Baryon-antibaryon production by disordered chiral condensates

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We investigate the production of baryons and antibaryons in the central rapidity region of high energy nuclear collisions within the framework of the Skyrme model taking into account the effects of explicit chiral symmetry breaking. We argue that the formation of disordered chiral condensates may lead to enhanced baryon-antibaryon production at low transverse momentum.

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The possibility of producing a quark-gluon plasma in relativistic heavy ion collisions is very exciting, especially from the point of view of observing the chiral/confinement phase transition (or rapid crossover) as the plasma expands and cools. Recently, many investigations have focused on the possibility of the formation of domains in a chiral phase transition where the chiral field may not be oriented along the true vacuum. Formation of a large domain with a *disoriented chiral condensate* (DCC) has been proposed by Anselm [1], by Blaizot and Krzywicki [2] and by Bjorken, Kowalski, and Taylor [3] in the context of high multiplicity hadronic collisions. It was argued in [3] and by Blaizot and Diakonov [4] that, as the chiral field relaxes to the true vacuum in such a domain, it may lead to coherent emission of pions. A motivation for this proposal comes from Centauro events in cosmic ray collisions [5]. In the context of quark-gluon plasma, Rajagopal and Wilczek proposed [6] that the nonequilibrium dynamics during the phase transition may produce DCC domains. They argued that long wavelength pion modes may get amplified leading to emission of coherent pions. Gavin, Gocksch, and Pisarski have argued [7] that large domains of DCC can arise if the effective masses of mesons are small, while Gavin and Müller propose [8] the annealing of smaller domains to give a large region of DCC's. Blaizot and Krzywicki [9] have pointed out that even if the average domain size is very small, random fluctuations could result in some subset of all nuclear collisions having a large domain of DCC's. Recently we have suggested that the effective potential of the strong interactions may have a second, local minimum at a chiral angle of π, which could mimic some of the effects of a first order phase transition even when the theory has no true thermodynamic phase transition of any order [10]. We referred to this as a proximal chiral phase transition. We further argued that this could lead to large domains of DCC.

In this paper we consider another signature which is expected from the formation of initial domain structure. This utilizes the Skyrmion picture of the nucleon [11] and is based on the ideas discussed in [12–14]. In these previous investigations, production of Skyrmions was studied in a manner analogous to the production of cosmic strings and monopoles in the early Universe. However, as we will elaborate later, the investigations in [12,13] did not take into account the boundary conditions which are needed for the specification of Skyrmions. Also, the studies in [12–14] corresponded to the situation when explicit chiral symmetry breaking is absent. As we will argue below, the formation of DCC's with the inclusion of chiral symmetry-breaking terms leads to enhanced production of baryons and antibaryons at low transverse momenta.

In this paper we use DCC domains to form Skyrmions whereas, in contrast, previous investigations [12–14] used non-DCC domains as defined by the thermal correlation length of the chiral field. It is quite likely that the details of the structure of the domains, whatever they are, are not important to the underlying picture of Skyrmion formation as discussed here and elsewhere. As we will see, the crucial assumptions are that the chiral field varies randomly from one domain to the next, and that the chiral field evolves according to field equations not dominated by fluctuations after the phase transition. The second assumption is more likely to be satisfied if we deal with DCC domains. However, to be as general as possible we define the correlation length ξ to be the distance beyond which the chiral field varies randomly without assigning any particular numerical value to it.

We use the σ model in its linear representation. The Lagrangian is expressed in terms of a scalar field σ and the pion field $\boldsymbol{\pi}$:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \boldsymbol{\pi}\right)^2 - \frac{\lambda}{4}\left(\sigma^2 + \boldsymbol{\pi}^2 - c^2/\lambda\right)^2 - V_{\rm SB}\,. \tag{1}$$

The piece of the Lagrangian which explicitly breaks chiral symmetry is $V_{\rm SB}$. In the absence of this term, the potential has the shape of the bottom of a wine bottle. Chiral symmetry is spontaneously broken in the vacuum, the pion is the massless Goldstone boson, and the σ meson gets a mass on the order of 1–2 GeV. For the present purpose it is sufficient to consider the symmetry-breaking potential to be as

$$V_{\rm SB} = -\epsilon\sigma\,. \quad (2)$$

We have then at our disposal three parameters in the effective Lagrangian: λ, c, and ϵ. These parameters must be restricted so as to give the proper pion mass, pion decay constant, a reasonable value for the σ mass, PCAC (partial conservation of axial vector current) in the weak field limit, and the condition that the ground state of the theory occur at $\sigma = f_\pi$ and $\boldsymbol{\pi} = 0$:

$$m_\pi^2 = \lambda f_\pi^2 - c^2\,, \quad (3)$$

$$m_\sigma^2 = 2\lambda f_\pi^2 + m_\pi^2\,, \quad (4)$$

$$f_\pi m_\pi^2 = \epsilon\,. \quad (5)$$

To obtain the effective potential, we expand the fields about an arbitrary point as

$$\sigma(x) = v\,\cos\theta + \sigma'(x)\,, \quad (6)$$

$$\boldsymbol{\pi}(x) = \mathbf{v}\,\sin\theta + \boldsymbol{\pi}'(x)\,, \quad (7)$$

where $v = |\mathbf{v}|$. The primes denote fluctuations about the given point. At one loop order, and in the high temperature approximation, the effective potential is

$$V(v,\theta;T) = \frac{\lambda}{4}v^4 - \frac{1}{2}\left(c^2 - \frac{\lambda T^2}{2}\right)v^2 - \epsilon v\cos\theta\,. \quad (8)$$

Here T is the temperature. Although the aforementioned approximations may not be quantitatively accurate, the above expression at least provides a basis for discussion. In particular, when $\epsilon > 0$ the bottom of the potential is tilted.

Now let us consider the Skyrmion. Quite simply, it is a configuration where the chiral field winds nontrivially around the manifold of degenerate minima of the effective potential at zero or finite temperature (neglecting for the time being the symmetry-breaking term), which is S^3 for the two flavor chiral model. An important aspect of the Skyrmion configuration is that it requires the chiral field to approach a constant value at large distances. This compactifies the local spatial region to a three-sphere S^3. As the chiral field, when restricted to the minimum of the potential, is valued in order-parameter space (which is S^3 for the two flavor case as mentioned above) one can have nontrivial winding number configurations on the spatially compactified S^3.

It was first suggested in [12], and later developed in [13], that a domain structure in quark-gluon plasma may lead to the formation of Skyrmions. However, the asymptotic boundary conditions which are required to define a Skyrmion were not considered in [12,13] which led to an overestimate of baryon production (for the case when there is no symmetry-breaking term present). Theoretical investigations with proper treatment of boundary conditions [14–16] show that it is practically impossible to form a full Skyrmion by domain formation which has integer winding number right at the time of formation (in contrast to the estimates in [12,13]). However, as discussed in [14–16], one can calculate the probability of forming partial winding number Skyrmions which could later evolve into a full Skyrmion. These studies showed that such *partial* Skyrmions form with a probability of about 0.05 in a suitable region. This region was taken to be equal to four elementary domains in [14]. The reason for taking four domains is due to the boundary conditions and corresponds to the extended nature of the Skyrmion [14]. Numerical simulations give the probability of the formation of (partial) textures, which are analogues of the Skyrmions in the context of the early Universe, to be about 0.04 [17]. As we have mentioned above, all these previous estimates corresponded to the case when there is no explicit symmetry-breaking term present in the effective potential. We will discuss this case first and then proceed to estimate Skyrmion production for the case when the effective potential is tilted.

The essential idea behind these estimates is based on a method first proposed by Kibble in the context of the production of topological defects during phase transitions in the early Universe [18]. It has been applied to analogous phase transitions in condensed matter physics [19] and to the study of the formation of glueballs and baryons or antibaryons in models based on QCD strings [20]. In the context of quark-gluon plasma, it leads to the following picture. As the phase transition proceeds, the chiral field settles to the minimum of the potential. In a small region the variation of the field will be small to avoid gradient energy. However, in regions which are far from each other, the orientation of the chiral field may differ. This situation can then be represented by triangulating the region containing the plasma in terms of elementary domains whose vertices are separated by a distance ξ. Here ξ represents some sort of correlation length beyond which the orientation of the chiral field varies randomly. For the chiral phase transition one could take this distance to be about 1 fm. Vertices of the domains then are associated with the chiral field which is roughly uniform close to a given vertex but has randomly varying orientation (but constant magnitude, assuming the phase transition is completed) from one vertex to another. In between any two adjacent vertices the variation of the chiral field will be smooth to minimize the gradient energy. This last requirement is generally called the geodesic rule as this requires that the chiral field traces the shortest path on the order parameter space S^3 between any two nearest vertices.

With these conditions we now estimate the probability of Skyrmion formation. To get a clear picture, consider first a lower dimensional example. A Skyrmion in two space dimensions arises when the field takes values in S^2. A Skyrmion configuration in this case corresponds to the field winding completely on S^2 in some area in the two-dimensional plane. Divide the two-dimensional space into triangular domains. One such domain is shown in Fig. 1. The triangle we want to consider has its center P_0 a distance ξ away from its vertices. The values of the chiral field at P_1, P_2, P_3 are $\alpha_1, \alpha_2, \alpha_3$ which define a patch, shown as the shaded area on S^2 in the figure, when these points are joined by the shortest path on S^2 and the resulting enclosed area is filled. Consider the image of this patch formed by diametrically opposed points of each point in this patch. If the value of the field α_0 at P_0 is such that it lies in this image then the variation

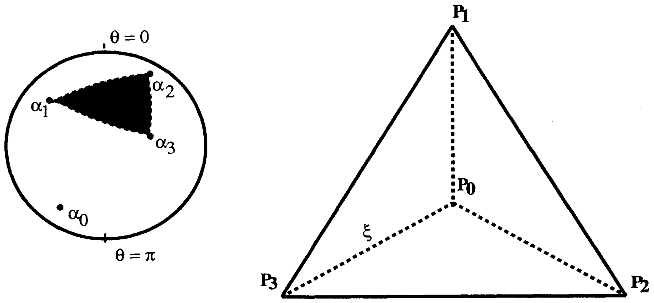

FIG. 1. A triangular domain for the two-dimensional case. P_1, P_2, P_3 are a distance ξ away from P_0. α_i represent the values of the chiral field on the manifold S^2. α_0 should fall in the diametrically opposite image of the triangular patch formed by $\alpha_1, \alpha_2, \alpha_3$ for the field configuration on the triangular domain to be able to evolve to a full Skyrmion.

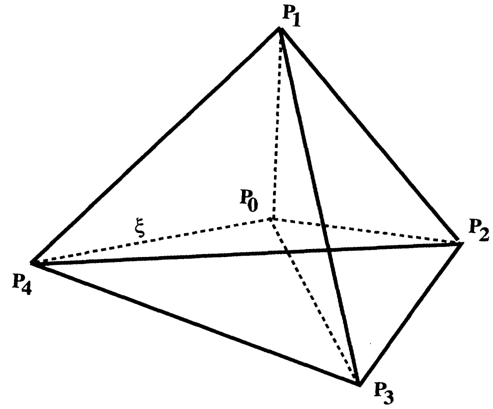

FIG. 2. A tetrahderal domain appropriate for the Skyrmion in three dimensions. All the vertices of the tetrahedron are a distance ξ away from the point P_0 at its center.

of the chiral field in the triangular area will cover more than half of S^2. For such a configuration it is natural to expect that the dynamics will force the field to develop full winding on S^2, resulting in a Skyrmion [14–17]. The probability for the formation of such a configuration can be estimated to be 1/8 because the average area of the patch is 1/8 of the total area of the two-sphere. To see this we note that the three points α_i trace three great circles on S^2 when these points are joined pairwise by the shortest geodesics. These three great circles divide S^2 into eight spherical triangles leading to the average area of one triangle equal to 1/8 of the surface area of the sphere [16]. We note that this is the same as the probability of getting a monopole configuration in three space dimensions corresponding to the winding of S^2 into S^2. Indeed, a Skyrmion in d space dimensions is just a stereographic projection of a field configuration corresponding to winding of S^d onto S^d in $d+1$ space dimensions. (Roughly similar results can be obtained by discretizing S^2 in terms of four points and requiring that the fields at P_0, P_1, P_2, P_3 all correspond to different points on this discretized S^2.) This probability of 1/8 is for a Skyrmion extended over the entire big triangle, which consists of three smaller triangles. Therefore the probability per unit area is $\xi^{-2}/6\sqrt{3}$.

It is important to note that Fig. 1 shows only a partial winding case; it is the dynamics which is supposed to evolve this configuration into a full integer winding number Skyrmion. Numerical simulations carried out in the context of textures in cosmology have shown that it indeed happens [17]. As we will show later, one of the effects of tilting the potential by adding explicit symmetry-breaking terms is to enforce such an evolution of the partial winding into the full winding configuration.

Next, let us consider Skyrmions in three space dimensions. Figure 2 shows the elementary domain: it is a tetrahedron whose vertices are a distance ξ away from its center P_0. Again, the values of the chiral field at the vertices P_1, P_2, P_3, P_4 of the tetrahedron define a volume ($\mathcal{V}$) on S^3 obtained by joining these values of the field along the shortest path and filling the enclosed region. The condition to get a partial winding which could evolve into a full winding Skyrmion is that the field at P_0 be such that it lies in the image of $\mathcal{V}$ obtained by considering diametrically opposite points of $\mathcal{V}$. The average volume of $\mathcal{V}$ is 1/16 of the total volume of S^3. Again, as in the two-dimensional case, this is because the four minimal surfaces formed by taking three P_i at a time divide S^3 into 16 spherical tetrahedra [16]. This gives the probability of the Skyrmion formation to be 1/16.

We mention here that the issue of estimating Skyrmion formation probability is not as straightforward as the estimation of other topological defects, such as strings and monopoles. For Skyrmions (and textures) dynamics plays a very important role. This is clear when we note that we are calculating the probability for only partial windings, and assuming that when this winding is more than some critical value (say 0.5) then the field will evolve into an integer winding Skyrmion. It could very well be that many partial winding Skyrmions may not be able to develop into full winding Skyrmions. Further, our analysis does not account for the field configuration in neighboring domains which could affect the evolution of the field in the domain under consideration. There are several estimates of this probability in the literature with resulting probabilities around 0.05 [14–16]. Numerical simulations show this probability to be about 0.04 [17]. From these considerations, we will take the Skyrmion formation probability to be about 0.04. This gives the probability of Skyrmion formation per unit volume as

$$\mathcal{P} \simeq 0.08\,\xi^{-3}\,. \tag{9}$$

Taking ξ to be 1 fm, the above probability gives the number of Skyrmions as about 20 in a region of volume equal to 250 fm^3, a typical quark-gluon plasma volume expected in heavy ion collisions. As the probability of baryon and antibaryon formations are equal, roughly half of these will be antibaryons. These antibaryons should have very small transverse momenta; in fact they should be practically at rest with respect to the local plasma.

It is important to realize that these baryons and antibaryons will be produced *in addition* to those produced in thermal processes. The production of topological objects due to domain formation is a nonequilibrium process and is independent of thermal production of such objects. The only effect of thermally produced objects is to affect the uniformity of neighboring domains. Therefore, thermal production will only affect our estimates by a factor roughly equal to the number of thermally produced objects per domain, which is much smaller than one for baryons. Another important aspect of this production mechanism is that it leads to strong correlation between baryon and antibaryon formations. This essentially arises from the fact that the formation of a Skyrmion fixes the winding in the region common with neighboring domains, enhancing the probability of formation of opposite winding in those domains; see [14] for details. Net baryon number of course should be fixed to be equal to the initial baryon number of the plasma. It is a nontrivial issue how to incorporate exactly the constraint of net baryon number within the framework of this mechanism for Skyrmion production. Skyrmion is a localized topological object, and one cannot just fix the winding at the boundary of the region to control the net Skyrmion number inside. In fact, it is very interesting to consider baryon-rich matter, with a relatively large net baryon number and associated chemical potential, and see how this affects our estimates of the production of baryons and antibaryons. Here we have restricted our attention to the central rapidity region which is expected to be nearly baryon-free. We hope to address these topics in a future work.

There will, of course, be constraints coming from the net energy available during the phase transition to form so many baryons. However, as discussed in [13], this constraint is not strong due to the strong temperature dependence of the baryon mass which decreases by about a factor of 0.45 as one approaches the phase transition temperature. It was also argued in [13] that annihilation of baryon-antibaryon pairs before coming out of the region may also not be very effective. We will not discuss these issues any further. This is because, as we have discussed above, the estimation of the probability of formation of the Skyrmion itself is a very nontrivial issue and to make more quantitative predictions requires detailed numerical simulation.

So far we have ignored explicit chiral symmetry breaking, which favors $\theta = 0$. We will now argue that this *increases* the probability of baryon production. Consider again a small tetrahedron such as the one shown in Fig. 2 but with sides equal to ξ. The chiral field at its vertices can be taken to be randomly varying, but we can no longer assume that at any point in the center of this triangle the chiral field will have independent orientation. If the potential was not tilted, then any distribution of the chiral field on these vertices would have been trivial in the sense that it will only cover a small region on the order parameter space S^3 and hence will shrink, decreasing the initial partial winding to zero. This would not result in the formation of a Skyrmion.

However, when the potential is tilted the situation is different. Assume that the four vertices of the tetrahedron are such that, when we use the geodesic rule to determine the field configuration for the inside of the tetrahedron, one of the interior points corresponds to $\theta = \pi$. As $\theta = 0$ is the absolute minimum, the chiral field at P_1, P_2, P_3, and P_4 will start evolving to approach $\theta = 0$. However, the interior of the tetrahedron covers the $\theta = \pi$ region and hence cannot evolve to the true minimum. (This is assuming that the patch on the order parameter space S^3 is not too small, otherwise it may shrink while rolling down. This depends on the magnitude of tilt of the potential. Such small patches will decrease the following estimate of the probability slightly. However, we will neglect it as we are interested in rough estimates. Clearly, if the tilt is very small then most of the patches will shrink and in the limit of zero tilt the results of [14,17,15,16] will be recovered.)

As the field at the four vertices approaches $\theta = 0$, the chiral field will be forced to wind around S^3 completely, giving us a full integer winding number Skyrmion configuration. This entire argument is essentially the same as the one used earlier for the case of untilted potential in the following sense. Previously, we required that the chiral field in the interior of the tetrahedron (at P_0) should lie in the image of the volume $\mathcal{V}$ formed by its diametrically opposed points. This led to the covering of most of S^3 formed by $\mathcal{V}$ and the field at P_0. Instead, we now require that $\mathcal{V}$ be such that a special fixed value ($\theta = \pi$) lies inside it rather than in its image. This means that a small patch on S^3 formed only by $\mathcal{V}$ is sufficient. Clearly it does not change the calculation of the probability. However, this probability now applies to a Skyrmion which extends over a *smaller* tetrahedron, leading to a *larger* probability per unit volume. (For the two-dimensional case this means taking a small triangle with sides ξ and requiring that the shaded area on S^2, corresponding to this triangle, cover the $\theta = \pi$ point. The probability calculation again is unchanged, but one gets larger probability per unit area.) Thus, for a tilted potential, we get the probability of Skyrmion formation per unit volume in three space dimensions as

$$\mathcal{P}' \simeq 0.33\, \xi^{-3} . \tag{10}$$

With $\xi = 1$ fm this leads to about 82 baryons and antibaryons in a region with volume equal to 250 fm^3. This probability is about four times bigger as compared to the case when the explicit chiral symmetry-breaking terms are absent. (As we discussed above this enhancement factor is applicable when the tilt is not too small. The detailed dependence of the probability on the magnitude of tilt is not easy to estimate, and one needs numerical simulations to address this issue.) We again emphasize that these baryons and antibaryons should be produced in addition to those which are thermally produced. These estimates of probability density depend on the choice of the elementary domains. It is possible that in certain situations different types of domains may be more appropriate, resulting in a different estimate of the probability density. For example, in a true first order phase transition proceeding by nucleation of bubbles, spherical domains may be more relevant. (In such a case, instead of calculating probability per unit volume, one should use probability per domain and multiply by the number of domains to get the net number of baryons.)

Explicit symmetry-breaking terms occur in various other models of topological defects too. Axionic strings in the early universe provide one example; condensed matter systems provide others. Our considerations can be generalized to these cases as well with similar conclusions [21].

We return to the importance of DCC domains in our picture of Skyrmion formation. As should be clear by now, Skyrmion formation is in one sense very different from the formation of other kinds of defects. For example, the formation of strings and monopoles occurs right after the phase transition, as soon as a nonzero winding number gets trapped in a region where thermal fluctuations are small. For an isolated defect the subsequent evolution of the field does not affect its existence. For Skyrmions, however, we must remember that we only computed the probability of formation of a partial winding number directly after the phase transition. Whether or not this evolves into a full integer winding number is only decided by the subsequent evolution of the field. Our final estimate for Skyrmion production was based on the assumption that the subsequent evolution of the chiral field is primarily controlled by the deterministic field equations and is not much affected by fluctuations. The effects of the pion mass term discussed here also require a rather classical evolution of the chiral field. This will be true for symmetry-breaking terms in other defects as well [21]. This type of field evolution is naturally expected for a DCC domain, and in fact one might even say helps to define it.

It is clear that, along with the formation of additional baryons and antibaryons at low transverse momentum, one should also expect the production of coherent pions as the chiral field rolls down to its equilibrium value. The number of coherent pions will decrease with decreasing correlation length ξ while the topological production of Skyrmions increases with decreasing ξ. This makes the observation of low transverse momentum baryons and antibaryons complementary to other proposed signals of DCC which require a large correlation length. It will be interesting to carry out the simulation of the formation of Skyrmions, such as in [17], but with the inclusion of explicit symmetry-breaking to check how the Skyrmion formation probability gets modified.

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