

QCD phase transition at finite temperature in the dual Ginzburg-Landau theory

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We study the pure-gauge QCD phase transition at finite temperatures in the dual Ginzburg-Landau theory, an effective theory of QCD based on the dual Higgs mechanism. We formulate the effective potential at various temperatures by introducing the quadratic source term, which is a new useful method to obtain the effective potential in the negative-curvature region. Thermal effects reduce the QCD-monopole condensate and bring a first-order deconfinement phase transition. We find a large reduction of the self-interaction among QCD monopoles and the glueball masses near the critical temperature by considering the temperature dependence of the self-interaction. We also calculate the string tension at finite temperatures.

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I. INTRODUCTION

It is believed that hadron physics is governed by quantum chromodynamics (QCD). Although high-energy phenomena such as deep inelastic scattering are described by perturbative QCD owing to asymptotic freedom [1], the theory becomes highly nonperturbative at low energy. Lattice QCD theory [2] was then developed for low-energy phenomena and demonstrated again the goodness of QCD. This gigantic numerical simulation method, however, does not tell us yet how low-energy phenomena such as quark confinement and chiral-symmetry breaking take place. We need therefore some effective theory, which incorporates the essence of the low-energy QCD physics and at the same time reproduces the observables with a few parameters in the effective theory, such as the Ginzburg-Landau theory in the superconductivity.

In recent years, Suzuki and co-workers [4] proposed the dual Ginzburg-Landau theory (DGL) as an attractive effective theory of nonperturbative QCD. The DGL theory incorporates the QCD monopole fields as essential ingredients for confinement of colored particles (quarks and gluons). The QCD monopole field has its clear origin through Abelian gauge fixing in the non-Abelian gauge theory in the manner of 't Hooft [5]. In this theory, the QCD vacuum is characterized by QCD monopole condensation, which provides a mass to the dual gauge field through the dual Higgs mechanism and hence the color electric field cannot freely develop in the QCD vacuum. Therefore, the color electric field originated from one colored object should be confined in a small vortexlike

tube to end at the other colored object [4,6]. This corresponds precisely to the Meissner effect of superconductivity. But here the role of the magnetic and the electric fields is reversed and this phenomenon is called as the dual Meissner effect. Such a picture for color confinement has been extensively investigated by recent studies [7,8] based on lattice gauge theory, and much evidence of QCD-monopole condensation in the nonperturbative QCD vacuum has been reported [7].

Suganuma, Sasaki, and Toki (SST) studied the DGL theory in further detail [6]. They formulated the confinement potential of heavy quarks in a natural way and obtained the linear potential in the same form as the vortexlike solution in the superconductivity. SST then showed QCD monopole condensation also induced the dynamical chiral-symmetry breaking ($D\chi SB$) [6,9,10]. Thus, the DGL theory describes both the confinement and $D\chi SB$ of QCD in the nonperturbative region. These two nonperturbative features, the confinement and $D\chi SB$, would be changed in the high-temperature system, which is expected to be realized as the quark-gluon plasma (QGP) in ultrarelativistic heavy-ion collisions or in the early Universe [12]. Nowadays, finite-temperature QCD including QGP physics is one of the most interesting subjects in intermediate-energy physics [12].

In this paper, we would like to develop the thermodynamics of the DGL theory [13] and study the change of the properties in the QCD vacuum with temperatures especially in terms of the deconfinement phase transition. To this end, we concentrate on the pure-gauge QCD case, where glueballs appear as physical excitation. Although such a pure-gauge system is different from the real world, it is regarded as a prototype of the real QCD and is well studied by using the lattice QCD simulation [2]. It is worth mentioning that our framework based on the DGL theory can be extended to include the dynamical quarks straightforwardly [4,6] keeping the chiral symmetry of the system, which is explicitly broken in the color-dielectric model [11] or in the lattice QCD with the Wilson fermion [2].

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II. EFFECTIVE POTENTIAL FORMALISM AT FINITE TEMPERATURE

The DGL Lagrangian [4,6,13] relevant for the QCD vacuum is written by the dual gauge field $\vec{B}_\mu = (B_\mu^3, B_\mu^8)$ and the QCD-monopole field χ_α ($\alpha = 1, 2, 3$):

$$\mathcal{L}_{\text{DGL}} = -\frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \sum_{\alpha=1}^3 [|(i\partial_\mu - g\vec{\epsilon}_\alpha \cdot \vec{B}_\mu)\chi_\alpha|^2 - \lambda(|\chi_\alpha|^2 - v^2)^2]. \quad (1)$$

Here, $\vec{\epsilon}_\alpha$ denotes the magnetic charge of the QCD-monopole field χ_α : $\vec{\epsilon}_1 = (1, 0)$, $\vec{\epsilon}_2 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, and $\vec{\epsilon}_3 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$. The dual gauge coupling constant g satisfies the Dirac condition, $eg = 4\pi$, with e being the gauge coupling constant. The strength λ for the self-interaction of the QCD-monopole field and the vacuum expectation value v are the parameters of the DGL theory. In principle, the values of these parameters can be extracted from the lattice QCD data, but it is practically difficult at present. Hence, these parameters are determined by fitting to various low-energy observables. The DGL Lagrangian (1) is obtained by integrating over \vec{A}_μ in the original DGL partition functional in the pure gauge case [4,6].

We investigate the effective potential in the path integral formalism. The partition functional is written as

$$Z[J] = \int \mathcal{D}\chi_\alpha \mathcal{D}\vec{B}_\mu \exp \left[i \int d^4x \left(\mathcal{L}_{\text{DGL}} - J \sum_{\alpha=1}^3 |\chi_\alpha|^2 \right) \right], \quad (2)$$

where we take the quadratic source term [14] instead of the standard linear source term [1,12]. As is well known

in the ϕ^4 theory [1,12], the use of the linear source term leads to an imaginary mass of the scalar field χ_α in the negative-curvature region of the classical potential, and therefore the effective action cannot be obtained there due to the appearance of ‘‘tachyons.’’ In this respect, there is an extremely advanced point in the use of the quadratic source term [14], because the mass of the scalar field χ_α is always real even in the negative-curvature region of the classical potential owing to the contribution of the source J to the scalar mass. [See Eq. (6).] Then, one obtains the effective action for the whole region of the order parameter without any difficulty of the imaginary-mass problem. Moreover, the effective action with the quadratic source can be formulated keeping the symmetry of the classical potential. Since this method with the quadratic source term is quite general, it is convenient to formulate the nonconvex effective potential in the ϕ^4 theory, the linear σ model, or the Higgs sector in the unified theory [1].

The vacuum expectation value of χ_α ($\alpha=1,2,3$) is the same value $\bar{\chi}$ due to the Weyl symmetry [4], and therefore we separate the QCD-monopole field χ_α into its mean field $\bar{\chi}$ and its fluctuation $\tilde{\chi}_\alpha$ as

$$\chi_\alpha = (\bar{\chi} + \tilde{\chi}_\alpha) \exp(i\xi_\alpha). \quad (3)$$

Here, the phase variables ξ_α have a constraint, $\sum_{\alpha=1}^3 \xi_\alpha = 0$, where two independent degrees of freedom remain corresponding to the dual gauge symmetry $[U(1)_3 \times U(1)_8]_m$ [4,6]. When QCD monopoles condense, the phase variables ξ_α turn into the longitudinal degrees of the dual gauge field \vec{B}_μ , which is the dual Higgs mechanism.

Since we are interested in the translational-invariant system as the QCD vacuum, we consider the x -independent constant source J , which leads to a homogeneous QCD-monopole condensate. In the unitary gauge, the Lagrangian with the source term is rewritten as

$$\begin{aligned} \mathcal{L}_{\text{DGL}} - J \sum_{\alpha=1}^3 |\chi_\alpha|^2 &= \mathcal{L}_{\text{cl}}(\bar{\chi}) - 3J\bar{\chi}^2 - 2\bar{\chi}[2\lambda(\bar{\chi}^2 - v^2) + J] \sum_{\alpha=1}^3 \tilde{\chi}_\alpha \\ &\quad - \frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \frac{1}{2}m_B^2 \vec{B}_\mu^2 + \sum_{\alpha=1}^3 [(\partial_\mu \tilde{\chi}_\alpha)^2 - m_\chi^2 \tilde{\chi}_\alpha^2] \\ &\quad + \sum_{\alpha=1}^3 \{g^2(\vec{\epsilon}_\alpha \cdot \vec{B}_\mu)^2(\tilde{\chi}_\alpha^2 + 2\bar{\chi}\tilde{\chi}_\alpha) - \lambda(4\bar{\chi}\tilde{\chi}_\alpha^3 + \tilde{\chi}_\alpha^4)\}, \end{aligned} \quad (4)$$

where $\mathcal{L}_{\text{cl}}(\bar{\chi})$ is the classical part,

$$\mathcal{L}_{\text{cl}}(\bar{\chi}) = -3\lambda(\bar{\chi}^2 - v^2)^2. \quad (5)$$

Here the masses of $\tilde{\chi}_\alpha$ and \vec{B}_μ are given by

$$m_\chi^2 = 2\lambda(3\bar{\chi}^2 - v^2) + J = 4\lambda\bar{\chi}^2, \quad m_B^2 = 3g^2\bar{\chi}^2, \quad (6)$$

where we have used the relation between the mean field $\bar{\chi}$ and the source J :

$$J = -2\lambda(\bar{\chi}^2 - v^2). \quad (7)$$

This relation is obtained by the condition that the linear term of $\bar{\chi}_\alpha$ vanishes. It is remarkable that the scalar mass m_χ is always real owing to the source J even in the negative-curvature region of the classical potential, $\bar{\chi} < v/\sqrt{3}$.

Integrating over \vec{B}_μ and $\bar{\chi}_\alpha$ by neglecting the higher-order terms of the fluctuations, we obtain the partition functional

$$Z[J] = \exp\left(i \int d^4x \{\mathcal{L}_{\text{cl}}(\bar{\chi}) - 3J\bar{\chi}^2\}\right) [\text{Det}(iD_B^{-1})]^{-1} [\text{Det}(iD_\chi^{-1})]^{-3/2}, \quad (8)$$

where the exponents, -1 and $-3/2$, originate from the numbers of the internal degrees of freedom. Here, D_B and D_χ are the propagators of \vec{B}_μ and $\bar{\chi}_\alpha$ in the QCD-monopole condensed vacuum:

$$D_B = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_B^2}\right) \frac{i}{k^2 - m_B^2 + i\epsilon}, \quad D_\chi = \frac{-i}{k^2 - m_\chi^2 + i\epsilon} \quad (9)$$

in the momentum representation. Hence, the effective action is given by the Legendre transformation [1]

$$\Gamma(\bar{\chi}) = -i \ln Z[J] + \int d^4x 3J\bar{\chi}^2 = \int d^4x \mathcal{L}_{\text{cl}}(\bar{\chi}) + i \ln \text{Det}(iD_B^{-1}) + \frac{3}{2} i \ln \text{Det}(iD_\chi^{-1}). \quad (10)$$

The functional determinants are easily calculable in the momentum space, and we obtain the formal expression of the effective potential [1]:

$$V_{\text{eff}}(\bar{\chi}) = -\Gamma(\bar{\chi}) / \int d^4x = 3\lambda(\bar{\chi}^2 - v^2)^2 + 3 \int \frac{d^4k}{i(2\pi)^4} \ln(m_B^2 - k^2 - i\epsilon) + \frac{3}{2} \int \frac{d^4k}{i(2\pi)^4} \ln(m_\chi^2 - k^2 - i\epsilon). \quad (11)$$

In the finite-temperature system [12], the partition functional Z is described by the Euclidean variables: $x_0 = -i\tau$, and the upper bound of the τ integration is $\beta = 1/T$ with T being the temperature. Then, the k_0 integration in the functional determinant becomes the infinite sum over the Matsubara frequency [12]. The effective potential at finite temperatures physically corresponds to the thermodynamical potential, and is given by

$$V_{\text{eff}}(\bar{\chi}; T) = 3\lambda(\bar{\chi}^2 - v^2)^2 + 3T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \ln\{(2n\pi T)^2 + k^2 + m_B^2\} \\ + \frac{3}{2} T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \ln\{(2n\pi T)^2 + k^2 + m_\chi^2\} \quad (12)$$

in the DGL theory. Performing the summation over n and the angular integration, we obtain the final expression of the effective potential at finite temperatures:

$$V_{\text{eff}}(\bar{\chi}; T) = 3\lambda(\bar{\chi}^2 - v^2)^2 + 3 \frac{T}{\pi^2} \int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2 + m_B^2}/T}\right) + \frac{3}{2} \frac{T}{\pi^2} \int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2 + m_\chi^2}/T}\right), \quad (13)$$

where m_B and m_χ are functions of $\bar{\chi}$ as shown in Eq. (6). Here, we have dropped the T -independent part (quantum fluctuation) because we are interested in the thermal contribution to the QCD vacuum [13].

III. NUMERICAL RESULTS AND DISCUSSIONS

We show in Fig. 1 the effective potential at various temperatures (thermodynamical potential), $V_{\text{eff}}(\bar{\chi}; T)$, as a function of the QCD-monopole condensate $\bar{\chi}$, an order parameter for the color confinement. The parameters $\lambda = 25$, $v = 0.126$ GeV, and $g = 2.3$ are extracted by fitting the static potential in the DGL theory to the Cornell

potential [6].¹ These values provide $m_B = 0.5$ GeV and $m_\chi = 1.26$ GeV at $T = 0$. The (local-) minimum point of $V_{\text{eff}}(\bar{\chi}; T)$ corresponds to the physical (meta)stable vacuum state. As the temperature increases, the broken dual gauge symmetry tends to be restored, and the QCD-monopole condensate in the physical vacuum, $\bar{\chi}_{\text{phys}}(T)$, is decreased. A first-order phase transition is found at the

¹We examined several possible parameter sets, and found a small parameter dependence on our results shown in this paper.

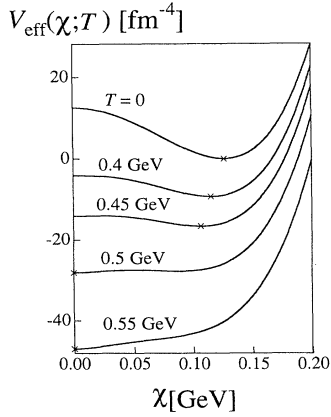


FIG. 1. The effective potentials at various temperatures as functions of the QCD-monopole condensate $\bar{\chi}$. The numbers beside each curve are the temperatures. The absolute minimum points of the effective potentials are shown by crosses.

thermodynamical critical temperature, $T_C \simeq 0.49$ GeV, and the QCD vacuum becomes trivial, $\bar{\chi}_{\text{phys}}(T) = 0$, for $T \geq T_C$. This phase transition is regarded as the deconfinement phase transition, because there is no confining force among colored particles in the QCD vacuum with $\bar{\chi}_{\text{phys}}(T) = 0$.

We show the behavior of the QCD-monopole condensate in the physical vacuum, $\bar{\chi}_{\text{phys}}(T)$, as a function of the temperature T in Fig. 2. One finds $\bar{\chi}_{\text{phys}} = 0.126$ GeV at $T = 0$, and the QCD-monopole condensate decreases monotonously up to $\bar{\chi}_{\text{phys}} = 0.07$ GeV at the upper critical temperature $T_{\text{up}} = 0.51$ GeV, where the minimum at finite $\bar{\chi}$ disappears in $V_{\text{eff}}(\bar{\chi}; T)$. On the other hand, the local minimum is developed at $\bar{\chi} = 0$ in $V_{\text{eff}}(\bar{\chi}; T)$ above

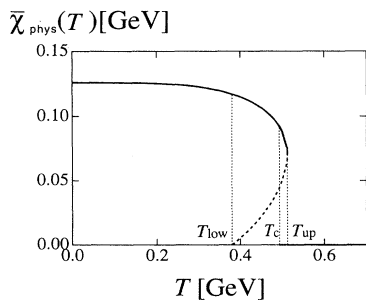


FIG. 2. The QCD monopole condensate $\bar{\chi}_{\text{phys}}(T)$ at minima of the effective potential as a function of the temperature T . The solid curve denotes $\bar{\chi}_{\text{phys}}(T)$ corresponding to the confinement phase, which is the absolute minimum up to $T_C = 0.49$ GeV and becomes a local minimum up to $T_{\text{up}} = 0.51$ GeV. A minimum appears at $\bar{\chi} = 0$ above $T_{\text{low}} = 0.38$ GeV and becomes the absolute minimum above $T_C = 0.49$ GeV. The dot-dashed curve denotes the value of $\bar{\chi}$ at the local maximum.

the lower critical temperature $T_{\text{low}} = 0.38$ GeV, which is analytically obtained by using the high-temperature expansion [12,13]:

$$T_{\text{low}} = 2v \left(\frac{6\lambda}{2\lambda + 3g^2} \right)^{1/2}. \quad (14)$$

The minimum value of $V_{\text{eff}}(\bar{\chi}; T)$ at $\bar{\chi} = 0$ becomes deeper than that at finite $\bar{\chi}$ above the thermodynamical critical temperature $T_C = 0.49$ GeV. Here, we get the first-order phase transition because we have considered full orders in $\bar{\chi}^2$ as shown in Eq. (13). On the other hand, Monden *et al.* [13] did not get the first-order phase transition due to the use of only the lowest order in $\bar{\chi}^2$ in the high-temperature expansion [12], and therefore they had to introduce the cubic term in χ_α in the Lagrangian.

Here, we consider the possibility of the temperature dependence on the parameters (λ, v) in the DGL theory. The critical temperature, $T_C = 0.49$ GeV, seems much larger than the one of the recent lattice QCD prediction, $T_C = 0.26\text{--}0.28$ GeV, which is, for instance, estimated from the relation, $T_C/\sqrt{k} \simeq 0.62$ [3] and the string tension $k = 0.89\text{--}1.0$ GeV/fm. However, we should remember that the self-interaction term of χ_α has been introduced phenomenologically in the DGL Lagrangian. In particular, the asymptotic freedom behavior of QCD leads to a possible reduction of the self-interaction among QCD monopoles at high temperatures. Hence, we use a simple ansatz for the temperature dependence on λ :

$$\lambda(T) = \lambda \left(\frac{T_C - aT}{T_C} \right), \quad (15)$$

keeping the other parameter v constant. Here, the constant a is determined as $a = 0.89$ so as to reproduce $T_C = 0.28$ GeV. [We take $\lambda(T) = 0$ for $T > T_C/a$.] The results for the monopole condensate $\bar{\chi}_{\text{phys}}(T)$ are shown in Fig. 3. The qualitative behavior is the same as in the above argument with a constant λ . We find a weak first-order phase transition in this case also. Here, we find a large reduction of the self-interaction of the QCD monopoles near the critical temperature

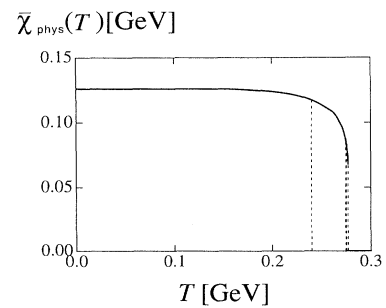


FIG. 3. The QCD monopole condensate $\bar{\chi}_{\text{phys}}(T)$ at minima of the effective potential as a function of the temperature in the case of variable $\lambda(T)$ so as to reproduce $T_C = 0.28$ GeV [3]. The meanings of the curves are the same as in Fig. 2.

T_C : $\lambda(T \simeq T_C) \simeq 2.7$ is considerably smaller than $\lambda(T=0) = 25$. It would be an interesting subject to examine such a large reduction of $\lambda(T)$ near T_C from the study of the monopole action in the lattice QCD [8].

Next, we investigate the variation of the masses of the dual gauge field \vec{B}_μ and the QCD-monopole field $\tilde{\chi}_\alpha$ at finite temperatures. Here, \vec{B}_μ and $\tilde{\chi}_\alpha$ would appear as the color-singlet glueball field with 1^+ and 0^+ , respectively [4,7,9]. The glueball masses, m_B and m_χ , at the finite temperature T are given by

$$\begin{aligned} m_B(T) &= \sqrt{3}g\tilde{\chi}_{\text{phys}}(T), \\ m_\chi(T) &= 2\sqrt{\lambda(T)}\tilde{\chi}_{\text{phys}}(T) \end{aligned} \quad (16)$$

as shown in Eq. (6). In Fig. 4, we show $m_B(T)$ and $m_\chi(T)$ as functions of the temperature T using variable $\lambda(T)$ in Eq. (15). (Their behaviors are almost the same as the case of a constant λ except for the difference of the value of T_C .) It is worth mentioning that $m_B(T)$ and $m_\chi(T)$ drop down to $m_B, m_\chi \sim T_C (=0.28 \text{ GeV})$ from $m_B, m_\chi \sim 1 \text{ GeV}$ near the critical temperature T_C . In other words, the QCD phase transition occurs at the temperature satisfying $m_B, m_\chi \simeq T$, which seems quite natural because the thermodynamical factor $1/\{\exp(\sqrt{k^2 + m^2}/T) \pm 1\}$ becomes relevant only for $m \lesssim T$. Thus, our result predicts a large reduction of the glueball masses, m_B and m_χ , near the critical temperature T_C . It is desirable to study the change of the glueball masses at finite temperatures, especially near T_C , in the lattice QCD simulation with the larger lattice size and the higher accuracy.

We investigate the string tension k at finite temperatures, since k is one of the most important variables for the color confinement, and controls the hadron properties through the interquark potential. We use the expression of the string tension $k(T)$ provided by SST [6]:

$$k(T) = \frac{e^2 m_B^2(T)}{24\pi} \ln \left(\frac{m_B^2(T) + m_\chi^2(T)}{m_B^2(T)} \right), \quad (17)$$

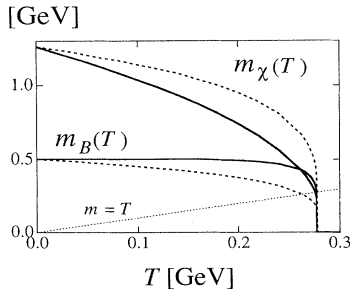


FIG. 4. The masses of the glueballs at various temperatures: $m_B(T)$ and $m_\chi(T)$. The solid lines denote the case of variable $\lambda(T)$ with a constant v . The dashed lines denote the case of variable $v(T)$ with a constant λ . A large reduction of these masses is found near the critical temperature. The dotted line denotes $m = T$. The phase transition occurs at the temperature satisfying $m_B, m_\chi \simeq T$.

where the glueball masses $m_B(T)$ and $m_\chi(T)$ are given by Eq. (16). The results are shown in Fig. 5 as a function of the temperature T . In the case of constant λ , the string tension $k(T)$ decreases very gradually up to the temperature, $T_{\text{up}} = 0.51 \text{ GeV}$. On the other hand, in the case of variable $\lambda(T)$, the string tension $k(T)$ decreases rapidly with temperature, and $k(T)$ drops down to zero around $T_C = 0.28 \text{ GeV}$. Hence, one expects a rapid change of the masses and the sizes of the quarkonia according to the large reduction of $k(T)$ at high temperatures. We plot also the pure-gauge lattice QCD results for the temperature dependence of the string tension by black dots [16], with $T_C = 0.28 \text{ GeV}$ [3]. We find our results with variable $\lambda(T)$ agree with the lattice QCD data.

We discuss further the temperature dependence of the parameters (λ, v) in the DGL theory. Definitely, we should follow the lattice QCD data for this determination as the case of the Ginzburg-Landau theory of superconductors extracting the temperature dependence from experiments. Since there exists the lattice QCD data on the string tension k [16], we try to reproduce k by taking a simple ansatz on λ and v . We try the ansatz

$$B(T) \equiv 3\lambda(T)v^4(T) = 3\lambda v^4 \left(\frac{T_C - aT}{T_C} \right), \quad (18)$$

where the constant a is determined so as to reproduce $T_C = 0.28 \text{ GeV}$. [We take $B(T) = 0$ for $T > T_C/a$.] The variable $B(T)$ corresponds to the bag constant, the energy-density difference between the nonperturbative vacuum ($|\chi_\alpha| \neq 0$) and the perturbative vacuum ($|\chi_\alpha| = 0$) in the DGL theory; see Eq. (13). The ansatz (18) suggests the reduction of the bag constant at high temperatures, which provides the swelling of hot hadrons by way of the bag-model picture. Since we have already examined a typical case for variable $\lambda(T)$ keeping v constant, we show here another typical case for variable $v(T)$ keeping λ constant. The string tension $k(T)$ in the variable $v(T)$ case with $a = 0.91$ is shown by the dashed line in Fig. 5. We find almost an identical result and find again a good agreement with the lattice QCD data.

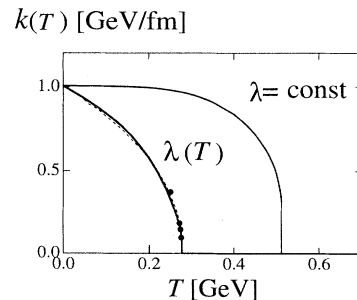


FIG. 5. The string tension $k(T)$ as a function of the temperature T . The solid and dashed lines correspond to the variable $\lambda(T)$ case with a constant v and the variable $v(T)$ case with a constant λ , respectively. The constant (λ, v) case is also shown by the thin line. The lattice QCD results in the pure gauge in [16] are shown by black dots near the critical temperature.

Other combinations on $\lambda(T)$ and $v(T)$ under the relation (18) also provide equally good results on $k(T)$.

Finally, we investigate the relation between the scalar glueball mass $m_\chi(T)$ and the string tension $k(T)$. For variable $\lambda(T)$ keeping v constant, one finds from Eq. (17) an approximate relation

$$\frac{m_\chi(T)}{\sqrt{k(T)}} \simeq \frac{(24\pi)^{1/2}}{e} \simeq 1.6, \quad (19)$$

near the critical temperature T_C . On the other hand, for variable $v(T)$ keeping λ constant, the glueball masses at finite temperatures, $m_B(T)$ and $m_\chi(T)$, are shown by the dashed line in Fig. 4, and Eq. (17) leads to a simple relation

$$\frac{m_\chi(T)}{\sqrt{k(T)}} = \left(\frac{2\lambda}{\pi \ln\{(3g^2 + 4\lambda)/3g^2\}} \right)^{1/2} \simeq 3.0, \quad (20)$$

for the whole region of T . Thus, the DGL theory suggests a proportional relation between the scalar glueball mass and the square root of the string tension at least near T_C . It is worth mentioning that Engels *et al.* [15] obtained a similar relation, $m_{GB}(T) = (1.7 \pm 0.5)\sqrt{k(T)}$, for the lowest scalar glueball at finite temperatures from the thermodynamical study on the SU(2) lattice gauge theory. Equations (19) and (20) can be examined from the thermodynamical study on the glueball mass in the lattice QCD, which may also reveal T dependence on the parameters in the DGL theory.

IV. SUMMARY AND CONCLUDING REMARKS

We have studied the properties of the pure-gauge QCD vacuum at finite temperatures in the dual Ginzburg-Landau (DGL) theory, where the color confinement is realized through the dual Higgs mechanism. We have formulated the effective potential at finite temperatures (thermodynamical potential) using the path integral formalism. We have used the quadratic source term instead of the linear source term. The use of the quadratic source term overcomes the problem of the imaginary scalar mass, which is encountered in the case of the linear source term as is well known in the ϕ^4 theory.

We have found the reduction of the QCD-monopole condensate at finite temperatures, and have found a

first-order deconfinement phase transition at the critical temperature $T_C \simeq 0.49$ GeV using the temperature-independent parameters. The QCD-monopole condensate vanishes and the broken dual gauge symmetry is restored above T_C . We have considered the temperature dependence of the QCD-monopole self-interaction noting $T_C = 0.28$ GeV as the lattice QCD simulation indicates. We have found a large reduction of the QCD-monopole self-interaction near the critical temperature. We have investigated the temperature dependence of the glueball masses, m_B and m_χ , and have found their large reduction near the critical temperature T_C : $m_B, m_\chi \sim T_C$. We have calculated also the string tension at finite temperatures. The results agree with the lattice QCD data both in the variable $\lambda(T)$ and in the variable $v(T)$ cases.

In particular, the glueball mass reduction at high temperatures would be an important ingredient in the QCD phase transition. In the pure gauge, there are only glueball excitations with the large masses ($\gtrsim 1$ GeV) at low temperatures [2,15], and therefore it seems unnatural that the QCD phase transition takes place at a small critical temperature, $T_C \simeq 0.28$ GeV. This problem would be explained by the large reduction of the glueball mass near the critical temperature as is demonstrated in this paper. In other words, this glueball-mass reduction may determine the magnitude of the critical temperature T_C in the QCD phase transition. This large reduction of the glueball masses at high temperature would serve as an interesting subject of the lattice QCD.

It is also very interesting to investigate the thermal effect of the dynamical quark in the DGL theory, especially in terms of the chiral phase transition [2,12]. The chiral-symmetry restoration as well as the deconfinement phase transition is expected to happen in the DGL theory, because our previous works [6,9,10] showed the close relation between the color confinement and the dynamical chiral-symmetry breaking in the DGL theory. With respect to this subject, we are studying now the restoration of chiral symmetry at high temperature in the DGL theory [17].

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