B_s -$\overline{B}_s$ mixing, CP violation, and extraction of CKM phases from untagged B_s data samples

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A width difference of the order of 20% has previously been predicted for the two mass eigenstates of the B_s meson. The dominant contributor to the width difference is the $b \to c\bar{c}s$ transition, with final states common to both B_s and $\overline{B}_s$. All current experimental analyses fit the time dependences of flavor-specific B_s modes to a single exponential, which essentially determines the average B_s lifetime. We stress that the same data sample allows even the measurement of the width difference. To see that, this article reviews the time-dependent formulas for tagged B_s decays, which involve rapid oscillatory terms depending on Δmt. In untagged data samples the rapid oscillatory terms cancel. Their time evolutions depend only on the much more slowly varying exponential falloffs. We discuss in detail the extraction of the two widths, and identify the large (small) CP-even (-odd) rate with that of the light (heavy) B_s mass eigenstate. It is demonstrated that decay length distributions of some *untagged* B_s modes, such as $\rho^0 K_S$, ωK_S, $D_s^{(*)\pm}K^{(*)\mp}$, can be used to extract the notoriously difficult CKM unitarity triangle angle γ. Sizable CP-violating effects may be seen with such untagged B_s data samples. Listing $\Delta\Gamma$ as an observable allows for additional important standard model constraints. Within the CKM model, the ratio $\Delta\Gamma/\Delta m$ involves no CKM parameters, only a hadronic uncertainty. Thus a measurement of $\Delta\Gamma$ (Δm) would predict Δm ($\Delta\Gamma$), up to the uncertainty. A large width difference would automatically solve the puzzle of the number of charmed hadrons per B decay in favor of theory. We also derive an upper limit of $(|\Delta\Gamma|/\Gamma)_{B_s} \lesssim 0.3$. Further, we must abandon the notion of branching fractions of $B_s \to f$, and instead consider $B(B^0_{L(H)} \to f)$, in analogy with the neutral kaons.

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I. INTRODUCTION

B physics has matured to the point that data samples of strange B mesons are currently being collected both at Fermilab [1] and at the CERN e^+e^- collider LEP [2–5]. More than 200 flavor-specific events and a few dozen $J/\psi\phi$ events have been recorded. It is believed that precision studies of B_s mesons requires a distinction between B_s and $\overline{B}_s$ mesons (henceforth denoted as "tagging") and superb vertex resolution so as to follow the rapid oscillatory behavior dependent upon Δmt. Then the observation of CP-violating phenomena and the extraction of fundamental Cabibbo-Kobayashi-Maskawa [6] (CKM) parameters can be contemplated [7,8].

It may not be imperative to trace the rapid Δmt oscillations. Time-dependent studies of *untagged* data samples of B_s's remove the rapid oscillatory behavior depending upon Δmt. What remains are two exponents $e^{-\Gamma_L t}$ and $e^{-\Gamma_H t}$, where the light and heavy B_s-mass eigenstates have an average lifetime of about $\tau_b \sim 1.6$ ps [9], and are expected to differ by about (10–30)% [10–18]. This could be sufficient for observation of B_s-$\overline{B}_s$ mixing (due to lifetime differences), CP violation, and the clean extraction of CKM parameters [19,20]. Tagging and time-resolving Δmt oscillations would of course allow many additional precision B_s measurements (for reviews see for instance Refs. [18,21,22]).

Lately there has been an emphasis on the predicted large mass mixing:

$$x_s = \left(\frac{\Delta m}{\Gamma}\right)_{B_s} . \tag{1.1}$$

The measurement of x_s requires tagged B_s data samples and superb vertex resolution for tracing the rapid Δmt oscillations [23]. The parameter x_s may turn out to be too large to be measured in the foreseeable future [23–25]. There exists, however, another clear measure of B_s-$\overline{B}_s$ mixing, namely, a width difference $\Delta\Gamma$ between the B_s-mass eigenstates. The estimate for $\Delta\Gamma/\Delta m$ suffers from no CKM uncertainty only from hadronic uncertainties [26,12,13,18]. Thus, large Δm values that are currently impossible to measure may imply values for $\Delta\Gamma$ that are currently feasible. It may happen that a width difference will be the first observed B_s-$\overline{B}_s$ mixing effect.

The implications of measuring a nonzero $\Delta\Gamma$ would be far reaching. Not only would B_s-$\bar{B}_s$ mixing be demonstrated, but Δm would perhaps be well estimated. The estimate would combine the predicted ratio $\Delta m/\Delta\Gamma$ with the more traditional approaches [24] to optimize our knowledge on Δm. A reliable estimate or measurement of Δm allows not only the extraction of the combination of decay constant and bag parameter $(B_{B_s} f^2_{B_s})$ [10], but even the planning of a multitude of CP-violating measurements and determinations of CKM parameters with *tagged* B_s-data samples [18,21,22]. (Conversely, if Δm were to be observed first, valuable information on $\Delta\Gamma$ would be available. In the long term, measurements of both Δm and $\Delta\Gamma$ allows us to probe the hadronic uncer-

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tainties arising in $\Delta\Gamma/\Delta m$.) Some of the central points of this article follow. First, a nonvanishing $\Delta\Gamma$ enables us to observe large CP-violating effects and to cleanly extract CKM parameters (for instance γ) from much more slowly varying time evolutions of some *untagged* B_s-data samples.

In contrast, the traditional methods that use B_s decays require tagging and the ability to trace the rapid Δmt oscillations. It is easy to explain why such measurements are possible for nonzero $\Delta\Gamma$ with some untagged data samples. Consider the creation of a B_s. The B_s state can be written as a linear superposition of the heavy B_H and light B_L eigenstates of the mass matrix. Because the two eigenstates have different lifetimes, suitably long times can be chosen where the longer lived B_H is highly enriched, $|B_H\rangle = p|B_s\rangle - q|\overline{B}_s\rangle$. Time is the tag here, in analogy to the neutral kaons.

Consider now any B_s mode f that can be fed from both a B_s and $\overline{B}_s$, and where the two unmixed amplitudes ($\langle f|B_s\rangle$ and $\langle f|\overline{B}_s\rangle$) differ in their CKM phase. Those modes then could harbor observable CP-violating effects. Further, it will become clear (by the end of this article) how to determine the CKM-phase difference. For instance, the CKM angle γ can be determined from the *untagged* $\rho^0 K_S$, ωK_S data samples if penguin amplitudes are negligible. Penguin diagrams may be sizable, in which case γ can be determined from untagged $D_s^{(*)\mp} K^{(*)\pm}$ data samples. This last determination assumes factorization for the color-allowed processes $B_s \to D_s^{(*)-} K^{(*)+}$, $D_s^{(*)-}\pi^+$.

To those who object to this factorization assumption, we offer the extraction of γ without any theoretical input from the *untagged* $D^0\phi, \overline{D}^0\phi$ and $D^0_{CP}\phi$ data samples. D^0_{CP} denotes that the D^0 or $\overline{D^0}$ is seen in a CP eigenmode, such as $\pi^0 K_S$, K^+K^-, $\pi^+\pi^-$. Clearly all those above-mentioned processes (and many more) could show sizable CP-violation effects, which we discuss.

Second, a large width difference would solve rather convincingly the charm deficit puzzle in favor of theory [28–31] because $B(b \to c\bar{c}s) \gtrsim (|\Delta\Gamma|/\Gamma)_{B_s}$. Third, if hadronic effects could be controlled and understood, f_{B_s} could be extracted from a measurement of $\Delta\Gamma$. Fourth, one would not be allowed to speak about branching fractions of an unmixed B_s to any final state f, but rather one would have to discuss $B(B_{H(L)} \to f)$.

The derivation of a reliable upper limit for $|\Delta\Gamma|/\Gamma \lesssim 0.3$ is also of some importance, because it informs us about the optimal size of such effects. Establishing a nonvanishing width difference is thus important, because of all the above-mentioned reasons.

Bigi *et al.* suggested the use of the $J/\psi\phi$ and $D_s^\mp \ell^\pm \nu$ data samples to extract the width difference [16,17]. This article reviews and refines that suggestion and discusses other determinations of $\Delta\Gamma$. What is intriguing is that $\Delta\Gamma$ could be measured from currently available data samples with more statistics, which are the *untagged*, flavor-specific modes of B_s. Such B_s modes time evolve as the sum of two exponentials [12,8]:

$$e^{-(\Gamma+\frac{\Delta\Gamma}{2})t} + e^{-(\Gamma-\frac{\Delta\Gamma}{2})t} . \tag{1.2}$$

A one parameter fit for $\Delta\Gamma$ determines the width difference. The average width Γ of B_s is well known. It can be obtained essentially from a one parameter fit of the time evolution of that same (untagged, flavor-specific B_s) data sample to a single exponential $\exp(-\Gamma t)$ [32]. Alternatively one can either use the prediction that Γ equals the B_d width to sufficient accuracy [16,17], or one can obtain Γ from the average b-hadron lifetime determined in high energy experiments. Several additional methods for extracting the width difference will become available in the future. This article discusses a few of them. A careful feasibility study will be reported elsewhere [32].

This report is organized as follows. Section II reviews B_s-$\overline{B}_s$ mixing phenomena. Section III lists a few ramifications of a sizable difference in widths, and derives an upper limit of $(|\Delta\Gamma|/\Gamma)_{B_s} \lesssim 0.3$. Section IV discusses time evolution of B_s mesons and finds that any rapid oscillatory behavior depending on Δmt cancels in untagged data samples. Suggestions for the experimental determination of $\Delta\Gamma$, CP violation, and CKM parameters with untagged B_s samples can be found in Sec. V. Section VI concludes.

II. PREDICTIONS FOR B_S-$\overline{B}_S$ MIXING

This section collects a few pertinent mixing formulas from the general treatment reviewed in Chap. 5 of Ref. [13]. An arbitrary neutral B_s-meson state

$$a \mid B_s\rangle + b \mid \overline{B}_s\rangle \tag{2.1}$$

is governed by the time dependent Schrödinger equation

$$i\,\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{H}\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} a \\ b \end{pmatrix} . \tag{2.2}$$

Here $\mathbf{M}$ and $\mathbf{\Gamma}$ are 2×2 matrices, with $\mathbf{M} = \mathbf{M}^+, \mathbf{\Gamma} = \mathbf{\Gamma}^+$. CPT invariance guarantees $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. We assume CPT throughout and obtain the eigenstates of the mass matrix as

$$\mid B_L\rangle = p \mid B_s^0\rangle + q \mid \overline{B}_s^0\rangle , \tag{2.3}$$

$$\mid B_H\rangle = p \mid B_s^0\rangle - q \mid \overline{B}_s^0\rangle , \tag{2.4}$$

with eigenvalues ($L =$ "light," $H =$"heavy")

$$\mu_{L,H} = m_{L,H} - \frac{i}{2}\,\Gamma_{L,H} . \tag{2.5}$$

Here $m_{L,H}$ and $\Gamma_{L,H}$ denote the masses and decay widths of $B_{L,H}$. Further, define

$$\Delta\mu \equiv \mu_H - \mu_L \equiv \Delta m - \frac{i}{2}\,\Delta\Gamma , \quad \Gamma \equiv \frac{\Gamma_L + \Gamma_H}{2} . \tag{2.6}$$

Within the CKM model, the dispersive M_{12} and absorptive Γ_{12} mass matrix elements satisfy [10,13]

$$\mid M_{12} \mid \gg \mid \Gamma_{12} \mid , \tag{2.7}$$

and thus [10,13]

$$\Delta m \approx 2 \mid M_{12} \mid \ . \tag{2.8}$$

M_{12} is by far dominated by the virtual $t\bar{t}$ intermediate state and

$$M_{12} \approx -c\, \xi_t^2 \ . \tag{2.9}$$

Here

$$\xi_q = V_{qb}\, V_{qs}^* \tag{2.10}$$

and c is a positive quantity under the phase convention

$$CP \mid B_s\rangle = + \mid \overline{B}_s\rangle \ . \tag{2.11}$$

The coefficients q/p satisfy

$$\frac{q}{p} = \frac{-\Delta\mu}{2(M_{12} - \frac{i}{2}\Gamma_{12})} \ . \tag{2.12}$$

The CKM model predicts

$$\left|\frac{q}{p}\right| = 1 + O(10^{-3} - 10^{-4}) \ . \tag{2.13}$$

The width difference is precisely [13]

$$\Delta\Gamma = \frac{4\ \mathrm{Re}\ (M_{12}\Gamma_{12}^*)}{\Delta m} \ . \tag{2.14}$$

Modes that are common to B_s and $\overline{B}_s$ contribute to Γ_{12} and thus determine $\Delta\Gamma$, see Eq. (2.14). The most dominant modes are governed by the CKM-favored $b \to c\bar{c}s$ transition, with the CKM-suppressed $b \to c\bar{u}s, u\bar{c}s, u\bar{u}s$ processes playing a minor role [10].

Box diagram calculations [10,11,15,17] yield a negative $\Delta\Gamma$:

$$\frac{\Delta\Gamma}{\Gamma} \sim (-0.2) \ . \tag{2.15}$$

In addition, Ref. [15] employed an orthogonal approach of summing over many exclusive modes governed by the $b \to c\bar{c}s$ process. Denote by $\Gamma_+(b \to c\bar{c}s)$ [$\Gamma_-(b \to c\bar{c}s)$] the CP-even [CP-odd] rate governed by the $b \to c\bar{c}s$ transition of the B_s meson. Reference [15] finds that $\Gamma_+(b \to c\bar{c}s)$ dominates $\Gamma_-(b \to c\bar{c}s)$, and again a width difference of $\sim 20\%$ results:

$$\begin{aligned} \Gamma_+(b \to c\bar{c}s) &\gg \Gamma_-(b \to c\bar{c}s), \\ \frac{\Gamma_+(b \to c\bar{c}s) - \Gamma_-(b \to c\bar{c}s)}{\Gamma_+(b \to c\bar{c}s) + \Gamma_-(b \to c\bar{c}s)} &= 0.97, \\ \frac{\Gamma_+(b \to c\bar{c}s) - \Gamma_-(b \to c\bar{c}s)}{\Gamma} &\sim 0.2 \ . \end{aligned} \tag{2.16}$$

Significant portions of the $b \to c\bar{c}s$ transition, such as $B_s \to D_s^{(r)-}DKX$, $D_s^{r'-}D_s^{(*)+}$, $D_s^{(*)+}D_s^{(*)-}\phi$, $\Xi_c^{(r)}\overline{\Xi}_c^{(r)}$, were not considered, however. Whereas the superscript (r) denotes either the particle or any of its resonances, $D_s^{r'-}$ denotes D_s resonances above the s-wave D_s and D_s^* mesons. The theoretical uncertainties in those predictions are so large that a vanishing width difference, although unlikely, cannot be ruled out.

CP-violating effects of B_s decays governed by the $b \to c\bar{c}s$ transition are tiny. Neglecting CP violation, the heavy and light mass eigenstates also have definite CP properties, [33]

$$\Gamma_H = \Gamma_-, \ \ \Gamma_L = \Gamma_+ \ . \tag{2.17}$$

The identification [Eq. (2.17)] will be seen from yet another viewpoint later on in Sec. V B. The box diagram calculation and the orthogonal approach of summing over many exclusive modes both predict the same sign for $\Delta\Gamma$.

III. CONSEQUENCES OF SIZABLE $(\Delta\Gamma)_{B_s}$

A large width difference $\Delta\Gamma$ would have important implications for several areas of the standard model [20]. We discuss only a few consequences such a $\Delta\Gamma$ measurement would make. First, within the CKM model, $\Delta m/\Delta\Gamma$ has no CKM ratio and can be estimated [26,12,13,18]. Whereas Δm is given by the matrix element of a local operator, the width difference is obtained from the matrix element of a nonlocal operator [10,11]. This difference may cause a significant uncertainty in predicting $\Delta m/\Delta\Gamma$. If QCD corrections are neglected, vacuum saturation employed and the approximation $m_{B_s} \approx m_b$ used, one obtains [26,12,13,18]

$$\frac{\Delta m}{\Delta\Gamma} \approx \frac{-2}{3\pi}\ \frac{m_t^2\ h(m_t^2/M_W^2)}{m_b^2} \times \left[\sqrt{1 - 4\frac{m_c^2}{m_b^2}}\left(1 - \frac{2}{3}\ \frac{m_c^2}{m_b^2}\right)\right]^{-1} , \tag{3.1}$$

where [34]

$$h(y) = 1 - \frac{3\,y\,(1+y)}{4\,(1-y)^2}\left\{1 + \frac{2\,y}{1-y^2}\ \ln(y)\right\} . \tag{3.2}$$

It is imperative to estimate $\Delta m/\Delta\Gamma$ as reliable as is presently feasible [11,27,17]. If the error does not turn out to be too large, then a measured $\Delta\Gamma$ implies an allowed range for Δm, or vice versa (depending upon which measurement comes first). If the ratio $\Delta m/\Delta\Gamma$ could be reliably calculated, then $|V_{td}/V_{ts}|^2$ could be determined by combining the measurement of $(\Delta\Gamma)_{B_s}$ with the B_d-$\overline{B}_d$ mixing parameter $(\Delta m)_{B_d}$ [35]. The ratio $(\Delta m/\Delta\Gamma)_{B_s}$ could become another standard model constraint.

Second, we have previously shown how to extract angles of the unitarity CKM triangle from time-dependent studies of B_s and/or B_d [36], assuming a vanishing width difference. If a nonzero $(\Delta\Gamma)_{B_s}$ were to be found, those studies would have to be modified. We are confident that the angles of the unitarity CKM triangle can still be extracted from those correlations. The demonstration of this fact goes beyond the scope of this article, however.

Third, a large width difference would solve the so-called puzzle of the number of charmed hadrons per B meson n_c by yielding a lower limit on $B(b \to c\bar{c}s)$, which we will demonstrate. Theoretically, we expect $n_c \approx 1.3$ [28–31], whereas the current world average is 1.11 ± 0.06 [37]. We wish to review the theoretical arguments for $n_c \approx 1.3$. Because rare processes, such as $b \to u$ transitions or penguin-induced decays, are negligible, the $b \to c$ transition almost always occurs:

$$B(b \to c) \approx 1 \,. \tag{3.3}$$

The number of charmed hadrons per B meson is thus given by

$$n_c \approx 1 + B(b \to c\bar{c}s') \,. \tag{3.4}$$

Here s' (and d' below) denotes the weak eigenstate $s' = d\sin\theta_C + s\cos\theta_C$ ($d' = d\cos\theta_C - s\sin\theta_C$) and $\sin\theta_C \approx \theta_C \approx 0.22$ is the Cabibbo angle. A reliable estimate (or measurement) of $B(b \to c\bar{c}s')$ thus determines n_c. A parton model calculation, which includes QCD corrections for a finite charm mass, obtains [38,31,30]

$$B(b \to c\bar{c}s) = 0.3 \pm 0.1. \tag{3.5}$$

The large error arises from the uncertainty in quark masses and from scale and scheme dependences.

It is therefore gratifying to note that $B(b \to c\bar{c}s)$ can be estimated in a complementary fashion without making any assumptions as to how $b \to c\bar{c}s$ hadronizes. We only assume that almost always $b \to c$ transitions occur and that the ratio $\Gamma(b \to c\bar{u}d')/\Gamma(b \to ce\nu)$ can be reliably estimated. The branching ratio of $b \to c\bar{c}s$ follows [29]:

$$\begin{aligned} B(b \to c\bar{c}s) &\approx |V_{cs}|^2 \Bigg(1 - \sum_\ell B(b \to c\ell\nu) \\ &\qquad -B(b \to c\bar{u}d')\Bigg) \\ &= |V_{cs}|^2 \Bigg(1 - \sum_\ell B(b \to c\ell\nu) \\ &\qquad -\frac{\Gamma(b \to c\bar{u}d')}{\Gamma(b \to ce\nu)}\, B(b \to ce\nu)\Bigg) \,, \end{aligned} \tag{3.6}$$

$$|V_{cs}|^2 \approx 1 - \theta_C^2. \tag{3.7}$$

The quantities on the right-hand side of Eq. (3.6) are all well known and thus $B(b \to c\bar{c}s)$ is estimated reliably, as we proceed to show. The highly involved α_s corrections for the $b \to c\bar{u}d'$ rate for a massive charm have been completed recently by Bagan *et al.* [39]; see also earlier work [38]. The ratio

$$\frac{\Gamma(b \to c\bar{u}d')}{\Gamma(b \to ce\nu)} = 3\,\eta_{\rm QCD} \approx 3 \times 1.35 \tag{3.8}$$

is thus well known theoretically [31], and the semileptonic branching fractions have been measured [40,41]:

$$B(B \to Xe\nu) = (10.7 \pm 0.5)\% \,, \tag{3.9}$$

$$B(B \to X\mu\nu) = (10.3 \pm 0.5)\% \,, \tag{3.10}$$

$$B(B \to X\tau\nu) = (2.8 \pm 0.6)\% \,, \tag{3.11}$$

$$\sum_\ell B(B \to X\ell\nu) = (23.8 \pm 0.9)\% \,. \tag{3.12}$$

Putting it all together, we estimate

$$B(b \to c\bar{c}s) \approx 0.31 \,, \tag{3.13}$$

$$B(b \to c\bar{c}s') \approx 0.33 \,. \tag{3.14}$$

Considerable improvements in measuring $B(b \to c\bar{c}s)$ can be made with presently existing B-data samples, as we now briefly discuss. First, the $\overline{B} \to D\,\overline{D}\,\overline{K}X$ transitions could be a sizable fraction of $b \to c\bar{c}s$ processes and cannot be ignored, as was done in recent analyses [42,37,43]. Second, we differ with the widely held view that the D_s yield in B decays originates dominantly from the virtual W [42,37,43]. There may be substantial D_s production in $b \to c$ transitions with $\bar{s}s$ fragmentation. The branching ratio for $b \to c\bar{c}s'$ can be obtained from the inclusive measurements

$$\begin{aligned} B(b \to c\bar{c}s') \approx\ & B(\overline{B} \to D_s^- X) + B(\overline{B} \to \overline{D}X) \\ & + B(\overline{B} \to \overline{\Lambda}_c X) \\ & + B(\overline{B} \to \overline{\Xi}_c X) + B(\overline{B} \to (c\bar{c})X) \,, \end{aligned} \tag{3.15}$$

where $(c\bar{c})$ denotes any charmonium resonance that is not seen in $D\overline{D}X$, such as $J/\psi, \psi', \chi_c, \eta_c$. The last term in Eq. (3.15) is probably small [42]:

$$B(\overline{B} \to (c\bar{c})X) \approx 0.02 \,. \tag{3.16}$$

The other four inclusive branching ratios can be determined from their inclusive yields in untagged B decays and by correlating those inclusive data samples with any conceivable tag [44], such as primary lepton, $K^\pm$, $\overset{(-)}{K}{}^*$, and cascade jet charge technique [45]. Care must be taken in correctly removing dilution and B^0-$\overline{B}^0$ mixing effects [44]. Recently, the CLEO collaboration conducted such a $\ell^\pm\Lambda_c$ correlation with the result [46]

$$B(\overline{B} \to \overline{\Lambda}_c X) \approx 0.01 \,. \tag{3.17}$$

At present, experiments can determine the right-hand side of Eq. (3.15) and thus $B(b \to c\bar{c}s')$. We expect experiment to agree with the theoretical prediction for $B(b \to c\bar{c}s')$. The so-called n_c puzzle will transform into the apparent puzzle of why almost all $\overline{B}$ decays are not governed by the $b \to c$ transition. At that point, it will be useful to obtain more precise inclusive yields of charmed hadrons in B decays. *The predicted number of charmed hadrons per B decay is about 1.3 ($n_c \approx 1.3$) as demonstrated by the discussion surrounding Eq. (3.6).*

It is amusing to note that a large value of $(-\Delta\Gamma/\Gamma)_{B_s}$ would give direct proof that $B(b \to c\bar{c}s)$ is large (here we neglect the tiny W-annihilation amplitude $b\bar{s} \to c\bar{c}$ and the small corrections that must be incorporated now that widths of the heavy and light B_s differ) because

$$B(b \to c\bar{c}s) \gtrsim \left(\frac{-\Delta\Gamma}{\Gamma}\right)_{B_s} \,. \tag{3.18}$$

Equation (3.18) follows from

$$B(b\to c\bar{c}s)=\frac{\Gamma(b\to c\bar{c}s)}{\Gamma}=\frac{\Gamma_+(b\to c\bar{c}s)+\Gamma_-(b\to c\bar{c}s)}{\Gamma}\geq\frac{\Gamma_+(b\to c\bar{c}s)-\Gamma_-(b\to c\bar{c}s)}{\Gamma}\approx\frac{-\Delta\Gamma}{\Gamma}\,, \tag{3.19}$$

where $\Gamma_+(b\to c\bar{c}s)$ $[\Gamma_-(b\to c\bar{c}s)]$ denotes the CP-even [CP-odd] width of the B_s modes governed by the one dominant CKM-favored $b\to c\bar{c}s$ transition. The inclusive width of B_s mesons governed by the $b\to c\bar{c}s$ process is denoted by $\Gamma(b\to c\bar{c}s)$ and satisfies [47]

$$\Gamma(b\to c\bar{c}s)=\Gamma_+(b\to c\bar{c}s)+\Gamma_-(b\to c\bar{c}s)\,. \tag{3.20}$$

This equation was used in the second step of Eq. (3.19). Thus a large width difference $\Delta\Gamma$ implies a large branching fraction for the $b\to c\bar{c}s$ transition, see Eq. (3.18). Conversely, an upper limit on $(|\Delta\Gamma|/\Gamma)_{B_s}$ can be derived by combining Eqs. (3.13) and (3.18):

$$(|\Delta\Gamma|/\Gamma)_{B_s}\lesssim B(b\to c\bar{c}s)\;\approx\;0.31\,. \tag{3.21}$$

Strictly speaking, however, it becomes meaningless to speak about branching fractions of B^0 to final states f because one does not know which width Γ_L or Γ_H is to be used in the denominator. The situation is completely analogous to the neutral kaons. We therefore will have to talk about the branching fractions of the heavy and light B_s mesons to final states f, i.e., $B(B_{H,L}\to f)$. For instance, the semileptonic widths satisfy

$$B(B_L\to D_s^{(*)-}\ell^+\nu)=\frac{\Gamma(B_L\to D_s^{(*)-}\ell^+\nu)}{\Gamma_L}=\frac{|p|^2\,\Gamma(B^0\to D_s^{(*)-}\ell^+\nu)}{\Gamma_L}\approx\frac{\Gamma(B^0\to D_s^{(*)-}\ell^+\nu)}{2\Gamma_L}\,, \tag{3.22}$$

$$B(B_H\to D_s^{(*)-}\ell^+\nu)=\frac{\Gamma(B_H\to D_s^{(*)-}\ell^+\nu)}{\Gamma_H}=\frac{|p|^2\,\Gamma(B^0\to D_s^{(*)-}\ell^+\nu)}{\Gamma_H}\approx\frac{\Gamma(B^0\to D_s^{(*)-}\ell^+\nu)}{2\Gamma_H}\,. \tag{3.23}$$

Whereas the numerators are identical, the denominators may differ substantially which causes different (in our example, semileptonic) branching fractions of the heavy and light B_s. Further, a sizable width difference allows CP-violating measurements [20] and the clean extraction of CKM phases with *untagged* B_s-data samples, which will be expanded upon below. Clearly, the observation of a large width difference in B_s mesons will have important ramifications for the standard model. Because establishing a nonvanishing width difference is so important, this note lists a few suggestions about how to measure $(\Delta\Gamma)_{B_s}$. To reach that goal, Sec. IV reviews time dependences of B^0 decays.

IV. TIME DEPENDENCES

This section gives a set of master equations from which one can read off desired time dependences. Denote by $B^0_{\rm phys}$ ($\overline{B}^0_{\rm phys}$) a time-evolved initially unmixed B^0 ($\overline{B}^0$):

$$|\,B^0_{\rm phys}(t=0)\rangle=|B^0\rangle\,. \tag{4.1}$$

Consider final states f which can be fed from both a B^0 and a $\overline{B}^0$, and define the interference terms

$$\lambda\equiv\frac{q}{p}\,\frac{\langle f\mid\overline{B}^0\rangle}{\langle f\mid B^0\rangle},\;\bar{\lambda}\equiv\frac{p}{q}\,\frac{\langle\bar{f}\mid B^0\rangle}{\langle\bar{f}\mid\overline{B}^0\rangle}. \tag{4.2}$$

Without any assumptions, the time-dependent rates are given by [8,13]

$$\Gamma(B^0_{\rm phys}(t)\to f)=\Gamma(B^0\to f)\{|\,g_+(t)\,|^2+|\,\lambda\,|^2\,|\,g_-(t)\,|^2+2{\rm Re}\,[\lambda\,g_-(t)\,g_+^*(t)]\}\,, \tag{4.3}$$

$$\Gamma(B^0_{\rm phys}(t)\to\bar{f})=\Gamma(\overline{B}^0\to\bar{f})\,\left|\frac{q}{p}\right|^2\{|\,g_-(t)\,|^2+|\,\bar{\lambda}\,|^2\,|\,g_+(t)\,|^2+2{\rm Re}\,[\bar{\lambda}\,g_+(t)\,g_-^*(t)]\}\,, \tag{4.4}$$

$$\Gamma(\overline{B}^0_{\rm phys}(t)\to\bar{f})=\Gamma(\overline{B}^0\to\bar{f})\,\{|\,g_+(t)\,|^2+|\,\bar{\lambda}\,|^2\,|\,g_-(t)\,|^2+2{\rm Re}\,[\bar{\lambda}\,g_-(t)\,g_+^*(t)]\}\,, \tag{4.5}$$

$$\Gamma(\overline{B}^0_{\rm phys}(t)\to f)=\Gamma(B^0\to f)\,\left|\frac{p}{q}\right|^2\{|\,g_-(t)\,|^2+|\,\lambda\,|^2\,|\,g_+(t)\,|^2+2{\rm Re}\,[\lambda\,g_+(t)\,g_-^*(t)]\}\,, \tag{4.6}$$

where

$$|\,g_\pm(t)\,|^2=\tfrac{1}{4}\{e^{-\Gamma_L t}+e^{-\Gamma_H t}\pm 2e^{-\Gamma t}\cos\Delta mt\}\,, \tag{4.7}$$

$$g_-(t)\, g_+^*(t) = \tfrac{1}{4}\{e^{-\Gamma_L t} - e^{-\Gamma_H t} + 2i\, e^{-\Gamma t} \sin \Delta m t\} \,. \tag{4.8}$$

Those equations make a very important point transparent. For $\left|\frac{q}{p}\right| = 1$, the rapid time-dependent oscillations dependent on Δmt cancel in untagged data samples:

$$\Gamma\,[f\,(t)] \equiv \Gamma(B^0_{\rm phys}\,(t) \to f) + \Gamma(\overline{B}^0_{\rm phys}\,(t) \to f) \;, \tag{4.9}$$

$$\Gamma\,[f\,(t)] = \frac{\Gamma(B^0 \to f)}{2} \,\{(1+|\lambda|^2)\; (e^{-\Gamma_L t} + e^{-\Gamma_H t}) + 2{\rm Re}\,\lambda\; (e^{-\Gamma_L t} - e^{-\Gamma_H t})\} \,, \tag{4.10}$$

$$\Gamma\,[\overline{f}\,(t)] = \frac{\Gamma(\overline{B}^0 \to \overline{f})}{2} \,\{(1+|\overline{\lambda}|^2)\; (e^{-\Gamma_L t} + e^{-\Gamma_H t}) + 2{\rm Re}\,\overline{\lambda}\; (e^{-\Gamma_L t} - e^{-\Gamma_H t})\} \,. \tag{4.11}$$

The only time dependences remaining are that of the two exponential falloffs, $e^{-\Gamma_{L,H} t}$, both of which are at the average b-lifetime scale. From the two time scales $1/\Delta m$ and $1/\Gamma$ governing time-dependent B_s decays, choosing untagged data samples removes any dependence on the much shorter $1/\Delta m$ scale:

$$1/\Delta m \ll 1/\Gamma \;.$$

This is of prime importance on several counts. First, at e^+e^- and $p\bar{p}$ colliders any B_s candidate belongs automatically to the untagged data sample. Tagging this event will cost in efficiency and in purity. Collecting an untagged data sample at pp colliders or fixed target experiments can be done but is more involved and will not be addressed here. Second, $\Delta m/\Gamma$ could turn out to be larger than what present technology can resolve, although there exists an intriguing expression of interest for a forward collider experiment [48] that claims to be able to study $\Delta m/\Gamma \lesssim 60$ which is above the upper CKM-model limit [24].

We wish to present some theorems which will be used throughout this article. For that purpose, define

$$|\,\overline{f}\rangle \equiv CP \mid f\rangle, |\,\overline{B}^0\rangle \;\equiv CP \mid B^0\rangle \,. \tag{4.12}$$

Suppose that a unique CKM combination governs $B^0 \to f$ and another unique one $\overline{B}^0 \to \overline{f}$, then the following theorems and consequences hold [18].

Theorem 1. If the amplitude for $B^0 \to f$ is denoted by

$$\langle f \mid B^0\rangle = G \mid a \mid e^{i\delta} \;, \tag{4.13}$$

then the CP-conjugated amplitude is

$$\langle \overline{f} \mid \overline{B}^0\rangle = G^* \mid a \mid e^{i\delta} \,. \tag{4.14}$$

Here G is the unique CKM combination, $|\,a\,|$ the magnitude of the strong matrix element, and δ a possible strong interaction phase.

Consequence 2.

$$|\langle f|B^0\rangle| = |\langle \overline{f}|\overline{B}^0\rangle| \,. \tag{4.15}$$

Consequence 3. If furthermore $\left|\frac{q}{p}\right| \approx 1$ is assumed, then

$$\lambda = |\lambda|\; e^{i(\phi+\Delta)} \;, \tag{4.16}$$

$$\overline{\lambda} = |\lambda|\; e^{i(-\phi+\Delta)} \,. \tag{4.17}$$

where ϕ denotes the CKM phase, and Δ the possible strong interaction phase difference.

Consequence 4. Consider final states f which are CP eigenstates governed by the same unique CKM combination. The sign of the interference term flips, depending on the CP parity of f:

$$\lambda_{CP=+} = -\lambda_{CP=-} \,. \tag{4.18}$$

Theorem 5. If in addition $\left|\frac{q}{p}\right| = 1$ is assumed, then, for a CP eigenstate f (either CP even or CP odd),

$$\overline{\lambda} = \lambda^*, \;\text{ and }\; |\lambda| = 1 \,. \tag{4.19}$$

Although the proofs of the theorems and consequences are well known [18], they will be rederived here for completeness sake and to illuminate what is exactly meant by final state phase differences. The proof of theorem 1 is based on the fact that CP violation occurs only due to complex-valued CKM elements within the CKM model. The Hamiltonian which governs $B^0 \to f$ decays can thus be factorized as

$$\mathcal{H} = Gh + G^* h^+ \,. \tag{4.20}$$

Here h is the sum of all relevant operators annihilating a B^0 and creating f, schematically written as (for example)

$$h = (\bar{b}c)_{V-A}\;\; (\overline{u}s)_{V-A} \,. \tag{4.21}$$

The Hermitian conjugate h^+ annihilates a $\overline{B}^0$ and creates $\overline{f}$. Since CP violation resides solely within the CKM elements, the h's satisfy

$$(CP)^+\; h\; CP = h^+, \;\; (CP)^+\; h^+\; CP = h \,. \tag{4.22}$$

Now, the amplitude of B^0 to f stands actually for

$$\langle f|B^0\rangle \equiv \langle f|\mathcal{H}|B^0\rangle = G\langle f|h|B^0\rangle = G|a|e^{i\delta} \,. \tag{4.23}$$

The strong matrix element is

$$\langle f|h|B^0\rangle = |a|e^{i\delta} \,. \tag{4.24}$$

The CP-conjugated amplitude satisfies [using Eqs. (4.20), (4.12), (4.22), and (4.24) in the second, third, fourth, and fifth step, respectively]

$$\langle \overline{f}|\overline{B}^0\rangle \equiv \langle \overline{f}|\mathcal{H}|\overline{B}^0\rangle = G^*\langle \overline{f}|h^+|\overline{B}^0\rangle$$
$$= G^*\langle f|(CP)^+h^+CP|B^0\rangle = G^*\langle f|h|B^0\rangle$$
$$= G^*|a|e^{i\delta} . \quad (4.25)$$

Theorem 1 is thus proven, and consequence 2 results immediately. Consequence 3 is proven as follows. Denote the amplitude of $B^0 \to f$ as

$$\langle f|B^0\rangle = G|a|e^{i\delta} , \quad (4.26)$$

and that of $B^0 \to \overline{f}$ as

$$\langle \overline{f}|B^0\rangle = K|b|e^{i\tau} , \quad (4.27)$$

where G, K are the unique CKM combinations, $|a|, |b|$ magnitudes of strong matrix elements, and δ, τ their respective strong phases. Theorem 1 informs us that

$$\langle \overline{f}|\overline{B}^0\rangle = G^*|a|e^{i\delta} , \quad (4.28)$$

$$\langle f|\overline{B}^0\rangle = K^*|b|e^{i\tau} . \quad (4.29)$$

From the definitions of the interference terms,

$$\lambda \equiv \frac{q}{p}\,\frac{\langle f|\overline{B}^0\rangle}{\langle f|B^0\rangle} = \frac{q}{p}\,\frac{K^*}{G}\,\frac{|b|}{|a|}\,e^{i(\tau-\delta)} , \quad (4.30)$$

$$\overline{\lambda} \equiv \frac{p}{q}\,\frac{\langle \overline{f}|B^0\rangle}{\langle \overline{f}|\overline{B}^0\rangle} = \frac{p}{q}\,\frac{K}{G^*}\,\frac{|b|}{|a|}\,e^{i(\tau-\delta)} . \quad (4.31)$$

Because $\left|\frac{q}{p}\right| = 1$, we get $p/q = (q/p)^*$ and

$$\lambda = \lambda_{\rm CKM}\, z , \quad \overline{\lambda} = \lambda^*_{\rm CKM}\, z . \quad (4.32)$$

The CKM combination of the interference term is denoted by

$$\lambda_{\rm CKM} = \frac{q}{p}\,\frac{K^*}{G} \equiv |\lambda_{\rm CKM}|\, e^{i\phi} , \quad (4.33)$$

whereas the ratio of strong matrix elements is

$$z \equiv \left|\frac{b}{a}\right|\, e^{i(\tau-\delta)} \equiv |z|\, e^{i\Delta} . \quad (4.34)$$

Consequence 3 is proven, where $\Delta \equiv \tau - \delta$ denotes the phase difference between the two strong matrix elements. To prove consequence 4, consider a CP eigenstate f_η with CP parity η $(= \pm 1)$. As before, define

$$\langle f_\eta|B^0\rangle = G|a|\, e^{i\delta} . \quad (4.35)$$

Theorem 1 yields

$$\eta\langle f_\eta|\overline{B}^0\rangle = G^*|a|\, e^{i\delta} , \quad (4.36)$$

and

$$\lambda_\eta = \frac{q}{p}\,\frac{\langle f_\eta|\overline{B}^0\rangle}{\langle f_\eta|B^0\rangle} = \eta\,\frac{q}{p}\,\frac{G^*}{G} . \quad (4.37)$$

That is,

$$\lambda_+ = \frac{q}{p}\,\frac{G^*}{G} , \quad \lambda_- = -\lambda_+ \quad (4.38)$$

and consequence 4 is proven. Proving theorem 5 is also straightforward. We get

$$\langle \overline{f}_\eta|B^0\rangle = \eta\,\langle f_\eta|B^0\rangle , \quad (4.39)$$

$$\langle \overline{f}_\eta|\overline{B}^0\rangle = \eta\,\langle f_\eta|\overline{B}^0\rangle . \quad (4.40)$$

Since

$$\lambda \equiv \frac{q}{p}\,\frac{\langle f_\eta|\overline{B}^0\rangle}{\langle f_\eta|B^0\rangle} , \quad (4.41)$$

and

$$\overline{\lambda} \equiv \frac{p}{q}\,\frac{\langle \overline{f}_\eta|B^0\rangle}{\langle \overline{f}_\eta|\overline{B}^0\rangle} = \frac{p}{q}\,\frac{\langle f_\eta|B^0\rangle}{\langle f_\eta|\overline{B}^0\rangle} = \frac{1}{\lambda} = \lambda^* . \quad (4.42)$$

The second and third steps in Eq. (4.42) occur because of Eqs. (4.39) and (4.40) and (4.41), respectively. The last step occurs because $|\lambda|^2 = 1$, which happens since $\left|\frac{q}{p}\right| = 1$ is assumed and $\left|\frac{\langle f_\eta|\overline{B}^0\rangle}{\langle f_\eta|B^0\rangle}\right| = 1$ due to Eqs. (4.35) and (4.36) or equivalently due to consequence 2.

Consider the situation under which the above-mentioned theorems and consequences hold (i.e., a unique CKM combination governs $B^0 \to f$ and another unique one $\overline{B}^0 \to f$) and assume $\left|\frac{q}{p}\right| = 1$, then the time-dependent rates simplify from Eqs. (4.3) – (4.6) to

$$\Gamma(B^0_{\rm phys}(t) \to f) = \Gamma\left(B^0 \to f\right)\left\{|g_+(t)|^2 + |\lambda|^2\,|g_-(t)|^2 + 2{\rm Re}\left[\lambda\, g_-(t)\, g_+^*(t)\right]\right\} , \quad (4.43)$$

$$\Gamma(B^0_{\rm phys}(t) \to \overline{f}) = \Gamma\left(B^0 \to f\right)\left\{|g_-(t)|^2 + |\lambda|^2\,|g_+(t)|^2 + 2{\rm Re}\left[\overline{\lambda}\, g_+(t)\, g_-^*(t)\right]\right\} , \quad (4.44)$$

$$\Gamma(\overline{B}^0_{\rm phys}(t) \to \overline{f}) = \Gamma\left(B^0 \to f\right)\left\{|g_+(t)|^2 + |\lambda|^2\,|g_-(t)|^2 + 2{\rm Re}\left[\overline{\lambda}\, g_-(t)\, g_+^*(t)\right]\right\} , \quad (4.45)$$

$$\Gamma(\overline{B}^0_{\rm phys}(t) \to f) = \Gamma\left(B^0 \to f\right)\left\{|g_-(t)|^2 + |\lambda|^2\,|g_+(t)|^2 + 2{\rm Re}\left[\lambda\, g_+(t)\, g_-^*(t)\right]\right\} . \quad (4.46)$$

The above four equations are our master equations. By considering different cases, the next section demonstrates how untagged data samples of B_s mesons could be used not only to extract the light and heavy widths, but even the unitarity angle γ and CP violation.

V. PHYSICS WITH MODES OF UNTAGGED B_S MESONS

Unless explicitly stated otherwise, this section supposes that the conditions hold under which the master equations, Eqs. (4.43)–(4.46), are satisfied—that is, $\left|\frac{q}{p}\right| = 1$ and unique CKM combinations govern the decays of the unmixed B_s and $\overline{B_s}$ to f. We analyze the time dependences for several cases of untagged B_s data samples. First, flavor-specific modes g of B_s are studied, such that an unmixed B_s decays to g, whereas an unmixed $\overline{B_s}$ is never seen in g, $\overline{B}_s \not\to g$. Examples for g are $D_s^{(*)-}\ell^+\nu, D_s^{(*)-}\pi^+, D_s^{(*)-}a_1^+, D_s^{(*)-}\rho^+$.

Second, time evolutions of CP eigenmodes of B_s mesons are scrutinized. Within the CKM model, CP eigenmodes of B_s decays driven by $b \to c\bar{c}s$ are governed by a single exponential decay law. In contrast, there are CP eigenmodes that are governed by two exponential decay laws, which signals CP violation [19]. A time-dependent study of the untagged $\rho^0 K_S$, ωK_S data samples extracts the angle γ of the CKM unitarity triangle, when penguin amplitudes can be neglected. The penguin amplitudes may be nonnegligible for $B_s \to \rho^0 K_S$, ωK_S.

We discuss thus next the extraction of γ from modes f that can be fed from both B_s^0 and $\overline{B_s^0}$, such as $D_s^{(*)-}K^{(*)+}, \overline{D}^{(*)0}\phi, \overline{D}^{(*)0}\eta$. Sizable CP-violating effects could be seen when untagged time evolutions of f are compared with those of $\overline{f}$ [20]. We then investigate what occurs when several CKM combinations contribute to the decay amplitude of an unmixed B_s. The last subsection combines all the information and spells out many methods for measuring a width difference from untagged B_s samples. Some of the methods are directly applicable to the current flavor-specific world data sample of B_s.

A. Flavor-specific modes of B_s

Since only the unmixed B^0 can be seen in g, but never the unmixed $\overline{B}^0$, one obtains

$$\lambda = \overline{\lambda} = 0 \,. \tag{5.1}$$

The time-dependent rates become [8,11–13]

$$\Gamma(B^0_{\text{phys}}(t) \to g) = \Gamma\left(B^0 \to g\right) |g_+(t)|^2 \,, \tag{5.2}$$

$$\Gamma(\overline{B}^0_{\text{phys}}(t) \to g) = \Gamma\left(B^0 \to g\right) |g_-(t)|^2 \,, \tag{5.3}$$

and

$$\Gamma\left[\overline{g}\left(t\right)\right] = \Gamma\left[g\left(t\right)\right] = \frac{\Gamma\left(B^0 \to g\right)}{2}\left\{e^{-\Gamma_L t} + e^{-\Gamma_H t}\right\} . \tag{5.4}$$

The untagged time-dependent rates for the process and CP-conjugated process are the same. The untagged data sample time evolves as the sum of two exponentials [12,49]. Examples for such flavor specific modes g are

$$D_s^{(*)-}\ell^+\nu,\ D_s^{(*)-}\pi^+,\ D_s^{(*)-}a_1^+,\ D_s^{(*)-}\rho^+ \,. \tag{5.5}$$

More than 200 such B_s events have been recorded at the Collider Detector at Fermilab (CDF) [1] and the LEP [2] experiments. Their time dependence has been fit to a single exponential, which essentially measures the average B_s width Γ [32]. This measurement for Γ could then be used to determine $\Delta\Gamma$ by fitting the time evolution of the same data sample to the correct functional form

$$e^{-(\Gamma+\frac{\Delta\Gamma}{2})t} + e^{-(\Gamma-\frac{\Delta\Gamma}{2})t} \,. \tag{5.6}$$

B. CP eigenstates

This subsection considers modes f of B_s that have definite CP parity. The CP-even (CP-odd) final state will sometimes be denoted as f_+ (f_-). We first describe how to determine Γ_L from the CP-even modes governed by the $b \to c\bar{c}s$ transition. The CP-odd modes driven by $b \to c\bar{c}s$ are governed by the $e^{-\Gamma_H t}$ exponent, and allow the determination of Γ_H, in principle. The CP-odd modes however are not only predicted to be rarer than the CP-even modes, but are harder to detect. One possible determination of $\Delta\Gamma$ could use the largest B_s data sample, that of flavor-specific decays of B_s, combined with the above-mentioned measurement of Γ_L to extract Γ_H. The CP-odd modes driven by $b \to c\bar{c}s$ are governed by the exponent $\exp(-\Gamma_H t)$ and may be used as a consistency check to determine Γ_H. Once a width difference between Γ_H and Γ_L has been established, interesting CP-violating effects and the clean extraction of fundamental CKM parameters become possible with untagged B_s-data samples.

CP invariance requires a single exponential decay law for tagged and untagged neutral B's seen in a CP eigenstate [19]. The CKM model predicts two different exponential decay laws for many CP eigenstates of B_s decays, such as $\rho^0 K_S$, ωK_S, $D^0_{CP}\phi$, K^+K^-. Not only can CP violation be exhibited, but even CKM phases can be extracted from time-dependent studies of *untagged* B_s data samples. For instance, the time evolution of the *untagged* $\rho^0 K_S$, ωK_S modes extracts $\cos(2\gamma)$ as shown below, when penguin contributions are neglected. Penguins may be sizable however, in which case one may use non-CP eigenmodes to extract γ as will be discussed in the next Subsection.

Suppose that a unique CKM combination governs the decay of B^0 to CP eigenstate f and that $\left|\frac{q}{p}\right| = 1$, then the time-dependent rates become

$$\Gamma(B^0_{\rm phys}(t) \to f) = \frac{\Gamma(B^0 \to f)}{2} \{e^{-\Gamma_L t} + e^{-\Gamma_H t} + {\rm Re}\lambda \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right) - 2\,{\rm Im}\lambda\, e^{-\Gamma t} \sin \Delta m t\} , \quad (5.7)$$

$$\Gamma(\overline{B}^0_{\rm phys}(t) \to f) = \frac{\Gamma(B^0 \to f)}{2} \{e^{-\Gamma_L t} + e^{-\Gamma_H t} + {\rm Re}\lambda \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right) + 2{\rm Im}\lambda\, e^{-\Gamma t} \sin \Delta m t\} . \quad (5.8)$$

As advertised, the rapid Δmt oscillations cancel in the time-dependent rate of the untagged data sample:

$$\Gamma\left[f\left(t\right)\right] = \Gamma\left(B^0 \to f\right) \{e^{-\Gamma_L t} + e^{-\Gamma_H t} + {\rm Re}\lambda \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\} . \quad (5.9)$$

CP-violating effects are predicted to be small for CP eigenmodes of B_s governed by $b \to c\bar{c}s$ [8,50,36]:

$$0.01 \lesssim {\rm Im}\lambda \lesssim 0.05 . \quad (5.10)$$

Since here $|\lambda| \approx 1$ to excellent accuracy, we obtain

$$0.999 \lesssim |{\rm Re}\lambda| < 1 . \quad (5.11)$$

Equation (5.11) tells us that the untagged data sample of CP eigenmodes of B_s governed by $b \to c\bar{c}s$ involves unobservably tiny CP-violating effects. In the absence of CP violation, the CP-even (CP-odd) interference term is

$$\lambda_+ = 1 \ \ (\lambda_- = -1) . \quad (5.12)$$

The time dependence of the untagged data sample is

$$\Gamma\left[f_+\left(t\right)\right] = 2\Gamma\left(B^0 \to f_+\right)\ e^{-\Gamma_L t} , \quad (5.13)$$

$$\Gamma\left[f_-\left(t\right)\right] = 2\Gamma\left(B^0 \to f_-\right)\ e^{-\Gamma_H t} , \quad (5.14)$$

and the CP-even rate is identified with Γ_L,

$$\Gamma_+ = \Gamma_L \quad (5.15)$$

and the CP-odd rate with Γ_H,

$$\Gamma_- = \Gamma_H . \quad (5.16)$$

This is consistent with the assignment made in Eq. (2.17). Aleksan *et al.* [15] claimed to have shown that $\Gamma_+ - \Gamma_- > 0$ from both a box diagram calculation and from a sum over many exclusive modes. Our addition, in that respect, is the identification $\Gamma_H = \Gamma_-$ and $\Gamma_L = \Gamma_+$. Examples of modes with even CP parity are $J/\psi\eta$, $D_s^+D_s^-$. It is not easy to come up with CP-odd modes, for example $J/\psi f_0(980)$, $J/\psi a_0(980)$. In contrast, $J/\psi\phi$, $D_s^{*+}D_s^{*-}$, $D_s^{*+}D_s^- + D_s^+D_s^{*-}$ are dominantly CP even [51,15], with possibly small CP-odd components. The evidence that the $J/\psi\phi$ mode is mainly CP even comes from the observed angular correlations of the $B \to J/\psi K^*$ mode [42] coupled with SU(3) flavor symmetry [36], or from an explicit calculation assuming factorization [15]. In any event, an angular correlation study separates in general the CP-even and CP-odd components [52,53]. Once the CP-even and CP-odd components have been separated, their different lifetimes could be determined [54]. In practice, however, the CP-odd modes occur much less frequently than the CP-even modes, and are harder to detect. Thus, Γ_L will be known well, whereas Γ_H could be obtained from the time evolution of untagged, flavor-specific modes g of B_s:

$$\Gamma\left[\overline{g}\left(t\right)\right] + \Gamma\left[g\left(t\right)\right] \sim e^{-\Gamma_H t} + e^{-\Gamma_L t} . \quad (5.17)$$

Examples of g have been listed in the previous subsection, in which D_s is dominantly featured. A discriminating feature between D_s and other charmed hadrons is the inclusive ϕ yield. Whereas the inclusive ϕ yield in D_s decays is about 20%, it is much smaller in D^+ and D^0 decays [40,44]. Mainly due to this large inclusive ϕ yield in D_s decays, and partly because ϕ even appears in the $B_s \to J/\psi\phi$ mode, we strongly support the use of a ϕ trigger in experimental studies [55].

Although $\rho^0 K_S$, ωK_S modes are CP-odd, they are in general not governed by a single exponential decay law, because their interference term satisfies [8,56]

$${\rm Re}\lambda = -\cos(2\gamma) , \quad (5.18)$$

when penguin amplitudes are neglected. Time dependences of *untagged* $\rho^0 K_S$, ωK_S events extract $\cos(2\gamma)$; see Eq. (5.9). They exhibit CP violation when more than one exponential decay law contributes [19]. Far reaching consequences on the CKM model would result, even if the $\rho^0 K_S$, ωK_S modes were governed by a single exponential decay law. The interference term would satisfy ${\rm Re}\lambda = \pm 1$. If ${\rm Re}\lambda = +1$, then the CP-odd $\rho^0 K_S$, ωK_S decay modes are governed by Γ_L. This constitutes a clear violation of CP, because the time evolution of the CP-odd modes $\rho^0 K_S$, ωK_S is governed by the same exponent Γ_L as the opposite CP-even modes driven by $b \to c\bar{c}s$ (and not by Γ_H governing CP-odd modes driven by $b \to c\bar{c}s$). On the other hand, if ${\rm Re}\lambda = -1$, then $\sin\gamma = 0$, contradicting what is currently known about $\sin\gamma$ in the CKM model [57],

$$0.5 \lesssim \sin\gamma \le 1 . \quad (5.19)$$

Penguin amplitudes may be significant however, in which case several CKM combinations contribute to the unmixed amplitude. The time-dependent, untagged decay rate (assuming $|\frac{q}{p}| = 1$) becomes

$$\Gamma\left[f\left(t\right)\right] = \Gamma\left(B^0 \to f\right) \left\{\frac{1}{2}\left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right)\left(1 + |\lambda|^2\right) + {\rm Re}\lambda\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\right\} . \quad (5.20)$$

This equation is relevant to, for instance, the $\rho^0 K_S$, ωK_S, $D^0_{CP}\phi$, $K^+K^-, \phi K_S$ modes of B_s. It shows that those CP eigenmodes will have in general two exponential decay laws, which demonstrates CP violation [19]. Other relevant, experimentally accessible modes are $\phi\phi$, $\rho^0\phi$. Angular correlations can separate their CP-even and CP-odd components [52,53]. If any component with definite CP parity has two exponential decay laws, CP violation occurs. CP violation may be seen not only in definite CP components, but in interference effects between different helicity amplitudes as well.

Because of a possible penguin contamination, the unitarity angle γ cannot be extracted cleanly from the time evolution of untagged $\rho^0 K_S$, ωK_S events. In contrast, a clean extraction is possible from non-CP eigenmodes which do not suffer from penguin contamination at all, as discussed next.

C. Modes common to B_s and $\overline{B_s}$

It is well known [58–61] that tagged, time-dependent studies (capable of observing the rapid Δmt oscillations) are able to extract the unitarity angle γ and observe CP violation from B_s modes governed by the $b \to c\bar{u}s, u\bar{c}s$ transitions, such as

$$f = D_s^{(*)-}K^{(*)+},\ \overline{D}^{(*)0}\phi,\ \overline{D}^{(*)0}\eta\ .$$

This subsection demonstrates that even *untagged*, time-dependent studies (now governed only by the two exponential decay laws) are able to extract the angle γ. Those untagged studies may observe CP violation for nonvanishing strong final-state phase differences. A nonzero strong final-state phase difference could arise from traditional rescattering effects or from resonance effects discovered recently by Atwood *et al.* in a different context [62,63]. For traditional rescattering effects, CP violation is probably more pronounced in color-suppressed modes, $\overline{D}^{(*)0}\phi$, $\overline{D}^{(*)0}\eta$, than in the color-allowed ones, $D_s^{(*)-}K^{(*)+}$. The reason is simple. Within the factorization approximation [64,65], rates of color-suppressed modes are tiny with respect to color-allowed ones. The latter may rescatter into the former causing possibly sizable strong phase differences Δ for the color-suppressed modes. In contrast, such large rescattering effects are not likely to occur for the color-allowed modes. It is reasonable to expect $\Delta \approx 0$ for the color-allowed modes.

In a nice series of papers, Atwood *et al.* have shown how CP violation can be enhanced by considering modes where several kaon or unflavored resonances contribute to the final state [62,63]. "Calculable" final-state phases are generated due to the different widths of the resonances. A straightforward application of this idea to untagged B_s modes such as $D_s^{(*)\mp}(K^{(*)}\pi)^\pm$, $D_s^{(*)\mp}(K\rho)^\pm$, enhances CP violation. Such "calculable" final-state phases ensure nonvanishing CP-violating effects for the B_s modes of interest here, which are governed by the $\bar{b} \to \bar{c}u\bar{s}$ transition. The *untagged* B_s modes, such as $D_s^{(*)\mp}(K^{(*)}\pi)^\pm$, $D_s^{(*)\mp}(K\rho)^\pm$, may be used to extract the CKM unitarity angle γ.

This subsection is divided into several parts. First, the angle γ is extracted from time dependences of *untagged* B_s data samples such as $D_s^{(*)\pm}K^{(*)\mp}$. The overall normalization is obtained by assuming factorization for the color-allowed processes $B_s \to D_s^{(*)-}K^{(*)+}, D_s^{(*)-}\pi^{(*)+}$, where π^{*+} denotes ρ^+, a_1^+, etc. One may object to the factorization assumption. We thus determine γ from time dependences of untagged $D^0\phi, \overline{D}^0\phi$, and $D^0_{CP}\phi$ modes. The determination does not involve any assumption beyond the validity of the CKM model. CP-violating effects are described next. By waiting long enough, essentially only the longer lived B_H survives:

$$|B_H\rangle = p|B_s\rangle - q|\overline{B}_s\rangle .$$

If the amplitudes $B_s \to f$ and $\overline{B}_s \to f$ are governed by different CKM phases, CP violation may occur. The relative CKM phase for B_s modes governed by $\bar{b} \to \bar{c}u\bar{s}$ is γ and is significant. Large CP-violating effects can be generated, either from traditional rescattering effects or from resonance effects.

1. CKM phase γ from B_s modes governed by $\bar{b} \to \bar{c}u\bar{s}$

The time evolutions of the untagged $\overset{(-)}{f}$ data samples are

$$\Gamma\left[\overset{(-)}{f}(t)\right] = \frac{\Gamma(B^0 \to f)}{2}\{(e^{-\Gamma_L t} + e^{-\Gamma_H t})(1+|\lambda|^2) + 2\mathrm{Re}\overset{(-)}{\lambda}(e^{-\Gamma_L t} - e^{-\Gamma_H t})\}\ . \qquad (5.21)$$

The rapidly oscillating terms of Δmt cancel again. A time-dependent fit extracts

$$\Gamma(B^0 \to f)\ (1+|\lambda|^2),\ \Gamma(B^0 \to f)\mathrm{Re}\lambda,\ \Gamma(B^0 \to f)\mathrm{Re}\overline{\lambda}\ . \qquad (5.22)$$

The overall normalization could be established from the flavor-specific data sample; see Eq. (5.4):

$$\Gamma[g(t)] = \frac{\Gamma(B^0 \to g)}{2}\{e^{-\Gamma_L t} + e^{-\Gamma_H t}\}\ . \qquad (5.23)$$

The ratio of the unmixed rates is well known from theory:

$$\frac{\Gamma(B_s^0 \to D_s^- K^+)}{\Gamma(B_s^0 \to D_s^- \pi^+)} \approx \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2 \text{(phase space)}\ . \qquad (5.24)$$

Here the factorization approximation is used for those color-allowed modes. The W-exchange amplitude contributing to $B_s^0 \to D_s^- K^+$ has been neglected [66] and has been estimated to be tiny [15]. It contributes the same unique CKM combination as the spectator graph [59]. Future precision studies would allow incorporation of even those effects. Analogously, other theoretically well-known ratios are, for instance,

$$\frac{\Gamma(B_s^0 \to D_s^{(*)-}K^{(*)+})}{\Gamma(B_s^0 \to D_s^{(*)-}\pi^{(*)+})}\ . \qquad (5.25)$$

Combining those well-known ratios with the observables in Eq. (5.22) and the measured $\Gamma(B^0 \to g)$ in Eq. (5.23) extracts:

$$1 + |\lambda|^2 \ \text{(that is, } |\lambda|) , \tag{5.26}$$

$$\text{Re}\lambda = |\lambda| \ \cos(\phi + \Delta) , \tag{5.27}$$

and

$$\text{Re}\overline{\lambda} = |\lambda| \ \cos(-\phi + \Delta) . \tag{5.28}$$

Here $\phi = -\gamma$ is the CKM phase of the interference term λ where γ is the CKM unitarity angle, and Δ the strong final state phase difference. Finally, the phases ϕ and Δ can be determined up to a discrete ambiguity from $\cos(\phi + \Delta)$ and $\cos(-\phi + \Delta)$. This implies the determination of the CKM unitarity-angle γ is possible from untagged data samples. More systematics may cancel by using the ratio

$$\frac{\Gamma\left[\overset{(-)}{f}(t)\right]}{\Gamma\left[g\,(t)\right]} = \frac{\Gamma(B^0 \to f)}{\Gamma(B^0 \to g)} \left\{ 1 + |\lambda|^2 + 2\text{Re} \overset{(-)}{\lambda} \times \tanh\left(\frac{\Delta\Gamma\, t}{2}\right) \right\} . \tag{5.29}$$

Theory provides the unmixed ratio $\Gamma(B^0 \to f)/\Gamma(B^0 \to g)$. The time-independent term yields $|\lambda|$, whereas the time-dependent one gives Reλ and Re $\overline{\lambda}$. Thus ϕ and Δ can be extracted.

A comment about the discrete ambiguity is in order. Two solutions for $\sin^2\phi$ exist:

$$\sin^2\phi = \frac{1 - c\bar{c} \pm \sqrt{1 + (c\bar{c})^2 - \bar{c}^2 - c^2}}{2}, \tag{5.30}$$

where the extracted cosines are denoted by

$$c = \cos(\phi + \Delta) , \ \ \bar{c} = \cos(-\phi + \Delta) . \tag{5.31}$$

One solution is the true $\sin^2\phi$ and the other is the true $\sin^2\Delta$. The CKM model predicts only large, positive $\sin(-\phi) = \sin\gamma$ [57]. Thus the twofold ambiguity in $\sin^2\phi$ stays a twofold ambiguity in $\sin\phi$, since $\sin\phi < 0$. Further, this twofold ambiguity can be easily resolved in several ways. First, various final states of B_s driven by the $\bar{b} \to \bar{c}u\bar{s}$ transition are governed by the universal CKM phase $\phi = -\gamma$. In contrast, they probably will differ in their strong phase difference Δ. Thus, by considering many such B_s modes, one can disentangle the universal from the nonuniversal phases. Second, if it were to happen that $\Delta \approx 0$ for all the many modes, then one solution for $\sin^2\phi$ would vanish contradicting Eq. (5.19). Only one solution for $\sin^2\phi$ would remain. This fact can be used to quantify the number of events required in a feasibility study. A third way uses resonance effects and is briefly mentioned below.

For the color-allowed modes, we believe that $\Delta \approx 0$, whereas for the color-suppressed modes, larger Δ's could occur. Thus, γ is probably more straightforwardly extracted from the color-allowed processes, because

$$\cos(\pm\gamma + \Delta) \approx \cos\gamma , \tag{5.32}$$

and there may be no need to disentangle γ from Δ.

2. CKM phase γ from $\overline{D}^0\phi, D^0\phi, D^0_{CP}\phi$

To extract the CKM phase γ, it was necessary to assume knowledge on a ratio of unmixed amplitudes, such as $\Gamma(B_s \to D_s^- K^+)/\Gamma(B_s \to D_s^- \pi^+)$.) Time-dependent studies of untagged data samples of $\overline{D}^0\phi, D^0\phi, D^0_{CP}\phi$ extract γ ($= -\phi$) without any assumptions, except the validity of the CKM model. They even determine $|\lambda|$ and the strong phase difference Δ. Denote by η ($+1$ or -1) the CP parity of D^0_{CP}. Thus the CP parity of the whole B_s mode $D^0_{CP}\phi$ is $(-\eta)$. The time dependences determine, respectively,

$$\frac{\text{Re}\lambda}{1 + |\lambda|^2} , \ \frac{\text{Re}\overline{\lambda}}{1 + |\lambda|^2} , \ \frac{\text{Re}\lambda_\eta}{1 + |\lambda_\eta|^2} , \tag{5.33}$$

where

$$\lambda \equiv \frac{q}{p} \frac{\langle \overline{D}^0\phi|\overline{B}_s\rangle}{\langle \overline{D}^0\phi|B_s\rangle} = |\lambda|\ e^{i(\phi+\Delta)} ,$$

$$\overline{\lambda} \equiv \frac{p}{q} \frac{\langle D^0\phi|B_s\rangle}{\langle D^0\phi|\overline{B}_s\rangle} = |\lambda|\ e^{i(-\phi+\Delta)} ,$$

$$\overline{\lambda} = \lambda\, e^{-2i\phi} ,$$

$$\lambda_\eta \equiv \frac{q}{p} \frac{\langle D^0_{CP}\phi|\overline{B}_s\rangle}{\langle D^0_{CP}\phi|B_s\rangle} = \frac{\eta\,\lambda\, -\, 1}{\eta - \overline{\lambda}} . \tag{5.34}$$

The three unknowns $|\lambda|, \phi$, and Δ can be determined from the three measurables, Eq. (5.33). The magnitude of the interference term $|\lambda|$ could be obtained alternatively by using theory on the ratio [see Eq. (5.26)]

$$\frac{\Gamma(B_s \to \overline{D}^0\phi)}{\Gamma(B_s \to \overline{D}^0\overline{K}^{*0})} . \tag{5.35}$$

We suspect, however, that theory cannot predict as reliably this ratio of rates, because rescattering effects may be more pronounced for the color-suppressed modes than for the color-allowed ones. A comparison of the two determinations of $|\lambda|$ therefore probes rescattering effects.

3. CP violation

Time dependences of untagged B_s modes governed by $\bar{b} \to \bar{c}u\bar{s}$ could show sizable CP-violating effects. CP invariance demands that

$$\Gamma\left[f\,(t)\right] = \Gamma\left[\overline{f}\,(t)\right] , \tag{5.36}$$

or equivalently,

$$\mathrm{Re}\lambda = \mathrm{Re}\overline{\lambda} \Longleftrightarrow \cos(\phi + \Delta) = \cos(-\phi + \Delta) \ . \quad (5.37)$$

Thus CP violation will be more pronounced for modes where Δ is more sizable. We expect the color-suppressed modes to show larger CP-violating effects than the color-allowed modes, where Δ is expected to be smaller.

It is very important to realize that the B_s meson harbors possibly large CP-violating effects, for which one is not required to distinguish an initial B_s and $\overline{B}_s$. Such CP-violating effects are the time-dependent or time-integrated asymmetries:

$$a(t) \equiv \frac{\Gamma[f(t)] - \Gamma[\overline{f}(t)]}{\Gamma[f(t)] + \Gamma[\overline{f}(t)]} \ , \quad (5.38)$$

$$A(t_0) \equiv \frac{\int_{t_0}^{\infty} dt \ \{\Gamma\left[f\left(t\right)\right] - \Gamma\left[\overline{f}\left(t\right)\right]\}}{\int_{t_0}^{\infty} dt \ \{\Gamma\left[f\left(t\right)\right] + \Gamma\left[\overline{f}\left(t\right)\right]\}} \ . \quad (5.39)$$

Equations (5.21) and (5.38) yield

$$a(t) = \frac{-2|\lambda| \sin\phi \ \sin\Delta \ \tanh\left(\frac{\Delta\Gamma t}{2}\right)}{1 + |\lambda|^2 + 2|\lambda|\cos\phi \ \cos\Delta \ \tanh\left(\frac{\Delta\Gamma t}{2}\right)} \ . \quad (5.40)$$

In the limit

$$\lim_{t\to\infty} \tanh\left(\frac{\Delta\Gamma \ t}{2}\right) = -1 \ , \quad (5.41)$$

which is satisfied in practice for

$$t \gtrsim \frac{2}{\Delta\Gamma} \ , \quad (5.42)$$

one finds

$$\lim_{t\to\infty} a(t) = \frac{2|\lambda| \sin\phi \ \sin\Delta}{1 + |\lambda|^2 - 2|\lambda|\cos\phi \ \cos\Delta} \ . \quad (5.43)$$

To demonstrate that large CP-violating effects are possible, proper decay times greater than about $2/\Delta\Gamma$ are used. Clearly, to optimize observation of CP violation and the extraction of CKM phases we recommend to *always* use all accessible proper times. Representative values for modes governed by $\bar{b} \to \bar{c}u\bar{s}$, such as $\overline{D^0}\phi$, $D_s^{(*)-}K^{(*)+}$, would be

$$|\lambda| = \frac{1}{3} \ , \ \sin\phi = -0.8 \ , \ \cos\phi = 0.6 \ . \quad (5.44)$$

For a large phase difference $\Delta = 30^0$, more relevant for $\overline{D^0}\phi$, we find

$$a(\infty) = -0.35 \ , \quad (5.45)$$

whereas for $\Delta = 5^0$, probably more in line for $D_s^{(*)-}K^{(*)+}$, we find

$$a(\infty) = -0.065 \ . \quad (5.46)$$

Even larger asymmetries can be envisioned. Such asymmetries would not be diluted by the many tagging inefficiencies and dilution effects encountered in asymmetries that require separation of B^0 and $\overline{B}^0$ mesons. Time is the tag here. By waiting long enough, the faster decaying of the two B_s mass eigenstates has vanished. What is seen is the remnant of the slower decaying B_s mass eigenstate.

We lose lots of statistics because we study decays at many lifetimes. But such long lived B's may harbor sizable effects, without any additional dilutions. One cannot but be struck by the comparison to the K_L and K_S mesons. Whereas there is no loss in statistics in separating K_L out from K^0, because $\tau_{K_L} \approx 600 \ \tau_{K_S}$, the involved CP-violating effects are minuscule and very hard to interpret in terms of the fundamental CKM parameters. In contrast, separating B_H out from B_s requires large statistics, because times $t \gtrsim \frac{2}{\Delta\Gamma}$ are used, but the CP-violating effects can be significant and the relevant CKM parameters can be extracted.

4. Resonance effects

Studies of B modes where several resonances contribute to the final state may enhance CP-violating effects as discussed by Atwood *et al.* They applied their method to final states governed by the $b \to s\gamma, d\gamma$ [62] transitions and by the $b \to s\overline{D}^0$, sD^0, sD^0_{CP} [63] transitions. Sizable CP-violating observables can be constructed for B_s modes such as $D_s^{(*)-}(K\pi)$, $D_s^{(*)-}(K^*\pi)$, $D_s^{(*)-}(K\rho)$, $D_s^{(*)-}(K\pi\pi)$, $(\bar{D}^{(*)})\bar{K}^{(*)}K$, $(\bar{D}^{(*)}\bar{K}^{(*)})(K\pi)$, $(\bar{D}^{(*)}\bar{K}^{(*)})(K\pi\pi)$, $\bar{D}^{(*)0}(K\bar{K})$, $(\bar{D}^{(*)}\pi)\phi(\bar{D}^{(*)}\pi)(K\bar{K})$, where the particles in parentheses originate from several interfering resonances. Those modes also extract the CKM phase γ and may eliminate a twofold ambiguity in the determination of $\sin\gamma$. A detailed study is underway [67].

To summarize, this subsection described the extraction of the CKM phase γ from time dependences of untagged B_s modes governed by $\bar{b} \to \bar{c}u\bar{s}$. CP-violating effects may be sizable and are enhanced by resonance effects.

D. Modes with several CKM contributions

Consider first flavor-specific modes g where several CKM combinations contribute to the unmixed decay amplitude,

$$B_s \to g, \ \overline{B}_s \not\to g, \ \lambda = \overline{\lambda} = 0 \ , \quad (5.47)$$

for example, $K^{(*)-}\pi^+$, $K^-\pi^+\pi^+\pi^-$, $J/\psi\overline{K}^{*0}(\to K^-\pi^+)$, $J/\psi K^-\pi^+, D_s^{(*)-}D^{(*)+}$. The untagged time evolution is given by

$$\Gamma\left[g\left(t\right)\right] = \frac{\Gamma(B^0 \to g)}{2} \ \{e^{-\Gamma_L t} + e^{-\Gamma_H t}\} \ , \quad (5.48)$$

$$\Gamma\left[\overline{g}\left(t\right)\right] = \frac{\Gamma(\overline{B}^0 \to \overline{g})}{2} \ \{e^{-\Gamma_L t} + e^{-\Gamma_H t}\} \ . \quad (5.49)$$

The modes g may show direct CP violation [68,69], where the CP-violating asymmetry is

$$A_g \equiv \frac{\Gamma(B^0 \to g) - \Gamma(\overline{B}^0 \to \overline{g})}{\Gamma(B^0 \to g) + \Gamma(\overline{B}^0 \to \overline{g})} . \quad (5.50)$$

The same asymmetry can be seen as either a time-dependent or a time-integrated effect:

$$A_g = \frac{\Gamma[g(t)] - \Gamma[\overline{g}(t)]}{\Gamma[g(t)] + \Gamma[\overline{g}(t)]} = \frac{\int_{t_0}^{\infty} dt \,\{\Gamma[g(t)] - \Gamma[\overline{g}(t)]\}}{\int_{t_0}^{\infty} dt \,\{\Gamma[g(t)] + \Gamma[\overline{g}(t)]\}} . \quad (5.51)$$

Modes common to B_s and $\overline{B}_s$ where several CKM combinations contribute to $B_s \to f$ may show direct CP violation $[\Gamma(B_s \to f) \neq \Gamma(\overline{B}_s \to \overline{f})]$ as well as CP violation due to mixing. CP invariance demands that $\Gamma[f(t)] = \Gamma[\overline{f}(t)]$. The time evolution of untagged modes f and $\overline{f}$ allows one to disentangle partially the various CP-violating effects. The B_s modes $K^+K^-, \phi K_S, \rho^0 K_S, \omega K_S, D^0_{CP}\phi, J/\psi K_S, \phi\phi, \rho^0\phi$, etc. all serve as examples.

E. Measuring the width difference

After having derived the time-dependent formulas in previous subsections, we are now in a position to list several suggestions for determing $\Delta\Gamma$. A detailed feasibility study will be presented elsewhere [32]. All the methods may be combined to optimize the determination.

The first two methods use the important observation that the average B_s width Γ is in fact already known [16,17]. Table I shows the predicted [16,17] and measured [9] ratios of lifetimes of b-flavored hadrons.

References [16,17] claim the following. The B^- lifetime is predicted to be longer than the B_d lifetime due to Pauli interference. For the neutral B mesons, the W-annihilation amplitudes ($b\overline{d} \to c\overline{u}, b\overline{s} \to c\overline{c}$) are helicity suppressed and unimportant numerically, which yields same lifetimes for the average B_s and B_d mesons. The Λ_b lifetime prediction still requires a careful theoretical analysis, but it is claimed that

$$0.9 \lesssim \frac{\tau(\Lambda_b)}{\tau(B_d)} < 1 . \quad (5.52)$$

References [16,17] must be critically reevaluated, however, because they obtain a too large semileptonic branching ratio and Λ_b lifetime, and too small an inclusive width for the $b \to c\overline{c}s$ transition in B decays [28–31]. Further, the W-annihilation amplitude interferes with different spectator decays. It interferes with the spectator decay $b \to c\overline{u}d, b \to c\overline{c}s$ for the $\overline{B}_d$, $\overline{B}_s$, respectively. We believe that the $b \to c\overline{c}s$ transition is the least understood theoretically. A detailed study, which estimates how different the B_d, B_s, and other b-hadron lifetimes can be, would be useful. Because such a critical reevaluation is still lacking, this subsection uses the predictions of Bigi *et al.* [16,17], with the understanding that their estimates require refinement.

The average decay width Γ of B_s could be determined essentially from a one parameter fit $\exp(-\Gamma t)$ of the time evolution of the untagged, flavor-specific data sample [32]. It could be deduced alternatively from the measured lifetimes of other b species. For instance, the B_d and average B_s $[\overline{\tau}(B_s) \equiv 1/\Gamma]$ lifetimes are claimed to be equal to excellent accuracy [70]. Thus the average decay width Γ of B_s is measured. The width Γ can also be obtained from inclusive b lifetime measurements. Denote by T a particle, collection of particles, or event topology, which characterizes b decay. Examples for T are detached J/ψ, primary leptons (i.e., leptons in $b \to c\ell$ processes) with an impact parameter, such primary leptons in coincidence with detached vertices, or detached multiprong vertices, where the whole event is consistent with being a b decay.

A single exponential fit of the proper (multiexponential) time distribution of this inclusive b-data sample determines the "average" b lifetime $\tau(b)$:

$$\begin{aligned} e^{-t/\tau(b)} \sim\; & p_d \, R(B_d \to TX) \, e^{-t/\tau(B_d)} \\ & + p_u \, R(B_u \to TX) \, e^{-t/\tau(B_u)} \\ & + p_s \, R(B_s \to TX) \, S(t) \\ & + p_{\Lambda_b} \, R(\Lambda_b \to TX) \, e^{-t/\tau(\Lambda_b)} . \end{aligned} \quad (5.53)$$

The production fractions for $\overline{B}_d$, B_u^-, $\overline{B}_s$, Λ_b are assumed to be [71]

$$p_d : p_u : p_s : p_{\Lambda_b} \approx 0.375 : 0.375 : 0.15 : 0.10 . \quad (5.54)$$

The inclusive yield of T in b-hadron decay is defined as

$$R(H_b \to TX) \equiv B(H_b \to TX) + B(\overline{H}_b \to TX) , \quad (5.55)$$

for $H_b = \overline{B}_d, B_u^-, \overline{B}_s, \Lambda_b$. The function $S(t)$ depends on which inclusive data sample is used. For flavor-specific T [such as $\ell^\pm X$],

$$S(t) = \frac{e^{-\Gamma_L t} + e^{-\Gamma_H t}}{2} , \quad (5.56)$$

whereas for flavor-nonspecific T [such as $J/\psi X$],

TABLE I. Predicted [16,17] and measured [9] lifetime ratios of b-flavored hadrons.

	Prediction	Data
$\tau(B^-)/\tau(B_d)$	$1 + 0.05 \left(\frac{f_B}{200\ \text{MeV}}\right)^2 [1 \pm O(10\%)]$	1.01 ± 0.09
$\overline{\tau}(B_s)/\tau(B_d)$	$1 \pm O(0.01)$	0.98 ± 0.12
$\tau(\Lambda_b)/\tau(B_d)$	~ 0.9	0.71 ± 0.10

$$S(t) = e^{-\Gamma_L t} . \quad (5.57)$$

Equation (5.57) assumes that the inclusive flavor-nonspecific T production in B_s decays [such as the prominent J/ψ] is dominated by CP-even modes.

It is instructive to approximate $\tau(b)$ for an inclusive flavor-specific data sample T as

$$\tau(b) \approx [p_d/\tau(B_d) + p_u/\tau(B_u) + p_s/\bar{\tau}(B_s) + p_{\Lambda_b}/\tau(\Lambda_b)]^{-1} . \quad (5.58)$$

This approximation uses the observation and prediction of small differences in separate b-hadron lifetimes and further assumes equal inclusive yields of T in all H_b decays. Using Table I and the assumed specific b-hadron production fractions, we get, from Eq. (5.58),

$$\tau(b) = \tau(B_d)\ [1 \pm O\,(0.01)] . \quad (5.59)$$

The truly inclusive b lifetime measures essentially the B_d lifetime, which in turn is essentially the average B_s lifetime. The average width Γ of B_s is thus known

$$\Gamma \approx 1/\tau(b) . \quad (5.60)$$

In summary, Γ is essentially known from either a single parameter fit of the untagged, flavor-specific B_s data sample, or from lifetime measurements of either B_d's or inclusive b decays. We are now ready to discuss several methods for extracting $\Delta\Gamma$.

Method 1. The proper time-dependence of untagged flavor-specific modes of B_s is given by

$$e^{-(\Gamma+\frac{\Delta\Gamma}{2})t} + e^{-(\Gamma-\frac{\Delta\Gamma}{2})t} . \quad (5.61)$$

The average width Γ is known and a one parameter fit of the measured time dependence determines $\Delta\Gamma$. More than 200 flavor specific B_s events have already been recorded at LEP and CDF.

Method 2. The CP-even [CP-odd] B_s modes driven by $b \to c\bar{c}s$ are governed by a single exponential decay law

$$e^{-\Gamma_L t} = e^{-(\Gamma-\frac{\Delta\Gamma}{2})t} \left[e^{-\Gamma_H t} = e^{-(\Gamma+\frac{\Delta\Gamma}{2})t}\right] . \quad (5.62)$$

Combining this determination of Γ_L [Γ_H] with the known Γ measures $\Delta\Gamma$.

For Methods 1 and 2, we may wish to parametrize our ignorance as to the exact value of Γ by a small parameter ϵ:

$$\Gamma \to \Gamma + \epsilon. \quad (5.63)$$

A two parameter fit would extract both $\Delta\Gamma$ and ϵ. In contrast to Methods 1 and 2, Methods 3–7 do not assume knowledge of Γ.

Method 3. This is basically the method advocated by Bigi *et al.* [16,17], which we reviewed and refined in previous subsections. The time evolutions of untagged, flavor-specific modes and of CP-even modes of B_s governed by $b \to c\bar{c}s$ are given by Eqs. (5.4) and (5.62), respectively. The CP-even modes determine Γ_L. A one parameter fit of the time evolution of the untagged, flavor-specific modes determines Γ_H, because Γ_L has been measured. Of course, the exponential decay law of the CP-odd B_s modes driven by $b \to c\bar{c}s$ can be used as a consistency check and must be governed by Γ_H.

Method 4. The time evolution of the CP-even and CP-odd eigenmodes driven by the $b \to c\bar{c}s$ transition are governed by $\Gamma_+ = \Gamma_L$ and $\Gamma_- = \Gamma_H$, respectively. A time-dependent study of untagged CP-even and CP-odd modes measures the width difference. The CP-even modes are expected to dominate over the CP-odd ones, and are probably also easier to detect. To increase usable data sets with definite CP, Ref. [54] suggested employing angular correlations [52,53] to decompose modes that are mixtures of CP-even and CP-odd parities (such as $J/\psi\phi, D_s^{*+}D_s^{*-}, J/\psi\phi\rho^0$) into definite CP components.

Method 5. Any mode governed by $b \to c\bar{c}s$, which is a mixture of CP-even and CP-odd parities (for example, $J/\psi\phi, D_s^{*+}D_s^{*-}, J/\psi\phi\rho^0$), allows the extraction of both Γ_H and Γ_L. This has been discussed in Ref. [54] by decomposing such modes into CP-even and CP-odd components and studying their different decay laws. The extraction of $\Delta\Gamma$ from such modes is optimized however by a complete study of angular correlations [53] combined with other relevant techniques (such as Dalitz plots, etc.), which we advocate. Time evolutions of interference terms will add valuable information on top of the time dependences of the definite CP components. Such a study truly optimizes the determination of $\Delta\Gamma$ from modes which are admixtures of CP-even and CP-odd parities.

Method 6. For a small width difference, one may be able to determine $\Delta\Gamma$ from CP-violating effects with untagged B_s-data samples, such as the asymmetries discussed in Eqs. (5.38)–(5.40). A time-dependent fit may be able to determine the argument of tanh and thus $\Delta\Gamma$, see for instance Eq. (5.40). The determination is facilitated by knowing $|\lambda|, \phi$ and Δ. $|\lambda|$ can be obtained as discussed in Sec. V C. The weak phase ϕ will be well known from other techniques by the time such a measurement of $\Delta\Gamma$ becomes feasible. As for the final-state phase Δ, it is calculable for B_s modes where several resonances contribute to the final state, such as $D_s^{(*)-}(K\pi)$, $D_s^{(*)-}(K^*\pi)$, $D_s^{(*)-}(K\rho)$, $D_s^{(*)-}(K\pi\pi)$, $(\bar{D}^{(*)}\bar{K}^{(*)})K$, $(\bar{D}^{(*)}\bar{K}^{(*)})(K\pi)$, $(\bar{D}^{(*)}\bar{K}^{(*)})(K\pi\pi)$, $\bar{D}^{(*)0}(K\bar{K})$, $(\bar{D}^{(*)}\pi)\phi$, $(\bar{D}^{(*)}\pi)(K\bar{K})$.

Method 7. There exist B_s modes with time evolutions that depend on both the sum and the differences of the two exponents:

$$e^{-\Gamma_L t} \pm e^{-\Gamma_H t} . \quad (5.64)$$

A fit to these time evolutions determines both Γ_L and Γ_H [32]. Within the CKM model, such modes are CKM suppressed and probably not competitive with other methods. However, if the CKM model is broken and CP eigenmodes of B_s driven by $b \to c\bar{c}s$ show two different exponential decay laws, then this method is one possible way to measure both widths.

Those are then some possible ways for extracting $\Delta\Gamma$. We wish to conclude this section with a suggestion of how to enrich a B-data sample with B_s mesons. The key is a ϕ

trigger [55]. The ϕ is seen in the K^+K^- mode about 50% of the time. This mode occurs close to threshold. For energetic ϕ's, the two charged kaons have roughly equal momenta and go in similar directions. This may simplify triggering on ϕ's. The inclusive ϕ yield is about 20% in D_s decays, whereas it is roughly an order magnitude less in other charmed hadron decays [40,44]. Thus, ϕ's discriminate well between D_s and other charmed hadrons. Further, it is believed that the inclusive yield of D_s in B_s decays is quite enhanced over that in B decays. Inclusive b decays with a D_s in the final state enrich the B_s content of that b sample. In fact, the DELPHI Collaboration used $\phi\ell X$ modes as an enriched B_s sample and extracted an average B_s lifetime from it [5]. We hope to see flavor-specific modes like $B_s \to \phi\ell X$ being used both at e^+e^- colliders and hadron accelerators to extract not only the average B_s lifetime, but the width difference $\Delta\Gamma$ as well.

VI. CONCLUSIONS

Theoretical predictions for a sizable lifetime difference between the light and heavy B_s mass eigenstates have existed for many years [10–17]. The observation of a nonvanishing $\Delta\Gamma$ would prove the existence of B_s-$\overline{B}_s$ mixing. How could such a width difference be determined experimentally? To that effect, we considered the time evolution of untagged data samples of B_s mesons. We found that the rapid oscillatory behavior Δmt cancels in all untagged samples, provided that $\left|\frac{q}{p}\right| \approx 1$ which is satisfied to $\sim 10^{-3} - 10^{-4}$ within the CKM model. The time evolution of untagged data samples are governed solely by the two exponential falloffs, $e^{-\Gamma_L t}$ and $e^{-\Gamma_H t}$, which enables $\Delta\Gamma$ to be measured in several ways; see Sec. V E. The exponentials are much more slowly varying functions of proper time than the rapid Δmt-oscillations. This allows us to conduct feasibility studies with presently existing technology [32].

Once the two widths are known and found to differ, CP violation can be seen with untagged, time-evolved data samples of B_s [19,20]. CP invariance demands that modes of B_s with definite CP parity (i.e., that are CP eigenstates) time evolve with a single exponential. Thus if the time evolution of CP eigenstates, such as $\rho^0 K_S$, $\omega K_S, D^0_{CP}\phi, K^+K^-$, has two nonvanishing exponential falloffs, CP violation has been demonstrated [19]. The demonstration can clearly already occur for *untagged* data samples.

The time evolution of the *untagged* $\rho^0 K_S$, ωK_S data samples is not only useful in observing CP violation but *even extracts* $\cos 2\gamma$, when penguin contributions are neglected. Those penguin effects could be sizable however, and thus we discussed next the extraction of the unitarity angle γ from time dependences of untagged B_s data samples governed by $\bar{b} \to \bar{c}u\bar{s}$. Penguin amplitudes are absent. The time evolution of untagged, for instance, $D_s^{(*)\pm}K^{(*)\mp}, \overset{(-)}{D^0}\ \phi\ B_s$ modes measure $\cos(\gamma + \Delta)$ and $\cos(-\gamma + \Delta)$, with the overall normalization being determined from flavor-specific modes, such as $D_s^{(*)\pm}\pi^\mp, D_s^{(*)\pm}\ell\nu,\ D^0 K^{*0}, \overline{D}^0\overline{K}^{*0}$. A twofold ambiguity in $\sin\gamma$ can be resolved, and both $\sin\gamma$ and $sin^2\Delta$ are extracted.

The above extraction of the phases γ and Δ involves the factorization assumption to determine the normalization. For those who object to this assumption, there exist a series of measurements that extracts γ without any theoretical input. The time evolutions of the untagged data samples $D^0\phi, \overline{D}^0\phi$ and $D^0_{CP}\phi$ determine $|\lambda|, \gamma$ and Δ without any theory. The measured $|\lambda|$ can then be cross checked with its measurement involving some theory input, which allows insights into rescattering effects of color-suppressed processes. Sizable CP-violating effects can occur with those untagged data samples for large enough proper times. A detailed study is underway which addresses the feasibility of all the above-mentioned measurements for a generic detector [32].

The ramifications of a large width difference for the B_s meson are far reaching. The ratio $\frac{\Delta m}{\Delta\Gamma}$ involves no CKM combination, only a hadronic uncertainty [26,12,13,18]. If a careful study finds that this ratio can be rather well estimated, then, by observing either $\Delta\Gamma$ or Δm first, the other difference will be known, as well (within the CKM model). The ratio $\frac{\Delta m}{\Delta\Gamma}$ may turn out to be an important standard model constraint. Second, a large width difference will prove that $B(b \to c\bar{c}s)$ is sizable and would transform the so-called puzzle of the number of charmed hadrons per B meson into the apparent puzzle of why almost all B decays do not proceed via the $b \to c$ transitions. We urge our experimental colleagues to measure directly $B(b \to c\bar{c}s')$ as discussed in Sec. III and to measure more precisely the inclusive charm yields in B decays. We derived a reliable complementary estimate that predicts the number of charmed hadrons per B to be around 1.3, in agreement with theoretical calculations [28–31]. Third, one will not be allowed to speak about branching fractions of $B_s^0 \to f$, but only about $B(B_{H,L} \to f)$.

An analogy to the neutral kaons is instructive. The K_L lives about 600 times as long as a K_S, thus a K^0 or $\overline{K}^0$ is essentially a K_L after a few K_S lifetimes, without having lost almost any K_L. The CP-violating effects are tiny and the extraction of the fundamental CKM parameters is messy because of large uncertainties in strong matrix elements.

In contrast, the B_s meson has comparable widths for the heavy and light mass eigenstates. They are expected to differ at the (10–30)% level. To guarantee a pure data sample of B_H requires events with decay lengths at least several times the B_s lifetime, costing tremendously in statistics. But then many exciting measurements become feasible, because the B_s proceeds to decay through several quark transitions into many possible final states. Sizable CP-violating effects and the clean extraction of fundamental CKM parameters may be possible with untagged data samples of B_s mesons. (Clearly, to optimize the measurements not only pure B_H data samples but rather all available proper times better be used.) As in the case of neutral kaons, time plays the role of the

"tag." Many more measurements can be contemplated than what is reported here, once $\Delta\Gamma$ is found to be nonvanishing.

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$$\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2 = 2 + O\left(\frac{m_c^4}{m_t^4}\right) .$$

Dedicated future precision measurements may observe the tiny Δmt oscillations in $\Gamma[g(t)]$ and $\Gamma[\overline{g}(t)]$. In contrast, such Δmt oscillations disappear to higher accuracy in the sum $\Gamma[g(t)] + \Gamma[\overline{g}(t)]$.

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