Spin effects in the radiative $b \to s\gamma g$ decay of a polarized b quark

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Using the general formalism of structure functions (SF's), the polarization effects in the rare radiative decay of polarized b quarks, $b \to s\gamma g$, are analyzed. Parametrizing the matrix element for the decay $b \to s\gamma g$ in terms of weak magnetic moments, which describe the transitions $b \to s\gamma$ and $b \to sg$, we find exact expressions for the SF's which are valid for a wide class of models for the FCNC's. We demonstrated that the different asymmetries in this decay are very sensitive to the model of FCNC's. The numerical calculations (with and without QCD corrections) show that asymmetry in $b \to \gamma(sg)$ is large in absolute value and its energy dependence is very sensitive to the choice of model. We also analyze the relative importance of different contributions to the asymmetry in $b \to \gamma(sg)$.

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I. INTRODUCTION

The $b \to s\gamma g$ decay, like the simpler decay $b \to s\gamma$, must be extremely sensitive to the structure of fundamental interactions at the electroweak scale. Being a typical flavor-changing neutral current (FCNC) process, it does not arise at the tree level in the standard model (SM), and takes place only at the one-loop level. It is possible that models beyond the standard model such as two Higgs doublet models, supersymmetry (SUSY), etc., having interesting implications for these decays.

The process $b \to s\gamma g$ is particularly interesting because its rate is intermediate between the simplest radiative decay $b \to s\gamma$, and the most of the other FCNC processes involving leptons, $b \to sl^+l^-$, or photons, $b \to s\gamma\gamma$.

A deeper theoretical investigation of the radiative B decays is motivated by the new experimental achievements in this direction. Recently the CLEO Collaboration presented experimental results on the inclusive $B \to \gamma X_s$ [1] and exclusive $B \to K^*\gamma$ [2] decays:

$$\mathcal{B}(B \to \gamma X_s) = (2.32 \pm 0.51 \pm 0.32 \pm 0.2) \times 10^{-4}\ , \quad (1)$$

$$\mathcal{B}(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}\ . \quad (2)$$

An attractive property of the radiative B decays is that the measurements of the photon energy spectra in their inclusive and exclusive decays [3] will provide independent measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ts} and V_{tb}, which can be tested against the ones measured from B-$\bar{B}$ mixing.

From this point of view, investigation of the $b \to s\gamma g$ decay becomes very interesting, since this decay gives a nontrivial contribution to the photon energy spectrum in inclusive $B \to \gamma X_s$ decay. For example, the analysis of (1) yields the following bound for CKM matrix elements [3,4]:

$$0.62 \le \frac{|V_{ts}|}{|V_{cb}|} \le 1.1 \quad (3)$$

at $m_t = 176$ GeV [5].

Measurements of only the total branching ratios for the radiative b decays do not allow one to distinguish between the SM and the two Higgs doublet model (2HDM) at $m_H = 600$ GeV, independently of the accuracy of future experiments [6]. Thus one must resort to the polarization effects in different FCNC processes. The simplest ones are the asymmetries of the polarized b-quark decays, $b \to s\gamma$ and $b \to s\gamma g$. It is well known that the b quarks are produced with 94% polarization at the CERN e^+e^- collider LEP in $e^+e^- \to b\bar{b}$, process at the Z peak (at $\sin^2\theta_W = 0.2321 \pm 0.0004$).

Despite the large polarization of the b quark, its measurement is not a simple task. Of course the P-odd longitudinal b-quark polarization can be measured in the weak decays of the b quark. But the main new problem is how the polarization transfer from the heavy quark produced in Z decay, $Z^0 \to b\bar{b}$, to the experimentally observed hadron with nonzero spin [7] takes place. One can assume that the Λ_b baryon, which accounts roughly for 10% of all b hadrons, retains the initial b quark spin, if produced directly. A naive spin-counting model predicts a 47–94 % range for the Λ_b polarization in the LEP conditions. Therefore, the measurement of Λ_b polarization can serve in the analysis of b-quark polarization.

The next problem is the measurement of the Λ_b polarization. But, using the exclusive Λ_b decays, measuring its degree of polarization is not without problems. Firstly, the b baryons are best observed inclusively at LEP, via an

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excess of jets containing a hard lepton and a charged (correlated) Λ_s. On the contrary, suggestions for measuring polarization in inclusive semileptonic Λ_b decay use only the electron spectrum, which is not sufficiently sensitive to Λ_b polarization; moreover, the fragmentation uncertainties introduce additional difficulties.

Another possible method is based on the measurement of the special ratio $y = \langle E_c \rangle / \langle E_\nu \rangle$ of the average energy of the produced leptons [8].

But in any case, the b-quark polarization is large and therefore polarization effects can give new information about the dynamics of the rare decays. The large b-quark polarization in $e^+e^- \to Z^0 \to b\bar{b}$ allows us to study the different asymmetries in the decay $b \to s\gamma g$. The corresponding amplitude for the decay $b \to s\gamma g$ is defined by two different magnetic moments, which correspond to the transitions $b \to s\gamma(\tilde{F}_2)$ and $b \to sg(F_2)$, and by the specific form factor of the box diagram (Fig. 1). Both functions F_2 and $\tilde{F}_2$ depend on the fundamental parameters, namely, the top quark mass in the SM, and the mass of the charged Higgs boson, m_H and $\tan\beta = v_1/v_2$ [9] (v_1 and v_2 are the vacuum expectations values for the Higgs sector of the 2HDM) in the 2HDM.

Although the polarization effects in $b \to s\gamma$ do not depend on the QCD corrections, this is not the case for the above-mentioned form factors in the decay $b \to s\gamma g$. For example, in SM, QCD corrections increase (in absolute value) both the photonic $\tilde{F}_2$ and the gluonic F_2 weak magnetic moments. As a result, the total probability of the $b \to s\gamma g$ is increased essentially [10–16]. And the final answer for different observable characteristics of the decay $b \to s\gamma g$ depends on the relative values of F_2 and $\tilde{F}_2$. Therefore, in the SM the process $b \to s\gamma g$ contains additional information. Thus the study of the energy spectra of produced particles in the different inclusive experiments, such as $b \to \gamma(sg)$ and $b \to g(s\gamma)$ or $b \to s(\gamma g)$ (where parentheses contain undetected particles), will be useful. The specific infrared behavior of both possible pole mechanisms (Fig. 1) of the decay $b \to s\gamma g$, whose amplitudes are defined by the corresponding magnetic moments of $b \to s\gamma$ and $b \to sg$ transitions, indicates the kinematical regions for the $b \to s\gamma g$ decay where only one (F_2 or $\tilde{F}_2$) form-factor contribution is important. For intermediate values of γ or g energies, the box-diagram contribution and different interference contributions are essential. Therefore, such regions are interesting for the determination of the relative signs of different form factors. Such information has a primary meaning not only for studying the QCD corrections but also for determining the correct FCNC model as well.

But the most sensitive observables that are sensitive to delicate details of the mechanisms of the decay $b \to s\gamma g$ must be those related to the polarization effects. All polarization effects are important for the reconstruction (partially or fully) of the complicated spin structure of the amplitude of the $b \to s\gamma g$ decay. In principle, the polarization effects allow us to check the relative contributions of different mechanisms which are predicted in the framework of the standard short-range calculations.

The simplest, and the cheapest, polarization effects in the rare b decays are the different asymmetries induced by the initial b-quark polarization. As mentioned above, such a situation is realized in the reaction $e^+e^- \to b\bar{b}$ at the Z peak. The P-odd longitudinal polarization of the produced b quarks induces at least two independent asymmetries, if two particles are detected in the final state of $b \to s\gamma g$. Inclusive decays $b \to \gamma(gs)$ or $b \to g(\gamma s)$ are characterized by only one asymmetry. We will show that such asymmetries are P odd, but T even, and they are the consequence of nonconservation of P parity in the decay $b \to s\gamma g$. The transversal (relative to the decay plane) b-quark polarization induces P-even, but T-odd asymmetry in the $b \to s\gamma g$ decay. For this decay there are at least three different sources of such T-odd asymmetry.

The complexity of the various elements of the CKM matrix.

The complexity of the decay amplitude for $t \geq 4m_c^2$ (t is the square of effective $s+g$ mass, m_c is the mass of c quark) due to the box diagram contribution. From the general point of view such complexity is a result of unitarity conditions in the γg channel, due to the chain of transitions: $b \to sc\bar{c} \to sg\gamma$.

The effects of final state interaction. The most important contribution is due to the strong $g+s$ interaction (without any threshold).

Note that transversally polarized b quark could be produced in $e^+e^- \to b\bar{b}$ reaction at relatively small energies, for example, in future B factories.

The polarization effects of produced photons in the unpolarized $b \to s\gamma g$ decay was studied in [17] and [18].

In this work we study the polarization effects induced by the longitudinal polarization of the b quark in the $b \to s\gamma g$ decay in the framework of the SM and two Higgs doublet models [9].

Our paper is organized as follows. In Sec. II we write the amplitude of the $b \to s\gamma g$ decay in terms of form factors $F_2(b \to sg)$ and $\tilde{F}_2(b \to s\gamma)$, and Q (the contribution of the box diagram). Both form factors F_2 and $\tilde{F}_2$ are calculated in the framework of the 2HDM as functions of m_t, m_H, and $\tan\beta$. We analyze the effects of the QCD corrections on the dependences of both form factors F_2

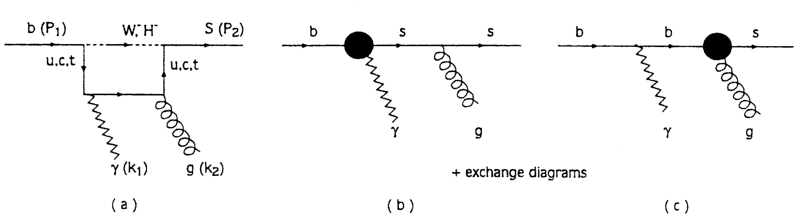

FIG. 1. (a) The box diagrams with W and H exchanges. The flavor-changing photon and gluon vertices are given by their weak magnetic form factors depicted as blobs in (b) and (c), respectively. The wave (spiral) line represents photon (gluon).

and $\tilde{F}_2$ to these parameters, and compare the predictions of SM with predictions of those 2HDM. The SF's which determine the asymmetries in the polarized b-quark decay (with the detection of two produced particles) are calculated in Sec. III. The expressions for these SF's in terms of the corresponding form factors are valid in the general case, and do not depend on the FCNC model. We obtain the expressions for different asymmetries in terms of SF's and discuss the consequences of the radiative zeros [19] in the decay amplitude corresponding to the massless vector bosons (γ or g). The inclusive P-odd asymmetries for the decays $b \to \gamma(gs)$ or $b \to g(\gamma s)$ are calculated in Sec. IV. Section V contains the discussion of results and the numerical calculations.

II. MATRIX ELEMENT AND WEAK MAGNETIC FORM FACTORS

The amplitude for the process $b \to s\gamma g$, corresponding to the Feynman diagrams in Fig. 1, can be written in the form

$$\begin{aligned} M &= \frac{eg_s}{2\pi^2}\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\epsilon_\mu^*(k_1)\epsilon_\nu^{(a)*}(k_2)\bar{s}(p_2)T_{\mu\nu}\frac{\lambda^a}{2}b(p_1)\,,\\ T_{\mu\nu} &= -\frac{2}{3}i[1+2Q(z_c)/z_c][(k_1-k_2)_\alpha\epsilon_{\alpha\mu\nu\beta}-\frac{1}{k_1k_2}\epsilon_{\rho\sigma\mu\beta}k_{2\rho}k_{1\sigma}k_{1\nu}+\frac{1}{k_1k_2}\epsilon_{\rho\sigma\mu\beta}k_{2\rho}k_{1\sigma}k_{2\mu}]\gamma_\beta L\\ &\quad+\tilde{F}_2\left(\frac{2p_{2\nu}+\gamma_\nu\hat{k}_2}{2k_2p_2}\sigma_{\mu\alpha}k_{1\alpha}-\sigma_{\mu\alpha}k_{1\alpha}\frac{2p_{1\nu}+\gamma_\nu\hat{k}_2}{2k_2p_1}(m_bR+m_sL)\right)\\ &\quad-\frac{1}{3}F_2\left(\frac{2p_{2\mu}+\gamma_\mu\hat{k}_1}{2k_1p_2}\sigma_{\nu\alpha}k_{2\alpha}-\sigma_{\nu\alpha}k_{2\alpha}\frac{2p_{1\mu}+\gamma_\mu\hat{k}_1}{2k_1p_1}(m_bR+m_sL)\right)\,. \end{aligned} \tag{4}$$

Here $\epsilon_\mu(k_1)$ is the four-vector of real photon polarization ($\epsilon\cdot k_1=0, k_1^2=0$), ϵ_ν^a is the four-vector of real gluon polarization ($\epsilon^a\cdot k_2=0, k_2^2=0$), λ^a are the SU(3) Gell-Mann matrices and $a=1,\ldots,8$ is the color index. Function $Q(z_c)$ corresponds to the box diagram with a c quark in the internal lines, $\tilde{F}_2=\tilde{F}_2^{\rm SM}+\tilde{F}_2^{\rm 2HDM}, F_2=F_2^{\rm SM}+F_2^{\rm 2HDM}$, and $\tilde{F}_2^{\rm SM}(F_2^{\rm SM})$ and $\tilde{F}_2^{\rm 2HDM}(F_2^{\rm 2HDM})$ describe weak magnetic moment of the $b\to s\gamma(b\to sg)$ transition in the SM and 2HDM models. The functions $F_s^{\rm SM}$ and $\tilde{F}_2^{\rm SM}$ were calculated in [4,20,21] and $\tilde{F}_2^{\rm 2HDM}$ and $F_2^{\rm 2HDM}$ in [9,10,22]:

$$Q(z_c) = -2\arctan^2\left(\frac{z_c}{4-z_c}\right)^{1/2},\quad z_c\le 4\,, \tag{5}$$

$$\begin{aligned} Q(z_c) &= -\frac{1}{2}\pi^2+2\ln^2\{\tfrac{1}{2}[z_c^{1/2}+(z_c-4)^{1/2}]\}-2i\pi\ln[\tfrac{1}{2}(z_c^{1/2}+(z_c-4)^{1/2})], z_c\ge 4\\ \tilde{F}_2^{\rm SM} &= \frac{x}{24(x-1)^4}[6x(3x-2)\ln x+(x-1)(7-5x-8x^2)]\,,\\ \tilde{F}_2^{\rm 2HDM} &= \frac{\tilde{x}}{24(\tilde{x}-1)^4}\left[v^2\left(2\tilde{x}(3\tilde{x}-2)\ln\tilde{x}+\frac{\tilde{x}-1}{3}(7-5\tilde{x}-8\tilde{x}^2)\right)+2vv'(\tilde{x}-1)[2(2-3\tilde{x})\ln\tilde{x}+3-8\tilde{x}+5\tilde{x}^2]\right]\,,\\ F_2^{\rm SM} &= -\frac{x}{8(x-1)^4}[6x\ln x+(x-1)(x^2-5x-2)]\,,\\ F_2^{\rm 2HDM} &= -\frac{1}{24(\tilde{x}-1)^4}\{v^2[6\tilde{x}\ln\tilde{x}+(\tilde{x}-1)(\tilde{x}^2-5\tilde{x}-2)]+6vv'(\tilde{x}-1)(-2\ln\tilde{x}-3+4\tilde{x}-5\tilde{x}^2)\}\,, \end{aligned} \tag{6}$$

where $z_c=2k_1k_2/m_c^2$, $x=m_t^2/m_W^2$, $\tilde{x}=m_t^2/m_H^2$; m_H is the mass of the charged Higgs boson, which appears in the 2HDM models.

Note that the complex nature of the form factor $Q(z_c)$ above $z_c=4$ is a result of the unitarity condition for the decay amplitude in the channel $b\to s(\gamma g)$.

In (4) we used the unitarity property of CKM matrices: namely,

$$V_{tb}V_{ts}^*+V_{cb}V_{cs}^*+V_{ub}V_{us}^*=0\,. \tag{7}$$

From direct experimental measurement it is known that the third term in (7) is negligible compared to the second, yielding

$$V_{cb}V_{cs}^*=-V_{tb}V_{ts}^*\,. \tag{8}$$

We would like to note that the Lagrangian describing the interaction between the quarks and the charged Higgs fields has the form [23]

$$L=\sqrt{\frac{4G_F}{\sqrt{2}}}(m_uv\bar{u}_iLd_j-m_dv'\bar{u}_iRd_j)V_{ij}H^++\text{H.c.}\,, \tag{9}$$

where m_u and m_d are masses of up and down quarks.

In the literature there are two widely used versions of the 2HDM, known as model I and model II. Model I and model II can be labeled in the form

$$v = \begin{cases} v' = \cot\beta \ \text{(model I)}, \\ -1/v' = \cot\beta \ \text{(model II)}. \end{cases} \tag{10}$$

Note that one recovers SM in the limit $v = v' = 0$.

Therefore, at least two new independent parameters appear in the general 2HDM model: namely, m_H and $\tan\beta$. It is important that only these parameters determine, as the tree level, all the phenomenology of the 2HDM scalar sector which is necessary for the physics at e^+e^- and the hadron colliders.

One can see that the matrix element (4) satisfies the gauge invariance conditions with respect to the photon and the gluon:

$$T_{\mu\nu}k_{1\mu} = T_{\mu\nu}k_{2\nu} = 0 \ . \tag{11}$$

Note that expression (4) has an interesting property. Namely, the resulting amplitude of the decay $b \to s\gamma g$ is determined by the same magnetic transition form factors which describe the $b \to s\gamma$ and $b \to sg$ decays. It is an illustration of the validity of the Low's low-energy theorem for a high-energy decay such as $b \to s\gamma g$ (see [22] and [24]).

Note that in Eq. (4) for the amplitude of $b \to s\gamma g$ decay we have not included the QCD corrections. The QCD corrections [10–15] can be described by the effective Hamiltonian

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{j=1}^{8} C_j(\mu)O(\mu) \tag{12}$$

where $C_j(\eta)$ are the Wilson coefficients at some scale η. The dominant operators for our problem are O_1, O_2, O_7, and O_8, where O_7 and O_8 describe the magnetic-moment-type operators of dimension 6:

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2}\bar{s}\sigma_{\mu\nu}(m_b R + m_s L)bF_{\mu\nu} \ , \\ O_8 &= \frac{g_s}{16\pi^2}\bar{s}\sigma_{\mu\nu}(m_b R + m_s L)\frac{\lambda^a}{2}bG^a_{\mu\nu} \ . \end{aligned} \tag{13}$$

The contribution of the two diagrams associated with the operator O_1 vanishes, due to the specific color behavior, i.e., due the proportionality of $\text{Tr}\lambda^a \to 0$. Therefore in the following, we need only three dominant coefficients, $C_2(\mu)$, $C_7(\mu)$, and $C_8(\mu)$. The complete leading logarithmic calculations for these coefficients lead to the result [6,12,15]

$$\begin{aligned} C_2(m_b) &= \tfrac{1}{2}[\eta^{-6/23} + \eta^{12/23}]C_2(M_W) \ , \\ C_7(m_b) &= \eta^{-16/23}\{C_7(M_W) + \tfrac{8}{3}[\eta^{2/23} - 1]C_8(M_W)\} \\ &\quad -0.175C_2(M_W) \ , \\ C_8(m_b) &= \eta^{-14/23}C_8(M_W) - 0.122C_2(M_W) \ , \end{aligned} \tag{14}$$

where $\eta = \alpha_s(m_b)/\alpha_s(M_W), \alpha_s = g_s^2/4\pi$. At the electroweak scale $\mu = M_W$, i.e., for $\eta = 1$, we have

$$\begin{aligned} C_2(M_W) &= 1 \ , \\ C_7(M_W) &= \tilde{F}_2 = \tilde{F}_2^{\text{SM}} + \tilde{F}_2^{\text{2HDM}} \ , \\ C_8(M_W) &= F_2 = F_2^{\text{SM}} + F_2^{\text{2HDM}} \ . \end{aligned} \tag{15}$$

Note that $C_7(M_W)$ and $C_8(M_W)$ are model dependent, but Eq. (14) holds for any model.

Let us now discuss the effect of QCD corrections on the values of the form factors F_2 and $\tilde{F}_2$ in SM and the two version of the 2HDM for the different values of the fundamental parameters $\tan\beta$ and m_H. Since its discovery, the t quark mass has been determined and is now known [5]. We will use, in our estimates, the central values of this mass, namely $m_t = 176$ GeV. For the estimation of $F_{2,\text{eff}} = C_8(m_b)$ and $\tilde{F}_{2,\text{eff}} = C_7(m_b)$, we use the following expression for $\alpha_s(m_b)$:

$$\begin{aligned} \alpha_s(m_b) &= \frac{12\pi}{(33 - 2N_F)\ln m_b^2/\Lambda^2} \\ &\quad \times \left(1 - \frac{6(153 - 19N_F)}{(33 - 2N_F)^2}\frac{\ln\ln(m_b^2/\Lambda^2)}{\ln(m_b^2/\Lambda^2)}\right) \end{aligned} \tag{16}$$

and $N_f = 5, \Lambda^{(5)} = 225$ MeV. The results of numerical estimates of all four form factors F_2, $\tilde{F}_2$, $F_{2,\text{eff}}$, and $\tilde{F}_{2,\text{eff}}$ are depicted in Figs. 2 and 3.

In SM, QCD corrections increase in absolute value the photon weak form factor $\tilde{F}_{2,\text{eff}}$ from $\tilde{F}_2 = 0.1969$ to $\tilde{F}_{2,\text{eff}} = 0.3112$ and the gluon form factor F_2, from $F_2 = 0.078$ to $F_{2,\text{eff}} = 0.18$. From these it follows that $|\tilde{F}_{2,\text{eff}}| \geq |F_{2,\text{eff}}|$.

The behavior of these form factors in the 2HDM de-

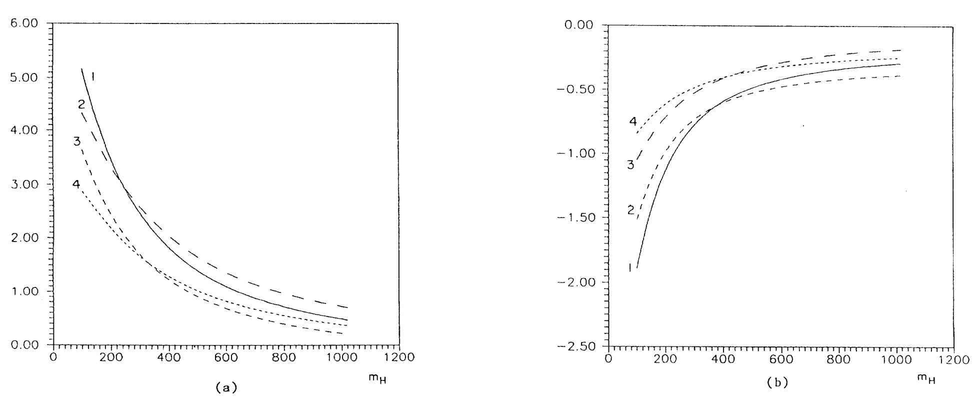

FIG. 2. The m_H dependence of the form factors $\tilde{F}_2$, F_2, $\tilde{F}_{2,\text{eff}}$, and $F_{2\text{eff}}$ calculated for $\tan\beta = 0.2$, $m_t = 176$ GeV; (a) model I and (b) model II. Here lines 1, 2, 3, and 4 correspond to the $\tilde{F}_2$, F_2, $\tilde{F}_{2,\text{eff}}$, and $F_{2,\text{eff}}$ respectively.

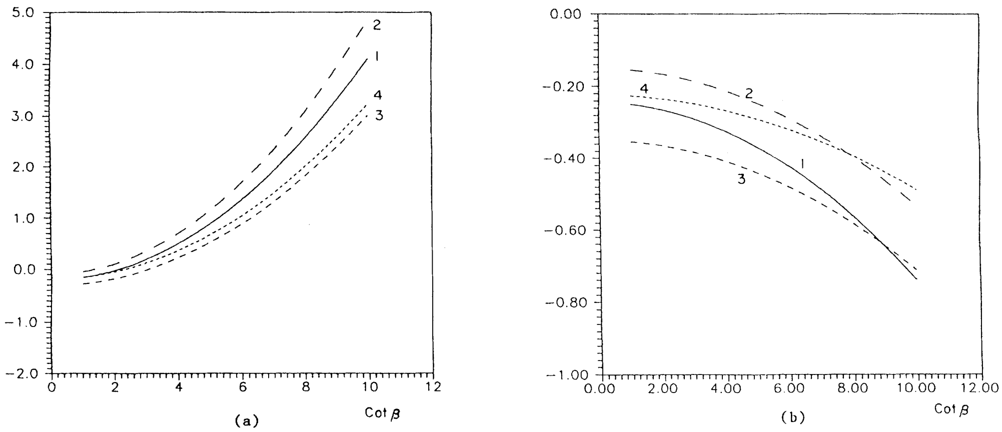

FIG. 3. $\cot\beta$ dependence of the form factors $\tilde{F}_2$, F_2, $\tilde{F}_{2,\text{eff}}$, and $F_{2,\text{eff}}$ at $m_t = 176$ GeV, $m_H = 700$ GeV; (a) model I and (b) model II.

pends on the versions of this model (namely, model I or model II) and on the values of the parameters $\tan\beta$ and m_H. For example, the most interesting predictions of model I are the following: for the suitable values of $\tan\beta$ and m_H all four form factors are positive and their values are larger than the corresponding values in SM; the form factors F_2, $\tilde{F}_2$, $F_{2,\text{eff}}$ and $\tilde{F}_{2,\text{eff}}$ are decreasing functions of m_H (at any value of $\tan\beta$); the QCD corrections diminish the form factors $\tilde{F}_2$ and F_2 (for all values of $\tan\beta$ and m_H), so $F_{2,\text{eff}} < F_2$ and $\tilde{F}_{2,\text{eff}} < \tilde{F}_2$; the relative values of $\tilde{F}_2(F_2)$ and $\tilde{F}_{2,\text{eff}}(F_{2,\text{eff}})$ depend on the values of m_H and $\tan\beta$, for instance $\tilde{F}_{2,\text{eff}} > F_{2,\text{eff}}$, if $m_H \leq m_H^0$ and $\tilde{F}_{2,\text{eff}} < F_{2,\text{eff}}$ if $m_H \geq m_H^0$, where the mass m_H^0 depends on $\tan\beta$, for example, for $\tan\beta = 0.2$, $m_H^0 = 300$ GeV; all four form factors $F_2, \tilde{F}_2, F_{2,\text{eff}}$, and $\tilde{F}_{2,\text{eff}}$ increase monotonically with $\cot\beta$ (for any values of m_H).

But in the case of model II we have totally different predictions for these form factors: all four form factors F_2, $\tilde{F}_2$, $F_{2,\text{eff}}$, and $\tilde{F}_{2,\text{eff}}$ are monotonically increasing functions of m_H (for any values of $\tan\beta$); for all values of $\tan\beta$ all these form factors are negative and satisfy the inequalities $|\tilde{F}_{2,\text{eff}}| > |F_{2,\text{eff}}|$ and $|\tilde{F}_2| > |F_2|$; the relative values of $\tilde{F}_2(F_2)$ and $\tilde{F}_{2,\text{eff}}(F_{2,\text{eff}})$ depend on m_H, at $m_H > 350$ GeV, $\tilde{F}_{2,\text{eff}}(F_{2,\text{eff}}) < \tilde{F}_2(F_2)$ and, for $m_H < 350$ GeV, $\tilde{F}_{2,\text{eff}}(F_{2,\text{eff}}) > \tilde{F}_2(F_2)$; all four form factors increase in absolute value with increasing $\cot\beta$.

These properties of the form factors $F_{2,\text{eff}}$ and $\tilde{F}_{2,\text{eff}}$ indicate a strong dependence on the parameters $\tan\beta$ and m_H. Moreover, the inclusion of QCD corrections change the relative values of different contributions to the amplitude of the decay $b \to s\gamma g$. All these must have an essential influence on the behavior of different polarization effects in this decay.

What can we say about the mass of m_H and $\tan\beta$? Of course the experimental results from the e^+e^- collider LEP [25] provide the strongest direct limit on the masses of charged Higgs bosons: $m_{H\pm} \geq m_Z/2$. The restrictions on the m_H and $\tan\beta$ can be obtained from the CLEO result on $\mathcal{B}(B_s \to X_s\gamma)$ (1) and our calculations. Assume that $\Gamma(B_s \to X_s\gamma) \sim \Gamma(b \to s\gamma) + \Gamma(b \to s\gamma g)$; so, for $\mathcal{B}(B \to X_s\gamma)$, we obtain (see also [4])

$$\mathcal{B}(B \to X_s\gamma) = 6\frac{\alpha}{\pi}\frac{|a_3^2|}{g(r)[1-2\alpha_2/3\pi f(r)]}0.107 + \frac{\alpha_s\alpha}{\pi^2}\frac{1}{g(r)[1-2\alpha_s/3\pi f(r)]}0.107\{-0.0269a_3 + 0.1389a_1 + 25.66|a_1^2| + 13.27a_1a_3 + 17.68|a_3|^2 + 0.265\}\ . \tag{17}$$

Here and in the future we will use the notation $a_1 = -\frac{1}{3}F_{2,\text{eff}}, a_2 = -\frac{2}{3}(1 + 2Q/z_c)$, and $a_3 = \tilde{F}_{2,\text{eff}}$, $r = m_c/m_b$, and function $g(r) = 1 - 8r^2 + 8r^6 - 24r^4 \ln r$ is the phase volume factor for $\Gamma(b \to cl\nu_l)$. The function $f(r)$ accounts for QCD corrections to the semileptonic $(b \to cl\nu_l)$ decay, $f(r) \equiv 2.51$ (see [26]), and the factor 0.107 is the measured semileptonic branching ratio $\mathcal{B}(B \to Xl\nu_e)$ [27]. In (17) the last term in curly brackets proportional to α_s corresponds to the box diagram contribution, and the first two terms describe the interference terms between box and Figs. 1(b) and 1(c). Using (1) and (17) we get a restriction on the $\tan\beta$ dependence of m_H (Fig. 4) From this figure it follows that in model II for $\tan\beta > 0.22$, the restriction to m_H practically does not depend on $\tan\beta$.

These limits are almost the same coming from $b \to s\gamma$ [28], are comparable to restrictions coming from $Z \to b\bar{b}$, and are stronger than restrictions from $b \to c\tau\nu_\tau$, $B_0 - \bar{B}_0$ mixing, and other FCNC processes [29,30].

III. THE STRUCTURE FUNCTIONS

Because the kinematics of any three-particle decay is determined by two energies of any pair of produced particles, the decay probability can be written in the form

(in the rest frame of decaying b quark)

$$\frac{d^2\Gamma}{dE_\gamma dE_g} = \frac{|M|^2}{64\pi^3 m_b} , \tag{18}$$

where E_γ and E_g are the energies of γ and g, and m_b is the b-quark mass.

Let us establish the dependence of the decay probability to the b-quark polarization. The polarization states of the b quark, as any free fermion with four-momentum p_1 and mass m_b, can be characterized by the four-pseudovector ξ; so the density matrix of the b quark is defined by the standard formula

$$\tfrac{1}{2}(\hat{p}_1 + m_b)(1 + \hat{\xi}\gamma_5) . \tag{19}$$

The moduli square of the decay matrix element can be written in the form

$$|\bar{M}^2| = N[X + Y(\xi)] , \tag{20}$$

where N is the normalization factor. Here the overbar of $\bar{M}^2$ describes the summing over polarizations of final particles in the decay $b \to s\gamma g$, and summing over color states of final g and s, with averaging over color states of initial b quark.

The general expression for Y_ξ (linear in ξ) can be written as [31]

$$\begin{aligned} Y(\xi) = {} & \xi \cdot k_1 S_1(s,t) + \xi \cdot k_2 S_2(s,t) \\ & + \epsilon_{\alpha\beta\gamma\delta}\xi_\alpha k_{1\beta} k_{2\gamma} p_{1\delta} S_3(s,t) , \end{aligned} \tag{21}$$

where $S_i(s,t), i = 1-3$, are real Lorentz-invariant structure functions (SF's), which depend in general on two independent variables: namely, $s = (p_1 - k_1)^2$ and $t = (p_2 - k_2)^2$ or $x_1 = \frac{2E_\gamma}{m_b}$ and $x_2 = \frac{2E_g}{m_b}$. In deriving Eq. (20) the relation $\xi \cdot p_1 = 0$ is used.

The SF's $S_1(s,t)$ and $S_2(s,t)$ describe the P-odd but T-even asymmetries in the decay $b \to s\gamma g$, which are induced by the components of $\vec{\xi}$ (in the b-rest frame) in the decay plane. Such asymmetries are nonzero even if the amplitude for $b \to s\gamma g$ decay is real.

The structure function $S_3(s,t)$ describes the P-even and T-odd decay asymmetry which corresponds, in b-quark rest frame, to the transversal (to the decay plane) polarization of b quarks with a resulting correlation $\vec{\xi} \cdot \vec{k} \times \vec{q}$, where $\vec{k}$ and $\vec{q}$ are three-momenta of γ and g. Such asymmetry can appear only in the kinematical region where the decay amplitude contains an imaginary part. For example, the box diagram [Fig. 1(c)] introduces a nonzero imaginary part in the corresponding contribution in the region with $s + t \geq m_b^2 + m_s^2 - 4m_c^2$.

Let us connect the SF's S_1 and S_2 with the decay asymmetries which are induced by the definite components of $\vec{\xi}$ along or normal to the three-momenta of the final particles. If the photon is produced along the b-quark spin direction then the corresponding asymmetry can be written in the form

$$\begin{aligned} A_\parallel^\gamma & \simeq E_\gamma S_1 + E_g \cos\theta_{\gamma g} S_2 \\ & = \frac{m_b}{2}(x S_1 + y \cos\theta_{\gamma g} S_2) , \end{aligned} \tag{22}$$

where we introduce the following dimensionless variables: $x = 2E_\gamma/m_b$, $y = 2E_g/m_b$, $z = 2E_s/m_b$, $x + y + z = 2$, $s = m_b^2(1-x)$, and $t = m_b^2(1-y)$, and $\theta_{\gamma g}$ is the angle between three-momenta of γ and g.

Any asymmetry in $b \to s\gamma g$ decay is defined by the ratio

$$A = \frac{|\bar{M}(\xi)|^2 - |\bar{M}(-\xi)|^2}{|\bar{M}(\xi)|^2 + |\bar{M}(\xi)|^2} . \tag{23}$$

Therefore the function X in (20) corresponds to the SF characterizing the decay of the unpolarized b quark.

If the photon in the decay $b \to s\gamma g$ is produced normally to the spin direction of the b quark, then only S_2 contributes to the corresponding asymmetry

$$A_\perp^\gamma \simeq E_g \sin\theta_{\gamma g} S_2 = \frac{m_b}{2} y \sin\theta_{\gamma g} S_2 \tag{24}$$

with the same coefficient of proportionality. The angle $\cos\theta_{\gamma g}$ is determined by

$$\cos\theta_{\gamma g} = 1 + 2\frac{1 - \Delta - x - y}{xy} = 1 + 2\rho , \tag{25}$$

where $\Delta = m_s^2/m_b^2$ and $\rho = (1 - \Delta - x - y)/xy$.

Similarly if the gluon is produced along (or normal) to the spin direction of the b quark, the corresponding asymmetries are defined by the combinations of $S_1(x,y)$ and $S_2(x,y)$:

$$\begin{aligned} A_\parallel^g & \simeq E_\gamma \cos\theta_{\gamma g} S_1 + E_g S_2 \\ & = \frac{m_b}{2}[(x + 2\rho)S_1 + y S_2] , \\ A_\perp^g & \simeq E_\gamma \sin\theta_{\gamma g} S_1 = m_b x\sqrt{-\rho(1+\rho)} S_1 . \end{aligned} \tag{26}$$

For completeness, let us find the combinations of the S_1 and S_2 which determine the asymmetries $A_\perp^s$ and $A_\parallel^s$ of s-quark production along ($\|$) or normal ($\perp$) to the b-quark spin directions:

$$\begin{aligned} A_\parallel^s & \simeq E_\gamma \cos\theta_{\gamma s} S_1 + E_g \cos\theta_{gs} S_2 , \\ A_\perp^s & \simeq E_\gamma \sin\theta_{\gamma s} S_1 + E_g \sin\theta_{gs} S_2 , \end{aligned} \tag{27}$$

where $\theta_{\gamma s}$ and θ_{gs} are the angles between γ and s, and g and s, which are determined by the formulas

$$\begin{aligned} \cos\theta_{\gamma s} & = \frac{xz + 2(1 + \Delta - x - z)}{x\sqrt{z^2 - 4\Delta}} , \\ \cos\theta_{gs} & = \frac{yz + 2(1 + \Delta - y - z)}{y\sqrt{z^2 - 4\Delta}} . \end{aligned} \tag{28}$$

Using these formulas it is easy to calculate any exclusive asymmetry in the decay of the polarized b quark, $b \to s\gamma g$.

To describe the kinematics of the decay $b \to s\gamma g$, it is convenient in some cases to change the set of independent kinematical variables. Namely, instead of two energies E_γ and E_g we can use some angle between the corresponding three-momenta and one of the energies E_γ or E_g, $d^2\Gamma/dE_\gamma d \cos_{\gamma g}$ or $d^2\Gamma/dE_g d \cos_{\gamma g}$. It is easy to obtain the relations

$$\frac{d^2\Gamma}{dE_\gamma d\cos\theta_{\gamma g}} = -\frac{d^2\Gamma}{dE_\gamma dE_g}\frac{m_b(m_b-2E_\gamma)E_\gamma}{2[m_b-E_\gamma(1-\cos\theta_{\gamma g})]^2},$$
$$\frac{d^2\Gamma}{dE_g d\cos\theta_{\gamma g}} = -\frac{d^2\Gamma}{dE_\gamma dE_g}\frac{m_b E_g(m_b-2E_g)}{2[m_g-E_g(1-\cos\theta_{\gamma g})]^2}. \tag{29}$$

The gluon energy must be eliminated from the right-hand side of Eq. (29) with the help of the relation

$$E_g = \frac{m_b E_b(m_b-2E_\gamma)}{2[m_b-E_g(1-\cos\theta_{\gamma g})]}. \tag{30}$$

In a similar manner one can obtain the relations

$$\frac{d^2\Gamma}{dE_\gamma d\cos\theta_{\gamma s}} = -\frac{d^2\Gamma}{dE_\gamma dE_s}|\vec{p}|\frac{m_b^2-2mE_s+m_s^2}{(m_b-E_s+|\vec{p}|\cos\theta_{\gamma s})^2}, \tag{31}$$

where on the right side the photon energy must be eliminated with the help of the relation

$$E_\gamma = \frac{(m_b-E_s)^2-\vec{p}^2}{2(m_b-E_s+|\vec{p}|\cos\theta_{\gamma s})}, \tag{32}$$

where $\vec{p}$ is the three-momentum of the s quark. Note that the zero in the denominator at $\cos\theta_{\gamma s}=-1$ is compensated by the corresponding zero in the numerator, so $E_\gamma(\cos\theta_{\gamma s}=-1)=m_b-E_s+|\vec{p}|$.

The formalism of SF's allows us to analyze the inclusive decays of the polarized b quark with the detection of only one particle in the final state. In this case the dependence of the decay probability on the transversal b polarizations disappears and such (inclusive) decay asymmetries depend on SF's S_1 and S_2 only.

So, for example, the spin-dependent contribution to the gluon spectrum is determined by the integral

$$\int(\xi\cdot k_1S_1+\xi\cdot k_2S_2)\delta(p_1-p_2-k_1-k_2)\frac{d^3k_1}{E_\gamma}\frac{d^3p_2}{E_s} = \xi_\alpha[p_{1\alpha}Y_1(t)+k_{2\alpha}Y_2(t)] = \xi\cdot k_2Y_2(t), t=(k_1-k_2)^2 \tag{33}$$

because $\xi\cdot p_1=0$.

The function $Y_2(t)$, which characterizes the asymmetry of the t distribution in the decay $b\to sg\gamma$ (more exactly the correlation of type $\vec{\xi}\vec{q}, \vec{q}$ is the three-momentum of the gluon), can be calculated using the formula

$$Y_2(t) = \frac{2\pi}{y}\int_{x_-(y)}^{x_+(y)}dx\left[\frac{1}{y}\left(x+2\frac{1-\Delta-x-y}{y}\right)S_1(x,y)+S_2(x,y)\right], \tag{34}$$

where the limits of integration are

$$x_+(y) = 1-\frac{\Delta}{1-y},\quad x_-(y)=1-\Delta-y. \tag{35}$$

The contributions to $Y_2(t)$, which is proportional to a_2^2, a_3^2, and a_2a_3, can be calculated analytically without any problems, but the contributions of the other possible three terms (which are proportional to a_1^2, a_1a_2, and a_1a_3) can be calculated only numerically.

However, for the calculation of the spin-dependent contribution to the s-quark energy spectrum, $b\to s\gamma g$, the exact expression for the form factor $Q(u), u=(k_1+k_2)^2$ is not needed, and all the integrations can be computed analytically:

$$\int\delta(p_1-p_2-k_1-k_2)\frac{d^3k_1}{E_\gamma}\frac{d^3k_2}{E_g}(\xi\cdot k_1S_1+\xi\cdot k_2S_2) = \xi_\alpha[p_{2\alpha}P_1(u)+p_{1\alpha}P_2(u)] = \xi\cdot p_2P_1(u). \tag{36}$$

For the SF $P_1(u)$ which characterizes the longitudinal (P-odd) asymmetry of the type $\vec{\xi}\cdot\vec{p}$, one gets the integral

$$P_1(u) = \frac{-2\pi}{\sqrt{(u-m_b^2-m_s^2)^2-4m_b^2m_s^2}}\int_{s-(u)}^{s+(u)}ds\left(-S_2+(S_1-S_2)\frac{us+m_b^2(u+s)-m_b^4-m_s^2(s-m_b^2)}{(u-m_b^2-m_s^2)^2-4m_b^2m_s^2}\right) \tag{37}$$
$$= \frac{2\pi}{\sqrt{z^2-4\Delta}}\int_{x_-(u)}^{x_+(u)}dx\left(-S_2+(S_1-S_2)\frac{xz+2(1+\Delta-x-z)}{z^2-4\Delta}\right),$$

where

$$s_\pm(u) = \frac{m_b^2+m_s^2-u}{2}\pm\frac{1}{2}\sqrt{(m_b^2+m_s^2-u)^2-4m_s^2m_b^2},\quad x_\pm(z)=1-\frac{z}{2}\pm\sqrt{z^2-4\Delta}. \tag{38}$$

These integration limits are simplified considerably in the limit $\Delta\to 0$:

$$s_\pm = \frac{m_b^2-u}{2}\pm\frac{m_b^2-u}{2},\quad 0\le s\le m_b^2-u. \tag{39}$$

Let us also write, for completeness, the spin-dependent contribution to the photon spectrum in the decay $b \to s\gamma g$. In this case it is necessary to calculate the integral

$$\int \xi \cdot k_2 S_2 \delta(p_1 - p_2 - k_1 - k_2) \frac{d^2k_2}{E_g} \frac{d^3p_2}{E_s} = \xi_\alpha [k_{1\alpha} Q_1(s) + p_{1\alpha} Q_2(s)] = (\xi \cdot k_1) Q_1(s) \ . \tag{40}$$

The SF $Q_1(s)$, which depends on one invariant variable only, is determined by the expression

$$\begin{aligned} Q_1(s) &= \frac{-2\pi}{(s-m_b^2)^2} \int_{t_-(s)}^{t_+(s)} dt \left(t + m_b^2 + 2m_b^2 \frac{t - m_b^2}{s - m_b^2} \right) S_2(t) \\ &= \frac{2\pi}{x^2} \int_{y_-(x)}^{y_+(x)} dy \left(y + 2\frac{1 - \Delta - x - y}{x} \right) S_2(x,y) \ , \end{aligned} \tag{41}$$

where $t_+(s) = m_b^2 + m_s^2 - s, t_- = m_s^2 \frac{m_b^2}{s}$ and $y_+(x) = 1 - \Delta/1 - x, y_-(x) = 1 - \Delta - x$.

The resulting asymmetry of the decay $b \to \gamma s g$, with the detection of a photon only, would be determined by the following combinations of SF's:

$$\xi \cdot k_1 \left(Q_1(s) + \frac{2\pi}{s - m_b^2} \int_{t_-(s)}^{t_+(s)} dt S_1(s,t) \right) = \xi \cdot k_1 \left(Q_1(x) + \frac{2\pi}{x} \int_{y_-(x)}^{y_+(x)} dy S_1(x,y) \right) \ . \tag{42}$$

Using the matrix element (4) we can obtain now the expression for the SF's $S_i(x,y)$ in terms of the form factors F_2, $\tilde{F}_2$, and Q. These expressions must be valid for different possible models of FCNC's, such as the SM or 2HDM.

After summing over polarizations of final particles in $b \to s\gamma g$ decay, and averaging over the color states of the initial b quark, and summing over the color states of final s quark and gluon, we can obtain the following expressions for the SF's $S_1, \ldots, S_3$ in terms of dimensionless variables x and y (which are valid in the limit $\Delta = 0$):

$$\begin{aligned} S_1(x,y) = m_b^3 \{ &|a_2|^2 + 2|a_1|^2 (x + y - 1)(xy - x + 1)/x^2(y-1) \\ &+ 2|a_3|^2 (x + y - 1)(2 - y + y^2)/y^2(x-1) \\ &+ \mathrm{Re} a_2 a_1 (-2x + 2xy + y^2 - 2y + 2)/(y-1)x \\ &+ 2\,\mathrm{Re} a_1 a_3 (x + y - 1)(2 - 2x + xy + xy^2 - y^2)/xy(1-x)(1-y) \\ &+ \mathrm{Re} a_2 a_3 (-2 + 2x + xy)/(x-1)y \} \ , \\ S_2(x,y) = m_b^3 \{ &|a_2|^2 + 2|a_1|^2 (x + y - 1)(2 - x + x^2)/x^2(y-1) \\ &+ 2|a_3|^2 (x + y - 1)(xy - y + 1)/y^2(x-1) + \mathrm{Re} a_1 a_2 (xy + 2y - 2)/x(y-1) \\ &+ 2\,\mathrm{Re} a_1 a_3 (x + y - 1)(2 - 2y + xy - x^2 + x^2 y)/xy(1-x)(1-y) \\ &+ \mathrm{Re} a_2 a_3 (x^2 + 2xy - 2x - 2y + 2)/y(x-1) \} \ , \\ S_3(x,y) = 2m_b &\mathrm{Im} a_2 \left(\frac{2-x}{y(x-1)} a_3 + \frac{2-y}{x(y-1)} a_1 \right) \ . \end{aligned} \tag{43}$$

From these expressions one can see that the SF's $S_1(x,y)$ and $S_2(x,y)$ contain all of the six possible contributions, which are induced by three different Feynman diagrams in Fig. 1: namely, three moduli squares and three interference terms. Each such contribution does not depend on the choice of gauge for the photon or gluon polarization vectors, because the contribution of each Feynman diagram satisfies the gauge invariance conditions separately. Nonzero contributions of each Feynman digram to P-odd SF's $S_1(x,y)$ and $S_2(x,y)$ are connected with the P-invariant violation for each mechanism of the $b \to s\gamma g$ decay. The three contributions to SF's $S_1(x,y)$ and $S_2(x,y)$, which are proportional to a_1^2, a_3^2, and $a_1 a_3$, go to zero along the line $x + y = 1$. This line is a part of the border of physical region for $b \to s\gamma g$ decay which corresponds to the production of γ and g with parallel three-momenta (collinear kinematics). Such behavior is the manifestation of the general properties of amplitudes for the process involving gauge bosons with zero mass, namely, a photon or (and) gluon, a phenomena which is known as radiative zeros [19].

The T-odd SF $S_3(x,y)$ contains the contributions of two specific interference terms: namely, $a_1 \mathrm{Im} a_2$ and $a_3 \mathrm{Im} a_2$. We can separate these contributions studying the behavior of the structure functions in the two limiting kinematical regions: namely, $x \to 0 (y \to 1)$ and $x \to 1 (y \to 0)$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We discuss here the results of numerical calculation for the asymmetries in the radiative decay of polarized b quark. For definiteness we choose the x behavior of the asymmetry of the process $b \to \gamma(sg)$ with the detection of photon only. We hope that the results for such an asymmetry will be illustrative enough and will demonstrate the main properties of the polarization effects in the decay $b \to s\gamma g$.

Let us define the corresponding asymmetry with the help of the expression

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = f_0(x)[1 + P_b \cos\theta A(x)] \tag{44}$$

where θ is the angle between spin direction of the b quark and the three-momentum of the photon, and P_b is degree of b-quark polarization. If b quarks are produced in the reaction $e^+e^- \to Z^0 \to \bar{b}b$, then θ is the angle between the three-momenta of b quark and γ. The matrix element square for the $b \to s\gamma g$ decay, summing over the polarization of produced particles, is written in the form

$$|M^2| = X_0(x,y) + \xi \cdot k_1 X_1(x,y) + \xi \cdot k_2 X_2(x,y) + \epsilon_{\alpha\beta\gamma\delta}\xi_\alpha k_{1\beta} k_{2\gamma} p_{1\delta} X_3(x,y) \, . \tag{45}$$

Then the decay asymmetry $A(x)$ can be written as

$$\begin{aligned} A(x) &= \frac{I_1(x)}{I_0(x)} \, , \\ I_0(x) &= \int_{y_-(x)}^{y^+(x)} dy X_0(x,y) \, , \\ I_1(x) &= -\frac{m_b x}{2} \int_{y_-(x)}^{y^+(x)} dy \left(X_1(x,y) + + \frac{1}{x^2}[2(1-x-y) + xy] X_2(x,y) \right) \, , \end{aligned} \tag{46}$$

where $y_-(x) = 1 - \Delta - x$, and $y_+ = 1 - \Delta/(1-x)$ and

$$\begin{aligned} X_0(x,y) = -m_b^4 \{ & |a_2|^2(x+y-2) + |a_1|^2(x+y-1)(2x^2y - x^2 - xy - x + 2)/x^2(y-1) \\ & + |a_3|^2(x+y-1)(2xy^2 - xy - y^2 - y + 2)/y^2(x-1) + \mathrm{Re} a_2 a_1 (x+y-1)(2x+y)/2x \\ & + \mathrm{Re} a_1 a_3 (x+y-1)(4x^2y^2 + 5x^2y - 3x^2 + 5xy^2 - 6xy + x - 3y^2 + y + 2)/2xy(1-x)(1-y) \\ & + \mathrm{Re} a_2 a_3 (x+y-1) \} \, . \end{aligned}$$

Let us also recall that the quantities $X_i(x,y), i = 1,3$, are proportional to the corresponding structure functions S_i.

Any asymmetry in the decay $b \to s\gamma g$ does not depend on the concrete values of CKM matrix elements. This is not an exact statement, however; its accuracy is same as the unitarity relation (9). This behavior of asymmetries is different from the differential decay probability where absolute values are very sensitive to the definite product of CKM matrix elements. On the contrary, all asymmetries which could be expressed as the ratios of the matrix element squares (with different b-quark spin orientations) are sensitive to other fundamental parameters such as m_t (in SM) or m_H and tanβ.

For numerical calculations we use $m_b = 5$ GeV, $m_c = 1.35$ GeV; for the top quark mass we use the central experimental value, namely, $m_t = 176$ GeV [5].

Numerical results for $A(x)$ which are obtained in the SM and in two versions of 2HDM (model I and model II) are presented in Figs. 5–10. Let us mention the most interesting peculiarities of these calculations.

The predicted asymmetry $A(x)$ is large in the absolute value and is characterized by strong x dependence. The asymmetry $A(x)$ is negative for small $x, x \leq 0.25$, and positive in the large part of physical region. This is a global property of $A(x)$ valid for both SM and the 2HDM (with and without QCD corrections).

The form of x dependence of $A(x)$, and its absolute values depend strongly on the model of FCNC, and on QCD corrections.

In SM (Fig. 5), the QCD corrections change the form of energy dependence of the asymmetry. Moreover, the point in x variable where the asymmetry $A(x)$ changes its sign is particularly sensitive to the QCD corrections: without QCD corrections the asymmetry changes sign at $x \sim 0.25$, but with QCD corrections this point is shifted to $x \sim 0.4$. It is interesting that the exact position of this point depends on the mass m_t. This property could be useful for more precise determination of m_t, because from the experimental point of view such points are very attractive.

Model II of 2HDM predicts an x dependence of $A(x)$ which is very similar to the SM case, with and without QCD corrections. The predicted behavior of $A(x)$ in model II is characterized by the weak dependence on the parameters as m_H and tanβ (Figs. 6 and 7).

Model I of the 2HDM is characterized by the strong dependence on the parameters m_H and tanβ (Fig. 8). From this figure we see that $A(x)$ is very sensitive to m_H. This is true for $A(x)$ with and without QCD corrections.

To understand the relative role of the contributions of different mechanisms in the resulting behavior of inclusive asymmetry $A(x)$, we have calculated the x dependence of all six terms in $A(x)$ (Figs. 9 and 10). We limit ourselves to the SM only ($m_t = 176$ GeV). The

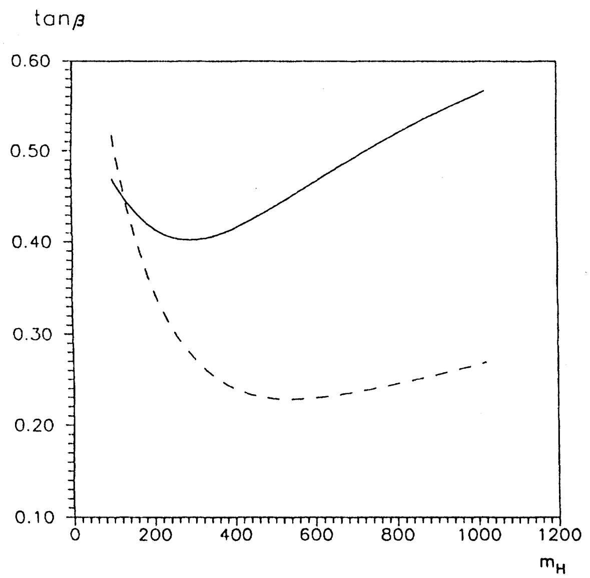

FIG. 4. Limit for the charged Higgs boson mass as a function of tanβ in the 2HDM. Here solid and dashed lines correspond to model I and model II, respectively.

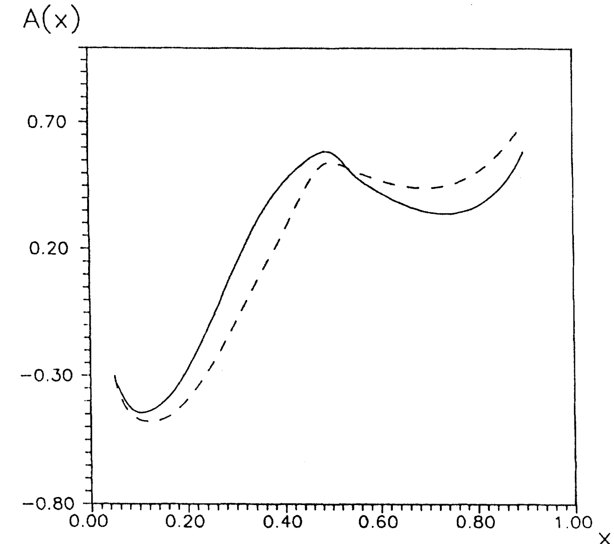

FIG. 5. The x dependence of the asymmetry $A(x)$ calculated in the framework of the SM, where solid and dashed lines correspond to without QCD corrections and with QCD corrections, respectively.

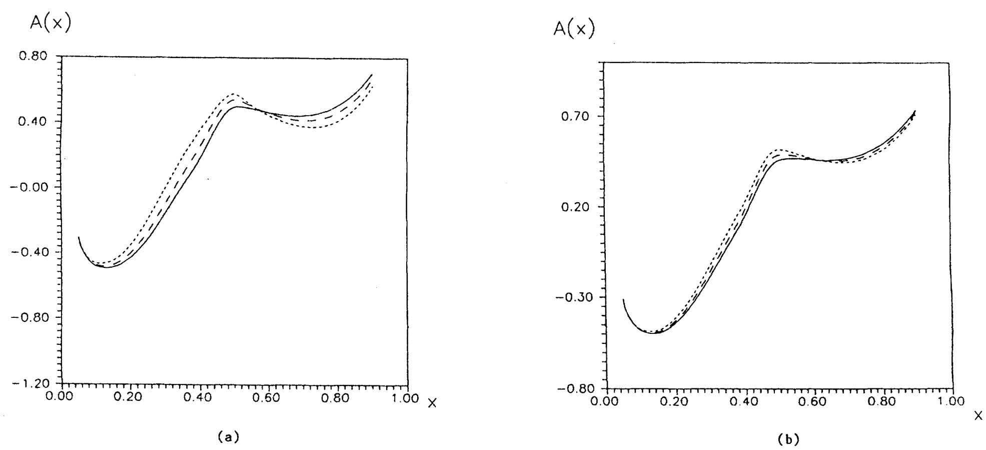

FIG. 6. The x dependence of the asymmetry $A(x)$ calculated in the framework of model II of 2HDM for $m_t = 176$ GeV and tan$\beta = 0.7$ (a) without QCD corrections and (b) with QCD corrections. The solid, long-dash, and short-dash lines correspond to m_H values 300, 500, and 1000 GeV, respectively.

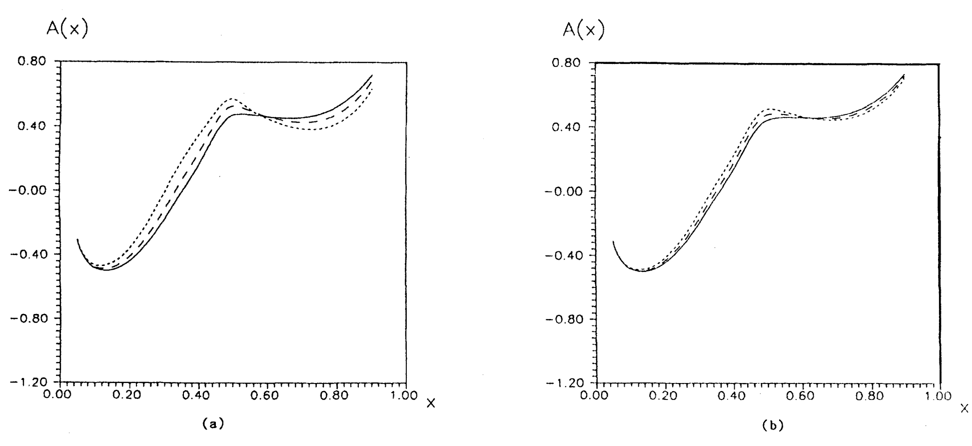

FIG. 7. The same as in Fig. 6, but for tan$\beta = 2$.

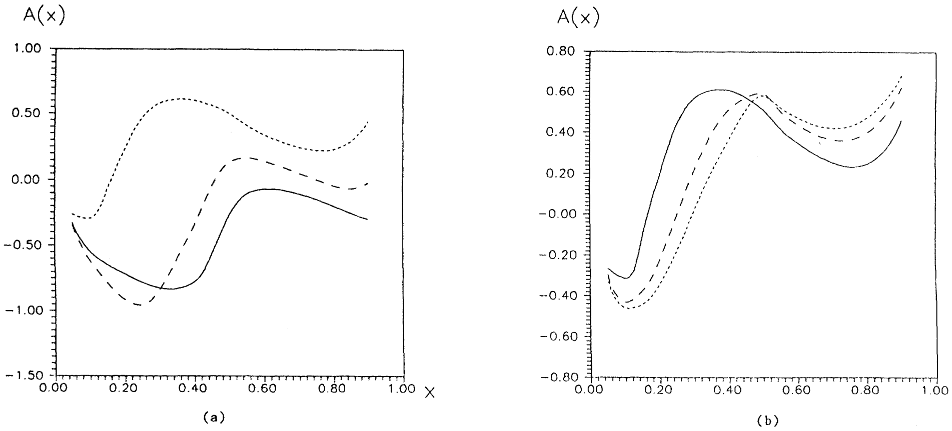

FIG. 8. The x dependence of the asymmetry $A(x)$ calculated in the framework of model I of the 2HDM for $m_t = 176$ GeV and $\tan\beta = 2$ (a) without QCD corrections and (b) with QCD corrections. The notation is as in Fig. 6.

different curves are noted by $a_i a_j (i, j = 1, 2, 3)$. These calculations are based on Eq. (46) with full denominators $I_0(x)$, but the numerator for each curve contains only one term, proportional to $a_i a_j$. In other words, the a_1^2, a_2^2, and a_3^2 curves characterize the contribution of one possible mechanism only, and the $a_1 a_2$, $a_1 a_3$, and $a_2 a_3$ curves characterize the contributions of the corresponding interference term of different mechanisms.

From the analysis of these figures, we deduce the following conclusions.

All six possible contributions to $A(x)$ are essential for formation of resulting asymmetry (with and without QCD corrections).

The contribution proportional to a_1^2 is always negative, and has a minimum at $x = 0.16$. This contribution is small for $x \geq 0.55$, and behavior with and without QCD correction is similar.

The contribution proportional to a_3^2 is always positive and is characterized by the strong dependence to the QCD corrections. This contribution is decisive for the behavior of $A(x)$ in the region $x \geq 0.25$. The negative values of $A(x)$ are determined by the a_1^2 contribution.

The interference contributions which are proportional to $a_1 a_3$, $a_2 a_1$, and $a_3 a_2$ are positive in the full x region; the location of the $a_1 a_2$ minimum and its value at the maximum depend essentially on the QCD corrections. QCD corrections increase the depth of this maximum and change its position, from $x \equiv 0.3$ (without QCD) to $x \sim 0.45$ (with QCD).

The change of the sign of the asymmetry is mainly due to the relative contributions of terms proportional to a_1^2 and a_3^2; when the term proportional a_3^2 dominates, then the asymmetry is positive and when a_1^2 term dominates, the asymmetry has a negative sign.

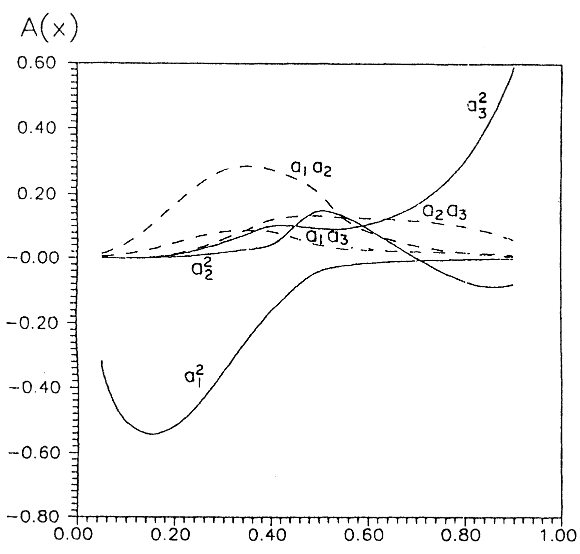

FIG. 9. The x dependence of different contributions to the asymmetry $A(x)$ calculated in the framework of the SM for $m_t = 176$ GeV without QCD corrections.

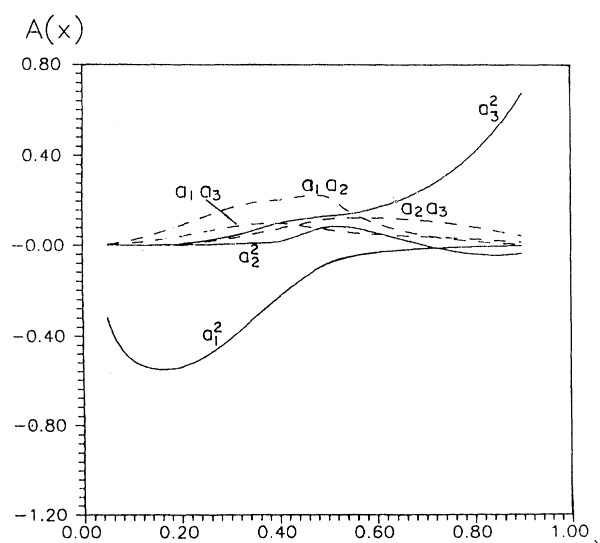

FIG. 10. The same as in Fig. 9, but with QCD corrections.

V. CONCLUSIONS

We analyzed the polarization effects in the decay $b \to s\gamma g$ induced by the polarization of the initial b quark. In the general case with the detection of two produced particles, such effects are characterized by three independent structure functions. Two SF's determine the P-odd and T-even decay asymmetries, which are due to the components of b-quark polarization vector in the decay plane, and one SF determines the P-even but T-odd decay asymmetry when the vector polarization of b quark is orthogonal to the decay plane. For the inclusive decay such as $b \to (sg)$ or $b \to g(s\gamma)$ with detection of one particle only, there is one P-odd asymmetry induced by the longitudinal polarization of the decaying quark. The calculation of all these asymmetries is very timely now, because the b quarks, which are copiously produced at LEP through the reaction $e^+e^- \to Z^0 \to b\bar{b}$ must have very large polarization, $\sim 94\%$.

It is shown that the decay amplitude is described in terms of the three form factors $F_{2,\text{eff}}$, $\tilde{F}_{2,\text{eff}}$, and Q, and this statement is model independent, with and without QCD corrections, since QCD corrections do not change the spin structure of the decay amplitude. But it is important to note that the relative role of different contributions to the total matrix element change essentially, when QCD corrections are taken into account.

As a result, the different asymmetries in the decay of polarized b quarks, $b \to s\gamma g$, are very sensitive to the following important ingredients of any description of FCNC: QCD corrections; choice of FCNC model; the concrete values of the corresponding fundamental parameters, such as m_t in SM, or m_H, tanβ in any version of the 2HDM.

We illustrated all this in the example of decay asymmetry of $b \to \gamma(sg)$, calculating the photon energy dependence of inclusive asymmetry in the framework of the SM and two popular versions of the 2HDM. The calculations (with and without QCD corrections) have shown that this asymmetry is large in absolute value. The asymmetry is characterized by a strong E_γ dependence, which is very sensitive to QCD corrections, on the particular model of FCNC and on the version of 2HDM.

We analyzed the relative importance of different contributions to this asymmetry to possible mechanisms of the decay $b \to s\gamma g$, demonstrating the effectiveness of future polarization effects for determination not only of the absolute values of such fundamental FCNC characteristics as the form factors F_2 and $\tilde{F}_2$, but their relative sign also. Of course, similar information must be significant for the search of the correct model of FCNC.

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