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Kaluza-Klein black holes within heterotic string theory on a torus

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We point out that in heterotic string theory compactified on a six-torus, after a consistent truncation of the 10-dimensional gauge fields and the antisymmetric tensor fields, four-dimensional black holes of Kaluza-Klein theory on a six-torus constitute a subset of solutions.

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In this paper we show that four-dimensional (4D) black holes (BH's) of Kaluza-Klein (KK) theory constitute a subset [1–5] of 4D BH solutions of an effective heterotic string theory compactified on a six-torus [6].

An effective 4D action for the massless bosonic sector of heterotic string vacua compactified on a 6D torus is obtained [7] by compactifying the (massless) bosonic part of $D=10$ $N=1$ supergravity coupled to $N=1$ super-Yang-Mills theory, containing the dilaton $\Phi^{(10)}$, the two-form field $B^{(10)}_{\hat{\mu}\hat{\nu}}$, and 16 Abelian gauge fields $A^{(10)\,I}_{\hat{\mu}}$ ($I=1,...,16$), on a six-torus [1]:

$$S=\int d^4x\sqrt{-G}e^{-\Phi}[\mathcal{R}_G+\partial_\mu\Phi\partial^\mu\Phi-\tfrac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} -F^a_{\mu\nu}(LML)_{ab}(F^b)^{\mu\nu}+\tfrac{1}{8}\mathrm{Tr}(\partial_\mu ML\partial^\mu ML)]\ , \qquad (1)$$

where $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu$ and $H_{\mu\nu\rho}=(\partial_\mu B_{\nu\rho}+2A^a_\mu L_{ab}F^b_{\nu\rho})$+permutations ($\{a,b\}=1,...,28$). $G\equiv\det G_{\mu\nu}$ and the Ricci scalar $\mathcal{R}_G$ is defined in terms of the string frame metric $G_{\mu\nu}$. M is the O(6,22) matrix of the following 28 scalar fields: the internal part of the 10D metric $\hat{G}_{mn}\equiv G^{(10)}_{m+3,n+3}$ ($\{m,n\}=1,...,6$), "antisymmetric" background fields $B_{mn}\equiv B^{(10)}_{m+3,n+3}$ ($\{m,n\}=1,...,6$), and "gauge" background fields $A^I_m\equiv A^{(10)\,I}_{m+3}$ ($m=1,...,6,\ \ I=1,...,16$). M has the properties

$$MLM^T=L,\qquad M^T=M,\qquad L=\begin{pmatrix}0 & I_6 & 0\\ I_6 & 0 & 0\\ 0 & 0 & -I_{16}\end{pmatrix}\ , \qquad (2)$$

where I_n is the $n\times n$ identity matrix. L is the matrix invariant under O(6,22) transformations. The 4D dilaton field $\Phi\equiv\Phi^{(10)}-\frac{1}{2}\ln\det\hat{G}$ is defined in terms of the 10D dilaton field $\Phi^{(10)}$ and determinant of the internal metric $\hat{G}_{mn}$. The gauge fields $A^m_\mu\equiv\frac{1}{2}\hat{G}^{mn}G^{(10)}_{n+3,\mu}$ ($\{m,n\}=1,...,6$) are related to the off-diagonal components of the 10D metric. The gauge fields A^a_μ with $a=7,...,28$ are related to the off-diagonal components of the 10D antisymmetric tensor $B^{(10)}_{\hat{\mu}\hat{\nu}}$ and the 4D space-time components of the 10D gauge fields $A^I_{\hat{\mu}}$.

We choose to set the 10D Abelian gauge fields and 10D two-form fields equal to zero; this choice is consistent with the equations of motion in the corresponding 10D supergravity theory, and thus with the equations of motion of the 4D effective action (1). Consequently, a consistent truncation of (1) corresponds to setting the antisymmetric tensor field $B_{\mu\nu}$ and a set of 4D gauge fields A^a_μ ($a=7,...,28$), as well as the scalar background fields B_{mn} and A^I_m to zero. The action (1) then reduces to the form

$$S=\int d^4x\sqrt{-G}e^{-\Phi}[\mathcal{R}_G+\partial_\mu\Phi\partial^\mu\Phi-F^a_{\mu\nu}(LML)_{ab}(F^b)^{\mu\nu}+\tfrac{1}{8}\mathrm{Tr}(\partial_\nu ML\partial^\mu ML)]\ , \qquad (3)$$

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[1]For notational conventions and the relationship of the 4D massless modes to the bosonic modes of the corresponding 10D $N=1$ supergravity theory, see, for example, Ref. [8].

where

$$M = \begin{pmatrix} \hat{G}^{-1} & 0 & 0 \\ 0 & \hat{G} & 0 \\ 0 & 0 & I_{16} \end{pmatrix} \tag{4}$$

depends only on $\hat{G}_{mn}$, a real symmetric (6×6) matrix of scalar fields associated with the internal metric of six-tori. The action (3) can now be written explicitly as

$$\begin{aligned} S &= \int d^4x\sqrt{-G}e^{-\Phi}\left[\mathcal{R}_G + \partial_\mu\Phi\partial^\mu\Phi - \frac{1}{4}\hat{G}_{mn}F^m_{\mu\nu}F^{n\mu\nu} + \frac{1}{4}\partial_\mu\hat{G}_{mn}\partial^\mu\hat{G}^{mn}\right] \\ &= \int d^4x\sqrt{-g}\left[\mathcal{R}_g - \frac{1}{2}\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{4}e^{\alpha\varphi}\rho_{mn}F^m_{\mu\nu}F^{n\mu\nu} + \frac{1}{4}\partial_\mu\rho_{mn}\partial^\mu\rho^{mn}\right] , \end{aligned} \tag{5}$$

where ρ_{mn} is the unimodular part of the metric $\hat{G}_{mn}$, $\varphi \equiv \frac{1}{\alpha}(\frac{1}{\tilde{n}}\ln\det\hat{G} - \Phi)$, and $\tilde{\varphi} \equiv \frac{1}{\alpha}(\sqrt{\frac{1}{2\tilde{n}}}\ln\det\hat{G} + \sqrt{\frac{2}{\tilde{n}}}\Phi) = \sqrt{\frac{2}{\tilde{n}+2}}\Phi^{(10)}$. Here, $\rho^{mn}\rho_{n\ell} = \delta^m_\ell$ and $\alpha = \sqrt{\frac{\tilde{n}+2}{\tilde{n}}}$ with $\tilde{n} = 6$. The scalar curvature $\mathcal{R}_g$ and $g = \det g_{\mu\nu}$ are expressed in terms of the Einstein-frame metric $g_{\mu\nu}$.

The action (5) is that of 11D KK theory compactified on a seven-torus, where the gauge field A^7_μ, associated with the seventh torus, is turned off. Consequently, the field $\tilde{\varphi} = \sqrt{\frac{2}{\tilde{n}+2}}\Phi^{(10)}$, parametrizing the size of the seventh torus, decouples (except for 4D gravity) from the other fields and can therefore be set to a constant. This result is obvious, once one realizes that the bosonic sector of 10D $N = 1$ supergravity with the 10D gauge fields and antisymmetric tensor field turned off corresponds to 11D KK theory compactified down to 10D with the 10D gauge field, associated with the compactified dimension, turned off.

Thus, the action (5) is effectively that of 10D KK theory compactified on a six-torus [9]. The corresponding 4D BH solutions of (5) are then those of $(4 + \tilde{n})$D ($\tilde{n}$=6) KK theory. In particular, with further consistent truncations of the gauge fields, i.e., $A^m_\mu = 0$ $[m = 1, ..., k(< 6)]$, (5) reduces to the effective action of $(10 - k)$D KK theory. Namely, the corresponding internal metric fields, *i.e.*, combinations of φ and ρ_{mn} ($\{m,n\} = 1, ..., k$), decouple from the other fields (except for 4D gravity) and can thus be set to constant values. Specifically, for the choice of $k = 5$ (only one nonzero gauge field) (5) reduces to the action of an effective 5D KK theory with the corresponding BH solutions [1], as discovered by Duff *et al.* [10].

Supersymmetric embedding of the bosonic action (5) allows one to derive the Bogomol'nyi bound for the Arnowitt-Deser-Misner (ADM) mass of the above class of spherically symmetric BH solutions. Among them the supersymmetric ones, i.e., those which preserve (constrained) supersymmetry, can be regarded as nontrivial vacuum configurations, since they saturate the corresponding Bogomol'nyi bounds. Supersymmetric embedding[2] of 4D Abelian KK BH's with a diagonal internal metric ansatz has been carried out [3] within $(4 + \tilde{n})$D KK theory ($1 \leq \tilde{n} \leq 11$), and can thus be applied to BH solutions of (5) as well. Such supersymmetric BH's have at most one magnetic (P) and one electric (Q) charge arising from different U(1)'s, thus corresponding to solutions in the effective 6D KK theory with internal isometry $\mathrm{U}(1)_M \times \mathrm{U}(1)_E$. Embedding of (5) in 10D $N = 1$ supergravity ensures [3] that the resulting vacuum configuration preserves one ($N = 1$) of $N = 4$ supersymmetries of the effective 4D action.

The corresponding nonextreme solutions with a diagonal internal metric [4,5] as well as a class of those with a nondiagonal internal metric [5] have also been found. The latter ones can be obtained [5] as solutions of (5), by performing SO($\tilde{n}$) ($\tilde{n} = 6$) rotations on the solutions with a diagonal internal metric. This SO(6) symmetry is realized as a subset of the O(6, 22) symmetry [12] of (1).

The class of solutions generated by the SO(6) transformations on the $\mathrm{U}(1)_M \times \mathrm{U}(1)_E$ BH solutions corresponds [5] to charged configurations $\{P^i, Q^i\}$ $[i = 1, ..., \tilde{n}(= 6)]$ subject to the constraint $\sum_{i=1}^{\tilde{n}} P^iQ^i = 0$. The most general solutions within this class, i.e., those with unconstrained charge configurations, are expected to be generated [13] by (one parameter) transformations, belonging to SO(2, $\tilde{n}$) ($\tilde{n} = 6$), on the former solutions. Here, SO(2, $\tilde{n}$) is a symmetry of the effective 3D Lagrangian density [2] for the spherically symmetric BH ansatz in $(4 + \tilde{n})$D KK theory.

Such explicit solutions for all static 4D KK BH's would allow for a study of their global space-time and thermal properties. They would in turn provide a sub-class of solutions for general 57- (or 58)-parameter dyonic (or rotating) BH solutions which could be generated [6] by [O(22,2)×O(6,2)]/[O(22) × O(6) × SO(2)] transformations on the 4D Schwarzschild (or Kerr) solution.

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