Baryon QCD sum rules in an external isovector-scalar field and baryon isospin mass splittings

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Within the QCD sum-rule approach in an external field, we calculate the baryon matrix element of isovector-scalar current, $H_B = \langle B|\overline{u}u - \overline{d}d|B\rangle/2M_B$, for octet baryons, which appears in the response of the correlator of baryon interpolating fields to a constant isovector-scalar external field. The sum rules are obtained for a general baryon interpolating field with an appropriate form for the phenomenological ansatz of the spectral density. The key phenomenological input is the response of the quark condensates to the external field. To first order in the quark mass difference $\delta m = m_d - m_u$, the nonelectromagnetic part of the baryon isospin mass splitting is given by the product of δm and H_B. Therefore, the QCD sum-rule calculation of H_B leads to an estimate of the octet baryon isospin mass splittings. The resulting values are comparable to the experimental values; however, the sum-rule predictions for H_B are sensitive to the values of the response of the quark condensates to the external source, which are not well determined.

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I. INTRODUCTION

To understand the observed properties of hadrons from the underlying theory of the strong interaction, quantum chromodynamics (QCD), is a challenging task since QCD remains intractable at low energies. Among the attempts made in dealing with the strong interactions at low energy scales is the QCD sum-rule approach [1], which has proved to be a useful tool of extracting qualitative and quantitative information about hadronic properties [1,2].

One of the extensions of the sum-rule methods made by Ioffe and Smilga [3] for external field problems enables one to calculate the baryon matrix elements of various bilinear quark operators. These include the matrix element of electromagnetic current to determine the magnetic moments [3–5], the matrix element of the axial vector current to find the renormalization of baryon axial vector coupling constant [6,7], the matrix element of the quark part of the energy-momentum tensor, which gives the momentum fraction carried by the up and down quarks in deep inelastic scattering [8,9], and the matrix element of isoscalar-scalar current for evaluating the nucleon σ term [10].

In this paper, we evaluate the baryon matrix elements of isovector-scalar current, $H_B = \langle B|\overline{u}u - \overline{d}d|B\rangle/2M_B$, within the external field QCD sum-rule approach. In Ref. [11], the proton matrix element $\langle p|\overline{u}u - \overline{d}d|p\rangle/2M_p$ has been calculated in the external field approach. However, one piece of the phenomenological representation has been omitted in the calculation. This has been pointed out recently by Ioffe [12]. In the present paper, we shall derive the appropriate phenomenological representation, which includes the piece neglected in Ref. [11]. We use this complete phenomenological representation and a general baryon interpolating field to calculate the matrix element H_B for octet baryons.

External field sum rules for baryons are based on the study of the correlation function of the baryon interpolating field in the presence of an external field. The appearance of the external field leads to specific new features in QCD sum rules which distinguish them from those in the absence of the external field. At the hadron level, the spectral parameters usually used in the parametrization of the spectral density, baryon masses, pole residues, and continuum thresholds, all respond to the external field. Consequently the phenomenological representation for the *response* of the correlation function contains a double pole at the baryon mass whose residue contains the matrix element of interest. This corresponds to the response of the pole position. The response of the pole residues gives rise to single pole terms, which contain information about the transition between the ground state baryon and excited states. The single pole contributions are not exponentially damped after Borel transformation relative to the double pole term and should be retained in a consistent analysis of the sum rules. In addition, there are terms corresponding to the response of the continuum thresholds, which should also be included in the calculation. At the quark level, the external field contributes in two different ways—by directly coupling to the quark fields in the baryon current and by polarizing the QCD vacuum. By equating these two different representations for the response of the baryon correlator, one obtains the external field sum rules, which relate the baryon matrix elements of various currents to QCD Lagrangian parameters, vacuum condensates, and the response of condensates to the external source.

The observed baryon isospin mass splitting has its origin in the electromagnetic interactions between quarks and in the different masses of the up and down current quarks. The contributions of the latter to first order in the quark mass difference $\delta m = m_d - m_u$ is given by the product of δm and the baryon matrix element of the isovector-scalar current. Therefore, the QCD sum-rule calculation of the baryon matrix elements of isovector-scalar current for octet baryons naturally leads to an estimate of the octet baryon isospin mass splittings. (The Σ^--Σ^0 and Σ^+-Σ^0 splittings will not be considered here as there is mixing of the Σ^0 with the Λ via isospin-violating interactions.)

The rest of this paper is organized as follows. In Sec. II, we establish the baryon QCD sum rules in an external isovector-scalar field. In Sec. III, we then analyze the sum rules and present the results. In Sec. IV, we estimate the isospin mass splittings of the octet baryons using the baryon matrix elements of isovector-scalar current calculated from QCD sum rules. Further discussion of our results are given in Sec. V.

II. BARYON QCD SUM RULES IN AN EXTERNAL ISOVECTOR-SCALAR FIELD

In this section, we establish the baryon QCD sum rules in the presence of an external isovector-scalar field. In previous works [3–12], the phenomenological representation for the correlator is usually obtained by analyzing a double dispersion relation. Here we present an alternative approach to derive the phenomenological representation. The operator product expansion (OPE) results can be easily obtained following the procedures outlined in Ref. [11]. We work to leading order in perturbation theory and to first order in the strange quark mass. Contributions proportional to the up and down current quark masses and the gluon condensate are neglected as they give numerically small contributions. We include condensates up to dimension 8.

Consider the correlator of the baryon interpolating field in the presence of a *constant* external isovector-scalar field S_V:

$$\Pi(S_V,q) \equiv i\int d^4x e^{iq\cdot x}\langle 0|T[\eta_B(x)\overline{\eta}_B(0)]|\rangle_{S_V} , \qquad (2.1)$$

where η_B is the interpolating field for the baryon under consideration. We consider baryon interpolating fields (currents) that contain no derivatives and couple to spin-$\frac{1}{2}$ states only. There are two linearly independent fields with these features, corresponding to a scalar or pseudoscalar diquark coupled to a quark. In this paper, we take a linear combination of these two fields:

$$\eta_p(x) = 2\epsilon_{abc}\left\{[u_a^T(x)Cd_b(x)]\gamma_5 u_c(x) + t[u_a^T(x)C\gamma_5 d_b(x)]u_c(x)\right\} , \qquad (2.2)$$

$$\eta_{\Sigma^+}(x) = 2\epsilon_{abc}\left\{[u_a^T(x)Cs_b(x)]\gamma_5 u_c(x) + t[u_a^T(x)C\gamma_5 s_b(x)]u_c(x)\right\} , \qquad (2.3)$$

$$\eta_{\Xi^0}(x) = 2\epsilon_{abc}\left\{[s_a^T(x)Cu_b(x)]\gamma_5 s_c(x) + t[s_a^T(x)C\gamma_5 u_b(x)]s_c(x)\right\} , \qquad (2.4)$$

where $u(x)$, $d(x)$ and $s(x)$ stand for the up, down, and strange quark fields, a, b, and c are the color indices, $C=-C^T$ is the charge conjugation matrix, and t is an arbitrary real parameter. The interpolating fields for the neutron, Σ^- and Ξ^-, can be obtained by changing $u(d)$ into $d(u)$. The interpolating fields with $t=-1$, advocated by Ioffe [13,14], have been used exclusively in previous papers on external field sum rules [3–12]. In principle, the sum-rule predictions are independent of the choice of t; in practice, however, the OPE is truncated and the phenomenological description is represented roughly. The goals in choosing the interpolating field for QCD sum-rule applications are to maximize the coupling of the interpolating field to the state of interest relative to other (continuum) states, while minimizing the contributions of higher-order terms in the OPE. These goals cannot be simultaneously realized. The optimal choice of the baryon interpolating field seems to be around Ioffe's choice. We refer the reader to Refs. [14,15] for more discussion about the choice of baryon interpolating fields. We shall consider the interval $-1.15 \le t \le -0.85$ here. For $t > -0.85$ the continuum contributions become large while for $t < -1.15$ the contributions from higher-order terms in the OPE become important relative to the leading-order terms.

The subscript S_V in Eq. (2.1) indicates the presence of the external field. Thus, the correlator should be calculated with an additional term

$$\Delta\mathcal{L} \equiv -S_V[\overline{u}(x)u(x) - \overline{d}(x)d(x)] , \qquad (2.5)$$

added to the usual QCD Lagrangian, and $-\Delta\mathcal{L}$ added to $\mathcal{H}_{\rm QCD}$. Since S_V is a scalar constant, Lorentz covariance and parity allow one to decompose $\Pi(S_V,q)$ into two distinct structures [11]:

$$\Pi(S_V,q) \equiv \Pi^1(S_V,q^2) + \Pi^q(S_V,q^2)\not{q} . \qquad (2.6)$$

To obtain QCD sum rules, one needs to construct a phenomenological representation for $\Pi(S_V,q)$ and evaluate $\Pi(S_V,q)$ using the OPE.

A. Dispersion relation and phenomenological spectral ansatz

To determine the correlator at the hadron level we use the dispersion relation

$$\Pi^i(S_V,q^2) = \int_0^\infty \frac{\rho^i(S_V,s)}{s-q^2}ds \qquad (2.7)$$

for each invariant function $\{i=1,q\}$, where $\rho^i(S_V,s) = \frac{1}{\pi}{\rm Im}\Pi^i(S_V,s)$ is the spectral density. Here we have omitted polynomial subtractions which will be eliminated by a subsequent Borel transformation. We have also omitted infinitesimal as we are only concerned with a large and spacelike q^2 in QCD sum rules.

In practical applications of QCD sum-rule approach, one usually parametrizes the spectral density by a simple pole representing the lowest energy baryon state of interest plus a continuum which is approximated by a perturbative evaluation of the correlator starting at an effective threshold [1,2,13]. When S_V is present, we add $-\Delta\mathcal{L}$ to $\mathcal{H}_{\rm QCD}$, which is equivalent to increasing m_u and m_d by S_V and $-S_V$, respectively. Consequently at the hadron level, the baryon spectrum will be shifted. Since we are concerned here with the linear response to the external source, S_V can be taken to be arbitrarily small (see below). Thus, there is no rearrangement of the spectrum, and we can use a pole plus continuum ansatz for the baryon spectral density

$$\rho^i(S_V,s) = \lambda_B^{*^2}\phi^i\delta(s-M_B^{*^2}) + \widetilde{\rho}^i(S_V,s)\theta(s-s_0^{*^i}) \ , \tag{2.8}$$

where $\phi^{*^i} = \{M_B^*, 1\}$ for $\{i=1,q\}$, and $\widetilde{\rho}^i(S_V,s)$ is to be evaluated in perturbation theory. Here λ_B^* is defined by $\langle 0|\eta_B|B\rangle_{S_V} = \lambda_B^* v_B^*$ with v_B^* the Dirac spinor normalized to $\overline{v}_B^* v_B^* = 2M_B^*$, M_B^* is the mass of the lowest baryon state, and $s_0^{*^i}$ is the continuum threshold in the presence of the external field.

Let us now expand both sides of Eq. (2.7) for small S_V:

$$\Pi_0^i(q^2) + S_V\Pi_1^i(q^2) + \cdots = \int_0^\infty \frac{\rho_0^i(s)}{s-q^2}ds + S_V\int_0^\infty \frac{\rho_1^i(s)}{s-q^2}ds + \cdots \ . \tag{2.9}$$

Since S_V is arbitrary, one immediately concludes that

$$\Pi_0^i(q^2) = \int_0^\infty \frac{\rho_0^i(s)}{s-q^2}ds \ , \tag{2.10}$$

$$\Pi_1^i(q^2) = \int_0^\infty \frac{\rho_1^i(s)}{s-q^2}ds \ . \tag{2.11}$$

Obviously, Eq. (2.10) leads to the baryon *mass* sum rules in vacuum which have been extensively studied [13,16,2,17]. Here we are interested in Eq. (2.11), which corresponds to the linear response of the correlator to the external source and contains the baryon matrix element under consideration (see below).

Expanding the right-hand side of Eq. (2.8), we find

$$\rho_0^i(s) = \lambda_B^2\phi_0^i\delta(s-M_B^2) + \widetilde{\rho}_0^i(s)\theta(s-s_0^i) \ , \tag{2.12}$$

$$\begin{aligned}\rho_1^i(s) = {} & -2H_B M_B\lambda_B^2\phi_0^i\delta'(s-M_B^2) + \Delta\lambda_B^2\,\phi_0^i\delta(s-M_B^2) \\ & +\Delta\phi^i\,\lambda_B^2\delta(s-M_B^2) - \Delta s_0^i\,\widetilde{\rho}_0^i(s)\delta(s-s_0^i) + \widetilde{\rho}_1^i(s)\theta(s-s_0^i) \ ,\end{aligned} \tag{2.13}$$

where we have defined

$$M_B^* = M_B + S_V H_B + \cdots \ , \tag{2.14}$$

$$\lambda_B^{*^2} = \lambda_B^2 + S_V\Delta\lambda_B^2 + \cdots \ , \tag{2.15}$$

$$s_0^{*^i} = s_0^i + S_V\Delta s_0^i + \cdots \ , \tag{2.16}$$

$$\phi^{*^i} = \phi_0^i + S_V\Delta\phi^i + \cdots \ , \tag{2.17}$$

$$\widetilde{\rho}^{*^i}(s) = \widetilde{\rho}_0^i(s) + S_V\widetilde{\rho}_1^i(s) + \cdots \ , \tag{2.18}$$

where the first terms are the vacuum spectral parameters in the absence of the external field. Note that $\Delta\phi^1 = H_B$ and $\Delta\phi^q = 0$. Treating S_V as a small parameter, one can use the Hellman-Feynman theorem [18,19] to show that

$$H_B = \frac{\langle B|\overline{u}u - \overline{d}d|B\rangle}{2M_B} \ , \tag{2.19}$$

where we have used covariant normalization $\langle k', B|k, B\rangle = (2\pi)^2 k^0\delta^{(3)}(\vec{k}' - \vec{k})$.

One notices that $\rho_1^i(s)$ has specific new features which distinguish it from $\rho_0^i(s)$. The first term in Eq. (2.13), which is *absent* in $\rho_0^i(s)$, gives rise to a double pole at the baryon mass whose residue contains the matrix element of interest. The second and third terms are single pole terms; the residue at the single pole contains information about the transition between the ground state baryon and the excited states. In terms of quantum mechanical perturbation, the double pole term corresponds to the energy shift while the single pole terms result from the response of baryon wave function to the external field. The fourth term is due to the response of the continuum threshold to the external source and the last term is the continuum contribution. As emphasized in the previous works, the single pole contributions are not exponentially damped after the Borel transformation relative to the double term and should be retained in a consistent analysis of the sum rules.

The fourth term has been neglected in Ref. [11]. The contribution of this term is suppressed in comparison with the single pole terms by a factor $e^{-(s_0^i-M_B^2)/M^2}$ [see Eqs. (2.29)–(2.36)]. If the response of the continuum threshold is small, one can neglect the contribution of the fourth term. However, if the response of the continuum threshold is strong, one needs to include the fourth term in the calculation. This point has been noticed recently by Ioffe in Ref. [12], where a double dispersion relation is considered for the vertex function

$$\Pi_1(q) = \int d^4x e^{iq\cdot x}\langle 0|T\eta_B(x)\left[\int d^4z[\overline{u}(z)u(z) - \overline{d}(z)d(z)]\right]\overline{\eta}_B(0)|0\rangle \tag{2.20}$$

in order to get the appropriate phenomenological representation. [This vertex function can be obtained by expanding the right-hand side of Eq. (2.1) directly.] We note that our discussion and Eq. (2.13) are consistent with those given in Ref. [12]. Substituting Eq. (2.13) into Eq. (2.11), one obtains the appropriate phenomenological representation.

B. QCD representation

The QCD representation of the correlator is obtained by applying the OPE to the time-ordered product in the correlator. When the external field is present, the up and down quark fields satisfy the modified equations of motion:

$$(i\not{D} - m_u - S_V)u(x) = 0 \ , \tag{2.21}$$

$$(i\not{D} - m_d + S_V)d(x) = 0 \ , \tag{2.22}$$

where $\not{D} = \gamma^\mu(\partial_\mu - ig_s\mathcal{A}_\mu)$ is the covariant derivative. (The equation of motion for the strange quark field does not change.) In the framework of the OPE, the external field contributes to the correlator in two ways: It couples directly to the quark fields in the baryon interpolating fields and it also polarizes the QCD vacuum. Since the external field in the present problem is a Lorentz scalar, nonscalar correlators cannot be induced in the QCD vacuum. However, the external field does modify the condensates already present in the QCD vacuum. To first order in S_V, the chiral quark condensates can be written as

$$\langle\overline{u}u\rangle_{S_V} = \langle\overline{u}u\rangle_0 - \chi S_V\langle\overline{u}u\rangle_0 \ , \tag{2.23}$$

$$\langle\overline{d}d\rangle_{S_V} = \langle\overline{d}d\rangle_0 + \chi S_V\langle\overline{d}d\rangle_0 \ , \tag{2.24}$$

$$\langle\overline{s}s\rangle_{S_V} = \langle\overline{s}s\rangle_0 - \chi_s S_V\langle\overline{s}s\rangle_0 \ , \tag{2.25}$$

where $\langle\hat{O}\rangle_0 \equiv \langle 0|\hat{O}|0\rangle$. The mixed quark-gluon condensates change in a similar way:

$$\langle g_s\overline{u}\sigma\cdot\mathcal{G}u\rangle_{S_V} = \langle g_s\overline{u}\sigma\cdot\mathcal{G}u\rangle_0 - \chi_m S_V\langle g_s\overline{u}\sigma\cdot\mathcal{G}u\rangle_0 \ , \tag{2.26}$$

$$\langle g_s\overline{d}\sigma\cdot\mathcal{G}d\rangle_{S_V} = \langle g_s\overline{d}\sigma\cdot\mathcal{G}d\rangle_0 + \chi_m S_V\langle g_s\overline{d}\sigma\cdot\mathcal{G}d\rangle_0 \ , \tag{2.27}$$

$$\langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_{S_V} = \langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_0 - \chi_{ms} S_V\langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_0 \ , \tag{2.28}$$

where $\sigma\cdot\mathcal{G} \equiv \sigma_{\mu\nu}\mathcal{G}^{\mu\nu}$ with $\mathcal{G}^{\mu\nu}$ the gluon field tensor. One can express χ, χ_s, χ_m, and χ_{ms} in terms of correlation functions (see Ref. [11]). Here we have assumed that the response of the up and down quarks is the same, apart from the sign. The Wilson coefficients can be calculated following the methods outlined in Ref. [11]. The results of our calculations for the invariant functions Π_1^1 and Π_1^q are given in the Appendix.

C. Sum rules

The QCD sum rules are obtained by equating the QCD representation and the phenomenological representation and applying the Borel transformation. The resulting sum rules in the proton case can be expressed as

$$\begin{aligned}\frac{c_1+6c_2}{2}M^8E_2L^{-8/9} - \frac{c_1+6c_2}{2}\chi aM^6E_1 + \frac{3c_2}{2}\chi_m m_0^2aM^4E_0L^{-14/27} + \frac{c_1+3c_2-c_3}{3}a^2M^2 \\ = \left[2H_p\,\widetilde{\lambda}_p^2M_p^2 - \Delta\widetilde{\lambda}_p^2\,M_pM^2 - H_p\,\widetilde{\lambda}_p^2M^2\right]e^{-M_p^2/M^2} \\ + \left[\frac{c_1-6c_2}{2}s_0^1aL^{-4/9} + \frac{3c_2}{2}m_0^2aL^{-26/27}\right]\Delta s_0^1M^2e^{-s_0^1/M^2} \ ,\end{aligned} \tag{2.29}$$

$$\begin{aligned}-\frac{4c_1-c_3}{4}aM^4E_0L^{-4/9} - \frac{c_4+c_5-6c_2}{12}m_0^2aM^2L^{-26/27} + \frac{2c_1}{3}\chi a^2M^2L^{4/9} - \frac{c_1+2c_2}{12}\chi m_0^2a^2L^{-2/27} \\ -\frac{c_1-2c_2}{12}\chi_m m_0^2a^2L^{-2/27} = \left[2H_p\,\widetilde{\lambda}_p^2M_p - \Delta\widetilde{\lambda}_p^2M^2\right]e^{-M_p^2/M^2} + \frac{c_3}{16}(s_0^q)^2\Delta s_0^qM^2L^{-8/9}e^{-s_0^q/M^2} \ ,\end{aligned} \tag{2.30}$$

where $a \equiv -4\pi^2\langle\overline{q}q\rangle_0$, $\widetilde{\lambda}_p^2 \equiv 32\pi^4\lambda_p^2$, $\Delta\widetilde{\lambda}_p^2 \equiv 32\pi^4\Delta\lambda_p^2$, and $m_0^2 \equiv \langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_0/\langle\overline{q}q\rangle_0$. Here we have ignored the isospin breaking in the vacuum condensates (i.e., $\langle\overline{u}\hat{O}u\rangle_0 \simeq \langle\overline{d}\hat{O}d\rangle_0 = \langle\overline{q}\hat{O}q\rangle_0$); the inclusion of the isospin breaking in vacuum condensates only gives small refinements of the results. We have also defined

$$E_0 \equiv 1 - e^{-s_0^i/M^2} ,$$

$$E_1 \equiv 1 - e^{-s_0^i/M^2}\left[\frac{s_0^i}{M^2}+1\right] ,$$

$$E_2 \equiv 1 - e^{-s_0^i/M^2}\left[\frac{(s_0^i)^2}{2M^4}+\frac{s_0^i}{M^2}+1\right] , \tag{2.31}$$

and

$$c_1 = (1-t)^2 , \quad c_2 = 1-t^2 , \quad c_3 = 5t^2+2t+5 ,$$
$$c_4 = t^2+10t+1 , \quad c_5 = t^2+4t+7 . \tag{2.32}$$

The anomalous dimensions of the various operators have been taken into account through the factor $L \equiv \ln(M^2/\Lambda_{\rm QCD}^2)/\ln(\mu^2/\Lambda_{\rm QCD}^2)$ [1,13]. We take the renormalization scale μ and the QCD scale parameter $\Lambda_{\rm QCD}$ to be 500 MeV and 150 MeV [13].

The sum rules in the Σ^+ case are given by

$$\begin{aligned}
&3c_2M^8E_2L^{-8/9} - 3c_2\chi aM^6E_1 + \frac{c_1}{2}\chi_s faM^6E_1 + (c_1-2c_3)m_saM^4E_0L^{-8/9} \\
&\quad -3c_2m_sfaM^4E_0L^{-8/9} + \frac{3c_2}{2}\chi_m m_0^2aM^4E_0L^{-14/27} - \frac{c_2}{4}m_sf_sm_0^2aM^2L^{-38/27} \\
&\quad -\frac{2c_1+3c_2-6c_3}{12}m_sm_0^2aM^2L^{-38/27} + \frac{c_1-2c_3}{3}fa^2M^2 + \frac{2c_3}{3}\chi m_sa^2M^2 \\
&\quad +c_2\chi m_sfa^2M^2 + c_2\chi_sm_sfa^2M^2 \\
&\quad = [2H_{\Sigma^+}\widetilde{\lambda}_{\Sigma^+}^2M_{\Sigma^+}^2 - \Delta\widetilde{\lambda}_{\Sigma^+}^2M_{\Sigma^+}M^2 - H_{\Sigma^+}\widetilde{\lambda}_{\Sigma^+}^2M^2]e^{-M_{\Sigma^+}^2/M^2} + \left[\frac{c_1}{4}m_s(s_0^1)^2L^{-4/3}\right. \\
&\quad \left. +\frac{c_1}{2}fas_0^1L^{-4/9} - 3c_2as_0^1L^{-4/9} + \frac{3c_2}{2}m_0^2aL^{-26/27}\right]\Delta s_0^1M^2e^{-s_0^1/M^2} ,
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
&3c_2m_sM^6E_1L^{-4/3} - \frac{2c_1-c_3}{2}aM^4E_0L^{-4/9} + 3c_2faM^4E_0L^{-4/9} - 3c_2\chi m_saM^4E_0L^{-4/9} \\
&\quad -\frac{c_3}{4}\chi_sm_sfaM^4L^{-4/9} - \frac{c_2}{12}m_0^2aM^2L^{-26/27} - \frac{5c_2}{4}f_sm_0^2aM^2L^{-26/27} \\
&\quad +\frac{7c_2}{4}\chi_mm_sm_0^2aM^2L^{-26/27} - \frac{c_5}{12}\chi_{ms}m_sf_sm_0^2aM^2L^{-2/27} + \frac{2c_1}{3}\chi a^2M^2L^{4/9} \\
&\quad -2c_2\chi fa^2M^2L^{4/9} - 2c_2\chi_sfa^2M^2L^{4/9} - c_2m_sa^2L^{-4/9} - \frac{c_3-2c_1}{6}m_sfa^2L^{-4/9} \\
&\quad -\frac{c_1}{12}\chi m_0^2a^2L^{-2/27} + \frac{5c_2}{12}\chi f_sm_0^2a^2L^{-2/27} + \frac{7c_2}{12}\chi_sfm_0^2a^2L^{-2/27} \\
&\quad -\frac{c_1}{12}\chi_mm_0^2a^2L^{-2/27} + \frac{5c_2}{12}\chi_{ms}f_sm_0^2a^2L^{-2/27} + \frac{7c_2}{12}\chi_mfm_0^2a^2L^{-2/27} \\
&\quad = [2H_{\Sigma^+}\widetilde{\lambda}_{\Sigma^+}^2M_{\Sigma^+} - \Delta\widetilde{\lambda}_{\Sigma^+}^2M^2]e^{-M_{\Sigma^+}^2/M^2} \\
&\quad +\left[\frac{c_3}{16}(s_0^q)^2 - 3c_2m_sa - \frac{c_3}{4}m_sfa\right]\Delta s_0^qM^2L^{-8/9}e^{-s_0^q/M^2} ,
\end{aligned} \tag{2.34}$$

where $f \equiv \langle\overline{s}s\rangle_0/\langle\overline{q}q\rangle_0$ and $f_s \equiv \langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_0/\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_0$. The sum rules in the Ξ^0 case are

$$\begin{aligned}
&-\frac{c_1}{2}M^8E_2L^{-8/9} + \frac{c_1}{2}\chi aM^6E_1 - 3c_2\chi_sfaM^6E_1 - 3c_2m_saM^4E_0L^{-8/9} \\
&\quad +(c_1-2c_3)m_sfaM^4E_0L^{-8/9} + \frac{3c_1}{2}\chi_{ms}f_sm_0^2aM^4E_0L^{-14/27} \\
&\quad -\frac{c_2}{4}m_sm_0^2aM^2L^{-38/27} - \frac{2c_1+3c_2-6c_3}{12}m_sf_sm_0^2aM^2L^{-38/27} - \frac{c_3}{3}f^2a^2M^2 \\
&\quad -c_2fa^2M^2 - (c_1-2c_3)\chi m_sfa^2M^2 - (c_1-2c_3)\chi_sm_sfa^2M^2 \\
&\quad = [2H_{\Xi^0}\widetilde{\lambda}_{\Xi^0}^2M_{\Xi^0}^2 - \Delta\widetilde{\lambda}_{\Xi^0}^2M_{\Xi^0}M^2 - H_{\Xi^0}\widetilde{\lambda}_{\Xi^0}^2M^2]e^{-M_{\Xi^0}^2/M^2}
\end{aligned}$$

$$+\left[-\frac{3c_2}{2}m_s(s_0^1)^2L^{-4/3}+\frac{c_1}{2}s_0^1aL^{-4/9}\right.$$
$$\left.-3c_2s_0faL^{-4/9}+\frac{3c_2}{2}f_sm_0^2aL^{-26/27}\right]\Delta s_0^1e^{-s_0^1/M^2}\ , \tag{2.35}$$

$$3c_2m_sM^6E_1L^{-4/3}+\frac{c_3}{4}aM^4E_0L^{-4/9}+3c_2faM^4E_0L^{-4/9}-3c_2\chi m_saM^4E_0L^{-4/9}$$
$$+\frac{2c_1-c_3}{2}\chi_sm_sfaM^4E_0L^{-4/9}+\frac{c_5}{12}m_0^2aM^2L^{-26/27}-\frac{7c_2}{4}f_sm_0^2aM^2L^{-26/27}$$
$$+\frac{5c_2}{4}\chi_mm_sm_0^2aM^2L^{-14/27}+\frac{c_4}{12}\chi_{ms}m_sf_sm_0^2M^2L^{-26/27}-2c_2\chi fa^2M^2L^{4/9}$$
$$-2c_2\chi_sfa^2M^2L^{4/9}+\frac{2c_1}{3}\chi_sf^2a^2M^2L^{4/9}-c_2m_sf^2a^2L^{-4/9}-\frac{c_3-2c_1}{6}m_sf_sa^2L^{-4/9}$$
$$+\frac{7c_2}{12}\chi f_sm_0^2a^2L^{-2/27}-\frac{c_1}{12}\chi_sff_sm_0^2a^2L^{-2/27}+\frac{5c_2}{12}\chi_sfm_0^2a^2L^{-2/27}$$
$$+\frac{5c_2}{12}\chi_mfm_0^2a^2L^{-2/27}-\frac{c_1}{12}\chi_{ms}ff_sm_0^2a^2L^{-2/27}+\frac{7c_2}{12}\chi_{ms}f_sm_0^2a^2L^{-2/27}$$
$$=[2H_{\Xi^0}\,\widetilde{\lambda}_{\Xi^0}^2M_{\Xi^0}-\Delta\widetilde{\lambda}_{\Xi^0}^2]e^{-M_{\Xi^0}^2/M^2}$$
$$+\left[\frac{c_3}{16}(s_0^q)^2-3c_2m_sa-\frac{c_3-2c_1}{2}m_sfa\right]\Delta s_0^qL^{-8/9}M^2e^{-s_0^q/M^2}\ . \tag{2.36}$$

III. SUM-RULE ANALYSIS

We now analyze the sum rules derived in the previous section and extract the baryon matrix elements of interest. Here we follow Ref. [11] and use only the sum rules (2.30), (2.34), and (2.36), which are more stable than the other three sum rules. The pattern that one of the sum rules (in each case) works well while the other does not has been seen in various external field problems [3,5,7,10,11]. This may be attributed to the different asymptotic behavior of various sum rules. As emphasized earlier, the phenomenological side of the external field sum rules contains single pole terms arising from the transition between the ground state and the excited states, whose contribution is *not* suppressed relative to the double pole term and thus contaminates the double pole contribution. The degree of this contamination may vary from one sum rule to another. The sum rule with smaller single pole contribution works better. We refer the reader to Refs. [7,10,11] for more discussion about the different behavior of various external field sum rules. In the analysis to follow, we disregard the sum rule Eqs. (2.29), (2.33), and (2.35), and consider only the results from the sum rules Eqs. (2.30), (2.34), and (2.36).

We adopt the numerical optimization procedures used in Refs. [17,20]. The sum rules are sampled in the fiducial region of Borel M^2, where the contributions from the high-dimensional condensates remain small and the continuum contribution is controllable. We choose

$$0.8\le M^2\le 1.4\,\mathrm{GeV}^2\quad\text{for the proton case}\ , \tag{3.1}$$

$$1.2\le M^2\le 1.8\,\mathrm{GeV}^2\quad\text{for the }\Sigma^+\text{ and }\Xi^0\text{cases}\ , \tag{3.2}$$

which have been identified as the fiducial region for the baryon mass sum rules [3,21]. Here we adopt these boundaries as the maximal limits of applicability of the external field sum rules. The sum-rule predictions are obtained by minimizing the logarithmic measure $\delta(M^2)=\ln[\text{maximum}\{\text{LHS,RHS}\}/\text{minimum}\{\text{LHS,RHS}\}]$ averaged over 150 points evenly spaced within the fiducial region of M^2, where LHS and RHS denote the left- and right-hand sides of the sum rules, respectively.

Note that the *vacuum* spectral parameters λ_B^2, M_B, and s_0^i also appear in the external field sum rules (2.29) and (2.30) and (2.33)–(2.36). Here we use the experimental values for the baryon masses and extract λ_B^2 and s_0^i from baryon mass sum rules using the same optimization procedure as described above. We then extract H_B, $\Delta\lambda_B^2$, and Δs_0^i from the external field sum rules.

For vacuum condensates, we use $a=0.55\,\mathrm{GeV}^3$ ($m_u+m_d\simeq 11.8$ MeV) [3,13], $m_0^2=0.8\,\mathrm{GeV}^2$ [3,16], and $f\simeq f_s=0.8$ [16,17]. We take the strange quark mass m_s to be 150 MeV [21]. The parameter χ has been estimated in Ref. [11]. The estimate in chiral perturbation theory gives $\chi\simeq 2.2\,\mathrm{GeV}^{-1}$. It is also shown that to the lowest order in δm, χ is determined by

$$\chi\delta m=-\gamma+O[(\delta m)^2]\ , \tag{3.3}$$

where $\gamma\equiv\langle\bar dd\rangle_0/\langle\bar uu\rangle_0-1$, and δm has been determined by Gasser and Leutwyler, $\delta m/(m_u+m_d)=0.28\pm0.03$ [22]. The value of γ has been estimated previously in various approaches [23–32] with results ranging from -1×10^{-2} to -2×10^{-3}, which upon using Eq. (3.3) and a median value for $\delta m=3.3\,\mathrm{MeV}$, corresponds to

$$0.5\,\mathrm{GeV}^{-1}\le\chi\le 3.0\,\mathrm{GeV}^{-1}\ . \tag{3.4}$$

We shall consider this range of χ values. We follow Ref. [11] and assume $\chi_m \simeq \chi$, which is equivalent to the assumption that m_0^2 is isospin independent.

The parameter χ_s measures the response of the strange quark condensate to the external field, which has not been estimated previously. Since $\bar{s}s$ is an isospin scalar operator, χ_s arises from the isospin mixing and we expect $\chi_s < \chi$. Following Ref. [11], one may express χ_s in terms of a correlation function and estimate it in chiral perturbation theory. It is easy to show that $\chi_s\langle\bar{s}s\rangle_0 = \frac{d}{d\delta m}\langle\bar{s}s\rangle_0$. So, one may determine χ_s by evaluating $\frac{d}{d\delta m}\langle\bar{s}s\rangle_0$ in effective QCD models. Here we shall treat χ_s as a free parameter and consider the values of χ_s in the range of $0 \leq \chi_s \leq 3.0\,\mathrm{GeV}^{-1}$. We also assume that $\chi_{ms} \simeq \chi_s$.

We first analyze the sum rules for Ioffe's interpolating field (i.e., $t=-1$). We start from the proton case. The optimized result for H_p as function of χ is plotted in Fig. 1. One can see that H_p varies rapidly with χ. Therefore, the sum-rule prediction for the proton matrix element H_p depends strongly on the response of the up and down quark condensates to the external source. (The sum rules in the proton case are independent of χ_s and χ_{sm}.) For moderate values of χ ($1.5\,\mathrm{GeV}^{-1} \leq \chi \leq 2.0\,\mathrm{GeV}^{-1}$), the predictions are

$$H_p \simeq 0.54\text{–}0.78\ . \tag{3.5}$$

On the other hand, for large values of χ ($2.4\,\mathrm{GeV}^{-1} \leq \chi \leq 3.0\,\mathrm{GeV}^{-1}$), we find $H_p \simeq 0.97$–1.25. For small values of χ ($\chi \leq 1.4\,\mathrm{GeV}^{-1}$), the continuum contribution is larger than 50%, implying that the continuum contribution is dominant in the Borel region of interest and the prediction is not reliable. The predictions for $\Delta\lambda_p^2$ and Δs_0^q also change with χ in the same way as H_p.

To see how well the sum rule works, we plot the LHS, RHS, and the individual terms of RHS of Eq. (2.30) as functions of M^2 with $\chi = 1.8\,\mathrm{GeV}^{-1}$ in Fig. 2 using the optimized values for H_p, $\Delta\lambda_p^2$, and Δs_0^q. We see that the solid (LHS) and long-dashed (RHS) curves are right on top of each other, showing a very good overlap. We also note from Fig. 2 that the first term of RHS (curve 1) is larger than the second (curve 2) and third (curve 3) terms. This shows that the double pole contribution is stronger than the single pole contribution and the predictions are thus stable. (Although the second and third terms are sizable individually, their sum is small.)

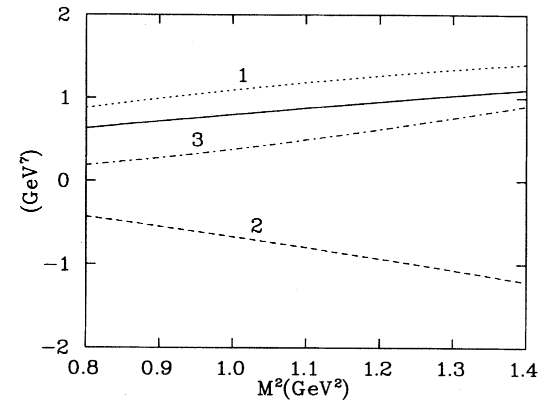

FIG. 2. The left-hand side (solid) and right-hand side (long-dashed) of Eq. (2.30) as functions of Borel M^2 for $t=-1$, with $\chi = 1.8\,\mathrm{GeV}^{-1}$ and the optimized values for H_p, $\Delta\lambda_p^2$, and Δs_0^q. Curves 1, 2, and 3 correspond to the first, second, and third terms on the right-hand side of Eq. (2.30).

In Fig. 3, we have displayed the predicted H_{Σ^+} as function of χ for three different values of χ_s. One notices that H_{Σ^+} is largely insensitive to χ_s, but strongly dependent on χ value. For χ values in the range of $2.2\,\mathrm{GeV}^{-1} \leq \chi \leq 3.0\,\mathrm{GeV}^{-1}$, we find

$$H_{\Sigma^+} \simeq 1.65\text{–}2.48\ . \tag{3.6}$$

For smaller χ, we obtain smaller values for H_{Σ^+}. The predictions for $\Delta\lambda_{\Sigma^+}^2$ and Δs_0^q change in a similar pattern. The sum rule works very well and the continuum contribution is small for all χ and χ_s values considered here.

The optimized H_{Ξ^0} as function of χ_s is shown in Fig. 4. [When $t=-1$, the sum rule Eq. (2.36) is independent of χ and χ_s.] We see that the result is very

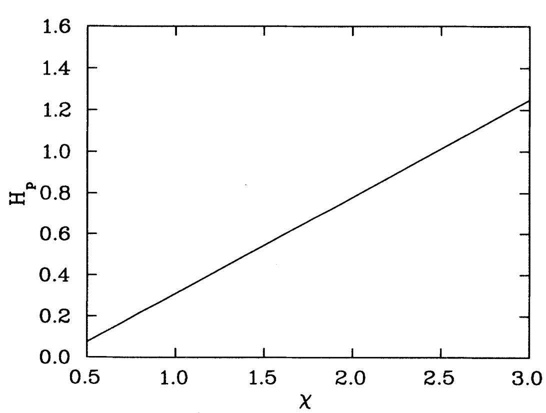

FIG. 1. Optimized sum-rule prediction for H_p as function of χ, with Ioffe's interpolating field (i.e., $t=-1$). The other input parameters are described in the text.

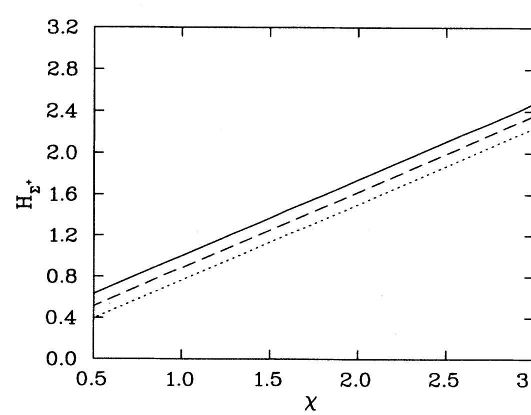

FIG. 3. Optimized sum-rule prediction for H_{Σ^+} as function of χ, with $t=-1$. The three curves correspond to $\chi_s = 0$ (solid line), $1.5\,\mathrm{GeV}^{-1}$ (dashed line), and $3.0\,\mathrm{GeV}^{-1}$ (dotted line). The other input parameters are the same as in Fig. 1.

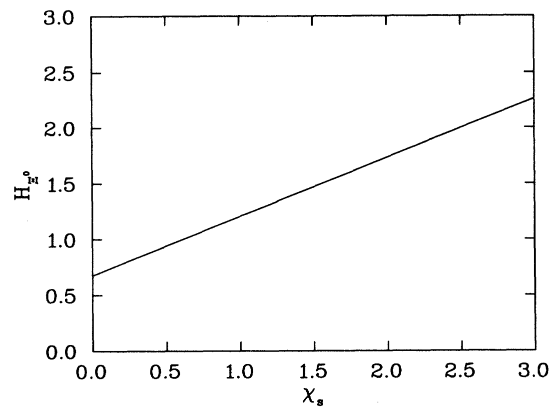

FIG. 4. Optimized sum-rule prediction for H_{Ξ^0} as function of χ_s, with $t=-1$. The other input parameters are the same as in Fig. 1.

sensitive to the χ_s value. Thus the prediction for H_{Ξ^0} has a strong dependence on the response of the strange quark condensate to the external field. For moderate χ_s ($1.7\,\mathrm{GeV}^{-1} \le x_s \le 2.2\,\mathrm{GeV}^{-1}$), we get

$$H_{\Xi^0} \simeq 1.57\text{–}1.84\ . \tag{3.7}$$

For larger (smaller) values of χ_s, we find larger (smaller) values for H_{Ξ^0}. At $\chi_s = 0$, we get $H_{\Xi^0} \simeq 0.68$. The results for $\Delta\lambda^2_{\Xi^0}$ and Δs^q_0 increase (decrease) as χ_s increases (decreases).

All of the results above use Ioffe's interpolating field (i.e., $t=-1$); we now present the results for general interpolating field. In Fig. 5, we have plotted the predicted H_p, H_{Σ^+}, and H_{Ξ^0} as functions of t for $\chi = 2.5\,\mathrm{GeV}^{-1}$ and $\chi_s = 1.5\,\mathrm{GeV}^{-1}$. As t increases, H_p, H_{Σ^+}, and H_{Ξ^0} all increase; the rate of increase is essentially the same for H_p and H_{Σ^+}, but somewhat smaller for H_{Ξ^0}. We note that the *vacuum* spectral parameters λ^2_B and s^q_0 decrease as t increases; this leads to a large variation of H_p, H_{Σ^+}, and H_{Ξ^0} with t.

The sensitivity of our results to the assumption of

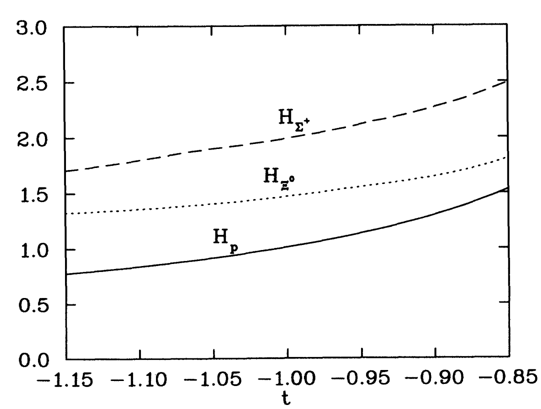

FIG. 5. Optimized sum-rule prediction for H_p, H_{Σ^+}, and H_{Ξ^0} as functions of t, with $\chi = 2.5\,\mathrm{GeV}^{-1}$ and $\chi_s = 1.5\,\mathrm{GeV}^{-1}$. The other input parameters are the same as in Fig. 1.

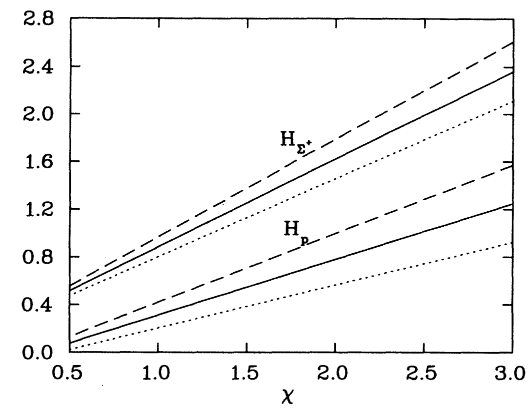

FIG. 6. Optimized sum-rule prediction for H_p and H_{Σ^+} as functions of χ, with $t=-1$ and $\chi_s = \chi_{ms} = 1.5\,\mathrm{GeV}^{-1}$. The three curves correspond to $\chi_m = \chi$ (solid), $\frac{1}{2}\chi$ (dashed), and $\frac{3}{2}\chi$ (dotted). The other input parameters are the same as in Fig. 1.

$\chi_m = \chi$ is displayed in Fig. 6, where t and $\chi_s (= \chi_{ms})$ are fixed at -1 and $1.5\,\mathrm{GeV}^{-1}$, respectively. The three curves are obtained by using $\chi_m = \chi$, $\frac{1}{2}\chi$, and $\frac{3}{2}\chi$, respectively. We note that H_p and H_{Σ^+} get larger (smaller) as χ_m becomes smaller (larger). The results are more sensitive to χ_m in the proton case than in the Σ^+ case. The prediction for H_p changes by about 25% while the prediction for H_{Σ^+} changes by about 15% when the χ_m value is changed by 50%. This implies that the terms proportional to χ_m in the sum rules give rise to sizable contributions. The sensitivity of our predictions to the assumption of $\chi_{sm} = \chi_s$ is illustrated in Fig. 7, with $t=-1$ and $\chi = \chi_m = 2.5\,\mathrm{GeV}^{-1}$. The three curves correspond to $\chi_{ms} = \chi_s$, $\frac{1}{2}\chi_s$, and $\frac{3}{2}\chi_s$, respectively. One can see that both H_{Σ^+} and H_{Ξ^0} are insensitive to changes in χ_{ms}. This indicates that the terms proportional to χ_{ms} give only small contributions to the sum rules. One also notices that H_{Σ^+} depends only weakly on χ_s. Finally, the effect of ignoring the response of continuum threshold is shown in Fig. 8. The solid (dashed)

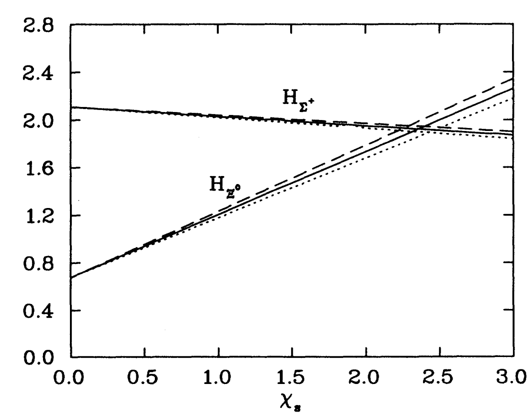

FIG. 7. Optimized sum-rule prediction for H_{Σ^+} and H_{Ξ^0} as functions of χ_s, with $t=-1$ and $\chi = \chi_m = 2.5\,\mathrm{GeV}^{-1}$. The three curves correspond to $\chi_{ms} = \chi_s$ (solid), $\frac{1}{2}\chi_s$ (dashed), and $\frac{3}{2}\chi_s$ (dotted). The other input parameters are the same as in Fig. 1.

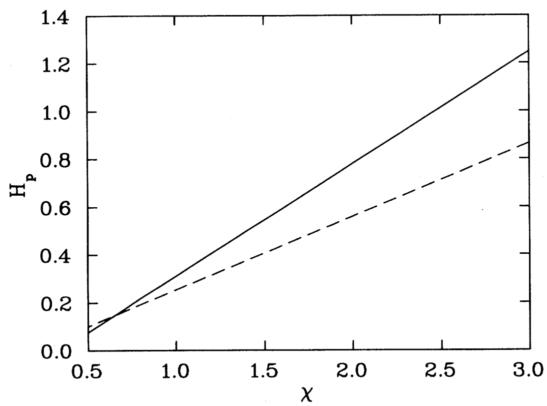

FIG. 8. Optimized sum-rule prediction for H_p as functions of χ, with $t = -1$. The solid curve is obtained by including all three terms on the RHS of Eq. (2.30), while the dashed curve is obtained by neglecting the third term on the RHS of Eq. (2.30).

curve is obtained by including (omitting) the third term on the RHS of Eq. (2.30). The difference between the two curves is large for moderate and large values of χ. This shows that the response of the continuum threshold can be sizable and should be included in the sum rules. Unfortunately, the response of the continuum thresholds has been omitted in all previous works on external field sum rules. This was first noticed by Ioffe [12].

IV. ESTIMATE OF BARYON ISOSPIN MASS SPLITTINGS

In this section we estimate the baryon isospin mass splittings using δm and the baryon matrix elements of isovector-scalar current calculated in the previous section.

The observed hadron isospin mass splittings arise from electromagnetic interaction and from the difference between up and down quark masses:

$$\delta m_h = (\delta m_h)_{\rm el} + (\delta m_h)_q \ , \tag{4.1}$$

where $(\delta m_h)_{\rm el}$ and $(\delta m_h)_q$ denote the contributions due to electromagnetic interaction and due to the up and down quark mass difference, respectively.[1] Following Ref. [11], one can treat δm as a small parameter and using the Hellman-Feynman theorem [18,19] to show that the octet baryon isospin mass splittings to first order in δm can be expressed as

$$M_n - M_p = (M_n - M_p)_{\rm el} + \delta m H_p \ , \tag{4.2}$$

$$M_{\Sigma^-} - M_{\Sigma^+} = (M_{\Sigma^-} - M_{\Sigma^+})_{\rm el} + \delta m H_{\Sigma^+} \ , \tag{4.3}$$

$$M_{\Xi^-} - M_{\Xi^0} = (M_{\Xi^-} - M_{\Xi^0})_{\rm el} + \delta m H_{\Xi^0} \ . \tag{4.4}$$

Note that $H_n = -H_p$, $H_{\Sigma^-} = -H_{\Sigma^+}$, and $H_{\Xi^-} = -H_{\Xi^0}$ to the lowest order in δm. Therefore, QCD sum rule predictions for H_p, H_{Σ^+}, and H_{Ξ^0}, along with the electromagnetic contributions [22]

$$(M_n - M_p)_{\rm el} = -0.76 \pm 0.30\,\text{MeV} \ , \tag{4.5}$$

$$(M_{\Sigma^-} - M_{\Sigma^+})_{\rm el} = 0.17 \pm 0.3\,\text{MeV} \ , \tag{4.6}$$

$$(M_{\Xi^-} - M_{\Xi^0})_{\rm el} = 0.86 \pm 0.30\,\text{MeV} \ , \tag{4.7}$$

will lead to an estimate of the baryon isospin mass splittings. Taking the experimental mass difference [33], one finds

$$(M_n - M_p)_q^{\rm expt} = 2.05 \pm 0.30\,\text{MeV} \ , \tag{4.8}$$

$$(M_{\Sigma^-} - M_{\Sigma^+})_q^{\rm expt} = 7.9 \pm 0.33\,\text{MeV} \ , \tag{4.9}$$

$$(M_{\Xi^-} - M_{\Xi^0})_q^{\rm expt} = 5.54 \pm 0.67\,\text{MeV} \ . \tag{4.10}$$

We have seen from last section that the uncertainties in our knowledge of the response of the quark condensates to the external field, χ and χ_s, leads to uncertainties in the sum-rule determination of the baryon matrix elements H_B. (There are also uncertainties in δm.) Therefore, our estimate here are only *qualitative.* For most of the values for t, χ, and χ_s considered here, the sum-rule analysis gives $0 < H_p < H_{\Xi^0} \le H_{\Sigma^+}$ (see Figs. 5, 6, and 7), which implies

$$0 < (M_n - M_p)_q < (M_{\Xi^-} - M_{\Xi^0})_q \le (M_{\Sigma^-} - M_{\Sigma^+})_q \ . \tag{4.11}$$

This qualitative feature is compatible with the experimental data. For the baryon interpolating fields with $t = -1$ and moderate χ and χ_s values ($1.6\,\text{GeV}^{-1} \le \chi \le 2.2\,\text{GeV}^{-1}$ and $1.3\,\text{GeV}^{-1} \le \chi_s \le 1.8\,\text{GeV}^{-1}$), we get

$$1.95\,\text{MeV} \le (M_n - M_p)_q \le 2.41\,\text{MeV} \ , \tag{4.12}$$

$$4.0\,\text{MeV} \le (M_{\Sigma^-} - M_{\Sigma^+})_q \le 6.3\,\text{MeV} \ , \tag{4.13}$$

$$4.5\,\text{MeV} \le (M_{\Xi^-} - M_{\Xi^0})_q \le 5.38\,\text{MeV} \ , \tag{4.14}$$

where we have used a median value $\delta m \simeq 3.3\,\text{MeV}$. These results are comparable to the experimental data, though the result in the Σ case is somewhat too small. Smaller and larger values of χ and χ_s lead to correspondingly smaller and larger values for the baryon isospin mass differences. As t increases (decreases), the results increase (decrease).

V. DISCUSSION

Our primary goal in the present paper has been to extract the baryon matrix element $H_B = \langle B|\overline{u}u - \overline{d}d|B\rangle/2M_B$ for octet baryons. We observe that the sum-

[1] This separation is renormalization scale dependent. However, this scale dependence is weak; it is thus meaningful to separate the contribution of quark mass difference from that due to electromagnetic interaction (see Ref. [11]).

rule predictions for H_B are quite sensitive to the response of quark condensates to the external isovector-scalar field, which is not well determined. This means that our conclusion about H_B can only be *qualitative* at this point. The most concrete conclusion we can draw from this work is that QCD sum rules predict positive values for H_p, H_{Σ^+}, and H_{Ξ^0} and $H_p < H_{\Xi^0} \leq H_{\Sigma^+}$. This qualitative feature is, for the most part, stable against variations of the response of the condensates to the external source and the choice of baryon interpolating fields.

We note that the inequality $H_p < H_{\Xi^0} \leq H_{\Sigma^+}$ indicates SU(3) symmetry violation in the baryon matrix elements of the isovector-scalar current. This arises mainly from the difference in the baryon interpolating fields used in the QCD sum rules and from the fact that the isovector-scalar current is not a SU(3) singlet. Clearly, it is a very interesting topic to check this inequality in other effective QCD models. At this stage, it is unclear whether the difference in the baryon interpolating fields is connected to the SU(3) symmetry breaking in the baryon wave functions.

In the present study, we derived and used a complete form for the phenomenological representation, which has also been given in Ref. [12]. This form includes the response of the continuum thresholds, which was ignored in Ref. [11]. We found that the neglect of the response of the continuum thresholds can have large effect on the extraction of the baryon matrix elements. This suggests that the contribution arising from the response of the continuum thresholds, neglected in previous works, should be accounted in the study of general external field sum rules (see Ref. [12] for estimates of the effects of this contribution on the extraction of various physical quantities).

The spectral parameters in the absence of the external source, M_B, λ_B^2, and s_0^i, appear in all external field sum rules. Unlike the mass, there are no experimental values for the coupling λ_B^2 and the thresholds s_0^i. One usually evaluates these parameters from the mass sum rules by fixing the mass at the experimental value. This means that the uncertainties associated with the vacuum spectral parameters will give rise to additional uncertainties in the determination of the baryon matrix elements of various current, in addition to the uncertainties in the external field sum rules themselves. This is a general drawback of the external field sum-rule approach. It is also worth pointing out that it is the product of λ_B^2 and the baryon matrix element appears in the external field sum rules [see Eqs. (2.29) and (2.36)]. So it is more suitable to determine the product of λ_B^2 and the baryon matrix element from the external field sum rules; one then needs a good knowledge of λ_B^2 in order to extract the baryon matrix element cleanly.

The sum-rule predictions are fairly sensitive to the choice of baryon interpolating fields. This sensitivity arises from *both* the dependence of the truncated OPE result *and* the dependence of the extracted parameters λ_B^2 and s_0^i on the choice of the baryon interpolating fields. We found that the latter has stronger dependence, and hence leads to larger contribution to the change of the predictions with t.

The nonelectromagnetic part of the baryon isospin mass difference is essentially given by the matrix element H_B multiplied by the light quark mass difference δm. Given the uncertainties in the determination of H_B mentioned above, our estimate of the isospin mass splittings for the octet baryons must be qualitative. It is found that the QCD sum-rule predictions yield $(M_n - M_p)_q < (M_{\Xi^-} - M_{\Xi^0})_q \leq (M_{\Sigma^-} - M_{\Sigma^+})_q$. This qualitative result is consistent with the experimental data and insensitive to the details of calculation. If we use a median value $\delta m = 3.3\,\mathrm{MeV}$ and moderate values for χ and χ_s, we obtain results comparable to the experimental values. However, since the response of various condensates to the external source and δm are not precisely known and the uncertainties from other sources cannot be accessed systematically, it is not wise to make a critical comparison with data or to attempt to extract χ [and hence γ through Eq. (3.3)] and χ_s by fitting the experimental data. Clearly, further study of the response of the quark condensates to external isovector-scalar field is important, along with more accurate determination of the vacuum spectral parameters. Effective QCD models may give some independent information on the response of the quark condensates while the lattice QCD may offer clean determination of the vacuum spectral parameters [15].

There have been several earlier papers that study the neutron-proton mass difference [25,30,31,34–36] and the baryon isospin mass splittings for other octet baryons [25,31], based on QCD sum-rule approach. In Ref. [25], the baryon mass differences were extracted directly from the baryon mass sum rules by including the quark mass difference and the isospin breaking in the quark condensates. The contributions of quark-gluon mixed condensates were ignored, and somewhat different values for the vacuum condensates and the strange quark mass were used. This can lead to large effects on the extraction of the isospin mass splittings. The procedure for analyzing the sum rules was also quite different from the one used in the present paper. In Ref. [34,30], the neutron-proton mass was extracted from the difference between the neutron and proton mass sum rules, but the continuum contributions were disregarded. In a later calculation [35], the authors of Ref. [34] have included the continuum contribution in the study of the density dependence of the neutron-proton mass difference in the medium. In these works, the contributions from the quark-gluon condensates and the change in the continuum thresholds were omitted. The study of neutron-proton mass difference in Ref. [36] was based on the mass sum rules directly. Apart from keeping the quark mass difference and the quark condensates difference, an attempt was made to incorporate the electromagnetic contribution also phenomenologically in the sum rules.

The analysis in Ref. [31] is more closely related to the present work. The goal of Ref. [31] was, however, to determine the parameters δm and γ by fitting all isospin mass splittings in the baryon octet. The sum rules were obtained for Ioffe's interpolating field by treating the quark mass and the isospin breaking in quark condensates as perturbations. On the phenomenological sides of the sum rules, all spectral parameters, mass, residue, and continuum thresholds, were allowed to change. Note that the sum rules derived by us in Sec. II can also be derived directly from the mass sum rules. Writing $m_u = \hat{m} - \delta m/2$, $m_d = \hat{m} + \delta m/2$ and assuming $\chi = -\gamma/\delta m$ [see Eq. (3.3)], one can differentiate the mass sum rules with respect to δm. For $t = -1$, one can then identify our sum rules Eqs. (2.29) and (2.30) and (2.33)–(2.36) with the sum rules given in Ref. [31]. This coincidence between the sum rules is not surprising, since the quark mass term in the QCD Lagrangian can also be regarded as a constant external scalar field. We observe, however, that the contributions from dimension eight condensates have not been included in Ref. [31]. We have seen in our analysis of the sum rules (see Figs. 6 and 7) that these contributions can be numerically significant. In addition, the authors of Ref. [31] directly used the Σ and Ξ mass sum rules from Ref. [16], where all the terms proportional to m_u or m_d were neglected. Consequently, some terms proportional to m_s were not taken into account in the Σ and Ξ cases and there was a factor two omitted in the contribution from four-quark condensates in the nucleon case.

We note that the authors of Ref. [31] took a very different procedure in analyzing the sum rules. They used both sum rules to eliminate $\Delta\lambda_B^2$ while we used only the more stable one. The continuum contribution in sum rules Eqs. (2.29), (2.33), and (2.35) is large. So these sum rules are likely to be dominated by the single pole terms and the predictions based on these sum rules may not be reliable. The size of the continuum contribution was not checked in Ref. [31]. Certain assumptions such as $\Delta s_0^i = 0$ were also used in some cases. We notice that the absence of continuum contribution in the external field sum rules does not necessarily imply $\Delta s_0^i = 0$. In fact, as long as there is continuum contribution in the mass sum rules, one must include Δs_0^i as an unknown quantity to be determined from the sum rules. Any assumption about Δs_0^i may bypass the information extracted for other quantities. The authors of Ref. [31] claimed that consistency of the two sum rules can be achieved for $\gamma = -(2\pm1)\times10^{-3}$, which is different from the values discussed in the present paper (see discussions in Sec. III). This discrepancy arises mainly from the difference in the procedures for analyzing the sum rules.

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APPENDIX

In this appendix, we give the OPE results for the invariant functions Π_1^1 and Π_1^q. The interpolating fields defined in Eqs. (2.2)–(2.4) are used in the calculation. We work to the leading order in the perturbation theory and to the first order in the strange quark mass m_s. The contributions proportional to up and down quark masses and to gluon condensates are neglected. Condensates up to dimension 8 are considered:

$$\begin{aligned}\text{proton}: \quad \Pi_1^1(q^2) = {} & \frac{c_1+6c_2}{128\pi^4}(q^2)^2\ln(-q^2) + \frac{c_1+6c_2}{16\pi^2}\chi\langle\bar{q}q\rangle_0 q^2\ln(-q^2) \\ & -\frac{3c_2}{16\pi^2}\chi_m\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\ln(-q^2) + \frac{c_1+3c_2-c_3}{6}\langle\bar{q}q\rangle_0^2\frac{1}{q^2}\,,\end{aligned} \tag{A1}$$

$$\begin{aligned}\Pi_1^q(q^2) = {} & \frac{4c_1-c_3}{32\pi^2}\langle\bar{q}q\rangle_0\ln(-q^2) + \frac{c_4+c_5-6c_2}{96\pi^2}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\frac{1}{q^2} \\ & +\frac{c_1}{3}\chi\langle\bar{q}q\rangle_0^2\frac{1}{q^2} + \frac{c_1+2c_2}{24}\chi\langle\bar{q}q\rangle_0\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\frac{1}{(q^2)^2} \\ & +\frac{c_1-2c_2}{24}\chi_m\langle\bar{q}q\rangle_0\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\frac{1}{(q^2)^2}\,;\end{aligned} \tag{A2}$$

$$\begin{aligned}\Sigma^+: \quad \Pi_1^1(q^2) = {} & \frac{3c_2}{64\pi^2}(q^2)^2\ln(-q^2) + \frac{3c_2}{8\pi^2}\chi\langle\bar{q}q\rangle_0 q^2\ln(-q^2) - \frac{c_1}{16\pi^2}\chi_s\langle\bar{s}s\rangle_0 q^2\ln(-q^2) \\ & -\frac{c_1-2c_3}{8\pi^2}m_s\langle\bar{q}q\rangle_0\ln(-q^2) + \frac{3c_2}{8\pi^2}m_s\langle\bar{s}s\rangle_0\ln(-q^2) \\ & -\frac{3c_2}{16\pi^2}\chi_m\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\ln(-q^2) + \frac{c_2}{32\pi^2}m_s\langle g_s\bar{s}\sigma\cdot\mathcal{G}s\rangle_0\frac{1}{q^2} \\ & +\frac{2c_1+3c_2-6c_3}{96\pi^2}m_s\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_0\frac{1}{q^2} + \frac{c_1-2c_3}{6}\langle\bar{q}q\rangle_0\langle\bar{s}s\rangle_0\frac{1}{q^2} \\ & +\frac{c_3}{3}\chi m_s\langle\bar{q}q\rangle_0^2\frac{1}{q^2} + \frac{c_2}{2}\chi m_s\langle\bar{q}q\rangle_0\langle\bar{s}s\rangle_0\frac{1}{q^2} + \frac{c_2}{2}\chi_s m_s\langle\bar{q}q\rangle_0\langle\bar{s}s\rangle_0\frac{1}{q^2}\,,\end{aligned} \tag{A3}$$

$$
\begin{aligned}
\Pi_1^q(q^2) = {} & \frac{3c_2}{32\pi^4} m_s q^2 \ln(-q^2) + \frac{2c_1 - c_3}{16\pi^2} \langle \overline{q}q\rangle_0 \ln(-q^2) - \frac{3c_2}{8\pi^2} \langle \overline{s}s\rangle_0 \ln(-q^2) \\
& + \frac{3c_2}{8\pi^2} \chi m_s \langle \overline{q}q\rangle_0 \ln(-q^2) + \frac{c_3}{32\pi^2} \chi_s m_s \langle \overline{s}s\rangle_0 \ln(-q^2) \\
& + \frac{c_4}{96\pi^2} \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{q^2} + \frac{5c_2}{32\pi^2} \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{q^2} \\
& - \frac{7c_2}{32\pi^2} \chi_m m_s \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{q^2} + \frac{c_5}{96\pi^2} \chi_{ms} m_s \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{q^2} \\
& + \frac{c_1}{3} \chi \langle \overline{q}q\rangle_0^2 \frac{1}{q^2} - c_2 \chi \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{q^2} - c_2 \chi_s \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{q^2} \\
& + \frac{c_2}{2} m_s \langle \overline{q}q\rangle_0^2 \frac{1}{(q^2)^2} + \frac{c_3 - 2c_1}{12} m_s \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{(q^2)^2} \\
& + \frac{c_1}{24} \chi \langle \overline{q}q\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} - \frac{5c_2}{24} \chi \langle \overline{q}q\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} \\
& - \frac{7c_2}{24} \chi_s \langle \overline{s}s\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} + \frac{c_1}{24} \chi_m \langle \overline{q}q\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} \\
& - \frac{5c_2}{24} \chi_{ms} \langle \overline{q}q\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} - \frac{7c_2}{24} \chi_m \langle \overline{s}s\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} \; ;
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
\Xi^0 : \quad \Pi_1^1(q^2) = {} & -\frac{c_1}{128\pi^4} (q^2)^2 \ln(-q^2) - \frac{c_1}{16\pi^2} \chi \langle \overline{q}q\rangle_0 q^2 \ln(-q^2) + \frac{3c_2}{8\pi^2} \chi_s \langle \overline{s}s\rangle_0 q^2 \ln(-q^2) \\
& + \frac{3c_2}{8\pi^2} m_s \langle \overline{q}q\rangle_0 \ln(-q^2) - \frac{c_1 - 2c_3}{8\pi^2} m_s \langle \overline{s}s\rangle_0 \ln(-q^2) \\
& - \frac{3c_2}{16\pi^2} \chi_{ms} \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \ln(-q^2) + \frac{c_2}{32\pi^2} m_s \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{q^2} \\
& + \frac{2c_1 + 3c_2 - 6c_3}{96\pi^2} m_s \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{q^2} - \frac{c_3}{6} \langle \overline{s}s\rangle_0^2 \frac{1}{q^2} - \frac{c_2}{2} \langle \overline{s}s\rangle_0 \langle \overline{q}q\rangle_0 \frac{1}{q^2} \\
& - \frac{c_1 - 2c_3}{6} \chi m_s \langle \overline{s}s\rangle_0 \langle \overline{q}q\rangle_0 \frac{1}{q^2} - \frac{c_1 - 2c_3}{6} \chi_s m_s \langle \overline{s}s\rangle_0 \langle \overline{q}q\rangle_0 \frac{1}{q^2} \; ,
\end{aligned}
\tag{A5}
$$

$$
\begin{aligned}
\Pi_1^q(q^2) = {} & \frac{3c_2}{32\pi^4} m_s q^2 \ln(-q^2) - \frac{c_3}{32\pi^2} \langle \overline{q}q\rangle_0 \ln(-q^2) - \frac{3c_2}{8\pi^2} \langle \overline{s}s\rangle_0 \ln(-q^2) \\
& + \frac{3c_2}{8\pi^2} \chi m_s \langle \overline{q}q\rangle_0 \ln(-q^2) - \frac{2c_1 - c_3}{16\pi^2} \chi_s m_s \langle \overline{s}s\rangle_0 \ln(-q^2) \\
& - \frac{c_5}{96\pi^2} \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{q^2} + \frac{7c_2}{32\pi^2} \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{q^2} \\
& - \frac{5c_2}{32\pi^2} \chi_m m_s \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{q^2} - \frac{c_4}{96\pi^2} \chi_{ms} m_s \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{q^2} \\
& - c_2 \chi \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{q^2} - c_2 \chi_s \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{q^2} + \frac{c_1}{3} \chi_s \langle \overline{s}s\rangle_0^2 \frac{1}{q^2} \\
& + \frac{c_2}{2} m_s \langle \overline{s}s\rangle_0^2 \frac{1}{(q^2)^2} + \frac{c_3 - 2c_1}{12} m_s \langle \overline{q}q\rangle_0 \langle \overline{s}s\rangle_0 \frac{1}{(q^2)^2} \\
& - \frac{7c_2}{24} \chi \langle \overline{q}q\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} + \frac{c_1}{24} \chi_s \langle \overline{s}s\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} \\
& - \frac{5c_2}{24} \chi_s \langle \overline{s}s\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} - \frac{5c_2}{24} \chi_m \langle \overline{s}s\rangle_0 \langle g_s \overline{q}\sigma \cdot \mathcal{G} q\rangle_0 \frac{1}{(q^2)^2} \\
& + \frac{c_1}{24} \chi_{ms} \langle \overline{s}s\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} - \frac{7c_2}{24} \chi_{ms} \langle \overline{q}q\rangle_0 \langle g_s \overline{s}\sigma \cdot \mathcal{G} s\rangle_0 \frac{1}{(q^2)^2} \; .
\end{aligned}
\tag{A6}
$$

Here c_1, c_2, c_3, c_4, and c_5 have been defined in Eq. (2.32), and we have ignored the isospin breaking in the vacuum condensates (i.e., $\langle \overline{u}\hat{O}u\rangle_0 \simeq \langle \overline{d}\hat{O}d\rangle_0 = \langle \overline{q}\hat{O}q\rangle_0$). All polynomials in q^2, which vanish under the Borel transformation, have been omitted in Eqs. (A1)–(A6).

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