Three-step two-body decay of J/ψ and the generalized moment analysis

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The three-step two-body decay process of J/ψ, $J/\psi \rightarrow V + X$, $X \rightarrow P_1 + Y$, $Y \rightarrow P_2 + P_3$ (where V and P_i represent vector and pseudoscalar mesons, respectively) is discussed by using the generalized moment analysis method. The spin-parity of the boson resonance X and the ratios of helicity amplitudes of the process can be determined in terms of measuring the corresponding moments except for a very special case.

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I. INTRODUCTION

The existence of glueballs (pure gluons bound states), hybrids (bound states which include quarks and gluons), and four quark states is an important prediction of quantum chromodynamics (QCD) theory [1]. The unambiguous identification of such new hadronic states would be an essential proof of the validity of QCD theory. But this identification has not been achieved as yet due to the complexity of this subject.

J/ψ decay process is considered to be an ideal place to search for these new hadronic states. Some of the reasons are as follows. First, some theoretical models, such as the potential model, bag model, and lattice gauge theory expect that the masses of these new hadronic states may lie in the range of 1 to 2.5 GeV [2]. Hence they may be produced in the J/ψ decay processes. Second, the perturbative QCD predicts that the J/ψ radiative decay processes proceed mainly via the following way: charm quark (or a anticharm quark) inside the J/ψ emits a photon, then annihilates into two gluons together with another anticharm quark (or charm quark) in the J/ψ, finally these two gluons become some hadrons through the final-state interaction. And the most of these final-state hadrons are the hadronic states with spin-parity $J^{PC} = 2^{++}$, 0^{-+}, and 0^{++} which just are the lower energy states of glueball [3]. Hence, the J/ψ radiative decay process is favorable to the production of glueballs containing two gluons. The branching ratio including first-order QCD corrections of J/ψ radiative decay processes is about 7% [2,4]. Third, QCD also predicts that the J/ψ hadronic decay processes proceed mainly via three-gluon decay, namely, the charm quark and anticharm quark inside the J/ψ annihilate each other into three gluons; then these gluons form various hadrons through hadronization. And the lowest diagram of these processes is the J/ψ decay into a hybrid in company with a usual meson. Therefore, the J/ψ hadronic decay process is favorable to the production of hybrids. The branching ratio of J/ψ hadronic decay processes is about 66% [2,4]. Fourth, for the particle J/ψ produced in e^+e^- collision, its energy and momentum are fixed $(E, \vec{p}) = (m_J, 0)$ in the center-of-mass system of e^+e^-. The J/ψ polarization state of the 0 component is suppressed strongly; i.e., the J/ψ is produced only with helicity ± 1, due to the vector nature of e^+e^- production mechanism. In addition, since J/ψ is a $c\bar{c}$ bound state it is a singlet state of $SU(3)_{\text{flavor}}$. All of these properties are favorable to determine the characteristics of the final-state hadrons produced in J/ψ decay processes.

Experimentally, three candidates of the new hadronic states have been discovered and confirmed by Mark II, Crystal Ball, Mark III, DM2, and BES in J/ψ radiative decays. They are ι/η (1440) [5], $\theta/f_2(1720)/f_0(1710)$ [6], and $\xi(2230)$ [7]. But many interesting questions need to be answered further. For example, the spin of $\theta(1720)$ is 0 or 2; in the mass region of about 1440 MeV there is only one state $\iota/\eta(1440)$ or there are three resonances at least [8]; the spin of $\xi(2230)$ is 2 or 4, and so on.

As pointed out in Ref. [9], in order to determine the properties of these new hadronic states there is a great need for systematic studies of J/ψ hadronic decay processes $J/\psi \rightarrow X + V$ (V denotes a vector meson) in addition to the J/ψ radiative decay processes. In Ref. [9] three-step two-body processes of J/ψ,

$$\begin{aligned} e^+ + e^- \rightarrow J/\psi \rightarrow \gamma + {} & X \\ & \hookrightarrow P_1 + Y \\ & \qquad \hookrightarrow P_2 + P_3 \,, \end{aligned} \tag{1}$$

and

$$\begin{aligned} e^+e^- \rightarrow J/\psi \rightarrow V + {} & X \\ & \hookrightarrow P_1 + Y \\ & \qquad \hookrightarrow P_2 + P_3 \end{aligned} \tag{2}$$

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(where P_i represents the pseudoscalar meson, Y is a known particle with spin-parity 0^+, 1^-, or 2^+, X is a boson resonance to be studied), have been studied more systematically by using the helicity amplitude method [10]. The helicity formalism of the angular distributions which can be used to perform the spin-parity analysis of the boson resonance X are presented.

It is the aim of this paper to study processes (1) and (2) by using the moment analysis. The moment analysis method is first used to perform the spin-parity analysis of the boson resonances B for the process

$$\begin{array}{llll} a+b \to B & & + & c \\ & \hookrightarrow 2P \text{ (or } 3P) & & \hookrightarrow 2P \text{ (or } 3P) \end{array}$$

(where P denotes the pseudoscalar meson) [11]. The Mark III and BES Collaborations have studied the J/ψ radiative decay $J/\psi \to \gamma + \theta(1720)$ and the J/ψ hadronic decay $J/\psi \to \omega + \theta(1720)$, respectively, by using the moment analysis to determine the spin of θ [12]. In Ref. [13] this method is used to determine the spin of the $\xi(2230)$ produced in J/ψ radiative decay process

$$\begin{array}{ll} J/\psi \to \gamma + \xi(2230) & \\ & \hookrightarrow K + \bar{K} \; . \end{array}$$

The moment analysis method is more effective than the helicity amplitude method when the statistics are lower and there are resonances with different spin overlap in the mass region under consideration [14]. Because when we use the helicity amplitude method to determine the spin of the resonance we first must assume all events within a given mass interval corresponding to an object with definite spin; i.e., the resonance is isolated then we perform the fit to the angular distribution. But the fact is not always so simple. For example, experiment shows that there are complicated structures near the $\iota(1440)$ and $\theta(1720)$ mass region. In Sec. II of this paper we derive the formalism of moments corresponding to the boson resonance X with the spin-parity 0^-, $1^\pm$, $2^\pm$ and the intermediate Y with spin-parity $0^+, 1^-, 2^+$ for the three-step two-body hadronic decay process (2) of J/ψ. Finally, a summary is given in Sec. III.

II. MOMENT ANALYSIS

We take the c.m. system of e^+e^-. The z axis is chosen to be along the direction of the incident positron and the vector meson V lies in the x-z plane. In this frame we have the decay amplitudes as [11]

$$\langle V_{\lambda_V} X_{\lambda_X} | T_1 | \psi_{\lambda_J} \rangle \sim A_{\lambda_V,\lambda_X} D^{1}_{\lambda_J,\lambda_V-\lambda_X}{}^{*}(0,\theta_V,0) \; ,$$

$$\langle P_1 Y_{\lambda_Y} | T_2 | X_{\lambda_X} \rangle \sim B^{J_X}_{\lambda_Y,0} D^{J_X}_{\lambda_X,\lambda_Y}{}^{*}(\phi_1,\theta_1,-\phi_1) \; , \qquad (3)$$

$$\langle P_2 P_3 | T_3 | Y_{\lambda_Y} \rangle \sim D^{J_Y}_{\lambda_Y,0}{}^{*}(\phi_2,\theta_2,-\phi_2) \; ,$$

and

$$I_{\lambda_J,\lambda'_J} \equiv \tfrac{1}{4}\sum_{r,r'} \langle \psi_{\lambda_J} | T | e^-_r e^+_{r'} \rangle \langle \psi_{\lambda'_J} | T | e^-_r e^+_{r'} \rangle^* \sim 2|\vec{p}_+|^2 \delta_{\lambda_J,\lambda'_J} \delta_{\lambda_J,\pm 1} \qquad (4)$$

for process (2), where $\lambda_J = \pm 1$, $\lambda_V = 0, \pm 1$, $\lambda_X = 0, \pm 1, \ldots, \pm J_X$, and $\lambda_Y = 0, \pm 1, \ldots, \pm J_Y$, are the helicities of the particles J/ψ, V, X, and Y, respectively. The function $D^J_{m,n}$ is the $(2J+1)$-dimensional representation of the rotation group. θ_V is the angle between the incident positron and the moving direction of the vector meson V. The decay angles (θ_1, ϕ_1) describe the direction of the momentum of particle Y in the rest frame of particle X where we choose the z_1 axis parallel to the momentum $\vec{p}_X$ direction of particle X in the rest frame of J/ψ and the y_1 axis may be taken to be along the normal to the production plane. (θ_2, ϕ_2) describes the direction of the momentum $\vec{p}_2$ of the pseudoscalar meson P_2 in the rest frame of particle Y where the corresponding z_2 axis is chosen to be along the moving direction of the particle Y in the rest frame of X and the y_2 axis is along $\vec{p}_X \times \vec{p}_Y$ (where $\vec{p}_Y$ is the momentum of the particle Y defined in the rest frame of X). r and r' are polarization indices of the electron and positron. $\vec{p}_+$ is the momentum of the positron in the rest frame of J/ψ. Finally, A_{λ_V,λ_X} and $B^{J_X}_{\lambda_Y,0}$ are the helicity amplitudes for the process $J/\psi \to X + V$ and $X \to P_1 + Y$, respectively. Because of the parity conservation for the decay processes these helicity amplitudes satisfy the symmetry relations [11]:

$$\begin{aligned} A_{\lambda_V,\lambda_X} &= P_X(-1)^{J_X} A_{-\lambda_V,-\lambda_X} \; , \\ B^{J_X}_{\lambda_Y,0} &= -P_X P_Y (-1)^{J_X - J_Y} B^{J_X}_{-\lambda_Y,0} \; . \end{aligned} \qquad (5)$$

Therefore, in the frame chosen above we obtain the following angular distribution of the three-step two-body decay process (2) of J/ψ:

$$\begin{aligned} W(\theta_V,\theta_1,\phi_1,\theta_2,\phi_2) \sim & \sum_{\lambda_J=\pm1,\lambda_V,\lambda_X,\lambda'_X,\lambda_Y,\lambda'_Y} A_{\lambda_V,\lambda_X} A^*_{\lambda_V,\lambda'_X} B^{J_X}_{\lambda_Y,0} \\ & \times B^{J_X}_{\lambda'_Y,0}{}^{*} D^{1*}_{\lambda_J,\lambda_V-\lambda_X}(0,\theta_V,0) D^{1}_{\lambda_J,\lambda_V-\lambda'_X}(0,\theta_V,0) D^{J_X}_{\lambda_X,\lambda_Y}{}^{*}(\phi_1,\theta_1,-\phi_1) \\ & \times D^{J_X}_{\lambda'_X,\lambda'_Y}(\phi_1,\theta_1,-\phi_1) D^{J_Y}_{\lambda_Y,0}{}^{*}(\phi_2,\theta_2,-\phi_2) D^{J_Y}_{\lambda'_Y,0}(\phi_2,\theta_2,-\phi_2) \; . \end{aligned} \qquad (6)$$

We now define the moments by

$$\begin{aligned} M(jlmin) = \int & d\theta_V \sin\theta_V d\phi_1 \, d\theta_1 \sin\theta_1 \, d\phi_2 \, d\theta_2 \, \sin\theta_2 \, D^j_{0,-m}(0,\theta_V,0) \\ & \times D^l_{m,n}(\phi_1,\theta_1,-\phi_1) D^i_{n,0}(\phi_2,\theta_2,-\phi_2) W(\theta_V,\theta_1,\phi_1,\theta_2,\phi_2) \; . \end{aligned} \qquad (7)$$

Using the integration formulas about D function (for example, see the Appendix of Ref. [11]) we have

$$\begin{aligned} M(jlmin) \sim & \sum_{\lambda_J=\pm1,\lambda_V,\lambda_X,\lambda'_X,\lambda_Y,\lambda'_Y} A_{\lambda_V,\lambda_X} A^*_{\lambda_V,\lambda'_X} B^{J_X}_{\lambda_Y,0} B^{J_X}_{\lambda'_Y,0}{}^* \\ & \times\langle 1\lambda_J j0|1\lambda_J\rangle\langle 1\lambda_V-\lambda'_X j-m|1\lambda_V-\lambda_X\rangle\langle J_X\lambda'_X lm|J_X\lambda_X\rangle \\ & \times\langle J_X\lambda'_Y ln|J_X\lambda_Y\rangle\langle J_Y\lambda'_Y in|J_Y\lambda_Y\rangle\langle J_Y 0i0|J_Y 0\rangle , \end{aligned} \tag{8}$$

where $\langle j_1m_1j_2m_2|jm\rangle$ are Clebsch-Gordan coefficients.

The discussion will proceed under the three cases of $J_Y^{P_Y}=0^+,1^-,2^+$ below.

A. $J_Y^{P_Y}=0^+$

In this case the relation (5) is reduced to

$$\begin{aligned} A_{\lambda_V,\lambda_X} &= P_X(-1)^{J_X}A_{-\lambda_V,-\lambda_X} , \\ B^{J_X}_{0,0} &= -P_X(-1)^{J_X}B^{J_X}_{0,0} . \end{aligned} \tag{9}$$

Hence process (2) is forbidden for the resonance X with $J_X^{P_X}=0^+$, 1^-, or 2^+. The probable lowest orbital states are ones with $J_X^{P_X}=0^-,1^+,2^-$.

When $J_X^{P_X}=0^-$ there is only one independent helicity amplitude $A_{1,0}$ for the process $J/\psi\to V+X$ and one helicity amplitude $B^0_{0,0}$ for the process $X\to P_1+Y$, respectively. From Eq. (8) we obtain only two moments:

$$\begin{aligned} M(00000) &\sim 4|A_{1,0}|^2|B^0_{0,0}|^2 , \\ M(20000) &\sim \tfrac{2}{5}|A_{1,0}|^2|B^0_{0,0}|^2 . \end{aligned} \tag{10}$$

When $J_X^{P_X}=1^+$ there are three independent helicity amplitudes $A_{1,0}$, $A_{1,1}$, and $A_{0,1}$ for the process $J/\psi\to V+X$ and there is only one helicity amplitude $B^1_{0,0}$ for the process $X\to P_1+Y$ according to the relation (9). Therefore, we have the following moments:

$$\begin{aligned} M(00000) &\sim 4(|A_{1,1}|^2+|A_{1,0}|^2+|A_{0,1}|^2)|B^1_{0,0}|^2 , \\ M(02000) &\sim -\tfrac{4}{5}(|A_{1,1}|^2-2|A_{1,0}|^2+|A_{0,1}|^2)|B^1_{0,0}|^2 , \\ M(20000) &\sim \tfrac{2}{5}(-2|A_{1,1}|^2+|A_{1,0}|^2+|A_{0,1}|^2)|B^1_{0,0}|^2 , \\ M(22000) &\sim \tfrac{2}{25}(2|A_{1,1}|^2+2|A_{1,0}|^2-|A_{0,1}|^2)|B^1_{0,0}|^2 , \\ M(22100) &\sim \tfrac{6}{25}\mathrm{Re}(A_{1,1}A^*_{1,0})|B^1_{0,0}|^2 , \\ M(22200) &\sim \tfrac{6}{25}|A_{0,1}|^2|B^1_{0,0}|^2 . \end{aligned} \tag{11}$$

Similarly, if $J_X^{P_X}=2^-$ there are four independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$, $A_{1,2}$, and $A_{0,1}$ for the process $J/\psi\to X+V$ and one helicity amplitude $B^2_{0,0}$ for the process $X\to P_1+Y$ due to relation (9). Hence when $J_Y^{P_Y}=0^+$ and $J_X^{P_X}=2^-$ we obtain the moments

$$\begin{aligned} M(00000) &\sim 4(|A_{1,2}|^2+|A_{1,1}|^2+|A_{1,0}|^2+|A_{0,1}|^2)|B^2_{0,0}|^2 , \\ M(02000) &\sim -\tfrac{4}{7}(2|A_{1,2}|^2-|A_{1,1}|^2-2|A_{1,0}|^2-|A_{0,1}|^2)|B^2_{0,0}|^2 , \\ M(20000) &\sim \tfrac{2}{5}(|A_{1,2}|^2-2|A_{1,1}|^2+|A_{1,0}|^2+|A_{0,1}|^2)|B^2_{0,0}|^2 , \\ M(22000) &\sim -\tfrac{2}{35}(2|A_{1,2}|^2+2|A_{1,1}|^2-2|A_{1,0}|^2-|A_{0,1}|^2)|B^2_{0,0}|^2 , \\ M(22100) &\sim \frac{2\sqrt{3}}{35}\mathrm{Re}(A_{1,1}A^*_{1,0}-\sqrt{6}\,A_{1,2}A^*_{1,1})|B^2_{0,0}|^2 , \\ M(22200) &\sim \tfrac{2}{35}[3|A_{0,1}|^2-2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0})]|B^2_{0,0}|^2 , \\ M(24000) &\sim \tfrac{2}{105}(|A_{1,2}|^2+8|A_{1,1}|^2+6|A_{1,0}|^2-4|A_{0,1}|^2)|B^2_{0,0}|^2 , \\ M(24100) &\sim \tfrac{2}{21}\left(\frac{3}{5}\right)^{1/2}\mathrm{Re}(\sqrt{6}\,A_{1,1}A^*_{1,0}+A_{1,2}A^*_{1,1})|B^2_{0,0}|^2 , \\ M(24200) &\sim \tfrac{4}{21}\left(\frac{3}{5}\right)^{1/2}\left[|A_{0,1}|^2+\left(\frac{3}{2}\right)^{1/2}\mathrm{Re}(A_{1,2}A^*_{1,0})\right]|B^2_{0,0}|^2 . \end{aligned} \tag{12}$$

Therefore, when $J_Y^{P_Y}=0^+$ we can easily determine the spin and parity $J_X^{P_X}$ of the boson resonance X from Eqs. (10)–(12). First, for the $J_X^{P_X}=0^-$, all of the moments with $l=,2,4$ are equal to zero, and there is a simple relation for the only two moments:

$$M(00000)=10M(20000) . \tag{13}$$

However, there are not these properties for resonances X with $J_x^{P_x}=1^+$ and $J_x^{P_x}=2^-$. Therefore, through the measuring moments in experiment the boson resonance X with $J_X^{P_X}=0^-$ can be distinguished from the resonances with $J_X^{P_X}=1^+$ and $J_X^{P_X}=2^-$. In order to distinguish the state with $J_X^{P_X}=1^+$ from the state with $J_X^{P_X}=2^-$ we measure the moments with $l=4$ first. This resonance must be one with $J_X^{P_X}=2^-$ provided there is one nonzero moment with $l=4$. If all of the moments with $l=4$ are equal to zero we can determine the spin-parity of X to be 1^+ or 2^- from one of the following two relations:

$$\frac{M(00000)-7M(02000)-10M(20000)+70M(22000)}{M(00000)-10M(20000)}=\begin{cases}0 & \text{for } J_X^{P_X}=2^- ,\\ \frac{12}{5} & \text{for } J_X^{P_X}=1^+ ,\end{cases} \tag{14}$$

$$\frac{M(00000)+5M(02000)-10M(20000)-50M(22000)}{7M(02000)-70M(22000)}=\begin{cases}\frac{12}{7} & \text{for } J_X^{P_X}=2^- ,\\ 0 & \text{for } J_X^{P_X}=1^+ ,\end{cases} \tag{15}$$

provided $A_{1,1}\neq 0$. If $A_{1,1}=0$ but $2|A_{1,0}|^2\neq|A_{0,1}|^2$ we can still distinguish the resonance with $J_X^{P_X}=1^+$ from the resonance with $J_X^{P_X}=2^-$ by using the relation

$$\frac{2M(00000)+5M(02000)-50M(22200)}{M(22000)}=\begin{cases}0 & \text{for } J_X^{P_X}=2^- ,\\ 100 & \text{for } J_X^{P_X}=1^+ .\end{cases} \tag{16}$$

We cannot distinguish the state with $J_X^{P_X}=1^+$ from the state with $J_X^{P_X}=2^-$ only in a very special case of all of moments with $l=4$ are equal to zero, $A_{1,1}=0$, and $2|A_{1,0}|^2=|A_{0,1}|^2$.

We now define the following independent helicity amplitude ratios x, y, z, z', ξ_J and η_J $(J=1,2)$ by

$$xe^{i\phi_x}=\frac{A_{1,1}}{A_{1,0}},\quad ye^{i\phi_y}=\frac{A_{1,2}}{A_{1,0}},\quad ze^{i\phi_z}=\frac{A_{0,0}}{A_{1,0}},\quad z'e^{i\phi'_z}=\frac{A_{0,1}}{A_{1,0}},\quad \xi_J e^{i\phi_J}=\frac{B^J_{0,0}}{B^J_{1,0}},\quad \eta_J e^{i\phi'_J}=\frac{B^J_{2,0}}{B^J_{1,0}} . \tag{17}$$

All of the helicity amplitude ratios are real if we suppose that process (2) is invariant under time reversal [15].

After determining the spin-parity of the resonance X, the helicity amplitude ratios can be obtained. For the resonance X with $J_X^{P_X}=1^+$ we have

$$x^2=\frac{M(00000)-10M(20000)}{M(00000)+5M(02000)},\quad z'^2=\frac{50M(22200)}{M(00000)+5M(02000)},\quad x\cos\phi_x=\frac{50M(22100)}{M(00000)+5M(02000)} , \tag{18}$$

from Eq. (11). For $J_X^{P_X}=2^-$ we obtain

$$\begin{aligned}
x^2&=\frac{245}{3T}[M(02000)-10M(22000)] ,\\
y^2&=\frac{14}{T}[M(00000)-7M(02000)+15M(24000)+20M(22000)] ,\\
z'^2&=\frac{70}{T}[5M(22200)+2\sqrt{15}\,M(24200)] ,\\
x\cos\phi_x&=\frac{70}{3T}[5\sqrt{3}\,M(22100)+9\sqrt{10}\,M(24100)] ,\\
xy\cos(\phi_y-\phi_x)&=\frac{70}{T}[\sqrt{15}\,M(24100)-5\sqrt{2}\,M(22100)] ,
\end{aligned} \tag{19}$$

where

$$T=7M(00000)-49M(02000)+630M(24000)+840M(22000) . \tag{20}$$

B. $J_Y^{P_Y}=1^-$

In this case we have the relations

$$A_{\lambda_V,\lambda_X}=P_X(-1)^{J_X}A_{-\lambda_V,-\lambda_X},\quad B^{J_X}_{\lambda_Y,0}=-P_X(-1)^{J_X}B^{J_X}_{-\lambda_Y,0} , \tag{21}$$

from Eq. (5). Therefore, only the process with $J_X^{P_X}=0^+$ is forbidden.

When $J_X^{P_X} = 0^-$ we have four moments:

$$\begin{aligned} &M(00000) \sim 4|A_{1,0}|^2|B_{0,0}^0|^2, \quad M(00020) \sim \tfrac{8}{5}|A_{1,0}|^2|B_{0,0}^0|^2 , \\ &M(20000) \sim \tfrac{2}{5}|A_{1,0}|^2|B_{0,0}^0|^2, \quad M(20020) \sim \tfrac{4}{25}|A_{1,0}|^2|B_{0,0}^0|^2 . \end{aligned} \tag{22}$$

When $J_X^{P_X} = 1^-$ there are four independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$, $A_{0,0}$, and $A_{0,1}$ for the process $J/\psi \to V+X$ and there is still one independent helicity amplitude $B_{1,0}^1$ for the process $X \to P_1 + Y$. From Eq. (8) we can obtain 16 moments. Below we only give the 12 of them which will be used in the latter calculation:

$$\begin{aligned} M(00000) &\sim 4(2|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + 2|A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(02000) &= -5M(02020) = \tfrac{5}{3}M(02022) \\ &\qquad \sim \tfrac{4}{5}(|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(20000) &\sim \tfrac{4}{5}(-2|A_{1,1}|^2 + |A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(22000) &= -5M(22020) = \tfrac{5}{3}M(22022) \\ &\qquad \sim -\tfrac{2}{25}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - 2|A_{0,0}|^2 - |A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(22100) &\sim \tfrac{6}{25}\mathrm{Re}[A_{0,1}A_{0,0}^* - A_{1,1}A_{1,0}^*]|B_{1,0}^1|^2 , \\ M(22200) &= -5M(22220) = \tfrac{5}{3}M(22222) \sim \tfrac{6}{25}|A_{0,1}|^2|B_{1,0}^1|^2 . \end{aligned} \tag{23}$$

When $J_X^{P_X} = 1^+$ we have $A_{0,0} = 0$ and there are two independent helicity amplitudes $B_{1,0}^1$ and $B_{0,0}^1$ for the process $X \to P_1 + Y$ from Eq. (21). In this case the total number of moments is 21. The nine moments below are useful for the latter calculation:

$$\begin{aligned} M(02020) &\sim -\tfrac{4}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 + |A_{0,1}|^2)(|B_{1,0}^1|^2 + 2|B_{0,0}^1|^2) , \\ M(02022) &\sim -\tfrac{12}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 + |A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(22020) &\sim \tfrac{2}{125}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - |A_{0,1}|^2)(|B_{1,0}^1|^2 + 2|B_{0,0}^1|^2) \\ M(22021) &\sim \tfrac{6}{125}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - |A_{0,1}|^2)\mathrm{Re}(B_{1,0}^1 B_{0,0}^{1}{}^*) \\ M(22022) &\sim \tfrac{6}{125}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - |A_{0,1}|^2)|B_{1,0}^1|^2 , \\ M(22122) &\sim \tfrac{18}{125}\mathrm{Re}(A_{1,1}A_{1,0}^*)|B_{1,0}^1|^2, \quad M(22220) \sim \tfrac{6}{125}|A_{0,1}|^2(|B_{1,0}^1|^2 + 2|B_{0,0}^1|^2) , \\ M(22221) &\sim \tfrac{18}{125}|A_{0,1}|^2\mathrm{Re}(B_{1,0}^1 B_{0,0}^{1}{}^*), \quad M(22222) \sim \tfrac{18}{125}|A_{0,1}|^2|B_{1,0}^1|^2 . \end{aligned} \tag{24}$$

When $J_X^{P_X} = 2^+$ there are five independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$, $A_{1,2}$, $A_{0,0}$, and $A_{0,1}$ for the process $J/\psi \to V + X$ and one helicity amplitude $B_{1,0}^2$ for the process $X \to P_1 + Y$. There are 28 moments in this case. Some of them are

$$\begin{aligned} M(00000) &\sim 4(2|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + 2|A_{0,1}|^2 + 2|A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(02000) &\sim \tfrac{4}{7}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(04000) &\sim \tfrac{16}{63}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 - 3|A_{0,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(04020) &= -\frac{2\sqrt{15}}{15}M(04022) \\ &\sim -\tfrac{16}{315}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 - 3|A_{0,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(20000) &\sim \tfrac{4}{5}(-2|A_{1,1}|^2 + |A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(22000) &= -\tfrac{5}{3}M(22022) \\ &\sim -\tfrac{2}{35}(2|A_{1,1}|^2 - 2|A_{1,0}|^2 + 2|A_{0,0}|^2 - |A_{0,1}|^2 + 2|A_{1,2}|^2)|B_{1,0}^2|^2 , \\ M(22200) &= -\tfrac{5}{3}M(22222) \sim -\tfrac{2}{35}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A_{1,0}^*) + 3|A_{0,1}|^2]|B_{1,0}^2|^2 , \\ M(22100) &\sim \frac{2\sqrt{3}}{35}\mathrm{Re}[A_{1,1}A_{1,0}^* - A_{0,1}A_{0,0}^* - \sqrt{6}A_{1,1}A_{1,2}^*]|B_{1,0}^2|^2 , \\ M(24100) &\sim -\tfrac{8}{21}\left(\frac{2}{5}\right)^{1/2}\mathrm{Re}\left[A_{1,1}A_{1,0}^* - A_{0,1}A_{0,0}^* + \frac{1}{\sqrt{6}}A_{1,2}A_{1,1}^*\right]|B_{1,0}^2|^2 , \end{aligned} \tag{25}$$

$$M(24200) = -5M(24220) = \frac{2\sqrt{15}}{3}M(24222)$$
$$\sim -\tfrac{8}{21}\left(\frac{2}{5}\right)^{1/2}\left[\mathrm{Re}(A_{1,2}A^*_{1,0}) - \frac{\sqrt{6}}{3}|A_{0,1}|^2\right]|B^2_{1,0}|^2 \ .$$

When $J_X^{P_X} = 2^-$, because of $A_{0,0} = 0$ there are four independent helicity amplitudes for the process $J/\psi \to V + X$ and two independent helicity amplitudes $B^2_{1,0}$ and $B^2_{0,0}$ for the process $X \to P_1 + Y$. There are 39 moments. The 14 moments of them are

$$\begin{aligned}
M(00000) &\sim 4(|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)(2|B^2_{1,0}|^2 + |B^2_{0,0}|^2) \ ,\\
M(00020) &\sim -\tfrac{8}{5}(|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)(|B^2_{1,0}|^2 - |B^2_{0,0}|^2) \ ,\\
M(02022) &\sim \tfrac{12}{35}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)|B^2_{1,0}|^2 \ ,\\
M(04020) &\sim -\tfrac{8}{315}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)(2|B^2_{1,0}|^2 + 3|B^2_{0,0}|^2) \ ,\\
M(04022) &\sim -\frac{8\sqrt{15}}{315}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)|B^2_{1,0}|^2 \ ,\\
M(22000) &\sim -\tfrac{2}{35}(2|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,1}|^2 + 2|A_{1,2}|^2)(|B^2_{1,0}|^2 + |B^2_{0,0}|^2) \ ,\\
M(22022) &\sim -\tfrac{6}{175}(2|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,1}|^2 + 2|A_{1,2}|^2)|B^2_{1,0}|^2 \ ,\\
M(22122) &\sim \frac{6\sqrt{3}}{175}\mathrm{Re}(A_{1,1}A^*_{1,0} - \sqrt{6}A_{1,1}A^*_{1,2})|B^2_{1,0}|^2 \ ,\\
M(22200) &\sim -\tfrac{2}{35}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2](|B^2_{1,0}|^2 + |B^2_{0,0}|^2) \ ,\\
M(22220) &\sim \tfrac{2}{175}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2](|B^2_{1,0}|^2 - 2|B^2_{0,0}|^2) \ ,\\
M(22221) &\sim -\frac{2\sqrt{3}}{175}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2]\mathrm{Re}(B^2_{1,0}B^{2\,*}_{0,0}) \ ,\\
M(22222) &\sim -\tfrac{6}{175}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2]|B^2_{1,0}|^2 \ ,\\
M(24020) &\sim \tfrac{4}{1575}(8|A_{1,1}|^2 + 6|A_{1,0}|^2 - 4|A_{0,1}|^2 + |A_{1,2}|^2)(2|B^2_{1,0}|^2 + 3|B^2_{0,0}|^2) \ ,\\
M(24021) &\sim \frac{2\sqrt{10}}{525}(8|A_{1,1}|^2 + 6|A_{1,0}|^2 - 4|A_{0,1}|^2 + |A_{1,2}|^2)\mathrm{Re}(B^2_{1,0}B^{2\,*}_{0,0})\\
M(24022) &\sim \frac{4\sqrt{15}}{1575}(8|A_{1,1}|^2 + 6|A_{1,0}|^2 - 4|A_{0,1}|^2 + |A_{1,2}|^2)|B^2_{1,0}|^2 \ ,\\
M(24122) &\sim \tfrac{4}{35}\left(\frac{2}{3}\right)^{1/2}\mathrm{Re}\left(A_{1,1}A^*_{1,0} + \frac{1}{\sqrt{6}}A_{1,1}A^*_{1,2}\right)|B^2_{1,0}|^2 \ ,\\
M(24200) &\sim -\tfrac{2}{21}\left(\frac{2}{5}\right)^{1/2}\left[\mathrm{Re}(A_{1,0}A^*_{1,2}) + \frac{\sqrt{6}}{3}|A_{0,1}|^2\right](4|B^2_{1,0}|^2 - 3|B^2_{0,0}|^2) \ ,\\
M(24220) &\sim \tfrac{4}{105}\left(\frac{2}{5}\right)^{1/2}\left[\mathrm{Re}(A_{1,0}A^*_{1,2}) + \frac{\sqrt{6}}{3}|A_{0,1}|^2\right](2|B^2_{1,0}|^2 + 3|B^2_{0,0}|^2) \ ,\\
M(24221) &\sim \tfrac{4}{35}\left[\mathrm{Re}(A_{1,0}A^*_{1,2}) + \frac{\sqrt{6}}{3}|A_{0,1}|^2\right]\mathrm{Re}(B^2_{1,0}B^{2\,*}_{0,0}) \ ,\\
M(24222) &\sim \tfrac{4}{35}\left(\frac{2}{3}\right)^{1/2}\left[\mathrm{Re}(A_{1,0}A^*_{1,2}) + \frac{\sqrt{6}}{3}|A_{0,1}|^2\right]|B^2_{1,0}|^2 \ .
\end{aligned} \tag{26}$$

From Eqs. (22)–(26) we can see that there is an obvious characteristic for the resonance X with $J_X^{P_X} = 0^-$, that is, all of the moments with $l > 0$, $m > 0$, and $n > 0$ are equal to zero and only four nonzero moments satisfy a simple relation:

$$2M(00000) = 5M(00020) = 20M(20000) = 50M(20020) \ . \tag{27}$$

But there is not this property for resonances X with $J_X^{P_X} = 1^\pm$ and $J_X^{P_X} = 2^\pm$. Therefore, 0^- state can be distinguished from the states with $1^\pm$ and $2^\pm$ through the measurement of moments experimentally. Then because all of the moments with $l > 2$ equal to zero for the state with $J_X = 1$, but there are 18 moments with $l > 2$ for 2^- state and 12 moments with $l > 2$ for 2^+ state. Hence so long as there is one measuring value of moments with $l > 2$

to be not equal to zero the spin of this boson resonance X must be not 1. After determining the spin the following three relations can be used to fix the parity of $J_X = 1$ boson resonance X:

$$\frac{M(22220)}{M(22222)} = \begin{cases} -\frac{1}{3} \text{ for } J_X^{P_X} = 1^- , \\ \frac{1}{3}(1+2\xi_1^2) \geq \frac{1}{3} \text{ for } J_X^{P_X} = 1^+ , \end{cases} \tag{28}$$

$$\frac{M(22020)}{M(22022)} = \begin{cases} -\frac{1}{3} \text{ for } J_X^{P_X} = 1^- , \\ \frac{1}{3}(1+2\xi_1^2) \geq \frac{1}{3} \text{ for } J_X^{P_X} = 1^+ , \end{cases} \tag{29}$$

and

$$\frac{M(02020)}{M(02022)} = \begin{cases} -\frac{1}{3} \text{ for } J_X^{P_X} = 1^- , \\ \frac{1}{3}(1+2\xi_1^2) \geq \frac{1}{3} \text{ for } J_X^{P_X} = 1^+ . \end{cases} \tag{30}$$

The following relations can be used to fix the parity of $J_X = 2$ boson resonance X:

$$\frac{M(22000)}{M(22022)} = \begin{cases} -\frac{5}{3} \text{ for } J_X^{P_X} = 2^+ , \\ \frac{5}{3}(1+\xi_2^2) \text{ for } J_X^{P_X} = 2^- , \end{cases} \tag{31}$$

$$\frac{M(04020)}{M(04022)} = \begin{cases} -\frac{2\sqrt{15}}{15} \text{ for } J_X^{P_X} = 2^+ , \\ \frac{\sqrt{15}}{15}(2+3\xi_2^2) \text{ for } J_X^{P_X} = 2^- , \end{cases} \tag{32}$$

$$\frac{M(22200)}{M(22222)} = \begin{cases} -\frac{5}{3} \text{ for } J_X^{P_X} = 2^+ , \\ \frac{5}{3}(1+\xi_2^2) \text{ for } J_X^{P_X} = 2^- , \end{cases} \tag{33}$$

and

$$\frac{M(24220)}{M(24222)} = \begin{cases} -\frac{2\sqrt{15}}{15} \text{ for } J_X^{P_X} = 2^+ , \\ \frac{\sqrt{15}}{15}(2+3\xi_2^2) \text{ for } J_X^{P_X} = 2^- . \end{cases} \tag{34}$$

The helicity amplitude ratios can be written in terms of the moments. For $J_X^{P_X} = 1^-$ we have

$$\begin{aligned} x^2 &= \frac{1}{T_1}[M(00000) + 5M(02000) - 50M(22200)] , \\ z^2 &= \frac{1}{T_1}[M(00000) + 50M(22000) - 50M(22200)] , \\ z'^2 &= \frac{50}{T_1}M(22200), \quad zz' \cos(\phi_z - \phi_z') - x \cos\phi_x = \frac{50}{T_1}M(22100) , \end{aligned} \tag{35}$$

where

$$T_1 = M(00000) + 5M(20000) - 50M(22200) . \tag{36}$$

When $J_X^{P_X} = 1^+$ we obtain

$$x^2 = \frac{25}{T_2}[10M(22022) - M(02022)], \quad z'^2 = \frac{250}{T_2}M(22222) ,$$

$$\xi_1^2 = \begin{cases} \frac{3}{2}\frac{M(22220)}{M(22222)} - \frac{1}{2} \text{ if } A_{0,1} \neq 0 , \\ \frac{3}{2}\frac{M(22020)}{M(22022)} - \frac{1}{2} \text{ if } A_{0,1} = 0 , \end{cases} \tag{37}$$

$$x \cos\phi_x = \frac{250}{T_2}M(22122), \quad \xi_1 \cos\phi_1 = \begin{cases} \frac{M(22221)}{M(22222)} \text{ if } A_{0,1} \neq 0 , \\ \frac{M(22021)}{M(22022)} \text{ if } A_{0,1} = 0 , \end{cases}$$

where

$$T_2 = 25[5M(22022) + M(02022) + 5M(22222)] . \tag{38}$$

For $J_X^{P_X} = 2^+$ the independent helicity amplitude ratios can be expressed as follows by using the moments

$$\begin{aligned}
x^2 &= \frac{5}{4T_3}[M(00000) - 10M(20000) - 7M(02000) + 70M(22000)] \;, \\
y^2 &= \frac{3}{16T_3}[4M(0000) - 40M(02000) - 9M(04000)] \;, \\
z^2 &= \frac{5}{4T_3}[-M(00000) + 10M(20000) + 14M(02000) - 140M(22000)] \;, \\
z'^2 &= \frac{105}{28T_3}[3\sqrt{15}\,M(24200) - 10M(22200)] \;, \\
y\cos\phi_y &= -\frac{35}{112T_3}[40\sqrt{6}\,M(22200) + 27\sqrt{10}\,M(24200)] \;, \\
xy\cos(\phi_y - \phi_x) &= -\frac{105}{56T_3}[3\sqrt{15}\,M(24100) + 20\sqrt{2}\,M(22100)] \;, \\
x\cos\phi_x - zz'\cos(\phi_z - \phi_z') &= \frac{5}{8T_3}[20\sqrt{3}\,M(22100) - 27\sqrt{10}\,M(24100)] \;,
\end{aligned} \tag{39}$$

where

$$T_3 = M(00000) - 5M(02000) - \tfrac{25}{4}M(20000) + \tfrac{175}{2}M(22000) - \tfrac{81}{16}M(04000) \;. \tag{40}$$

Finally, when $J_X^{P_X} = 2^-$ we can obtain the relations

$$\begin{aligned}
x^2 &= \frac{1225}{12T_4}[M(02022) - 10M(22022)] \;, \\
y^2 &= \frac{7}{8T_4}[4M(00000) - 10M(00020) - 100M(02022) + 9\sqrt{15}\,M(04022)] \;, \\
z'^2 &= \frac{175}{2T_4}[9M(24222) + 5M(22222)] \;, \\
\xi_2^2 &= \begin{cases} \frac{S}{315M(24222)+175M(22222)} & \text{if } A_{0,1} \neq 0 \\ \frac{\sqrt{15}M(24020)}{3M(24022)} - \frac{2}{3} & \text{if } A_{0,1} = 0 \;, \end{cases} \\
x\cos\phi_x &= \frac{175}{12T_4}[10\sqrt{3}\,M(22122) + 27\sqrt{6}\,M(24122)] \;, \\
y\cos\phi_y &= \frac{175\sqrt{6}}{24T_4}[-8M(22200) + 20M(22220) + 27M(24222)] \;, \\
\xi_2\cos\phi_2 &= \begin{cases} \frac{3\sqrt{6}M(24221)+5\sqrt{3}M(22221)}{9M(24222)+5M(22222)} & \text{if } A_{0,1} \neq 0 \;, \\ \sqrt{2/3}\,\frac{M(24021)}{M(24022)} & \text{if } A_{0,1} = 0 \;, \end{cases}
\end{aligned} \tag{41}$$

where

$$T_4 = \frac{7}{4}M(00000) - \frac{35}{8}M(00020) + \frac{175}{4}M(02022) + \frac{189\sqrt{15}}{8}M(04022) \tag{42}$$

and

$$S = 70\sqrt{15}\,M(24220) + 14\sqrt{15}\,M(24200) + 35M(22200) + 175M(22220) \;. \tag{43}$$

C. $J_Y^{P_Y} = 2^+$

When $J_Y^{P_Y} = 2^+$, the process (2) is still forbidden if the spin-parity of resonance X is 0^+ and there are more moments for $J_X^{P_X} = 0^-$, $1^\pm$, and $2^\pm$. Because of the limited space we do not intend to give the expressions of moments for every case. We only give the final results below. There are only six moments for the resonance X with $J_X^{P_X} = 0^-$ and they satisfy the relations

$$2M(00000) = 7M(00020) = 7M(00040) = 20M(20000) = 70M(20020) = 70M(20040) \;. \tag{44}$$

Therefore, the 0^- state can be distinguished from the states with $1^\pm$ and $2^\pm$. Similarly, because all of the moments with $l > 2$ equal to zero for the state with $J_X = 1$, but there are 40 moments with $l > 2$ for 2^- state and 39 moments with $l > 2$ for 2^+ state. Therefore, as long as there is one measuring value of moments with $l > 2$ to be not equal to zero the spin of this boson resonance must be not 1. After fixing the spin of the resonance X we can use the following three relations to distinguish the parity of $J_X = 1$ boson resonance X:

$$\frac{M(22240)}{M(22242)} = \begin{cases} -\frac{2\sqrt{15}}{15} & \text{for } J_X^{P_X} = 1^- \\ \frac{2\sqrt{15}}{15}(1+\frac{3}{2}\xi_2^2) & \text{for } J_X^{P_X} = 1^+ \end{cases} \quad \text{if } A_{0,1} \neq 0 , \tag{45}$$

$$\frac{M(22140)}{M(22142)} = \begin{cases} -\frac{2\sqrt{15}}{15} & \text{for } J_X^{P_X} = 1^- \\ \frac{2\sqrt{15}}{15}(1+\frac{3}{2}\xi_2^2) & \text{for } J_X^{P_X} = 1^+ \end{cases} \quad \text{if } A_{0,1} = 0, A_{1,1} \neq 0 , \tag{46}$$

and

$$\frac{M(02040)}{M(02042)} = \begin{cases} -\frac{2\sqrt{15}}{15} & \text{for } J_X^{P_X} = 1^- \\ \frac{2\sqrt{15}}{15}(1+\frac{3}{2}\xi_2^2) & \text{for } J_X^{P_X} = 1^+ \end{cases} \quad \text{if } A_{0,1} = A_{1,1} = 0 . \tag{47}$$

Similarly, the parity of $J_X = 2$ state can be determined by using the four relations

$$\frac{M(04020)}{M(04044)} = \frac{M(24040)}{M(24044)} = \frac{M(24140)}{M(24144)} = \frac{M(24240)}{M(24244)}$$
$$= \begin{cases} -\frac{16}{35}(1+\frac{1}{16}\eta_2^2) & \text{for } J_X^{P_X} = 2^+ \\ \frac{16}{35}(1+\frac{9}{8}\xi_2^2+\frac{1}{16}\eta_2^2) & \text{for } J_X^{P_X} = 2^- . \end{cases} \tag{48}$$

After determining the spin-parity of the boson resonance X the helicity amplitude ratios can be written in terms of the moments. For $J_X^{P_X} = 1^-$ we have

$$x^2 = \frac{1}{T_1}[M(00000) + 5M(02000) - 50M(22200)] ,$$
$$z^2 = \frac{1}{T_1}[M(00000) + 50M(22000) - 50M(22200)] ,$$
$$z'^2 = \frac{50}{T_1}M(22200), \quad zz'\cos(\phi_z - \phi_z') - x\cos\phi_x = \frac{50}{T_1}M(22100) , \tag{49}$$

where

$$T_1 = M(00000) + 5M(20000) - 50M(22200) . \tag{50}$$

When $J_X^{P_X} = 1^+$ we obtain

$$x^2 = \frac{1}{T_2}\left(-\frac{35}{12}M(02022) + \frac{175}{6}M(22022)\right), \quad z'^2 = \frac{175}{6T_2}M(22222) ,$$

$$\xi_1^2 = \frac{14M(00020) - 2M(00000)}{2M(00000) - 7M(00020)}, \quad x\cos\phi_x = \frac{175}{6T_2}M(22122) ,$$

$$\xi_1\cos\phi_1 = \begin{cases} \frac{\sqrt{3}M(22121)}{M(22122)} & \text{if } A_{1,1} \neq 0 \\ \frac{\sqrt{3}M(22221)}{M(22222)} & \text{if } A_{0,1} \neq 0 \\ \frac{\sqrt{3}M(02021)}{M(02022)} & \text{if } A_{1,1} = A_{0,1} = 0 , \end{cases} \tag{51}$$

where

$$T_2 = \tfrac{1}{4}M(00000) - \tfrac{7}{8}M(00020) + \tfrac{35}{12}M(02022) . \tag{52}$$

In the case of $J_X^{P_X} = 2^+$ the expressions of the independent helicity amplitude ratios in terms of the moments are

$$x^2 = \frac{1}{T_3}\left(\frac{2}{45}M(00000) + \frac{7}{45}M(00020) - \frac{5}{18}M(20000) - \frac{35}{36}M(20020) - \frac{245}{36}M(22022)\right.$$
$$\left. + \frac{7}{18}M(02022) + \frac{189}{200}M(04042) - \frac{35}{12}M(22222) - \frac{21}{4}M(24222)\right) ,$$

$$y^2 = \frac{1}{T_3}\left(\frac{1}{30}M(00000) + \frac{7}{60}M(00020) + \frac{7}{6}M(02022) - \frac{189}{400}M(04042)\right) ,$$

$$z^2 = \frac{1}{T_3}\left(- \frac{1}{30}M(00000) - \frac{7}{60}M(00020) + \frac{245}{18}M(22022) - \frac{7}{9}M(02022) - \frac{189}{100}M(04042)\right.$$
$$\left. + \frac{35}{6}M(22222) + \frac{21}{2}M(24222)\right) ,$$

$$z'^2 = \frac{1}{T_3}\left(\frac{35}{6}M(22222) + \frac{21}{2}M(24222)\right), \quad \eta_2^2 = \frac{M(00000) - 7M(00020)}{2M(00000) + 7M(00020)} , \tag{53}$$

$$y\,\cos\phi_y = \frac{1}{T_3}\left(\frac{35\sqrt{6}}{18}M(22222) - \frac{21\sqrt{6}}{8}M(24222)\right) ,$$

$$xy\,\cos(\phi_y - \phi_x) = \frac{1}{T_3}\left(\frac{35\sqrt{2}}{6}M(22122) - \frac{21}{4}M(24122)\right) ,$$

$$x\,\cos\phi_x - zz'\cos(\phi_z - \phi_z') = \frac{1}{T_3}\left(-\frac{35\sqrt{3}}{18}M(22122) - \frac{21\sqrt{6}}{4}M(24122)\right) ,$$

$$\eta_2\cos\phi_2' = \frac{1}{12T_3}\left(- \frac{21\sqrt{30}}{5}M(04021) + \frac{35}{6}M(22021) + \frac{7}{6}M(02021)\right.$$
$$\left. - \frac{35}{2}M(22221) + 63\sqrt{2}\,M(24221) - 21\sqrt{30}\,M(24021)\right) ,$$

where

$$T_3 = \tfrac{1}{30}M(00000) + \tfrac{7}{60}M(00020) - \tfrac{245}{36}M(22022) - \tfrac{7}{36}M(02022) - \tfrac{189}{400}M(04042)$$
$$-\tfrac{35}{12}M(22222) - \tfrac{21}{4}M(24222) . \tag{54}$$

Finally, when $J_X^{P_X} = 2^-$ there are the following relations between the independent helicity amplitude ratios and moments:

$$x^2 = \frac{1}{T_4}\left(\frac{1}{60}M(00000) + \frac{1}{24}M(00020) - \frac{1}{10}M(00040) - \frac{1}{6}M(20000) - \frac{5}{12}M(20020) + M(20040)\right) ,$$

$$y^2 = \frac{1}{T_4}\left(\frac{1}{50}M(00000) + \frac{1}{20}M(00020) - \frac{3}{25}M(00040) - \frac{1}{2}M(02022) - \frac{\sqrt{15}}{5}M(02042)\right.$$
$$\left. + \frac{27}{100}M(04042) + \frac{9\sqrt{15}}{200}M(04022)\right) ,$$

$$z'^2 = \frac{1}{T_4}\left(\frac{5}{2}M(22222) + \sqrt{15}M(22242) + \frac{9}{2}M(24222) + \frac{9\sqrt{15}}{5}M(24242)\right) ,$$

$$\xi_2^2 = \frac{2M(00000) + 10M(00020) + 18M(00040)}{2M(00000) + 5M(00020) - 12M(00040)} ,$$

$$\eta_2^2 = \frac{2M(00000) - 10M(00020) + 3M(00040)}{2M(00000) + 5M(00020) - 12M(00040)} ,$$

$$x \cos\phi_x = \frac{1}{T_4}\left(\frac{5\sqrt{3}}{6}M(22122) + \sqrt{5}\,M(22142) + \frac{27\sqrt{6}}{12}M(24122) + \frac{27\sqrt{10}}{10}M(24142)\right) ,$$

$$y \cos\phi_y = \frac{1}{T_4}\left(-\frac{5\sqrt{6}}{6}M(22222) - \sqrt{10}M(22242) + \frac{9\sqrt{6}}{8}M(24222) + \frac{27\sqrt{10}}{20}M(24242)\right) ,$$

$$xy \cos(\phi_x - \phi_y) = \frac{1}{T_4}\left(\frac{9}{4}M(24122) + \frac{9\sqrt{15}}{10}M(24142) - \frac{5\sqrt{2}}{2}M(22122) - \sqrt{30}M(22142)\right) ,$$

$$\xi_2 \cos\phi_2 = \frac{1}{T_4}\left(\frac{27}{20}M(04041) + \frac{3\sqrt{30}}{40}M(04021) + \frac{5}{12}M(02021) + \frac{\sqrt{30}}{4}M(02041) - \frac{35}{12}M(22021)\right.$$
$$\left. + \frac{7\sqrt{30}}{4}M(22041) + \frac{21\sqrt{2}}{4}M(24221) + \frac{9\sqrt{15}}{10}M(24243) + \frac{5}{4}M(22221) + \frac{3\sqrt{30}}{4}M(22241)\right) ,$$

$$\eta_2 \cos\phi_2' = \frac{1}{T_4}\left(\frac{27}{20}M(04041) - \frac{9\sqrt{30}}{20}M(04021) + \frac{5}{12}M(02021) - \frac{\sqrt{30}}{24}M(02041) - \frac{35}{12}M(22021) + \frac{7\sqrt{30}}{24}M(22041)\right.$$
$$\left. + \frac{9\sqrt{15}}{10}M(24243) + \frac{5}{4}M(22221) - \frac{\sqrt{30}}{8}M(22241)\right) , \quad (55)$$

where

$$T_4 = \frac{1}{100}M(00000) + \frac{1}{40}M(00020) - \frac{3}{50}M(00040) + \frac{1}{4}M(02022)$$
$$+ \frac{\sqrt{15}}{10}M(02042) + \frac{81}{100}M(04042) + \frac{81\sqrt{15}}{700}M(04022) . \quad (56)$$

The discussion above for the three-step two-body hadronic decay process (2) of J/ψ is also applicable to the corresponding J/ψ radiative decay process (1) provided the independent helicity amplitude ratios z and z' appearing in the formulas are fixed to zero.

III. SUMMARY

We discuss the three-step two-body hadronic and radiative decay processes (1) and (2) of J/ψ by using the moment analysis. The moments for the cases of the boson resonance X with spin-parity $J_X^{P_X} = 0^-$, $1^\pm$, and $2^\pm$, and the intermediate state Y with spin-parity $J_Y^{P_Y} = 0^+$, 1^-, and 2^+ have been given. In every case each independent helicity amplitude ratio can be expressed in terms of the moments. The results show that the spin, parity of the boson resonance X, and the ratios of helicity amplitudes for the corresponding processes can be determined except for a very special case in terms of these theoretical formulas and the measuring of the corresponding moments experimentally. The moment analysis method is more effective than the helicity amplitude method when the statistics are lower and there are resonances with different spin overlap in the mass region under consideration. Although Mark II, Crystal Ball, Mark III, DM2, and BES have accumulated about 2.7×10^7 samples of

J/ψ experimentally up to now. It is still not enough to study the new hadronic state, such as, glueball, hybrid states. Therefore, a systematic study of J/ψ radiative and hadronic decay processes in terms of the moment analysis method may provide a useful approach to search for the new hadronic states.

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