Gluon propagator in non-Abelian Weizsäcker-Williams fields

Alejandro Ayala, Jamal Jalilian-Marian, and Larry McLerran

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

Raju Venugopalan

Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195

We carefully compute the gluon propagator in the background of a non-Abelian Weizsäcker-Williams field. This background field is generated by the valence quarks in very large nuclei. We find contact terms in the small fluctuation equations of motion which induce corrections to a previously incorrect result for the gluon propagator in such a background field. The well-known problem of the Hermiticity of certain operators in the light cone gauge is resolved for the Weizsäcker-Williams background field. This is achieved by working in a gauge where singular terms in the equations of motion are absent and then gauge transforming the small fluctuation fields to the light cone gauge.

I. INTRODUCTION

In a recent work, two of the authors have argued that parton distribution functions may be computed for very large nuclei at small x [1]. They found that the valence quarks in these nuclei are the sources of non-Abelian Weizsäcker-Williams fields. This work is similar in spirit to that of Mueller who considers the distribution functions generated by heavy quark charmoniumlike bound states [2]. In [1] the general form of these fields was found. It was argued that to compute the distribution functions, one should compute the fields associated with these valence quarks and then color average over these fields treating the valence quarks as classical charges. The weight for this averaging was shown to be

$$\exp\left(-\frac{1}{2\mu^2}\int d^2x_t\rho^2(x_t)\right), \tag{1}$$

where $\rho(x_t)$ is the distribution of transverse valence quark color charge and μ^2 is a parameter corresponding to the valence quark color charge squared per unit area.

The current for the valence quarks was found to be

$$J_a^\mu(x) = \delta^{\mu+}\delta(x^-)\rho_a(x_t, x^+). \tag{2}$$

Here the light cone variables are

$$y^\pm = (y^0 \pm y^3)/\sqrt{2}. \tag{3}$$

The approximation which leads to this form of the current is that $x \ll A^{-1/3}$ so that the small x partons see a nucleus which is essentially Lorentz contracted to a thin sheet. This is the origin of the factor $\delta(x^-)$ above. Further, since the small x partons are weakly coupled to the source, the source is "static" and only the + component of the current is important.

The charge distribution in transverse coordinates is essentially random with a Gaussian weight as described

above. This is true so long as we look at transverse resolution scales which are large enough that there are many valence quarks per unit area, and small enough that it is much less than the confinement scale of 1 fm. In momentum space, this requires that we only consider transverse momenta which are in the range

$$\Lambda_{\rm QCD} \ll k_t \ll \mu \sim A^{1/6}. \tag{4}$$

The x^+ (light cone time) dependence of the charge density is a consequence of the extended current conservation law

$$D_\mu J^\mu = 0. \tag{5}$$

The current may be taken to be time independent if we fix the residual gauge freedom by the constraint that $A^-(x^- = 0) = 0$. This point will be discussed further later.

Under these conditions, it is easy to guess the form of the Weizsäcker-Williams field. The charge distribution is confined to an infinitesimally thin sheet. Therefore, one expects that the fields on either side of the sheet might be a gauge transform of vacuum. We deduce therefore that [1]

$$\begin{aligned} A^+ &= 0, \\ A^- &= 0, \\ A^i &= \theta(x^-)\alpha^i(x_t), \end{aligned} \tag{6}$$

where

$$\alpha^j(x_t) = \frac{i}{g}U(x_t)\nabla^jU^\dagger(x_t). \tag{7}$$

The field α is a gauge transform of the vacuum. It generates zero magnetic field. There is a transverse electric field which is confined to the sheet $x^- = 0$. Gauss's law shows that this field solves

$$\nabla \cdot \alpha = g\rho. \tag{8}$$

One can verify that the light cone Hamiltonian satisfies

$$P^- = 0 \tag{9}$$

so that these classical fields have vanishing light cone energy.

These fields are gauge transforms of vacuum fields with a discontinuity which is adjusted so as to reproduce the correct valence charge distribution. Because it is a pure gauge transform of the vacuum on either side of the infinitesimally thin sheet, we expect that the propagators are simple. In a previous paper, two of the authors argued that the fields which generate the propagator are just gauge transforms of the ordinary small fluctuation plane wave fields [3]. This argument is correct for matter fields such as scalars and fermions, but as we shall see is not quite correct for gluon fields.

The problem with the gluon fields is that there can be contact terms in the gluon small fluctuation equations of motion associated with the thin sheet of valence charge. These contact terms can generate contributions to the gluon field which are constant or grow linearly at large distances away from the sheet. The linearly growing terms will give rise to a constant electric field. In any case, neither of these terms were allowed for in the previous analysis which claimed to have determined the gluon propagator.

The terms which were ignored in the previous analysis are very singular. A direct consequence of their bad behavior at infinity is the non-self-adjoint character of the light cone gauge equations of motion.

The present work is organized as follows: in Sec. II we will see that the above-mentioned terms are a consequence of the small fluctuation equations of motion in light cone gauge ($A^+ = 0$). To resolve the ambiguities inherently associated with the singular behavior of the small fluctuation fields, we choose to work in $A^- = 0$ gauge. In Sec. III, we show that this gauge is nonsingular and that the propagator can be computed directly. It turns out that the propagator in this gauge is precisely that previously computed by McLerran and Venugopalan [3]. The propagator in light cone gauge is derived from the propagator in the non-singular gauge. Details of this derivation are the subject of Sec IV. In Sec. V, we discuss the regularization prescriptions which enable us to generate a properly defined propagator in light cone gauge. We write down an explicit expression for this propagator. The propagator will be used in a sequel to this work to calculate the gluon distribution function to next order in α_s. In Sec. VI, we summarize our results.

II. THE SMALL FLUCTUATION EQUATIONS IN LIGHT CONE GAUGE

The conventional approach to computing a Green's function is to write down the equations for the small fluctuation field in a fixed gauge. Denoting the small fluctuation field by $\delta\phi$, the generic small fluctuation equation is

$$G^{-1}\delta\phi_\lambda = \lambda\delta\phi_\lambda, \tag{10}$$

and the propagator is

$$G(x,y) = \sum \frac{\delta\phi_\lambda(x)\delta\phi_\lambda^\dagger(y)}{\lambda}. \tag{11}$$

To derive the equations for the small fluctuation field in light cone gauge ($A^+ = 0$) we first look at Eq. (10) with $\lambda = 0$. Expanding around our background field, we find that the equations for the gluon field become

$$\left[D(A)^2 g^{\mu\nu} - D^\mu(A)D^\nu(A)\right]\delta A_\nu - 2F^{\mu\nu}\delta A_\nu = 0. \tag{12}$$

In this equation, A is the background field which according to Eq. (6) is nonvanishing only for its transverse components. $D^\mu(A)$ is the covariant derivative with A as the gauge field (notice that $D^\pm = \partial^\pm$). We have assumed the gauge condition $\delta A^-(x^- = 0) = 0$ so that we can ignore any time evolution of the source of valence quark charge. If this gauge condition were not chosen, then the extended current conservation condition would require that the source evolve in time, and this would generate a correction to the above equation. This background field generates no magnetic field, but it does generate a nonzero electric field F^{+i}:

$$F^{i+} = \delta(x^-)\alpha^i(x_t). \tag{13}$$

On either side of $x^- = 0$, the background field is a gauge transform of the vacuum. We might therefore think that the solutions to the above equation, in the two different regions of x^-, are simply different gauge transforms of the plane wave small fluctuation field one would get in zero background field. Further, continuity at the origin could be assured by a matching condition. This is precisely what was done in the previous work of McLerran and Venugopalan [3].

Unfortunately, this procedure will not work here due to the presence of the background electric field term. Before seeing how this works, let us first express the small fluctuation equations in a slightly more transparent form (hereafter latin indices refer to transverse variables):

$$D^2(A)\delta A^\mu - D(A)^\mu D(A) \cdot \delta A$$

$$= -2\delta^{\mu+}\delta(x^-)\alpha^i\delta A_i - 2\delta^{\mu j}\delta(x^-)\alpha^j\delta A^-.$$

The terms on the right-hand side of the above equation are called contact terms; they appear when the derivative $\partial/\partial x^-$ acts on the classical field which is proportional to $\theta(x^-)$. Notice that the last term vanishes in the gauge where $\delta A^- |_{x^-=0} = 0$.

The small fluctuation equation for the plus component of the above equations becomes

$$\partial^+[\partial^+\delta A^- - D(A)^i\delta A_i] = -2\delta(x^-)\alpha^i\delta A_i. \tag{14}$$

This equation requires that

$$\partial^+ \delta A^- - D(A)^i \delta A_i$$

$$= -2\theta(x^-)\alpha^i \delta A_i \mid_{x^-=0} + h(x^+, x_t) , \quad (15)$$

where h is some as yet undetermined function. We see immediately that this implies that the fields have singular behavior at infinity. We might remove the singular behavior at say $x^- \to -\infty$ but due to the step function cannot remove it as $x^- \to +\infty$. In fact, the fields will behave like x^- as x^- goes to infinity. This singular behavior is sufficient to guarantee that the equations of motion are not self-adjoint on this space of functions. Further, the small fluctuation fields are discontinuous at the origin ($x^- = 0$) and matching the fields there is more difficult. The singular behavior at infinity and the discontinuity of the fields at the origin were absent from the previous analysis of McLerran and Venugopalan [3]. This was because they neglected to include the contact term at the origin which (as we have seen) is present in the equation of motion in light cone gauge.

Because of this lack of self-adjointness on this space of functions, we have found no consistent way of directly integrating the equations of motion in light cone gauge. The most obvious problem is that the Green's function will not solve the constraint imposed by symmetry and by the Hermiticity of the underlying fields. Without a better way of defining the equations of motion, we are at a quandary and do not know how to properly construct the Green's function in light cone gauge. This problem with light cone gauge is well known in the relevant literature and various schemes have been adopted to circumvent it [4].

However, and this is the core of the present analysis, we are compelled to compute the Green's function in light cone gauge. There are several reasons for this.

It is well known that only in light cone gauge does the parton model have the most direct and manifest physical realization [5]. Only in light cone gauge is it possible to construct a simple and intuitive Fock space basis on which our formalism relies heavily. Light cone gauge (like any other axial gauge) is free of ghosts and negative-norm gauge boson states which makes the theory simpler [6].

In addition, there are further advantages in choosing the $A^+ = 0$ gauge. It can be shown that in this gauge J^+ is the only large component of the fermionic current. The transverse and minus components of the current are proportional to $\frac{1}{p^+}$ and are therefore small [1]. It is much easier to eliminate the dependent fermionic degrees of freedom in this gauge since we do not need to worry about inverting covariant derivatives. Finally, the gluon distribution function is most simply related to physical quantities measured in deep inelastic scattering experiments only in $A^+ = 0$ [7].

We will demonstrate in the following sections that the Green's functions in light cone gauge can be obtained by the following procedure: (i) first, solve the small fluctuation equations in an alternative gauge ($A^- = 0$) and then (ii) gauge transform the Green's functions thus obtained to light cone gauge. In the following section, we will show that the Green's function may be constructed in $A^- = 0$ gauge and that the equations of motion for

the Green's function are self-adjoint. In subsequent sections we discuss the mechanics of how the Green's function may then be gauge transformed to light cone gauge. When this transformation is carried through, there is in general a singularity of the transformation function. This singularity is regulated using the well-known Leibbrandt-Mandelstam prescription [8]. We find that the Green's function in light cone gauge does not satisfy the constraints which arise from self-adjointness of the gluon field. This lack of self-adjointness is not irretrievably damaging and has been dealt with in the past for Green's functions of gluon fields in light cone gauge [9]. Presumably the precise nature of the singularity is not important for gauge invariant quantities. When one chooses a prescription for regulating the singularity, if performed consistently in all quantities, it will give gauge invariant results for physical quantities. We are not aware that there is a proof of this, but believe it to be so.

III. THE SMALL FLUCTUATIONS PROPAGATOR IN $A^- = 0$ GAUGE

What is the virtue of writing down the equations of motion in the $A^- = 0$ gauge? In the small fluctuation equations for the transverse fields, the source term vanishes (the equation reduces to the sourceless Klein-Gordon field equation):

$$\left[-2\partial^+ \partial^- + D_t^2(A) \right] \delta A_i = 0 . \quad (16)$$

This equation is obtained by appropriately fixing the residual gauge freedom in $A^- = 0$ gauge. One of the other equations is also sourceless, and is the Gauss law constraint which must be imposed on initial field configurations. This constraint commutes with the equations of motion. It is

$$\partial^- (\partial^- \delta A^+ - D_i(A)\delta A_i) = 0 . \quad (17)$$

We must now check that the remaining equation of motion is properly satisfied. If we use the Gauss law constraint that $D^\mu(A)\delta A_\mu = 0$, this equation is

$$\left[-2\partial^+ \partial^- + \vec{D}^2(A) \right] \delta A^+ = -2\delta(x^-)\alpha^i \delta A^i . \quad (18)$$

Now from the Gauss law constraint,

$$\delta A^+ = \frac{1}{\partial^-} \vec{D}(A) \cdot \delta \vec{A} . \quad (19)$$

In deriving the above equation, we have used the condition that A^+ does not blow up at $x^+ = \pm\infty$ to eliminate arbitrary constants that arise in integrating Eq. (17). Although δA^i is continuous at $x^- = 0$, the spatial covariant derivative operator is not. The discontinuity gives precisely the source term on the right-hand side of the equation, and the equations are consistent.

We can now solve the above equations of motion in the following way. We assume that the transverse field on the side where the background field vanishes is a simple plane wave. On the other side of the boundary where the background field is a gauge transform of the vacuum,

we take the small fluctuation field to be a linear superposition of plane waves. The linear superposition is done in a manner to ensure that the fields are continuous across $x^- = 0$. This is possible because the equations above have no δ function singularity at $x^- = 0$.

The expression for the transverse components of the fluctuation field in the $A^- = 0$ gauge is

$$\delta \tilde{A}_i^{\alpha\beta} = e^{ipx} \eta_i \left\{ \theta(-x^-)\tau^{\alpha\beta} + \theta(x^-) \int \frac{d^2 q_t}{(2\pi)^2} e^{iq_t x_t} \right.$$

$$\times \exp\left[-i\frac{2p_t q_t + q_t^2}{2p^-} x^- \right] \int d^2 z_t e^{-iq_t z_t} \left[U(x_t)U^\dagger(z_t)\tau U(z_t)U^\dagger(x_t) \right]^{\alpha\beta} \left. \right\}. \tag{20}$$

This solution is continuous at $x^- = 0$ and was fully discussed in an earlier paper by McLerran and Venugopalan [3].

The above expression can be substituted in Eq. (11) to obtain the gluon propagator in $A^- = 0$ gauge. There are four separate pieces for $G_{ij}^{\alpha\beta;\alpha'\beta'}(x,y)$ which depend upon the signs of x^- and y^-. The overall expression for G is

$$G_{ij}^{\alpha\beta;\alpha'\beta'}(x,y) = -\delta_{ij} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - i\epsilon} \left\{ \theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} + \theta(x^-)\theta(y^-)F_a^{\alpha\beta}(x_t)F_a^{\alpha'\beta'}(y_t) + \int \frac{d^2 q_t}{(2\pi)^2} d^2 z_t \right.$$

$$\times [\theta(x^-)\theta(-y^-)e^{i(p_t-q_t)(z_t-x_t)}e^{-i(q_t^2-p_t^2)x^-/2p^-} F_a^{\alpha\beta}(x_t)F_a^{\alpha'\beta'}(z_t)$$

$$+ \theta(-x^-)\theta(y^-)e^{i(p_t-q_t)(y_t-z_t)}e^{i(q_t^2-p_t^2)y^-/2p^-} F_a^{\alpha\beta}(z_t)F_a^{\alpha'\beta'}(y_t)] \left. \right\}, \tag{21}$$

where $F_a^{\alpha\beta}(x_t) = (U(x_t)\tau_a U^\dagger(x_t))^{\alpha\beta}$. It can be easily checked that this Green's function satisfies

$$\left[-2\partial^+\partial^- + D_t^2(A) \right]_{ik} G_{kj}(x,y) = \delta_{ij}\, \delta^{(4)}(x-y)$$

as it must. This construction of the Green's function was previously performed by McLerran and Venugopalan in [3]. However, they incorrectly interpreted it to be the result for $A^+ = 0$ gauge, while we have shown here that it applies to $A^- = 0$ gauge only.

For the pieces of the propagator which involve the component δA^+ we have

$$\langle \delta A^i \delta A^+ \rangle = \left\langle \delta A^i \frac{1}{\partial^-} \vec{D}(A) \cdot \vec{\delta A} \right\rangle \tag{22}$$

and

$$\langle \delta A^+ \delta A^+ \rangle = \left\langle \frac{1}{\partial^-} \vec{D}(A) \cdot \vec{\delta A} \frac{1}{\partial^-} \vec{D}(A) \cdot \vec{\delta A} \right\rangle - i\delta G^{++}, \tag{23}$$

where in our convention $\langle A_\mu A_\nu \rangle = -i G_{\mu\nu}$ and $\langle A_\mu A_\nu \rangle$ represents the correlation of fields in the path integral sense.

IV. ON DERIVING THE $A^+ = 0$ PROPAGATOR FROM THE $A^- = 0$ PROPAGATOR

In the previous section we derived an explicit expression for the small fluctuation Green's function in the $A^- = 0$ gauge. In this section, we discuss general arguments which allow us to compute expectation values of operators in one gauge in terms of the expectation values of operators in another gauge. The formal arguments will be illustrated by the specific example of the free gluon propagator. We are of course interested

in these arguments because we want to find the gauge transform which will relate the Green's function in the light cone gauge to the Green's function in the $A^- = 0$ gauge.

Let $O(A)$ be some function of the field operators evaluated in the light cone gauge, $A^+ = 0$. We will here consider QED and later generalize to the case at hand. We have to evaluate the expectation value

$$\langle O(A) \rangle = \frac{\int [dA]\, \delta(A^+)\, e^{iS} O(A)}{\int [dA]\, \delta(A^+)\, e^{iS}}. \tag{24}$$

We now introduce the identity in the form

$$1 = \frac{\int [d\Lambda]\, \delta(A^- + \partial^-\Lambda)}{\int [d\Lambda]\, \delta(A^- + \partial^-\Lambda)}. \tag{25}$$

Inserting this into the equation for $O(A)$, we find that

$$\langle O(A) \rangle = \frac{\int [dA][d\Lambda]\, \delta(A^- + \partial^-\Lambda)\, \delta(A^+)\, e^{iS} O(A)}{\int [dA][d\Lambda]\, \delta(A^- + \partial^-\Lambda)\, \delta(A^+)\, e^{iS}}. \tag{26}$$

We now perform the gauge transformation

$$\tilde{A}^\mu = A^\mu + \partial^\mu \Lambda. \tag{27}$$

Since both the measure and the action are invariant under the gauge transformation, we see that

$$\langle O(A) \rangle$$

$$= \frac{\int [d\tilde{A}][d\Lambda]\, \delta(\tilde{A}^-)\, \delta(\tilde{A}^+ - \partial^+\Lambda)\, e^{iS} O(\tilde{A} - \partial\Lambda)}{\int [d\tilde{A}][d\Lambda]\, \delta(\tilde{A}^-)\delta(\tilde{A}^+ - \partial^+\Lambda)e^{iS}}. \tag{28}$$

Now doing the integration over Λ, we get

$$\langle O(A) \rangle = \frac{\int [d\tilde{A}] \, \delta(\tilde{A}^-) \, e^{iS} O\left[\tilde{A} - \partial\left(\frac{\tilde{A}^+}{\partial^+}\right)\right]}{\int [d\tilde{A}]\delta(\tilde{A}^-)e^{iS}}. \tag{29}$$

Before proceeding any further, we will be more explicit and illustrate the above approach by an example: we show that the free propagator in the absence of a background field in $A^+ = 0$ gauge can be obtained as a gauge transformation of the same quantity in $A^- = 0$ gauge.

Let us first review the construction of the propagator in $\tilde{A}^- = 0$ gauge. Henceforth all fields in this gauge will be denoted by an $\tilde{A}$ to distinguish them from the corresponding fields in light cone gauge. The action for the vector field is

$$S = \frac{1}{2}(\partial^- \delta\tilde{A}^+ - \nabla \cdot \delta\tilde{A})^2 + \frac{1}{2}\delta\tilde{A}^i \delta_{ij} \square \, \delta\tilde{A}^j, \tag{30}$$

where the operator $\square$ is the D'Alambertian in light cone coordinates. From the above expression for the action, we distinguish two classes of small fluctuation fields that satisfy the equations of motion. We can see this from the following argument. To find the solution that minimizes the action let us perform the shift

$$\delta\tilde{A}^+ \to \delta\tilde{A}^+ + \frac{1}{\partial^-}\nabla \cdot \delta\tilde{A}, \tag{31}$$

to obtain a purely quadratic action in $\delta\tilde{A}^+$ and $\delta\tilde{A}_i$. The minimum thus corresponds to the fields that make the action vanish. We label the fields as follows: Type-I fields satisfy

$$\partial^- \delta\tilde{A}^+ - \nabla \cdot \delta\tilde{A} = 0, $$
$$\square \delta\tilde{A}^i = 0, \tag{32}$$

whereas type-II fields satisfy

$$\delta\tilde{A}^i = 0, $$
$$\partial^- \partial^- \delta\tilde{A}^+ = 0. \tag{33}$$

The contributions of different components to the free propagator are then

$$\langle \delta\tilde{A}^i \delta\tilde{A}^j \rangle = i\frac{\delta^{ij}}{k^2}, $$
$$\langle \delta\tilde{A}^i \delta\tilde{A}^+ \rangle = i\frac{k^i}{k^- k^2}, $$
$$\langle \delta\tilde{A}^+ \delta\tilde{A}^+ \rangle = i\left(\frac{k_t^2}{(k^-)^2 k^2} - \frac{1}{(k^-)^2}\right). \tag{34}$$

The second term in the parentheses above corresponds to the contribution of the expectation value $\langle \delta\tilde{A}_{II}^+ \delta\tilde{A}_{II}^+ \rangle$ to $\langle \delta\tilde{A}^+ \delta\tilde{A}^+ \rangle$. Notice that the contribution from terms like $\langle \delta\tilde{A}_I^+ \delta\tilde{A}_{II}^+ \rangle$ vanish. If we now combine all the above terms together we obtain the familiar expression for the gluon propagator,

$$G^{\mu\nu} = -\frac{1}{k^2}\left(g^{\mu\nu} - \frac{n^{*\mu}k^\nu + k^\mu n^{*\nu}}{k \cdot n^*}\right), \tag{35}$$

where $n^{*\mu} = \delta^{\mu+}$.

Now perform the gauge transformation

$$\delta A^\mu = \delta\tilde{A}^\mu + \partial^\mu \Lambda. \tag{36}$$

Since we are gauge transforming to the light cone gauge, we require

$$\partial^+ \Lambda = -\delta\tilde{A}^+. \tag{37}$$

We therefore obtain

$$\delta A^+ = 0, $$
$$\delta A^- = -\frac{\partial^-}{\partial^+}\delta\tilde{A}^+, $$
$$\delta A^i = \delta\tilde{A}^i - \frac{\partial^i}{\partial^+}\delta\tilde{A}^+. \tag{38}$$

We now use Eq. (29) and the previous expression for the propagator in $\tilde{A}^- = 0$ gauge to derive

$$G^{\mu\nu} = -\frac{1}{k^2}\left(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{k \cdot n}\right), \tag{39}$$

where $n^\mu = \delta^{\mu-}$. This result is the well-known expression for the free gluon propagator in the light cone gauge.

For the case at hand, small fluctuations around a background field, the analysis goes through almost exactly as before. There are two essential differences. The first is that for the type of background field we consider the gauge transformation property of the transverse field is

$$\delta A^i = \delta\tilde{A}^i + D^i(A)\Lambda. \tag{40}$$

One should note that the particular form of the gauge transform above is valid only because we are interested in small fluctuations. We are throwing away terms which correspond to higher powers of the small fluctuation field. For the plus and minus components, the transformation involves ordinary derivatives. We see that the analysis above goes through in the same way it did for QED.

The other difference from the QED case is that the structure of the small fluctuations analysis is more complicated. There are modes (labeled type I) which involve both $\delta\tilde{A}^i$ and $\delta\tilde{A}^+$ as before but these modes now couple to the background field [see Eq. (38) with ∂^i replaced by $D^i(A)$]. There are also modes (type II) which involve only $\delta\tilde{A}^+$ and therefore do not couple to the background field since $\delta\tilde{A}^i = 0$. The contribution of the type-I modes to the propagator arises from the distortion of the small fluctuation fields by the background field. The contribution of the type-II modes to the gluon propagator in momentum space is

$$\delta G_{(II)}^{++} = \frac{1}{(k^-)^2} $$
$$= \frac{1}{k^2}\left(\frac{k_t^2}{(k^-)^2} - 2\frac{k^+}{k^-}\right). \tag{41}$$

The second of the two expressions in the above equations is algebraically equivalent to the first but corresponds to a different way of regulating the singular behavior of the type-II modes at $k^- = 0$. In the next section, we

will see that the latter choice is the one aptly suited to regularization.

V. REGULARIZATION PRESCRIPTIONS AND RESULTS

We shall now use the method described in the previous section to derive the gluon propagator in the light cone gauge from the propagator in $\tilde{A}^- = 0$ gauge [Eq. (21)]. Note that the gluon propagator here is the gluon propagator in the background field. It will shortly become clear that we need to choose an appropriate regularization prescription to obtain the final expression for the propagator.

If we have a fluctuation field in $\tilde{A}^- = 0$ gauge, the fields in $A^+ = 0$ gauge are given in terms of the tilde fields as

$$\delta A^+ = 0, \tag{42}$$

$$\delta A^- = -\frac{\partial^-}{\partial^+}\delta \tilde{A}^+, \tag{43}$$

$$\delta A^i = \delta \tilde{A}^i - D^i(A)\frac{1}{\partial^+}\delta \tilde{A}^+. \tag{44}$$

To derive these equations, we note that a small gauge transformation with a fixed background field A, where A has only transverse components, is

$$\delta A^+ \to \delta A^+ + \partial^+\Lambda, \tag{45}$$

$$\delta A^- \to \delta A^- + \partial^-\Lambda, \tag{46}$$

$$\delta A^i \to \delta A^i + D(A)^i\Lambda. \tag{47}$$

We have used the fact that in $A^- = 0$ gauge

$$\partial^-\delta \tilde{A}^+ = D(A)_i\delta \tilde{A}_i. \tag{48}$$

The transverse fluctuation field in the light cone gauge is then Eq. (44) which we rewrite in matrix component notation as

$$\delta A_i^{\alpha\beta}(x) = \left[\delta_{ik} - D_i\frac{1}{\partial_-\partial_+}D_k\right]^{\alpha\gamma}\delta \tilde{A}_k^{\gamma\beta}(x). \tag{49}$$

To see how the covariant derivative acts on the field, notice that $D_k\delta \tilde{A}_k(x) = \partial_k\delta \tilde{A}_k - ig[A_k, \delta \tilde{A}_k]$ where $A_k(x) = \theta(x^-)\frac{i}{g}U(x_t)\partial_k U^\dagger(x_t)$ is the background field. Then the commutator of the background field with $\delta \tilde{A}_k$ will exactly cancel the derivative acting on U's. The result is that the covariant derivative will reduce to a simple derivative which acts only on the exponential part of the field.

Following the discussion leading to Eq. (29), we obtain an implicit expression for the Green's function in the light cone gauge which we can write as the sum of the contributions of the type-I and type-II modes discussed in the previous section. The former can be written as

$$G_{ij(\mathrm{I})}^{\alpha\beta;\alpha'\beta'}(x,y) = \left\{\delta_{ik}^{\alpha\gamma}\delta_{jl}^{\alpha'\gamma'} - \delta_{jl}^{\alpha'\gamma'}\left(D_i\frac{1}{\partial_-\partial_+}D_k\right)_{(x)}^{\alpha\gamma} - \delta_{ik}^{\alpha\gamma}\left(D_j\frac{1}{\partial_-\partial_+}D_l\right)_{(y)}^{\alpha'\gamma'}\right.$$
$$\left. + \left(D_i\frac{1}{\partial_-\partial_+}D_k\right)_{(x)}^{\alpha\gamma}\left(D_j\frac{1}{\partial_-\partial_+}D_l\right)_{(y)}^{\alpha'\gamma'}\right\} \tilde{G}_{kl(\mathrm{I})}^{\gamma\beta;\gamma'\beta'}(x,y), \tag{50}$$

where $\tilde{G}_{kl(\mathrm{I})}^{\gamma\beta;\gamma'\beta'}(x,y)$ is the Green's function in $A^- = 0$ gauge given by Eq. (21). The subscript I in the above equation denotes the contribution of the type-I class of small fluctuation fields.

We must add to this expression the contribution from the gauge transform of $\delta \tilde{G}_{(\mathrm{II})}^{++}$. This can be expressed as

$$G_{ij(\mathrm{II})}^{\alpha\beta;\alpha'\beta'}(x,y)$$

$$= \left(D^i\frac{1}{\partial^+}\right)_{(x)}^{\alpha\gamma}\left(D^j\frac{1}{\partial^+}\right)_{(y)}^{\alpha'\gamma'}(\delta \tilde{G}^{++})_{(\mathrm{II})}^{\gamma\beta;\gamma'\beta'}(x,y), \tag{51}$$

where

$$(\delta \tilde{G}^{++})_{(\mathrm{II})}^{\gamma\beta;\gamma'\beta'}(x,y) = (\tau_a)^{\gamma\beta}(\tau_a)^{\gamma'\beta'}\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{(k^-)^2 - i\epsilon}. \tag{52}$$

It is now straightforward to construct similar expres-

sions for nondiagonal components of the Green's function. Even though only transverse components of the light cone gauge Green's function are needed to compute the gluon distribution function, we will give an implicit expression for all components for the sake of completeness. They are

$$G^{-i}(x,y) = \left[\frac{1}{\partial_-}D_j\right]_{(x)}\left[\delta_{ki} - D_i\frac{1}{\partial_-\partial_+}D_k\right]_{(y)}\tilde{G}_{jk}(x,y)$$

$$-\left[\frac{\partial_+}{\partial_-}\right]_{(x)}\left[D_i\frac{1}{\partial_-}\right]_{(y)}\delta \tilde{G}_{(\mathrm{II})}^{++}(x,y)$$

and

$$G^{--}(x,y) = \left[\frac{1}{\partial_-}D_i\right]_{(x)}\left[\frac{1}{\partial_-}D_j\right]_{(y)}\tilde{G}_{ij(\mathrm{I})}(x,y)$$

$$+\left[\frac{\partial_+}{\partial_-}\right]_{(x)}\left[\frac{\partial_+}{\partial_-}\right]_{(y)}\delta \tilde{G}_{(\mathrm{II})}^{++}(x,y),$$

where $\tilde{G}_{ij(\mathrm{I})}$ is the Green's function in the $A^- = 0$ gauge given by Eq. (21) while $\delta \tilde{G}_{(\mathrm{II})}^{++}$ is the contribution of type-

II solutions to the Green's function as given by Eq. (52). From here on, we will concentrate on the transverse components only.

It is evident from the above equations that in relating the fields in the two gauges, one has to define $1/\partial^+$ and $1/\partial^-$. These operators are not well defined unless some prescription is adopted for treating the singularity at $k^+ = 0$ and at $k^- = 0$. Presumably, there is no physics in these singularities—when one computes gauge invariant quantities, different prescriptions should give the same results. It has however proven most convenient to choose a Feynman-like $i\epsilon$ prescription for regularizing this singularity, the so-called Leibbrandt-Mandelstam prescription [8]. This prescription however spoils the Hermiticity of the field δA^i. This is not a new problem since even in the absence of our background field, such a prescription will destroy the reality properties of A^- in $A^+ = 0$ gauge. It is indeed unpleasant, but it does make for a Green's function which is manifestly causal. The price to be paid for a small lack of Hermiticity of the fields is small compared to the price of dealing with noncausal Green's functions, albeit all such ambiguities must come to naught when computing gauge invariant quantities. For instance, a similar problem exists with the principal value prescription but it has been used successfully previously to compute gauge invariant quantities (see p. 115 in Ref. [6]).

We must now establish our prescriptions for defining the inversion of $\partial^\pm$. We first begin with $\frac{1}{k^-}$. For this operator we use the ordinary Leibbrandt-Mandelstam prescription

$$\frac{1}{k^-} \to \frac{1}{k^- + i\epsilon/k^+}. \tag{53}$$

For the operator $1/\partial^+$, we choose a prescription which guarantees that $\delta A^-\,|_{x^-=0} = 0$. This integration choice is in fact a further specification of the gauge and fixes the residual gauge freedom completely. This is because if we work in $A^+ = 0$ gauge, we can still make x^- independent

gauge transformations. We can therefore adjust the value of A^- at any fixed value of x^- which we choose to be $x^- = 0$. The virtue of this particular gauge choice is that it makes the source of charge time independent, since in this case the extended current conservation law is

$$\partial^- J^+ = 0. \tag{54}$$

Our prescription will therefore be

$$\frac{1}{\partial_-} F(x^-) = \int_0^{x^-} dx'^- F(x'^-). \tag{55}$$

Let us see what this corresponds to in momentum space. If we define

$$\tilde{F}(k^+) = \int dx^- e^{ik^+ x^-} F(x^-) \tag{56}$$

then

$$\int_0^{x^-} dx'^- F(x'^-) = \int \frac{dk^+}{2\pi} \frac{1}{-ik^+} (e^{-ik^+ x^-} - 1) \tilde{F}(k^+). \tag{57}$$

We see that with this integration prescription, there is never any singularity in Fourier space as $k^+ \to 0$.

Under the additional gauge constraint $A^-(x^- = 0) = 0$, the free propagator in the light cone gauge is no longer the usual free propagator given by Eq. (39). To see why this is so, gauge transform the free propagator to a gauge where $A^-(x^- = 0)$:

$$A'^- = A^- - A^-(x^- = 0). \tag{58}$$

Hence, $\partial^- \Lambda = A^-(x^- = 0) \Rightarrow \Lambda = \frac{1}{\partial^-} A^-(x^- = 0)$. Since $A'^+ = A^+ - \partial^+ \Lambda$ and Λ has no functional dependence on x^-, $A'^+ = A^+ \equiv 0$. The Green's function in the light cone gauge with the additional gauge fixing is given by

$$G'_{ij} = i\langle A'^i A'^j \rangle = i \left\langle \left(A^i - \partial^i \frac{1}{\partial^-} A^-(x^- = 0) \right) \left(A^j - \partial^j \frac{1}{\partial^-} A^-(y^- = 0) \right) \right\rangle. \tag{59}$$

Fourier transforming Eq. (39) to coordinate space, and substituting the results in the above equation, we obtain the following expression for the free Green's function in the gauge $A'^+ = 0, A'^-(x^- = 0) = 0$:

$$G_{ij}^{ab} = -\delta^{ab} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2 - i\epsilon} \left[\delta_{ij} - \frac{k_i k_j}{k^- k^+} \left(e^{-ik^+ y^-} + e^{ik^+ x^-} - 2e^{ik^+(x^- - y^-)} \right) \right]. \tag{60}$$

We will show later that the small fluctuation Green's function reduces to the above expression in the absence of an external source.

The reason why the prescription $A^-(x^- = 0) = 0$ is not often employed is because the types of fields one ordinarily considers are oscillatory at infinity, and therefore vanish in the Riemann-Lebesgue sense. In such cases it is more convenient to choose $1/\partial^+ = \int_{-\infty}^{x^-}$ or $1/\partial^+ = -\int_{x^-}^{\infty}$ since in the sense described above one can ignore the contribution from infinity [10]. In the Leibbrandt-Mandelstam prescription, positive frequencies propagate forward in time and negative frequencies propagate backward in time. For our problem where we have a nucleus acting as a source of charge, the fields do not vanish at $x^- = \pm\infty$. There is therefore no advantage to choosing an integration prescription with one of its integration limits at either $\pm\infty$.

We can now combine terms together to get an expression for the small fluctuations propagator in the external

background field. The derivation of $G_{ij(\mathrm{II})}$ from Eq. (51) is rather subtle and we shall elaborate a little on it. We can easily do the k^+ and k_t integrations in Eq. (52) to get

$$\delta \tilde{G}^{++}_{(\mathrm{II})}(x,y) = \tau_a \tau_a \delta^{(2)}(x_t - y_t)\delta(x^- - y^-) \int \frac{dk^-}{2\pi} \frac{e^{-ik^-(x^+-y^+)}}{(k^-)^2 - i\epsilon} .$$

It is clear that we have locality in both transverse and minus coordinates. Hence there will not be any contributions from the regions $\theta(x^-)\theta(-y^-)$ and $\theta(-x^-)\theta(y^-)$. Then

$$(D^i)_{(x)}(D^j)_{(y)}\delta \tilde{G}^{++}_{(\mathrm{II})} = [\theta(-x^-)\theta(-y^-)\tau_a\tau_a \; \partial_i\partial_j \delta^{(2)}(x_t - y_t) + \theta(x^-)\theta(y^-) F_a(x_t)F_a(y_t) \; \partial_i\partial_j\delta^{(2)}(x_t - y_t)]$$

$$\times \delta(x^- - y^-) \int \frac{dk^-}{2\pi} \frac{e^{-ik^-(x^+-y^+)}}{(k^-)^2 - i\epsilon} . \tag{61}$$

We now write the δ functions in their Fourier representations and replace

$$\frac{1}{(k^-)^2} \to \frac{1}{k^2}\left(\frac{k_t^2}{(k^-)^2} - 2\frac{k^+}{k^-}\right) . \tag{62}$$

Finally, making use of the identity

$$\frac{1}{\partial_-} e^{ik^+x^-}\theta(x^-) = \theta(x^-)\frac{(e^{ik^+x^-} - 1)}{ik^+} , \tag{63}$$

which we also used in deriving $G_{ij(\mathrm{I})}$, we can now write Eq. (51) as

$$G^{\alpha\beta;\alpha'\beta'}_{ij(\mathrm{II})}(x,y) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2 - i\epsilon}\left(\frac{p_t^2}{(p^-p^+)^2} - \frac{2}{p^+p^-}\right) p_i p_j \left(e^{ip^+x^-} - 1\right)\left(e^{-ip^+y^-} - 1\right)[\theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'}$$

$$+ \theta(x^-)\theta(y^-)F_a^{\alpha\beta}(x_t)F_a^{\alpha'\beta'}(y_t)] . \tag{64}$$

The derivation of an analogous expression for $G_{ij(\mathrm{I})}$ in Eq. (50) follows directly from the prescription rules discussed above. Our final expression for the Green's function in the light cone gauge is then

$$G^{\alpha\beta;\alpha'\beta'}_{ij}(x,y) = -\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2 - i\epsilon}\left\{\left[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(2e^{ip^+(x^--y^-)} - e^{-ip^+y^-} - e^{ip^+x^-})\right]\right.$$

$$\times [\theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} + \theta(x^-)\theta(y^-) F_a^{\alpha\beta}(x_t)F_a^{\alpha'\beta'}(y_t)]$$

$$+ \theta(-x^-)\theta(y^-) \int \frac{d^2q_t}{(2\pi)^2}d^2z_t \; e^{i(q^+-p^+)y^-} e^{i(p_t - q_t)(y_t - z_t)}F_a^{\alpha\beta}(z_t)F_a^{\alpha'\beta'}(y_t)\left[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(e^{ip^+x^-} - 1)\right.$$

$$+ \frac{q_iq_j}{p^-q^+}(e^{-iq^+y^-} - 1) + \left.\frac{p_iq_jp_t \cdot q_t}{(p^-p^+)(p^-q^+)}(e^{ip^+x^-} - 1)(e^{-iq^+y^-} - 1)\right]$$

$$+ \theta(x^-)\theta(-y^-) \int \frac{d^2q_t}{(2\pi)^2}d^2z_t \; e^{i(p_t - q_t)(z_t - x_t)}e^{-i(q^+-p^+)x^-}$$

$$\times F_a^{\alpha\beta}(x_t)F_a^{\alpha'\beta'}(z_t)\left[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(e^{-ip^+y^-} - 1) + \frac{q_iq_j}{p^-q^+}(e^{iq^+x^-} - 1)\right.$$

$$+ \left.\left.\frac{p_jq_ip_t \cdot q_t}{(p^-p^+)(p^-q^+)}(e^{-ip^+y^-} - 1)(e^{iq^+x^-} - 1)\right]\right\} , \tag{65}$$

where $q^+ = p^+ + \frac{q_t^2 - p_t^2}{2p^-}$. Equation (65) is the main result of this paper. In the limit where the source vanishes, $U = 1$, $q_t \to p_t$, and $q^+ \to p^+$, one can use the identity $p^2 = -2p^+p^- + p_t^2$ to show that the above result reduces to the expression in Eq. (60), as it should. Notice further, that our expression for the gluon propagator in the background field does not obey the parity and translational symmetries. This is not surprising because the background field itself explicitly breaks these symmetries.

In a forthcoming work, the small fluctuation Green's function above will be used to compute the gluon distribution function in the Weizsäcker-Williams background field to $O(\alpha_s^2)$. There we will show explicitly that when computing the gluon distribution function from expression (65), the only prescription needed to deal with poles is the Feynman (causal) $i\epsilon$ prescription. In other words, the nature of the additional p^- poles (besides the usual one in $p^2 - i\epsilon$) in the Green's function is unimportant. Keep in mind that there are no additional poles in p^+

since their residues vanish. As a result, we never have to use any additional $i\epsilon$ prescription to compute the gluon distribution function to this order. We are not sure if this happens in all orders of perturbation theory or if it is an amusing coincidence specific to $O(\alpha_s^2)$.

VI. SUMMARY

In this paper we have computed the propagator for transverse gluons in the light cone gauge $A^+ = 0$ and in the presence of a background Weizsäcker-Williams field produced by the valence quarks of a heavy nucleus. We have argued that such a propagator can be obtained from the propagator in the $A^- = 0$ gauge by a gauge transformation. This method has the advantage of avoiding the difficulties introduced by the singular behavior of the fields at $x^- = \pm\infty$ caused by the presence of the sources. The gauge transformation requires a choice for the inverse of the operators ∂^-, ∂^+. We chose these to be such that the gluon fields satisfy the boundary condition $\delta A^-|_{x^-=0} = 0$ and the usual Feynman causal behavior. The expression for the gluon propagator in $A^+ = 0$ gauge is necessary if we want to extract from it the information about the gluon density which can be compared to experimental measurements.

In the above analysis, there are still many subtle questions remaining about the independence of physical quantities from the precise choice of the regularization prescription for the small k^- and k^+ singularities. We have shown that there is a gauge where there are no singularities and that the subsequent singularities in the light cone gauge Green's function arise from the gauge transformation relations. Then, it is only sensible that the physical quantities computed here will be independent of the nature of these singularities.

At the very least it seems we have a consistent prescription. It should be noted that, because of the gauge choice, there is no ambiguity about the limit $k^+ \to 0$. The propagator is smooth in this limit as a consequence of the way we have specified the residual gauge degree of freedom. If it were only the free transverse propagator which was of interest, we would have a nonsingular propagator at small k^+. The problem we have with the distribution function is however at small k^-. These singularities have been regularized in the only causal way possible, by an $i\epsilon$ prescription. The question of physical interest is whether these small k^-, or correspondingly, large k^+ singularities cause any problems at higher orders. It has been argued extensively in the literature that the prescription we chose for k^- is essential to prove the renormalizability of Yang-Mills theories in the light cone gauge [11].

ACKNOWLEDGMENTS

[1] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994).
[2] A. H. Mueller, Nucl. Phys. **B415**, 373 (1994).
[3] L. McLerran and R. Venugopalan, Phys. Rev. D **50**, 2225 (1994).
[4] A. C. Kalloniatis and H. C. Pauli, Heidelberg Report No. MPIH-V15-1993 (unpublished).
[5] A. H. Mueller, Lectures at *NATO Advanced Study Institute: Frontiers in Particle Physics*, Cargese, France, 1994 [Columbia University Report No. CU-TP-658].
[6] S. J. Brodsky and G. P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989), p. 93.
[7] S. J. Brodsky and H.-C. Pauli, in *Recent Aspects of Quantum Fields*, Proceedings of the 30th Schladming Winter School in Particle Physics, Schladming, Austria, 1991, edited by H. Mitter and H. Gausterer, Lecture Notes in Physics Vol. 396 (Springer, Berlin, 1991).
[8] G. Leibbrandt, Phys. Rev. D **29**, 1699 (1984); S. Mandelstam, Nucl. Phys. **B213**, 149 (1983).
[9] A. Bassetto, M. Dalbosco, I. Lazzizzera, and R. Soldati, Phys. Rev. D **31**, 2012 (1985).
[10] J. B. Kogut and D. H. Soper, Phys. Rev. D **8**, 2901 (1970).
[11] G. Leibbrandt, Rev. Mod. Phys. **59**, 1067 (1987); A. Bassetto, G. Nardelli, and R. Soldati, *Yang-Mills Theories in Algebraic Non-Covariant Gauges* (World Scientific, Singapore, 1991).