Breakup of hadron masses and the energy-momentum tensor of QCD

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Hadron masses are shown to be separable in QCD into contributions of quark and gluon kinetic and potential energies, quark masses, and the trace anomaly. The separation is based on a study of the structure of the QCD energy-momentum tensor and its matrix elements in hadron states. The paper contains two parts. In the first part, a detailed discussion of the renormalization properties of the energy-momentum tensor is given. In the second part, a mass separation formula is derived and then applied to the nucleon, pion, and the QCD vacuum. Implications of the results on hadron structure and nonperturbative QCD dynamics are discussed.

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I. INTRODUCTION

For any physical system, a good knowledge about its mass structure is helpful in understanding the underlying dynamics of the system. The ground state energy of nuclei as a function of the mass number reveals the nuclear shell structure and the saturation property of nuclear force [1]. An organized pattern of hadron masses led to the speculation of the quark substructure of hadrons [2]. Likewise, any knowledge about the structure of hadron masses in terms of its underlying constituents, quarks, and gluons, can be useful to unlocking the physics of quantum chromodynamics (QCD) in the strong coupling region.

At the classical level, the Lagrangian of chromodynamics is invariant under scale transformation if quark masses are neglected, and thus hadron masses necessarily vanish. Radiative quantum effects break the scale symmetry [3] and introduce a dimensional parameter $\Lambda_{\rm QCD}$ through dimensional transmutation [4]. (For a classical discussion on the scale anomaly, see Ref. [3].) Although $\Lambda_{\rm QCD}$ is well determined through scale-breaking effects in high-energy processes [5], the physical mechanism for generating the scale at low energy is not quite clear. (It is encouraging, though, that the scale determined from hadron spectrum using lattice QCD is consistent with that determined at high energy [6].)

In the past, scale generation at low energy is largely understood in two seemingly unrelated pictures, and so are masses of hadrons. The first picture emphasizes the aspect of color confinement, through which quarks in hadrons are confined to a cavity of radius ~ 1 fm. A simple, representative model is the MIT bag, in which the mass of the nucleon is generated from the quark kinetic energy and vacuum pressure [7]. A dimensional analysis shows that the quark kinetic energy accounts for 3/4 of the mass and the vacuum pressure accounts for the remaining 1/4. In some loose sense, the vacuum pressure models effects of long-wavelength gluons. The second picture for scale generation emphasizes chiral symmetry breaking. Through such phase transition in the QCD vacuum, light quarks acquire a constituent mass of order 300 MeV. Masses of hadrons are then approximately the sum of the constituent masses and quark kinetic and potential energies [8]. Both pictures work quite well for hadron spectra and other physical observables. It is not known, however, which picture or which combination of the two is closer to reality.

In a recent Letter [9], this author showed that insight into the mass structure of hadrons can be obtained through a study of the energy-momentum tensor of QCD. The result is a separation of hadron masses into contributions from quark and gluon kinetic and potential energies, the current quark masses, and the trace anomaly. The last part is a direct result of scale symmetry breaking, and is analogous to the vacuum pressure empirically introduced in the MIT bag model. Though the scale of other contributions is determined by the anomaly, relative magnitudes reflect important aspects of the low-energy quark-gluon dynamics. The separation here is analogous to the *virial theorem* for a simple harmonic oscillator and the hydrogen atom.

The goal of this paper is to provide more field theoretical basis for the mass separation and to extend its application to other hadrons in addition to the nucleon. The field theoretical discussion mainly answers questions such as the following: Why is the QCD Hamiltonian finite? Why can it be separated into gauge-invariant and finite pieces? How does the renormalization scale dependence affect the separation? Why is there an extra term in the Hamiltonian from the trace anomaly? An application of the mass separation to a specific hadron requires knowledge of two matrix elements: the momentum fraction of the hadron carried by quarks in the finite momentum frame and the quark scalar charges. The matrix elements are known to a good accuracy in the nucleon and pion. However, there is no firm estimate of these for other hadrons. Nonetheless, a plausible assumption leads to a crude picture for a general mass partition.

The paper consists of two main sections. In Sec. II, we start with the separation of the QCD energy-momentum tensor into the traceless and trace parts. Then we consider renormalization of both parts in dimensional regularization and covariant gauge fixing. We present a derivation of the trace anomaly from the point of view of operator mixing. We also discuss the structure of the mixing matrix for the traceless part of the energy-momentum tensor. As a by-product, we derive the dilatation current in QCD and study its anomalous Ward identities (Callan-Symanzik equation). For a phenomenology oriented reader, this section can be skipped. In Sec. III, we study the hadron matrix element of the tensor and derive a separation of the QCD Hamiltonian and hence hadron masses. For the application to the nucleon, we focus on discussing physical significance of the different contributions. The result for the pion strongly supports the concept that the pion is a collective vacuum excitation. Finally, comments are made about general features of the mass separation for other hadrons and the QCD vacuum.

II. RENORMALIZATION OF THE QCD ENERGY-MOMENTUM TENSOR

In this section, we discuss renormalization properties of the QCD energy-momentum tensor. A similar but less extensive discussion was first made by Freedman, Muzinich, and Weinberg [10] for non-Abelian gauge theory. Since then, many papers relevant to the subject have appeared in the literature. The present discussion is not a review of the subject. In particular, there are still open questions that are under current debate. Rather, we will focus on the following three aspects that are most useful for our discussion of hadron masses. First, the energy-momentum tensor can be uniquely separated into the trace and traceless parts which are to be separately renormalized. Second, the trace part of the tensor is shown to be finite according to the fact that renormalized Green's functions are finite functions of renormalized masses and couplings. The result is equivalent to a new derivation of the trace anomaly originally given in Refs. [11,12]. Third, the traceless part of the tensor is shown to be finite using Ward identities related to space-time translational symmetry. A study of scale symmetry and related Ward identities (Callan-Symanzik equation) is included in the end of this section as a by-product of the previous discussion.

The fundamental QCD Lagrangian reads

$$\mathcal{L}_{\rm QCD} = \bar\psi(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ , \tag{1}$$

where ψ are quark fields, carrying implicit flavor, color, and Dirac indices, $D^\mu = \partial^\mu + igA^\mu$ is a covariant derivative with $A^\mu = A^{\mu a}t^a$ being the gauge potential $({\rm Tr}t^at^b = \delta^{ab}/2)$, m is the quark mass matrix in flavor space. The gluon field strength has the usual non-Abelian expression

$$F^{\mu\nu a} = \partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} - gf^{abc}A^{\mu b}A^{\nu c} \ , \tag{2}$$

where f^{abc} is the structure constant of the gauge group. To simplify notations, we shall omit color indices if no confusion arises. All quantities without further specification are bare ones.

To discuss renormalization, we work in dimensional regularization and (modified) minimal subtraction scheme. An important virtue of this scheme is that it does not modify the basic QCD Lagrangian except all Lorentz indices and space-time integrations are taken to be d dimensional. (The Pauli-Villars regularization, some momentum cutoff schemes, or lattice cutoff all modify the Lagrangian in a more significant way.) We use perturbation theory to explore large momentum behavior, and thus a gauge fixing is needed. We choose the usual prescription for use of the covariant gauge, which introduces two extra terms in the Lagrangian: the gauge-fixing term

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2 \ , \tag{3}$$

and the Faddeev-Popov ghost term

$$\mathcal{L}_{\rm gs} = \partial^\mu\bar\omega D_\mu\omega \ . \tag{4}$$

Therefore the total effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm GS} \tag{5}$$

enters the remaining discussion.

The energy-momentum tensor can be derived from the fact that the action $S = \int d^dx\mathcal{L}_{\rm eff}$ is invariant under space-time translation. The result is well known [10]:

$$\begin{aligned} T^{\mu\nu} = &-g^{\mu\nu}\mathcal{L}_{\rm eff} - F^{\mu\alpha}F^\nu{}_\alpha + \frac{1}{2}\bar\psi iD^{(\mu}\gamma^{\nu)}\psi + \frac{1}{2}\bar\psi i\overleftarrow{D}^{(\mu}\gamma^{\nu)}\psi - g^{\mu\nu}\xi^{-1}\partial^\alpha(A_\alpha\partial\cdot A) + 2\xi^{-1}A^{(\mu}\partial^{\nu)}(\partial\cdot A) \\ &+2\partial^{(\mu}\bar\omega D^{\nu)}\omega - \frac{\delta S}{\delta A_\mu}A_\nu - \frac{\delta S}{\delta\psi}\frac{1}{8}[\gamma^\mu,\gamma^\nu]\psi - \bar\psi\frac{1}{8}[\gamma^\mu,\gamma^\nu]\frac{\delta S}{\delta\bar\psi} \ , \end{aligned} \tag{6}$$

where $(\mu\nu)$ means symmetrization of the indices and $\overleftarrow{D}^\mu = -\overleftarrow{\partial}^\mu + igA^\mu$. There are two standard methods to derive the above expression. First, one can write down the canonical Noether current associated with space-time translational symmetry and then improve it with the Belinfante procedure [13]. Second, one can follow Ref. [10], deriving a current coupling to gravity. Both methods yield the same tensor for QCD, though not for the ϕ^4 theory.

The last three terms in (6) are usually ignored in other references because they vanish when the equations of motion for gluon and quark fields are used. When they are inserted into a Green's function, their role is to change polarizations of external lines, and thus they are finite operators. When they are included in the tensor, the Ward identities related to space-time translation symmetry take a simpler form [12]. Without these terms, the energy-momentum tensor is symmetric:

$$T^{\mu\nu} = T^{\nu\mu} \ . \tag{7}$$

In the following discussion, we concentrate on the symmetric part of the tensor only.

The symmetric part of the energy-momentum tensor can be split into a sum of traceless and trace parts. Although a splitting like this is not unique in general, we define the splitting in the following unambiguous way:

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \ , \tag{8}$$

where the traceless ($\bar{T}^{\mu\nu}$) and trace parts ($\hat{T}^{\mu\nu}$) are

$$\begin{aligned} \bar{T}^{\mu\nu} &= T^{\mu\nu} - \frac{1}{d} g^{\mu\nu} T^{\alpha}{}_{\alpha} \ , \\ \hat{T}^{\mu\nu} &= \frac{1}{d} g^{\mu\nu} T^{\alpha}{}_{\alpha} \ . \end{aligned} \tag{9}$$

Notice that we have defined the trace consistent with dimensional regularization, under which tensors of different ranks do not mix. Therefore the necessary and sufficient condition for the energy-momentum tensor to be finite is that the traceless and trace parts are separately finite. This is indeed the case as we shall see below.

A. The trace part of the energy-momentum tensor

Let us first consider the trace-part of the energy-momentum tensor,

$$\begin{aligned} T^{\alpha}{}_{\alpha} = {} & \bar{\psi} m \psi + \epsilon [O_F + \bar{O}_{\rm GF}] \\ & -(2-\epsilon)\partial^{\alpha}[\xi^{-1} A_{\alpha} \partial \cdot A + \bar{\omega} D_{\alpha} \omega] \\ & -(3-\epsilon)\bar{\psi}\frac{\delta S}{\delta \bar{\psi}} - (2-\epsilon)\bar{\omega}\frac{\delta S}{\delta \bar{\omega}} \\ & - \left(1 - \frac{\epsilon}{2}\right) A_{\mu} \frac{\delta S}{\delta A_{\mu}} \ , \end{aligned} \tag{10}$$

where we have defined

$$\begin{aligned} O_F &= -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} \ , \\ O_{\rm GF} &= -(2\xi)^{-1} (\partial \cdot A)^2 \ , \\ \bar{O}_{\rm GF} &= O_{\rm GF} - \frac{1}{2} A_{\mu} \frac{\delta S}{\delta A_{\mu}} \ . \end{aligned} \tag{11}$$

In Refs. [11,12], it was shown that the trace can be expressed in terms of a linear combination of a set of renormalized operators with finite coefficients. Here we would like to derive the same result from the operator mixing point of view [14], by extending the discussion in Ref. [15].

First consider the equations-of-motion-related operators

$$\begin{aligned} O_A &= A_{\mu} \frac{\delta S}{\delta A_{\mu}} \ , \\ O_{\psi} &= \bar{\psi} \frac{\delta S}{\delta \bar{\psi}} = \bar{\psi}(i \not{D} - m)\psi \ , \\ O_{\omega} &= \bar{\omega} \frac{\delta S}{\delta \bar{\omega}} = \bar{\omega} \partial^{\alpha} D_{\alpha} \omega \ . \end{aligned} \tag{12}$$

A zero momentum insertion of these operators into a renormalized Green's function produces the number of external lines. Therefore all these operators are finite and need no renormalization.

The total derivative operator in the third term in Eq. (10) is a Becchi-Rouet-Stora-Tyutin (BRST) variation of $\partial_{\alpha}(A^{\alpha}\bar{\omega})$: i.e.,

$$\partial^{\alpha} \delta_{\rm BRST}(A_{\alpha}\bar{\omega}) = \partial^{\alpha}[\xi^{-1} A_{\alpha} \partial \cdot A + \bar{\omega} D_{\alpha} \omega] \ . \tag{13}$$

As such it has vanishing physical matrix elements at nonzero momentum [16,17]. At zero momentum, the operator vanishes identically as it contains no poles. Since other dimension-four operators do not mix with it, it is a finite operator by itself.

Renormalization of the rest three operators can be worked out by studying the renormalized Green's function generating functional,

$$Z(J,\eta,\bar{\eta},\chi,\bar{\chi}) = \frac{\int D(A^R,\psi^R,\bar{\psi}^R,\omega^R,\bar{\omega}^R) \exp\left\{ i \left[S + \int (J^{\mu} A^R_{\mu} + \bar{\eta}\psi^R + \bar{\psi}^R \eta + \bar{\chi}\omega^R + \bar{\omega}^R \chi) \right] \right\}}{\int D(A^R,\psi^R,\bar{\psi}^R,\omega^R,\bar{\omega}^R) \exp(iS)} \ , \tag{14}$$

where the renormalized fields are related to bare fields by the multiplicative renormalization constants,

$$\psi = Z_2^{1/2} \psi^R, \quad \omega = \tilde{Z}^{1/2} \omega^R, \quad A_{\mu} = Z_3^{1/2} A^R_{\mu} \ . \tag{15}$$

Since $Z(J,\eta,\bar{\eta},\chi,\bar{\chi})$ is finite, its derivatives with respect to the renormalized quark masses m^R, the gauge coupling constant g^R, and the gauge-fixing parameter ξ^R are also finite. In the dimensional regularization and minimal subtraction scheme, all the renormalization constants are independent of quark masses, and the renormalization factors for the gauge coupling Z_g and the quark masses Z_m are independent of the gauge-fixing parameter [18]. According to these, we find the following quantities are finite:

$$O_m = \bar{\psi} m \psi \ ,$$
$$\bar{O}^R_{\rm GF} = \left(1+\xi^R\frac{\partial \ln Z_3}{\partial \xi^R}\right)\bar{O}_{\rm GF} - \xi^R\frac{\partial \ln Z_2}{\partial \xi^R}O_\psi - \xi^R\frac{\partial \ln \tilde{Z}}{\partial \xi^R}O_\omega \ ,$$
$$O^R_F = \left(1+g^R\frac{\partial \ln Z_g}{\partial g^R}\right)O_F + \left(g^R\frac{\partial \ln Z_g}{\partial g^R} - \xi^R\frac{\partial \ln Z_3}{\partial \xi^R} + \frac{1}{2}g^R\frac{\partial \ln Z_3}{\partial g^R}\right)\bar{O}_{\rm GF}$$
$$+\left(\xi^R\frac{\partial \ln Z_2}{\partial \xi^R} - \frac{1}{2}g^R\frac{\partial \ln Z_2}{\partial g^R}\right)O_\psi + \left(\xi^R\frac{\partial \ln \tilde{Z}}{\partial \xi^R} - \frac{1}{2}g^R\frac{\partial \ln \tilde{Z}}{\partial g^R}\right)O_\omega + \frac{1}{2}g^R\frac{\partial \ln Z_m}{\partial g^R}O_m \ . \tag{16}$$

From the above equations, we can solve O_F and $\bar{O}_{\rm GF}$ in terms of the renormalized ones:

$$\epsilon(O_F + \bar{O}_{\rm GF}) = (-2\beta/g^R)O^R_F + (-2\beta/g^R + \gamma_3)\bar{O}^R_{\rm GF} - \tilde{\gamma}O_\omega - \gamma_2 O_\psi + \gamma_m O_M \ , \tag{17}$$

where we have defined the anomalous dimensions

$$\gamma_{2,3,m} = \mu\frac{d\ln Z_{2,3,m}}{d\mu}, \quad \tilde{\gamma} = \mu\frac{d\ln\tilde{Z}}{d\mu}, \quad \beta = -g^R\mu\frac{d\ln Z_g}{d\mu} \ . \tag{18}$$

Since all these quantities have no $1/\epsilon$ poles, the right-hand side of Eq. (17) is finite.

Inserting Eq. (17) into Eq. (10), we find the trace part of the energy-momentum tensor expressed in terms of finite, renormalized operators:

$$T^\alpha{}_\alpha = (-2\beta/g^R)(O^R_F + \bar{O}^R_{\rm GF}) + (1+\gamma_m)O_m - 2\partial^\alpha\delta_{\rm BRST}(A_\alpha\omega)$$
$$+\gamma_3\bar{O}^R_{\rm GF} - \left(1+\frac{\gamma_3}{2}\right)O_A - (2+\tilde{\gamma})O_\omega - (3+\gamma_2)O_\psi \ . \tag{19}$$

Two comments are in order at this point. First, composite operators defined through path-integral formalism have implied subtraction of their vacuum expectations. Thus they are said to be "normal ordered," but not in the usual sense of relative to perturbative vacuum. Second, although the individual term in the above equation depends on the renormalization scale (μ^2), the sum does not.

According to the Joglekar-Lee theorems [16,17], the matrix elements of O_A, O_ψ, and O_ω, and BRST exact operators vanish in a physical state. Thus as far as hadron matrix elements are concerned, we effectively have

$$T^\alpha{}_\alpha = (-2\beta/g^R)O^R_F + (1+\gamma_m)O_m \ . \tag{20}$$

This suggests we write the trace part of the energy-momentum tensor as

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_a(\mu^2) + \hat{T}^{\mu\nu}_m(\mu^2) \ , \tag{21}$$

where $T^{\mu\nu}_a = -(2\beta/g^R)O^R_F(g^{\mu\nu}/4)$ and $T^{\mu\nu}_m = (1+\gamma_m)O_m(g^{\mu\nu}/4)$.

B. The traceless part of the energy-momentum tensor

The traceless part of the energy-momentum tensor can be written as a sum of four parts:

$$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_g + \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_{\rm GV} + E^{\mu\nu} \ . \tag{22}$$

The gauge-invariant gluon part $\bar{T}^{\mu\nu}_g$ is

$$T^{\mu\nu}_g = -F^{(\mu\alpha}F^{\nu)}{}_\alpha \ , \tag{23}$$

where and henceforth the symbol $(\mu\nu)$ also means subtraction of the trace. The gauge-invariant quark part is

$$T^{\mu\nu}_q = \frac{1}{2}\bar{\psi}iD^{(\mu}\gamma^{\nu)}\psi + \frac{1}{2}\bar{\psi}i\overleftarrow{D}^{(\mu}\gamma^{\nu)}\psi \ . \tag{24}$$

The gauge variant part is

$$T^{\mu\nu}_{\rm GV} = \xi^{-1}A^{(\mu}\partial^{\nu)}\partial\cdot A + \partial^{(\mu}\bar{\omega}D^{\nu)}\omega \ , \tag{25}$$

which is the BRST variation of $\partial^{(\mu}\bar{\omega}A^{\nu)}$. Finally, the gluon-equation-of-motion-related operator is

$$E^{\mu\nu} = -A^{(\mu}\frac{\delta S}{\delta A_{\nu)}} \ , \tag{26}$$

which is finite.

According to the Joglekar and Lee theorems [16], the above four operators close under renormalization. Furthermore, the mixing matrix takes the form

$$\begin{pmatrix} T^{\mu\nu}_q \\ T^{\mu\nu}_g \\ T^{\mu\nu}_{\rm GV} \\ E^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{qq} & Z_{qg} & Z_{qa} & Z_{qe} \\ Z_{gq} & Z_{gg} & Z_{ga} & Z_{ge} \\ 0 & 0 & Z_{aa} & Z_{ae} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T^{\mu\nu}_q \\ T^{\mu\nu}_g \\ T^{\mu\nu}_{\rm GV} \\ E^{\mu\nu} \end{pmatrix}^R \ . \tag{27}$$

That is, the gauge-variant operators do not mix with gauge-invariant operators and the equations of motion operators do not mix with any other operators.

To find the Z's, one needs to study Green's functions with one insertion of the composite operators in perturbation theory. However, we can find constraints among the renormalization constants by studying Ward identities related to space-time translational symmetry. Calculating the divergence of the energy-momentum tensor without using the equations of motion, we find

$$\partial_\mu T^{\mu\nu} = -\frac{\delta S}{\delta A_\mu}\partial^\nu A^\mu - \frac{\delta S}{\delta\psi}\partial^\nu\psi - \frac{\delta S}{\delta\bar{\psi}}\partial^\nu\bar{\psi} - \frac{\delta S}{\delta\omega}\partial^\nu\omega - \frac{\delta S}{\delta\bar{\omega}}\partial^\nu\bar{\omega} \ . \quad (28)$$

When the above equation is inserted into a renormalized Green's function (Ward identity), the right-hand side simply replaces the elementary fields by their derivatives, and thus is finite. Hence the divergence of the energy-momentum tensor is a finite operator. The only remaining counterterm for the tensor itself is of form

$$(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)f(A,\omega,\bar{\omega}) \ , \quad (29)$$

where f is a dimension-two function of its arguments. However, since the trace part of the energy-momentum tensor is finite, Lorentz symmetry allows only counterterm tensors that are symmetric and traceless. Thus the traceless part of the tensor must be finite by itself [12].

The finite traceless part of the energy-momentum tensor imposes the following constraints on the renormalization constants in minimal subtraction scheme:

$$\begin{aligned} Z_{qq} + Z_{gq} &= 1 \ , \\ Z_{qg} + Z_{gg} &= 1 \ , \\ Z_{qa} + Z_{ga} + Z_{aa} &= 1 \ , \\ Z_{qe} + Z_{ge} + Z_{ae} &= 0 \ . \end{aligned} \quad (30)$$

Thus in the scheme, we have

$$T_q^{\mu\nu} + T_g^{\mu\nu} + T_{\rm GV}^{\mu\nu} + E^{\mu\nu} = T_q^{\mu\nu R}(\mu^2) + T_g^{\mu\nu R}(\mu^2) + T_{\rm GV}^{\mu\nu R}(\mu^2) + E^{\mu\nu} \ . \quad (31)$$

Although the individual term on the right-hand side depends on the renormalization scale, the sum does not. Again, according to the Joglekar-Lee theorems, $T_{\rm GV}^{\mu\nu R}(\mu^2)$ and $E^{\mu\nu}$ have vanishing physical matrix elements, and for practical purposes, we can keep only the gauge-invariant quark and gluon contributions in the traceless part of the energy-momentum tensor,

$$\bar{T}^{\mu\nu} = \bar{T}_q^{\mu\nu R}(\mu^2) + \bar{T}_g^{\mu\nu R}(\mu^2) \ . \quad (32)$$

Recently, some questions have been raised in the literature about validity of the Joglekar and Lee theorems [19,20], in particular, regarding the form of the mixing matrix appearing in Eq. (27). Harris and Smith [21] pointed out that so long as one is working with composite operators at nonzero momentum transfer, all the theorems should remain valid. Collins and Scalise [20], on the other hand, have worked at zero momentum transfer. They showed that the on-shell limit for gauge bosons is subtle and potentially causes problems. However, it appears that if one works with off-shell Green's functions or physical hadron states, the operator mixing shall follow the standard theorems. More work is certainly needed in this direction to clear up the issue.

C. Scale symmetry and the anomalous Ward identities

The above discussion on mixing of dimension-four operators and the trace of the energy-momentum tensor permits a simple explanation for the anomalous breaking of scale symmetry and a derivation of Callan-Symanzik equation (or the anomalous Ward identity). Consider the scale (dilatation) transformation in d dimensions:

$$\begin{aligned} x &\to \lambda x \ , \\ \psi(x) &\to \lambda^{(d-1)/2}\psi(\lambda x) \ , \\ A^\mu(x) &\to \lambda^{d/2-1}A^\mu(\lambda x) \ , \\ \omega(x) &\to \lambda^{d/2-1}\omega(\lambda x) \ . \end{aligned} \quad (33)$$

The change in the effective QCD Lagrangian is a total derivative plus symmetry-breaking terms. There are two types of symmetry-breaking effects: quark masses and the gauge coupling, the latter has dimension $\epsilon/2$ in dimensional regularization. Thus,

$$\delta\mathcal{L}_{\rm eff} = \partial^\mu(x_\mu\mathcal{L}_{\rm eff}) + \bar{\psi}m\psi - \frac{\epsilon}{2}g\frac{\partial\mathcal{L}_{\rm eff}}{\partial g} \ . \quad (34)$$

The last term is clearly not gauge invariant. A simple rearrangement reveals that it consists of operators O_F and $\bar{O}_{\rm GF}$ defined in the previous section plus a total derivative term:

$$-g\frac{\partial\mathcal{L}_{\rm eff}}{\partial g} = \partial^\mu(F_{\mu\nu}A^\nu + \xi^{-1}(\partial\cdot A)A_\mu) + 2O_F + 2\bar{O}_{\rm GF} \ . \quad (35)$$

Since O_F and $\bar{O}_{\rm GF}$ diverges like $1/\epsilon$, the scale symmetry is broken anomalously at the quantum level.

On the other hand, with use of the equations of motion, one has

$$\delta\mathcal{L}_{\rm eff} = \partial^\mu\left(\frac{\partial\mathcal{L}_{\rm eff}}{\partial\partial_\mu\phi_i}\delta\phi_i\right) \ , \quad (36)$$

where ϕ_i is a generic notation for all the fields. According to this, we can define a dilatation current corresponding to the scale transformation:

$$J_D^\mu = \frac{\partial\mathcal{L}_{\rm eff}}{\partial\partial_\mu\phi_i}\delta\phi_i - x^\mu\mathcal{L}_{\rm eff} - \frac{\epsilon}{2}[F^{\mu\nu}A_\nu + \xi^{-1}(\partial\cdot A)A^\mu] \ . \quad (37)$$

The symmetry-breaking terms now appear as a divergence of the current:

$$\partial_\mu J_D^\mu = \bar{\psi}m\psi + \epsilon(O_F + \bar{O}_{\rm GF}) \ . \quad (38)$$

To prove this, use of the equations of motion is essential.

The dilatation current has a simple relation with the energy-momentum tensor in Belinfante's form. To see this, we use the definition of the canonical energy-momentum tensor $T_C^{\mu\nu}$ to write

$$J_D^\mu = T_C^{\mu\nu} x_\nu + \frac{\partial \mathcal{L}_{\rm eff}}{\partial \partial_\mu \phi_i} d_{\phi_i} \frac{\epsilon}{2}[F^{\mu\nu}A_\nu + \xi^{-1}(\partial \cdot A)A^\mu] \ , \tag{39}$$

where d_ϕ is the canonical dimension for ϕ_i field. Using the Belinfante improvement, one has

$$J_D^\mu = T^{\mu\nu}x_\nu + \frac{\partial \mathcal{L}_{\rm eff}}{\partial \partial_\mu \phi_i}\left(\Sigma^{\mu\rho} + g^{\mu\rho}d_\phi\right)_{ij}\phi_j - \frac{\epsilon}{2}[F_{\mu\nu}A^\nu + \xi^{-1}(\partial\cdot A)A_\mu] \ , \tag{40}$$

where the second term is called field virial, which has a part that cancels the last gauge noninvariant term,

$$J_D^\mu = T^{\mu\nu}x_\nu + \left(1-\frac{\epsilon}{2}\right)(\partial^\mu\bar\omega\omega + \bar\omega D^\mu\omega) + (2-\epsilon)\xi^{-1}(\partial\cdot A)A^\mu \ . \tag{41}$$

The remaining terms cannot be written as a total derivative. According to Callan, Coleman, and Jackiw [3], this means that conformal symmetry present in the basic QCD Lagrangian is broken by the gauge fixing. This fact is of course well known in perturbation theory where a gauge-fixed gluon propagator does not have conformal symmetry. However, the breaking term is a BRST-exact operator plus the ghost equation of motion and thus has no physical consequences.

To derive the anomalous Ward identities associated with scale transformation, we consider the corresponding change in the Green's function generating functional $Z(J_\mu, \eta, \bar\eta, \chi, \bar\chi)$. A simple derivation yields the equation

$$\sum_i \left(d_{\phi_i} + x_i\frac{\partial}{\partial x_i}\right) G^R(x_j) + G^R(x_j, \partial^\mu J_{D\mu}) = 0 \ , \tag{42}$$

where $G^R(x_i)$ is a renormalized Green's function with n_A external gluon lines, n_ψ quark (antiquark) lines, and n_ω ghost (antighost) lines. $G^R(x_i, \partial^\mu J_{D\mu})$ is the same Green's function inserted with divergence of the dilatation current. In momentum space, we have

$$\left[-\sum_i p_i\frac{\partial}{\partial p_i} - 4(n_A + n_\psi + n_\omega - 1) + \left(n_A + \frac{3}{2}n_\psi + n_\omega\right)\right] G^R(p_i) + G^R(p_i, \partial^\mu J_{D\mu}) = 0 \ . \tag{43}$$

Here momenta p_i are conjugation of all x_i except for one which is taken to be zero. On the other hand, a simple dimensional analysis yields

$$\left[\sum_i p_i\frac{\partial}{\partial p_i} + \mu\frac{\partial}{\partial\mu} + m^R\frac{\partial}{\partial m^R} + 4(n_A + n_\psi + n_\omega - 1) - \left(n_A + \frac{3}{2}n_\psi + n_\omega\right)\right] G^R(p_i) = 0 \ , \tag{44}$$

where μ is a renormalization scale. Combining the above two equations, we get

$$\left[\mu\frac{\partial}{\partial\mu} + m^R\frac{\partial}{\partial m^R}\right] G^R(p_i) + G^R(p_i, \partial^\mu J_{D\mu}) = 0 \ . \tag{45}$$

According to Ref. [11], an insertion of the divergence of the dilatation current can be replaced by derivatives with respect to the gauge coupling, quark masses, and the gauge-fixing parameter. Thus, we find the anomalous Ward identity (Callan-Symanzik equation)

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g^R} - \gamma_m m^R\frac{\partial}{\partial m^R} - \gamma_3\xi^R\frac{\partial}{\partial\xi^R} + n_A\frac{\gamma_3}{2} + n_\psi\frac{\gamma_2}{2} + n_\omega\frac{\tilde\gamma}{2}\right] G^R(p_i) = 0 \ . \tag{46}$$

This is just the well-known renormalization group equation which can be derived independently by studying the dependence of the renormalized Green's function on the renormalization scale.

III. BREAKUP OF THE HADRON MASSES

Let us first recapitulate the main results obtained in the last section. First of all, we have shown explicitly that the QCD energy-momentum tensor $T^{\mu\nu}$ (with vacuum subtraction) is a finite composite operator and thus a physical observable. Second, the tensor can be separated uniquely into the traceless and trace parts, each of which is a finite and scale-independent composite operator:

$$T^{\mu\nu} = \bar T^{\mu\nu} + \hat T^{\mu\nu} \ . \tag{47}$$

So the separation at this level is completely physical. Finally, in physical states the traceless part of the tensor is effectively a sum of the renormalized, gauge-invariant quark and gluon contributions,

$$\bar T^{\mu\nu} = \bar T_q^{\mu\nu R}(\mu^2) + \bar T_g^{\mu\nu R}(\mu^2) \ , \tag{48}$$

and the trace part is a sum of the gauge-invariant quark mass and trace anomaly contributions,

$$\hat T^{\mu\nu} = \hat T_a^{\mu\nu R}(\mu^2) + \hat T_m^{\mu\nu R}(\mu^2) \ . \tag{49}$$

The separation at this second level is renormalization and regularization scheme dependent.

In this section, we study a breakup of hadron masses according to the above properties of the energy-momentum tensor. We first discuss matrix elements of various parts of the tensor in hadron states. Then we write down the finite form of the QCD Hamiltonian and drive a mass separation formula. We reexamine its application to the nucleon and discuss implications of the result on the nucleon's quark-gluon substructure and on the low-energy dynamics of QCD. Application to the pion is studied in a separated subsection, where we show that the color electric and magnetic fields in the pion is the same as those in the QCD vacuum in chiral limit. We also discuss the color fields in the QCD vacuum in a Lorentz covariant, nonperturbative regularization scheme.

A. Matrix element of the energy-momentum tensor in hadrons

Let us consider the forward matrix element of the energy-momentum tensor in a hadron state, $|P\rangle$, where P^μ is the four-momentum of the state. We assume the state is normalized according to $\langle P|P\rangle = (E/M)(2\pi)^3\delta^3(\mathbf{0})$, where $E = P^0$ is the energy of the state and M is the mass of the hadron. It is well known that

$$\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \ . \tag{50}$$

A simple derivation of the above equation goes like this: According to Lorentz symmetry, one has

$$\langle P|T^{\mu\nu}|P\rangle = aP^\mu P^\nu + bg^{\mu\nu} \ , \tag{51}$$

where a and b are scalar constants. On the other hand, the Hamiltonian of the system is $H = \int d^3\vec{x}T^{00}$, which has the following matrix element in the hadron state:

$$\langle P|H|P\rangle = (E^2/M)(2\pi)^3\delta^3(\mathbf{0}) \ . \tag{52}$$

Comparing Eq. (52) with Eq. (51), we obtain Eq. (50).

Equation (50) allows us to obtain the matrix elements of the traceless and trace parts of the energy-momentum tensor separately. In fact, decomposing both sides of Eq. (50) into trace and traceless parts, we have

$$\begin{aligned}\langle P|\bar{T}^{\mu\nu}|P\rangle &= \left(P^\mu P^\nu - \frac{1}{4}g^{\mu\nu}M^2\right)\Big/M \ ,\\ \langle P|\hat{T}^{\mu\nu}|P\rangle &= \frac{1}{4}g^{\mu\nu}M \ .\end{aligned} \tag{53}$$

The right-hand sides of both equations are independent of the renormalization scale, as required by Lorentz symmetry. The second line in (53) is not new; it has appeared before in the literature [22–24].

We use Lorentz symmetry again to define two scheme-dependent matrix elements below. First, we define the matrix element of the quark operator appearing in the traceless tensor,

$$\langle P|\bar{T}_q^{\mu\nu}|P\rangle = a(\mu^2)\left(P^\mu P^\nu - \frac{1}{4}g^{\mu\nu}M^2\right)\Big/M \ , \tag{54}$$

where $a(\mu^2)$ has an explicit scale dependence. From Eq. (48) and the first line in Eq. (53), we have

$$\langle P|\bar{T}_g^{\mu\nu}|P\rangle = [1-a(\mu^2)]\left(P^\mu P^\nu - \frac{1}{4}g^{\mu\nu}M^2\right)\Big/M \ . \tag{55}$$

Second, we define the matrix element of the quark-mass operator:

$$\langle P|\hat{T}_m^{\mu\nu}|P\rangle = b(\mu^2)\frac{1}{4}g^{\mu\nu}M \ , \tag{56}$$

where the renormalization scale dependence comes entirely from the anomalous dimension γ_m of the mass operator, which depends on the renormalized gauge coupling. From Eq. (49) and the second line in (53), we have

$$\langle P|\hat{T}_a^{\mu\nu}|P\rangle = [1-b(\mu^2)]\frac{1}{4}g^{\mu\nu}M \ . \tag{57}$$

Thus the matrix elements of the four parts of the energy-momentum tensor are entirely determined by two parameters $a(\mu^2)$ and $b(\mu^2)$.

The matrix element $a(\mu^2)$ can be extracted from hadron structure functions measured in deep inelastic scattering. According to operator product expansion, the twist-two operator $\bar{T}_q^{\mu\nu}$ appears in the expansion of product of two vector or axial-vector currents. Using a dispersion relation, one can relate the matrix element of $\bar{T}_q^{\mu\nu}$ to the first moment of quark distributions:

$$a(\mu^2) = \sum_f \int_0^1 dx\, x[q_f(x,\mu^2) + \bar{q}_f(x,\mu^2)] \ , \tag{58}$$

where q_f and $\bar{q}_f$ are quark and antiquark distributions in the hadron. The physical meaning of $a(\mu^2)$ is the momentum fraction of the hadron carried by quarks in the infinite momentum frame [25].

The matrix element $b(\mu^2)$ is proportional to the nucleon's scalar charge $\langle P|\bar{\psi}\psi|P\rangle$, which cannot be measured directly from an experiment. However, the operator $\bar{\psi}m\psi$ is a part of the QCD Lagrangian which breaks the chiral symmetry explicitly. Therefore, useful information about $b(\mu^2)$ may be obtained by studying physical effects of chiral symmetry breaking.

B. A separation of hadron masses

The QCD Hamiltonian is defined as

$$H_{\rm QCD} = \int d^3\vec{x}T^{00}(0,\vec{x}) \ . \tag{59}$$

From the discussion of the last section, $H_{\rm QCD}$ is a finite operator. (Interestingly, however, the Lagrangian density has $1/\epsilon$ divergences.) According to the separation of the energy-momentum tensor, we have the partition of the Hamiltonian operator,

$$H_{\rm QCD} = H_q' + H_g + H_m' + H_a \ , \tag{60}$$

where various terms are

$$\begin{aligned} H_q' &= \int d^3\vec{x}\left[\psi^\dagger(-i\mathbf{D}\cdot\alpha)\psi + \frac{3}{4}\bar{\psi}m\psi\right] \ ,\\ H_g &= \int d^3\vec{x}\frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) \ ,\\ H_m' &= \int d^3\vec{x}\frac{1}{4}(1+\gamma_m)\bar{\psi}m\psi \ ,\\ H_a &= \int d^3\vec{x}\frac{1}{4}\beta(g)(\mathbf{E}^2-\mathbf{B}^2) \ . \end{aligned} \tag{61}$$

Rearranging the mass term by defining

$$H_q = \int d^3\vec{x}\psi^\dagger(-i\mathbf{D}\cdot\alpha)\psi \ ,$$
$$H_m = \int d^3\vec{x}\left(1+\frac{1}{4}\gamma_m\right)\bar{\psi}m\psi \ , \quad (62)$$

we have Eq. (60) without the primes. Note that the above expression implicitly contains the renormalization counterterms, which we omit for simplicity. The physical meaning of the various pieces of the Hamiltonian is clear: H_q represents the quark kinetic plus potential energy (the static color interaction is not included); H_g is the gluon kinetic and potential energy; H_m is the quark mass contribution; and finally, H_a is the anomaly contribution whose significance will become clear later.

In field theory, masses of bound states are usually defined as poles in Green's functions (implicitly so in lattice calculations). In Hamiltonian formalism, one can define masses as eigenvalues of a Hamiltonian as in nonrelativistic quantum mechanics. In the following discussion, we assume that hadron masses are calculated as expectation values of the Hamiltonian operator in the rest frame of hadrons:

$$M = \left.\frac{\langle P|H_{\rm QCD}|P\rangle}{\langle P|P\rangle}\right|_{\rm rest\ frame} \ . \quad (63)$$

We view the above equation as a mass probe into the structure of quark and gluon configurations in hadron states, since different pieces of the Hamiltonian have different sensitivity to various components of hadron wave functions. At this point, the reader might recall the recent investigation in the literature about the spin structure of the nucleon [22], where one is interested in how the spin of the nucleon is made of the spin and orbital angular momenta of quarks and gluons. The same question can be asked of the mass structure of hadrons.

According to Eqs. (61), (62), and (63), masses of hadrons can be separated into various contributions:

$$M = M_q + M_g + M_m + M_a \ . \quad (64)$$

Relating the matrix elements of different pieces of the Hamiltonian to those of the energy-momentum tensor, we have

$$M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right)M \ ,$$
$$M_g = \frac{3}{4}(1-a)M \ ,$$
$$M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}bM \ ,$$
$$M_a = \frac{1}{4}(1-b)M \ . \quad (65)$$

Therefore, knowing the parameters a and b, we can determine a complete separation of the masses. In the following subsections, we shall apply the separation to several cases and discuss physical significance of the result.

C. The mass structure of the nucleon

The mass structure of the nucleon was analyzed in detail in the Letter [9]. Let us briefly summarize the main result here and discuss its physical implications more thoroughly. In the end, we shall make some remarks for the mass separation of other hadrons following a rather plausible assumption.

The matrix element $a(\mu^2)$ for the nucleon has been measured quite accurately in various high-energy scattering processes involving nucleons. A recent extraction gives [26]

$$a_{\overline{\rm MS}}(1\ {\rm GeV}^2) = 0.55 \ , \quad (66)$$

where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme. In the Letter, two estimates of $b(1\ {\rm GeV}^2)$, in the limits of chiral SU(3) ($m_s \to 0$) and heavy strange quark ($m_s \to \infty$), were given. They are

$$b(m_s \to 0) = 0.17 \ ,$$
$$b(m_s \to \infty) = 0.11 \ . \quad (67)$$

Since the two limits do not lead to qualitatively different conclusions, we focus on the chiral limit below. The numerical numbers in this case are

$$M_q = 270\ {\rm MeV} \ ,$$
$$M_m = 160\ {\rm MeV} \ ,$$
$$M_g = 320\ {\rm MeV} \ ,$$
$$M_a = 190\ {\rm MeV} \ . \quad (68)$$

According to the above result, the quark kinetic and potential energies contribute about 1/3 of the nucleon mass. The further separation into the two is not gauge invariant. However, the practice does have an intuitive appeal at the phenomenological level. If there are three massless quarks confined to a spherical cavity of radius 1 fm, as in the MIT bag model, the total kinetic energy is 600–700 MeV. Thus, the color-current interaction between quarks and gauge fields contributes −300 to −400 MeV to the mass, which is consistent with the magnitude of N-Δ splitting. Such large current interaction is intrinsic to a truly relativistic theory. It does not occur for instance in low-energy QED, where the static Coulomb potential plays a dominant role. The strong current interaction certainly induces quark interactions of Nambu–Jona-Lasino type, and is perhaps at the origin of the chiral symmetry breaking.

The quark energy can be further separated into contributions of different flavors. The parameter a measures the fraction of the nucleon momentum carried by quarks, which is known separately for each flavor. For instance, 0.38 fraction of the nucleon momentum is carried by up quarks, which can be translated into the up quark contribution to the nucleon mass 250 MeV. Likewise, down quarks contribute 105 MeV mass, and strange quark pairs contribute −85 MeV. One might try to break the contribution from each flavor into these from valence and sea quarks. However, since the valence and sea contributions to the matrix element b are unknown, the separation is not possible. Nonetheless, for up and down quarks, the contribution from b is not large, and may be neglected. From the momentum fractions carried by the sea, we find that the up or down sea contribution to the nucleon mass is on the level of 30 MeV. The small number indicates that there are not many quark and antiquark pairs in the wave function. Thus the nucleon seems to have a simpler Fock expansion than the small current quark masses imply.

The quark mass term contributes about 1/8 of the nucleon mass. About half of which comes from the strange quark pairs. The strange quark contribution here is definitely less certain than two light flavors. One might hope that future lattice measurement of the strange scalar charge may reduce the uncertainty. The implied magnitude of the scalar charges $\langle P|\bar{u}u|P\rangle$, $\langle P|\bar{d}d|P\rangle$, which are charge conjugation odd quantities, is another indication that the number of quark-antiquark pairs is small.

The normal gluon energy contributes about 1/3 of the nucleon mass. This energy includes the color-static Coulomb energy between quarks. The gluon part of the trace anomaly contributes about 1/4 of the mass. From these, we deduce the color-electric and color-magnetic fields in the nucleon separately (taking $\alpha_s(1\ \mathrm{GeV}^2) = 0.4$),

$$\begin{aligned}\langle P|\mathbf{E}^2|P\rangle &= 1700\ \mathrm{MeV}\ ,\\ \langle P|\mathbf{B}^2|P\rangle &= -1050\ \mathrm{MeV}\ .\end{aligned} \tag{69}$$

The second line indicates that the color magnetic field in the nucleon is smaller than that in the vacuum. This property of the magnetic field has long been suspected phenomenologically. The present result lends strong support for the educated guess. Clearly, this behavior of color fields is closely related to color confinement.

In the chiral limit, the trace anomaly contribution is analogous to the vacuum energy in the MIT bag model. In fact, the trace part of the energy-momentum tensor in the bag is $Bg^{\mu\nu}$, where B is the energy density of the "perturbative vacuum." The role of such energy density is to confine quarks. Thus the scale symmetry breaking is explicitly connected to quark confinement. It is essential then to include the effects of the trace anomaly in phenomenological hadron models.

A final comment is about the role of strange quarks. They contribute −60 MeV to the mass through the trace anomaly because the β function depends on the number of flavors. The kinetic and potential energy contributes about −85 MeV. Adding these to the strange mass contribution of 115 MeV, one gets a total of −30 MeV, roughly three percent of the total mass. Therefore the uncertainty in the separation is largely limited to the strange sector.

What shall be the general feature of the mass separation for other hadrons for which there are no data? First of all, so long as one is concerned with nonstrange hadrons, the contribution of the quark mass term is presumably small, and we may neglect b. Second, that gluons carry about half of the nucleon momentum in infinite momentum frame is perhaps approximately true for all hadrons, e.g., ρ or Δ. Thus we further assume $a = 0.5$. Given these guesses, we have

$$M_q = \frac{3}{8}M,\quad M_g = \frac{3}{8}M,\quad M_a = \frac{1}{4}M\ . \tag{70}$$

This is a heuristic way to sum up the main result of the mass separation.

D. The mass structure of the pion and the vacuum

The mass structure of the pion is particularly interesting because, according to the Goldstone theorem, the pion is intrinsically different from ordinary hadrons: it is a collective mode in the QCD vacuum. As we shall see, the mass structure indeed reflects this.

The matrix element $a(\mu^2)$ can be extracted from the quark distributions measured in π-N Drell-Yan processes. The quality of the available data [27], however, is much less satisfactory compared with that of the quark distributions in the nucleon. Nonetheless, it seems safe to conclude that $a(1\ \mathrm{GeV}^2)$ is known at ten percent level, with a central value similar to that of the nucleon:

$$a(1\ \mathrm{GeV}^2) = 0.55 \pm 0.05\ . \tag{71}$$

(Note that the precision of the data is not good enough to discern radiative corrections at the subleading-logarithmic level, so we have suppressed the scheme label.) Thus as far as high-energy probes are concerned, the pion is not dramatically different from other hadrons. This is also true for the π-N total cross section at high energy, for which the quark counting rule appears valid.

The matrix element b can be calculated through study of the pion mass in chiral perturbation theory. To avoid kinematic singularity in chiral limit, we adopt the normalization of the π state, $\langle P|P\rangle = 2E(2\pi)^3\delta^3(\mathbf{0})$. Therefore the matrix element b is

$$bm_\pi^2 = \left\langle P|\sum_{f=u,d,s} m_f\bar{\psi}_f\psi_f|P\right\rangle\ . \tag{72}$$

On the other hand, the first-order chiral perturbation theory predicts [25]

$$m_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle / f_\pi^2\ , \tag{73}$$

where f_π is the pion decay constant and $|0\rangle$ is the QCD vacuum. The above equation tells us two things. First, strange quarks do not contribute to the pion mass in the first-order perturbation, and matrix element $\langle P|\bar{s}s|P\rangle$ is strongly suppressed. Second, $m_\pi^2 \sim m_u, m_d$, and so to first-order accuracy, one can take the chiral limit of the pion wave function on the right-hand side of Eq. (72).

The pion mass can also be calculated by using the ordinary first-order perturbation theory [28],

$$\frac{1}{2}m_\pi^2 = (P|m_u\bar{u}u + m_d\bar{d}d|P)\ , \tag{74}$$

where $|P)$ is the pion wave function in chiral limit. Comparing the above equation with Eq. (72), we have

$$b = 1/2 \ . \tag{75}$$

So the first-order chiral perturbation gives a clean prediction.

Using the above matrix elements, we roughly have

$$M_q = 0, \ \ M_g = \frac{3}{8}m_\pi, \ \ M_m = \frac{1}{2}m_\pi, \ \ M_a = \frac{1}{8}m_\pi \ . \tag{76}$$

Two comments can be made immediately with regard to the above mass partition. First, the quark kinetic and potential energies cancel almost exactly. This fact is difficult to reproduce in quark models for the pion, where quarks carry large kinetic energy when confined to a small region of space. Second, the color electric and magnetic fields in the pion approach those in the vacuum in chiral limit. This strongly indicates that the pion is a collective excitation of the QCD vacuum.

It is tempting to use the above formalism to study the color electric and magnetic fields in the QCD vacuum. However, the energy-momentum tensor without vacuum subtraction ("normal ordering") is not finite and it is dangerous to work with divergent quantities. Nonetheless, if one finds a Lorentz covariant and nonperturbative regularization scheme to define the energy-momentum tensor without the subtraction, one can make statements about the vacuum color fields, except, of course, discussion is regularization scheme dependent and is meaningful only in a carefully defined context. (Perturbative regularization schemes are usually plagued with the infrared renormalon problem, and hence are not useful in this context [29].)

For simplicity, let us neglect quarks and study pure non-Abelian gauge theory. Since the vacuum is not characterized by any four-vector, the matrix element of the traceless part of the energy momentum tensor vanishes. That is,

$$\langle 0|\bar{T}^{\mu\nu}|0\rangle = 0 \ . \tag{77}$$

Taking $\mu = \nu = 0$, one gets

$$\langle 0|\mathbf{E}^2|0\rangle = -\langle 0|\mathbf{B}^2|0\rangle \ . \tag{78}$$

The color electric field is negative of the color magnetic field in vacuum. This appears to be a quite dramatic statement at first. However, it is trivially true, with some caveats, in lattice QCD calculation. On lattice, the color electric and magnetic fields can be defined as the average of the trace of elementary plaquettes in space-time and space-space planes. Because of hypercubic symmetry on lattice, the two different plaquettes have the same expectation value in vacuum. Remembering that the electric field in Minkowski space is related to that in Euclidean space by a factor of i, the imaginary unit, one gets the above equation immediately.

The trace part of the energy-momentum tensor contains only the anomaly. According to Lorentz symmetry,

$$\langle 0|\hat{T}^{\mu\nu}|0\rangle = g^{\mu\nu}\rho \ , \tag{79}$$

where ρ is the vacuum energy-density in a particular regularization scheme. Using the trace anomaly, one has

$$\langle 0|(-2\beta/g^R)F^{\alpha\beta}F_{\alpha\beta}|0\rangle = 4\rho \ . \tag{80}$$

The above equation shows that the vacuum energy density is related to the expectation value of F^2, which is called the gluon condensate in the literature. Knowing the condensate in a particular scheme, one obtains the color electric and magnetic fields of the vacuum, and hence the vacuum energy density.

Unfortunately, no one has yet proposed a natural Lorentz covariant, nonperturbative regularization scheme for the unsubtracted energy-momentum tensor operator. However, there is a phenomenological definition of the vacuum condensate used in the QCD sum rule calculation [30]. The magnitude of the condensate has been determined through fitting to hadron spectra:

$$\left\langle 0|\frac{\alpha_s}{\pi}F^2|0\right\rangle = (0.35 \text{ GeV})^4 \ . \tag{81}$$

This translates to

$$\langle 0|\mathbf{B}^2|0\rangle = -\langle 0|\mathbf{E}^2|0\rangle = 3.68 \text{ GeV/fm}^3 \ . \tag{82}$$

It is a rather large number from the phenomenological point of view. However, one should not forget its definition when it is used in a physical context.

IV. DISCUSSIONS AND COMMENTS

The structure of the nucleon is a subject that has been discussed for many years. The nonrelativistic quark model has the virtue that it is simple and captures many important aspects of physics. Unfortunately, to improve our understanding, we must solve QCD in the nonperturbative region. Although lattice QCD provides an effective method to calculate many observables, it provides little insight about the physics.

The measurement of the spin structure functions of the nucleon is a milestone in motivating new explorations of the quark-gluon structure of the nucleon. It points to our deficiency in traditional modeling of hadrons and to a need for unquenched lattice calculations. On the other hand, it also urges a better physical description for hadrons. In light of this, any rigorous information about the nucleon properties is useful.

The mass separation of the nucleon is certainly one step towards a better understanding of the quark and gluon dynamics of the nucleon. As was discussed, it has many implications about the physics of gluons and quarks, and their interactions. What is particularly interesting is the anomaly contribution. If the reader is familiar with the "spin crisis," he/she might recall that the axial anomaly was considered as one of the contributions to the nucleon's spin [31]. Unfortunately, there the separation between the anomaly and normal contributions are not quite clear. In particular, the question of gauge invariance and factorization has not been solved satisfactorily. Here the anomaly contribution to the nucleon mass is unambiguous, and its physics interpretation is quite clear.

One may ask where one goes from here. First, one can confirm the present result by doing lattice calculations. One can measure different pieces of the Hamiltonian in the nucleon state. Such a calculation in the end may help to test lattice approximations. Second, one can try to build models which are consistent with the present mass separation. One important conclusion here is that the anomaly term must be added to models such as the Nambu–Jona-Lasino model. Without this term, it is difficult to include the confinement effects.

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