^R} + e^2\Pi_{\rho\sigma}^{\perp R} \ . \tag{34}$$

Equation (32) can be imposed at any order in the NJL calculation and this, together with the tranversality of the vacuum polarization, keeps the photon mass zero. The gap equation itself is not much of a constraint on the coupling, simply relating the dimensionful vector coupling to the high, but arbitrary, cutoff Λ much as in Eq. (3) for the scalar sector. After wave function renormalization only the dimensionless vector couplings remain; E is replaced by e.

As we noted in the Introduction it may be possible to rederive our results from a standardlike model at a very high scale near the cutoff, with the photon a gauge particle throughout. Then the effective four-fermion theory could be viewed as just a calculational tool for finding the positions of the bound W, Z, and Higgs bosons which are present near $2m_f$. Interestingly, a connection is developed in Eqs. (4) and (6) between the effective composite boson-fermion couplings and the cutoff scale [linear in $\ln(\Lambda)^{-1}$], where "new" physics might arise.

Finally, there is the question of the total number of low-mass fermions. In Ref. [1], we indicated that a fourth generation, with massive quarks $m_{t'} \sim m_{b'} \sim m_t$, implies a top quark mass near 115 GeV. Such a constraint arises from the sum rule [Eq. (19)] for m_W^2. Present data at Fermilab appear to rule out this possibility, and, although the present work gives no reason for the existence of just three generations, it is neatly consistent with such a counting.

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[1] D. E. Kahana and S. H. Kahana, Phys. Rev. D **43**, 2361 (1991); D. E. Kahana, in *Proceedings of the Workshop on QCD Vacuum Structure*, Paris, 1992, edited by H. M. Fried and B. Müller (World Scientific, Singapore, 1993); D. E. Kahana and S. H. Kahana, in *The Fermilab Meeting*, Proceedings of the Meeting of the Division of Particles and Fields of the American Physical Society, Batavia, Illinois, 1992, edited by Carl H. Albright, Peter H. Kasper, Rajendran Raja, and John Yoh (World Scientific, Singapore, 1993), Vol. 1, p. 477.

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[3] J. D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963).

[4] W. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[5] H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D **15**, 480 (1977).

[6] T. Eguchi, Phys. Rev. D **14**, 2755 (1976).

[7] M. Bando, T. Kugo, and K. Yanawaki, Phys. Rep. **164**, 210 (1988); M. Suzuki, Phys. Rev. D **37** 210 (1988).
[8] V. A. Miranski, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1043 (1989).
[9] CDF Collaboration, Avi Yagil *et al.*, in *The Fermilab Meeting* [1], Vol. 1, and other contributions therein; D0 Collaboration, Ronald J. Mahas, *et al.*, *ibid.*, and other contributions therein; D0 Collaboration, in *Lepton and Photon Interactions*, Proceeding of the 16th International Conference, Ithaca, New York, 1993, edited by P. Drell and D. Rubin, AIP Conf. Proc. No. 302 (AIP, New York, 1994).
[10] Samuel Ting, in The Fermilab Meeting [1], Vol. 1, and other contributions therein; LEP Summary, in *Lepton and Photon Interactions* [9].
[11] G. S. Guralnik, K. Tamvakis, Nucl. Phys. **B148**, 283 (1979).
[12] W. Marciano, Phys. Rev. Lett. **62**, 2793 (1989).
[13] W. Marciano, Phys. Rev. D **41**, 219 (1990); W. Marciano and A. Sirlin, *ibid.* **22**, 2695 (1980).
[14] John F. Gunion, Howard E. Haber, Gordon Kane, and Sally Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, New York, 1990).
[15] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. Quinn, and S. Weinberg, *ibid.*, **33**, 451 (1974).
[16] U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1984); LEP Collaborations, in *The Fermilab Meeting* [1].
[17] S. Fanchiotti, Bernd Kniehl, and A. Sirlin, Phys. Rev. D **48**, 307 (1993).
[18] S. Bethke, J. Phys. G **17**, 1455 (1991).
[19] L. Rolandi, in *Proceedings of the XXVI International Conference on High Energy Physics*, Dallas, Texas, 1992, edited by J. Sanborn, AIP Conf. Proc. No. 272 (AIP, New York, 1993).
[20] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **73**, 225 (1995); **74**, 2626 (1995).
[21] D0 Collaboration, S. Abachi *et al.*, Phys. Rev. Lett. **74**, 2632 (1995).
[22] LEP Collaboration and the LEP Electroweak Working Group, B. Pietrzyk, Laboratoire de Physique de Particule, Report No. LAPP-Exp-94.07, 1994 (unpublished).