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Tensor charge of the nucleon

Hanxin He* and Xiangdong Ji
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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We seek to understand the physical significance of the nucleon's tensor charge and make estimates of its size in phenomenological models and the QCD sum rule.

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The nucleon's *tensor charge* $\delta\psi$ ($\psi = u, d, s, \ldots$) is defined as the forward matrix element of the tensor current $T^{\mu\nu} = \bar{\psi}\sigma^{\mu\nu}\psi$ in the nucleon state:

$$\langle PS|\bar{\psi}\sigma^{\mu\nu}\psi|PS\rangle = \delta\psi\, \bar{U}(PS)\sigma^{\mu\nu}U(PS)\ , \qquad (1)$$

where P is the nucleon's four-momentum, S is a polarization vector, and $U(PS)$ is a Dirac spinor. Because of the γ-matrix identity $\sigma^{\mu\nu}\gamma_5 = (i/2)\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$, one can also define the tensor charge in terms of the operator $\bar{\psi}\sigma^{\mu\nu}i\gamma_5\psi$, and then the right-hand side of Eq. (1) becomes $2\delta\psi(P^\mu S^\nu - P^\nu S^\mu)$. Throughout the paper, we adopt the notations of Itzykson and Zuber [1].

Like other nucleon charges (*baryon charge* defined by the matrix element of $\bar{\psi}\gamma^\mu\psi$, *axial charge* by $\bar{\psi}\gamma^\mu\gamma_5\psi$, and *scalar charge* by $\bar{\psi}\psi$), the tensor charge is one of the fundamental parameters that characterize properties of the nucleon. So far, however, little is known about its value and its implication on the structure of the nucleon. In this paper we seek to understand the physical significance of the tensor charge and make estimates in the MIT bag model and the QCD sum rule.

The main reason for the lack of studies about the tensor charge is that it is difficult to access experimentally. There are no fundamental probes that couple directly to the tensor current. (Before the $V - A$ weak interaction was firmly established, physicists had entertained the possibility of weak scalar and tensor couplings.) However, the situation has changed fundamentally when the factorization theorems in high-energy scattering are shown to be valid on quite general ground [2]. The theorems provide a firm basis for the general parton-model result that the perturbative scattering in hard processes effectively provides a versatile probe into the structure of hadrons. One recent example of such an application is the measurement of the nucleon's axial charge from polarized lepton-nucleon scattering [3].

It was discussed by Ralston and Soper [4] that the transversely polarized Drell-Yan scattering can probe a new quark distribution of the nucleon, the transversity distribution $h_1(x)$. What is the $h_1(x)$ distribution? Consider a nucleon traveling in the z direction with its polarization in the x direction. The polarization of quarks and antiquarks in the nucleon can be classified in terms of the transversity eigenstates $|\uparrow\downarrow\rangle = (|+\rangle \pm |-\rangle)/\sqrt{2}$, where $|\pm\rangle$ are the usual helicity eigenstates. If one uses $N_\uparrow(x)$ $[N_\downarrow(x)]$ to represent the density of quarks with polarization $|\uparrow\rangle$ $[|\downarrow\rangle]$, then

$$h_1(x) = N_\uparrow(x) - N_\downarrow(x)\ , \qquad (2)$$

and likewise for antiquarks. The $h_1(x)$, together with the unpolarized quark distribution $q(x)$ and the quark helicity distribution $g_1(x)$, forms a complete set for describing the quark state inside the nucleon in the leading-order hard processes. It was demonstrated by Jaffe and Ji [5] that the first moment of $h_1(x)$ is related to the nucleon's tensor charge:

$$\int_{-1}^{1} h_1(x)dx = \int_0^1 [h_1(x) - \bar{h}_1(x)]dx = \delta\psi\ , \qquad (3)$$

where $h_1(x)$ at negative x is the negative of the antiquark distribution $\bar{h}_1(-x)$. Given no fundamental tensor coupling, the integral may be the best hope to gain knowledge about the tensor charge. In Ref. [6], other possible experiments of measuring the transversity distribution are examined. The BNL Relativistic Heavy Ion Collider (RHIC) Spin Collaboration and the HERMES Collaboration have proposed a first measurement of $h_1(x)$ in the future [7].

According to Eq. (24), the nucleon's tensor charge measures the *net number of transversely polarized valence quarks* (quarks minus antiquarks) in a transversely polarized nucleon. One would argue that this number should be the same as the net number of longitudinally polarized valence quarks in a longitudinally polarized nucleons (which is related to the axial charge), since, after all, a polarization of the nucleon in its rest frame can be said to be longitudinal, or transverse, or a combination of both. This argument would be correct if the nucleon were made of free quarks. During high-energy scattering, quarks in the nucleon do appear to be free. However, rotational invariance now becomes nontrivial because high-energy processes select a special direction. In fact, in the so-called infinite momentum frame, where the parton model was originally formulated, the rotational operators explicitly involve interactions [8]. Thus, the difference between the tensor and axial charges has a dynamical origin.

*Permanent address: China Institute of Atomic Energy, Beijing, China.

Unlike the baryon or axial charges, the tensor charge is renormalization-scale dependent. A simple calculation of the anomalous dimension for $\bar{\psi}\sigma^{\mu\nu}\psi$ yields

$$\gamma = 2C_F \frac{g^2}{16\pi^2} + \cdots , \quad (4)$$

where $C_F = 4/3$, g is the strong-coupling constant, and the ellipsis represents higher-order terms in the coupling. Thus, $\delta\psi$ scales according to

$$\delta\psi(\mu^2) = \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right)^{\frac{4}{33-2n_f}} \delta\psi(\mu_0^2) , \quad (5)$$

where n_f is the number of flavors. As $\mu^2 \to \infty$, $\delta\psi$ vanishes. This contrasts with the nucleon's scalar charge $\langle P|\bar{\psi}\psi|P\rangle$, which scales as $(\alpha(\mu^2)/\alpha(\mu_0^2))^{-\frac{12}{33-2n_f}}$, and blows up as $\mu^2 \to \infty$.

Now, let us consider the size of the tensor charge in nonrelativistic quark models [5]. In the limit of $m_q \to \infty$, the transverse-spin operator commutes with a free-quark Hamiltonian, and so the transverse polarized quarks are in the transverse-spin eigenstates. Then rotational invariance implies

$$\delta\psi = \Delta\psi , \quad (6)$$

where $\Delta\psi$ is a conventional notation for the axial charge. Or

$$\delta u = \frac{4}{3} , \quad \delta d = -\frac{1}{3} , \quad \delta s = 0 . \quad (7)$$

This result can also be obtained from the fact that the tensor operator $\bar{\psi}\sigma^{0i}i\gamma_5\psi$ differs from the axial vector current $\bar{\psi}\gamma^i\gamma_5\psi$ by a γ_0 factor, which reduces to 1 in the nonrelativistic limit.

In the MIT bag model, the tensor charge can be expressed in terms of the upper and lower components (f and g) of the quark wave function [5]:

$$\delta u = \frac{4}{3}\int\left(f^2 + \frac{1}{3}g^2\right) , \quad \delta d = -\frac{1}{3}\int\left(f^2 + \frac{1}{3}g^2\right) . \quad (8)$$

This differs from the expressions for the nucleon's axial charge by a sign in front of g^2, because of the same γ_0 factor mentioned above. Instead of trying to find the best bag parameters, we demand $\Delta u - \Delta d = 1.257$ and use the normalization $\int(f^2 + g^2) = 1$; then the tensor charge is uniquely fixed:

$$\delta u = 1.17 , \quad \delta d = -0.29 . \quad (9)$$

These numbers are closer to the nonrelativistic quark model result than are the nucleon's axial charge in the bag. In other words, the nonrelativistic quark model prediction for the tensor charge appears to be less susceptible to relativistic effects than for the axial charge.

Of course, these estimates in phenomenological models are very crude and provide only guidance at best. In particular, the matching between QCD quarks and constituent quarks used in models is a subtle and unsolved problem. This is reflected by the fact that model calculations have no explicit reference to any scale, although one would generally believe that these models live in a scale somewhere in between $\Lambda_{\rm QCD}$ and the nucleon mass. More reliable estimates can be made with QCD-based approaches in which one deals with QCD quarks directly. One approach is the lattice QCD. The recent progress in calculating axial and scalar charges on a lattice shows that the lattice QCD becomes increasingly competitive with other methods in computing hadron observables [9]. Another approach is the QCD sum rule. In the past 15 years, this method has produced a large number of interesting results, which are largely consistent with hadron phenomenology [10]. In the remainder of this paper, we present a QCD sum-rule estimate of the tensor charge.

There exist in the literature several equivalent formulations of the QCD sum-rule technique for calculating forward hadron matrix elements. Following the approach initiated by Balitsky and Yang [11], we consider the three-point correlation function

$$W^{\mu\nu} = i^2 \int d^4x d^4y e^{ip\cdot x} \langle 0|T[\bar{\psi}\sigma^{\mu\nu}\psi(y)\eta(x)\bar{\eta}(0)]|0\rangle , \quad (10)$$

where η is the nucleon interpolating field, $\eta = \epsilon^{abc}u_a^T C\gamma_\mu u_b \gamma_5\gamma^\mu d_c$, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix. We calculate $W^{\mu\nu}$ at large Euclidean $-p^2$ using the operator-product-expansion technique on the one hand, and using resonance saturation on the other. The tensor charge is extracted by matching the two results at a certain kinematic domain where both methods are supposed to be valid.

In resonance saturation, $W^{\mu\nu}$ contains the nucleon double pole, single pole, and other resonance contributions:

$$W^{\mu\nu} = \delta\psi \left(\not{p}\,\sigma^{\mu\nu}\not{p} + m_N^2\sigma^{\mu\nu} + m_N\{\not{p},\sigma^{\mu\nu}\}\right)\frac{\lambda^2}{(p^2 - m_N^2)^2} + \cdots = W_1\not{p}\,\sigma^{\mu\nu}\not{p} + W_2\sigma^{\mu\nu} + W_3\{\not{p},\sigma^{\mu\nu}\} + \cdots . \quad (11)$$

Here we have shown only the double pole term, in which λ^2 is the coupling of the nucleon with the interpolating field, $\langle 0|\eta(0)|p\rangle = \lambda U(p)$. Other terms are neglected because they either vanish or are suppressed after being multiplied by $(m_N^2 - p^2)$ and the Borel transformation. There are three different Dirac structures emerging from the double-pole term: chiral-odd ones with coefficients W_1 and W_2 and chiral-even one with coefficient W_3, each of which can be used to construct a sum rule and extract $\delta\psi$. In principle, one has to obtain the same result from each of them before one trusts the final answer. Otherwise, one can obtain any desired result by making different combinations of the sum rules. In practice, however, some sum rules are better approximated by leading power corrections than others. Thus, choosing the right sum rule to extract the physical observable is a very delicate issue.

Depending upon momentum flow in the Feynman diagrams, the operator-product expansion for $W^{\mu\nu}$ has three distinct classes of contributions: perturbative, local, and bilocal power corrections. The perturbative contribution comes from large momentum flow through *all* internal lines of diagrams. The local-power contribution refers to diagrams in which some particle lines are condensed into a vacuum and the momentum flowing through the composite operator $\bar\psi\sigma^{\mu\nu}\psi$ is large. The bilocal power contribution is similar to the local one except the momentum flowing through the composite operator is infrared. To evaluate the bilocal contribution, one needs two-point correlation functions at zero momentum:

$$\int d^4x \langle 0|T\left[O_n(0)\bar\psi\sigma^{\mu\nu}\psi(x)\right]|0\rangle\,, \tag{12}$$

where O_n are local operators from the operator-product expansion of $T\eta(x)\bar\eta(0)=\sum_n C_n(x^2)O_n(0)$. These two-point functions can be evaluated either in terms of the QCD sum rule, or, in some cases, with QCD equations of motion. The bilocal contribution is similar in spirit to the contribution from vacuum susceptibility introduced by Ioffe [12].

Now we present the sum-rule results of the tensor charges for the up and down quarks separately. For the u quark, the leading large-momentum contribution to W_1 and W_2 comes from the power corrections with a dimension-six condensate:

$$W_1 = \frac{2}{p^4}\langle\bar uu\rangle\langle\bar dd\rangle+\cdots\,,$$
$$W_2 = -\frac{2}{3p^2}\langle\bar uu\rangle\langle\bar dd\rangle+\cdots, \tag{13}$$

where W_1 receives contributions from both local and bilocal power terms, whereas W_2 receives a contribution from a bilocal power term alone. Following the standard procedure of multiplying by $m_N^2-p^2$, making a Borel transformation, and matching it with the corresponding term from Eq. (11), we find, for the W_1 sum rule,

$$\delta u = \frac{2}{\lambda^2}\langle\bar uu\rangle^2 e^{m_N^2/M^2}\,. \tag{14}$$

As the Borel mass M^2 changes from m_N^2 to $2m_N^2$, δu changes by about 50%, so the sum rule is reasonably stable. Taking $\langle\bar uu\rangle=-(240\text{ MeV})^3$, $M^2=m_N^2$, $\lambda^2=7.0\times10^{-4}\text{ GeV}^6$, we get

$$\delta u = 1.0\text{-}1.5\,. \tag{15}$$

On the other hand, the result from the W_2 sum rule is smaller by a factor of 3. Without calculating higher-order terms, it is difficult to determine which one is more reliable. However, experience with other sum rules indicates that the result from W_1 with a nonvanishing local contribution is more stable against higher-order corrections.

The contribution to the chiral-even W_3 comes from the dimension-three and five power corrections:

$$W_3 = \frac{1}{2\pi^2}\ln(-p^2)\langle\bar uu\rangle + \frac{1}{24\pi^2p^2}\ln\left(\frac{-p^2}{\mu^2}\right)\langle\bar u gG\cdot\sigma u\rangle+\cdots\,, \tag{16}$$

where μ^2 is an infrared cutoff that can be taken to be $\Lambda_{\rm QCD}^2$. After the Borel transformation, we get, at $M^2=m_N^2$,

$$\delta u = -\frac{m_N s_0}{2\pi^2\lambda^2}e^{1-s_0/m_N^2}\langle\bar uu\rangle - \frac{em_0^2m_N}{24\pi^2\lambda^2}\left[\ln\left(\frac{m_N^2}{\mu^2}\right)-1\right]\langle\bar uu\rangle\,. \tag{17}$$

Taking $s_0=(1.5\text{ GeV})^2$ and $m_0^2=0.8\text{ GeV}^2$, we get

$$\delta u = 0.94\,. \tag{18}$$

Combining the above results, we conclude that the leading-order sum-rule calculation gives

$$\delta u = 1.0\pm0.5 \tag{19}$$

at the scale of $\mu^2=m_N^2$.

Next, we consider the d-quark tensor charge. Due to its chiral-even property, W_3 receives local power corrections only from odd-dimensional condensates. A simple consideration shows that such contributions start with the dimension-nine condensate, $\langle\bar\psi\psi\rangle^3$. This suggests that the d-quark tensor charge is quite small. This suspicion is confirmed by the consideration of the other two chiral-odd sum rules.

For W_2, the leading contribution comes from a perturbative term, followed by a power correction associated with the dimension-four condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$. Neglecting the latter, we have

$$W_2 = \frac{1}{32\pi^4}p^4\ln -p^2+\cdots, \tag{20}$$

which yields

$$\delta d = 0.3\,, \tag{21}$$

at $M^2=m_N^2$, a number indeed quite small. The leading contribution to W_1 comes from a bilocal correlator $\Pi(0)$,

$$W_1 = -\frac{1}{144\pi^2}\ln(-p^2)\Pi(0)+\cdots, \tag{22}$$

where $\Pi(0)$ is

$$\Pi(0) = i\int d^4x\langle 0|T[\bar d\sigma^{\alpha\beta}d(0)\bar d\sigma_{\alpha\beta}d(x)]|0\rangle\,. \tag{23}$$

Using a dispersion relation, we write

$$\Pi(0) = \sum_n \frac{1}{m_n^2} \langle 0|\bar{d}\sigma^{\alpha\beta} d(0)|n\rangle \langle n|\bar{d}\sigma_{\alpha\beta} d(0)|0\rangle \ . \tag{24}$$

Since $\bar{d}\sigma^{\alpha\beta}d(0)$ belongs to the (1,0)+(0,1) representations of the Lorentz group and is charge conjugation odd, the states with quantum numbers 1^{+-} and 1^{--} contribute to the sum in Eq. (24). Assuming the $\rho(1^{--})$ and $B(1^{+-})$ meson dominance, we find

$$\Pi(0) = 3(f_B^2 - f_\rho^2) \tag{25}$$

with

$$\langle 0|\bar{d}\sigma^{\mu\nu} d|\rho\rangle = \frac{f_\rho}{\sqrt{2}}(P^\mu \epsilon^\nu - P^\nu \epsilon^\mu) \ ,$$

$$\langle 0|\bar{d}\sigma^{\mu\nu} d|B\rangle = \frac{f_B}{\sqrt{2}} i\epsilon^{\mu\nu\rho\sigma} P_\rho \epsilon_\sigma \ . \tag{26}$$

Estimating the coupling constants f_ρ and f_B again in the QCD sum rule using the information in [13], we find

$$\Pi(0) \sim (0.15 \ \text{GeV})^2 \ . \tag{27}$$

The small $\Pi(0)$ result comes from the cancellation of the two resonances, and thus the theoretical error on the estimate is large. The W_1 sum rule produces

$$\delta d = \frac{\Pi(0)}{144\pi^2\lambda^2} M^2 (m_N^2 - M^2) e^{m_N^2/M^2} \ . \tag{28}$$

Depending upon of a choice of the Borel parameter, δd is in the range of 0.0–0.1, Given uncertainties with different sum rules, we conclude that

$$\delta d = 0.0 \pm 0.5 \tag{29}$$

at $\mu^2 = m_N^2$. This is consistent with a recent QCD sum-rule calculation for the transversity distribution $h_1(x)$ [14].

To recapitulate, the leading-order QCD sum rule suggests $\delta u = 1.0 \pm 0.5$ and $\delta d = 0.0 \pm 0.5$ at the scale of about 1 GeV^2. A recent SU(3)-symmetric, leading-order large-N_c analysis [15] shows that $\delta u + \delta d$ is of the order of $1/N_c$, relative to $\delta u - \delta d$. This result on the flavor structure also applies to the axial charge, for which an analysis of a recent measurement [3] yields $\Delta u = 0.78$ and $\Delta d = -0.46$, a favorable comparison with the large N_c. If the true value of δd is indeed rather small, as the QCD sum rule indicates, the large-N_c analysis has, perhaps, little relevance for the tensor charge.

In summary, we discussed in this paper various aspects of the nucleon's tensor charge. We focused on its numerical value in the MIT bag model and the QCD sum rule. With various caveats, both results seem consistent. Admittedly, the QCD sum-rule calculation is done only at the leading order, and one must show that the results are stable against higher-order power corrections and that all sum rules for the same quantity yield the same answer. Nonetheless, we believe our result is qualitatively reliable. Clearly, a lattice QCD calculation or a direct experimental measurement of the tensor charge will produce a more definitive determination of this interesting observable.

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