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How large are the rates of the CP-violating $\eta, \eta' \to \pi\pi$ decays?

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The rates of $\eta, \eta' \to \pi\pi$ are computed in the framework of the standard electroweak model. The results $B(\eta \to \pi\pi) \leq 2 \times 10^{-27}$ and $B(\eta' \to \pi\pi) \leq 3 \times 10^{-29}$ imply that a search for these decays at the existing and planned $\eta(\eta')$ factories would indeed be a search for CP violation in strong interactions or unconventional mechanisms of CP violation.

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I. INTRODUCTION

In order to clarify the nature of CP violation it is very important that in addition to studying flavor-changing processes also to look for CP effects in flavor-conserving transitions such as the decays $\eta \to \pi\pi$ [1] or in the interactions of the baryons with the electromagnetic field through their electric dipole moments (for a review see, for example, [2, 3]). As these effects are expected to be very tiny, in the framework of the standard electroweak model (SM [4]), any unconventional source of CP violation, whatever it may be, gets a golden opportunity to exhibit itself.

In the case of the electric dipole moment (EDM) of the neutron the predicted value within the electroweak model, $D_n \leq 10^{-32} e\,\mathrm{cm}$ [2], looks beyond the experimental reach in the near future [3]. For the observation of the decays $\eta \to \pi\pi$, in principle, all that is needed is a very intensive source of η mesons, provided that the rate is sufficiently large.

In addition to the CP violation naturally residing in the electroweak model, at least two more sources of CP violation could contribute to $\eta \to \pi\pi$ transitions. One is the so-called θ term in QCD [5] and the other spontaneous breakdown of CP symmetry triggered by scalar fields ([6, 7]).

In order to draw any conclusions on the mechanism of CP violation from data on $\eta \to \pi\pi$ decays one needs to know the corresponding rates predicted in the SM as well as in the alternative theories. In this paper we restrict ourselves to a study of the predictions of the standard electroweak model. An estimate of the $\eta \to \pi\pi$ rate due to the θ term was given in Ref. [8]. In a forthcoming paper we shall present results obtained in the framework of spontaneous CP violation.

Our results in this paper will be expressed in terms of the coefficients of the Wilson operator expansion of the $|\Delta S| = 1$ effective weak nonleptonic Lagrangian. For completeness we also present the rates for the decays $\eta' \to \pi\pi$.

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II. METHOD OF CALCULATION

The minuteness of CP-violating effects in flavor-conserving processes is connected with the necessity of having a two step change of flavor—a first step for obtaining a product of the elements of the quark mixing matrix which contains the appropriate imaginary part and a second one for returning to the intial flavor state.

The amplitudes of CP-violating effects, considered here, are of the form

$$A(CP = -1) = A_{fn} B_{ni} - A_{fn^*} B_{n^* i}, \tag{1}$$

where A_{fn} and B_{ni} correspond to the above-mentioned first and second step transitions, respectively, and n and n^* are intermediate states with opposite strangeness (or charm or b flavor). Evidently, a nonzero contribution to $A(CP = -1)$ can only arise from intermediate states where the phases of A_{fn} and B_{in} are different.

For the processes with light mesons the main contribution to $A(CP = -1)$ arises from the intermediate states with opposite strangeness because the corresponding contributions from charm and b quarks are suppressed by factors m_l/m_h where l stands for a light quark, u, d, s, and h denotes the heavy quarks c or b. For this reason in the calculation of $A(\eta \to \pi\pi)$ we may limit ourselves to taking into account the successive transitions

$$\eta \to \{K^0, \bar{K^0}\} \quad \to \pi\pi$$

and

$$\eta \to \{K\pi\} \to \pi\pi. \tag{2}$$

Note that the indirect CP violation via mixing of K_1^0 and K_2^0 states does not play any significant role in our case, contrary to the case of $K_L \to \pi\pi$ decays where the on-shell parameter ϵ is given by the expression

$$\epsilon \approx \frac{m_K(\langle \bar{K^0}|K^0\rangle - \langle K^0|\bar{K^0}\rangle)}{P_{K_L}^2 - M_S^2}$$

evaluated at $P_{K_L}^2 = M_L^2$. Here $M_X = m_X - \frac{i}{2}\Gamma_X$, $X = L, S$. The quantity ϵ owes its significance to the fact that the denominator in the above equation is very small.

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In our case, however, the quantity $P^2_{K_L}$ of the virtual long-lived kaon equals P^2_η and thus the denominator is appreciable and gives

$$\frac{m_K(\langle \bar{K^0}|K^0\rangle - \langle K^0|\bar{K^0}\rangle)}{P^2_\eta - M^2_S} \approx \epsilon \frac{m_L - m_S}{m_\eta - m_K} \approx 10^{-13}\epsilon.$$

For the calculation of the amplitude of $\eta \to \pi\pi$ we shall use the effective strangeness-changing Lagrangian given in Ref. [10]:

$$L(|\Delta S| = 1) = \sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_{j=1}^{6} (c_j O_j + c_j^* O_j^\dagger), \tag{3}$$

where

$$O_1 = (\bar{s_L}\gamma_\mu d_L)(\bar{u_L}\gamma_\mu u_L) - (\bar{s_L}\gamma_\mu u_L)(\bar{u_L}\gamma_\mu d_L) \quad (\{8_f\}, \Delta I = 1/2),$$
$$O_2 = 2(\bar{s_L}\gamma_\mu d_L) \sum_{q=u,d,s} (\bar{q_L}\gamma_\mu q_L) - O_1 \quad (\{8_f\}, \Delta I = 1/2),$$
$$O_3 = O_2 - 5(\bar{s_L}\gamma_\mu d_L)(\bar{s_L}\gamma_\mu s_L) \quad (\{27\}, \Delta I = 1/2),$$
$$O_4 = (\bar{s_L}\gamma_\mu d_L)(\bar{u_L}\gamma_\mu u_L) + (\bar{s_L}\gamma_\mu u_L)(\bar{u_L}\gamma_\mu d_L) - (\bar{s_L}\gamma_\mu d_L)(\bar{d_L}\gamma_\mu d_L) \quad (\{27\}, \Delta I = 3/2),$$
$$O_5 = (\bar{s_L}\gamma_\mu \lambda^a d_L)\left(\sum_{q=u,d,s} \bar{q_R}\gamma_\mu \lambda^a q_R\right) \quad (\{8\}, \Delta I = 1/2),$$
$$O_6 = (\bar{s_L}\gamma_\mu d_L)\left(\sum_{q=u,d,s} \bar{q_R}\gamma_\mu q_R\right) \quad (\{8\}, \Delta I = 1/2).$$

As was recognized a few years ago [11–14], the above set of operators is not sufficient for the calculation of the ratio ϵ'/ϵ in $K_L \to \pi\pi$ decays, but, in our case, as is shown in the Appendix, the additional operators, representing the so-called electroweak penguin and box diagrams, can give only a correction of the order of a few percents which is comparable with the next-order in SU(3)-breaking corrections. But the latter corrections introduce an insignificant modification of the result obtained in the leading approximation, and thus we neglect them in our calculations.

An additional hypothesis used by us here is factorization whereby a product of quark currents translates into a product of mesonic currents. It has been shown in Refs. [10] and [15] that this approximation works rather well for the description of the decays $K \to 2\pi$, $K \to 3\pi$ and nonleptonic decays of the hyperons.

In accordance with the discussion above, we shall use the effective chiral Lagrangian [16]

$$L^{\rm eff} = \frac{f_\pi^2}{4}\left(\mathrm{Tr}(D_\mu U D_\mu U^\dagger) + r\mathrm{Tr}[m(U+U^\dagger)] - \frac{r}{\Lambda^2}\mathrm{Tr}[m(D^2U + D^2U^\dagger)]\right), \tag{4}$$

where m is the quark mass matrix and

$$U = \exp(i\sqrt{2}\,\hat{\pi}/f_\pi), \tag{5}$$

$$r = \frac{2m_\pi^2}{m_u + m_d}, \tag{6}$$

$$\hat{\pi} = \begin{pmatrix} \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \frac{\pi_0}{\sqrt{3}} - 2\frac{\pi_8}{\sqrt{6}} \end{pmatrix}. \tag{7}$$

The parameter Λ^2 in Eq. (4) can be found from a comparison of the results obtained using the above $L^{\rm eff}$ and the matrix U in the form (5) with the corresponding results in the framework of a linear σ model where

$$U = \hat{\sigma} + i\hat{\pi}.$$

Here $\hat{\sigma}$ is the three-by-three matrix containing scalar partners of the pseudoscalar mesons. Then [17]

$$\Lambda^2 = m^2_{a_0(980)} - m_\pi^2 \approx 0.946 \text{ GeV}^2. \tag{8}$$

The individual quark currents entering into operators O_j, $j = 1-4$, may be expressed in terms of mesonic field operators using the relation

$$\bar{q_L}^j\gamma_\mu q_L^i = \frac{if_\pi^2}{4}\{(\partial_\mu U)U^\dagger - U(\partial_\mu U^\dagger) - \frac{r}{\Lambda^2}[m(\partial_\mu U^\dagger) - (\partial_\mu U)m]\}_{ij}. \tag{9}$$

The operators O_5 and O_6 may be expressed in the form

$$O_5 = -\frac{32}{9}(\bar{s_L}q_R)(\bar{q_R}d_L) + \cdots , \quad (10)$$

$$O_6 = -\frac{2}{3}(\bar{s_L}q_R)(\bar{q_R}d_L) + \cdots , \quad (11)$$

where the ellipses stand for products of colored currents. The quark bilinears appearing here above may be rewritten in terms of mesonic field operators using

$$\bar{q_R}^j q_L^k = -\frac{1}{4}f_\pi^2 r \left[U - \frac{1}{\Lambda^2}\partial^2 U\right]_{kj} . \quad (12)$$

Using these relations it is not difficult to derive, in the tree approximation, the relations

$$\langle\pi^+\pi^-|L^{\text{eff}}(\Delta S=-1)|K^0\rangle$$
$$= -\frac{i}{2}f_\pi G_F \sin\theta_C \cos\theta_C (p_K^2 - p_\pi^2) \times(c_1 - c_2 - c_3 + \tilde{\beta}c_5 - c_4), \quad (13)$$

where

$$\tilde{\beta} = \left(2m_\pi^4/[\Lambda^2(m_u+m_d)^2]\right)\frac{32}{9}\left(1+\frac{3c_6}{16c_5}\right). \quad (14)$$

For the $K^0 \to \pi^0\pi^0$ the result reads

$$\langle\pi^0\pi^0|L^{\text{eff}}(\Delta S=-1)|K^0\rangle$$
$$= -\frac{i}{2}f_\pi G_F \sin\theta_C \cos\theta_C (p_K^2 - p_\pi^2) \times(c_1 - c_2 - c_3 + \tilde{\beta}c_5 + 2c_4). \quad (15)$$

Note that in Eqs. (13) and (15) the contributions proportional to c_4 correspond to $\Delta I = 3/2$ transition and the remaining ones to $\Delta I = 1/2$ transition. When the rescattering of the final pions is taken into account the coefficient c_4 must be taken with the relative phase factor $\exp[i(\delta_2 - \delta_0)]$. From Eq. (2) follows that we need to know the amplitude of $\eta \to K^0$ transition. It is given by

$$\langle K^0|L^{\text{eff}}(\Delta S=1)|\eta\rangle = \frac{1}{2}f_\pi^2 p_\eta^2(\sqrt{2}G_F\sin\theta_C\cos\theta_C)\left[c_1^\star\left(-\frac{\sin\theta_P}{\sqrt{3}}+\frac{\cos\theta_P}{\sqrt{6}}\right)+c_2^\star\left(-5\frac{\sin\theta_P}{\sqrt{3}}-\frac{\cos\theta_P}{\sqrt{6}}\right) + c_3^\star\frac{9\cos\theta_P}{\sqrt{6}} + \tilde{\beta}c_5^\star\left(2\frac{\sin\theta_P}{\sqrt{3}}+\frac{\cos\theta_P}{\sqrt{6}}\right)\right]. \quad (16)$$

Thus the total contribution of K^0 and $\bar{K}^0$ intermediate states to $\eta \to \pi^+\pi^-$ is given by

$$\langle\pi^+\pi^-|L^{\text{eff}}(\Delta S=-1)|K^0\rangle\langle K^0|L^{\text{eff}}(\Delta S=1)|\eta\rangle + \langle\pi^+\pi^-|L^{\text{eff}}(\Delta S=+1)|\bar{K}^0\rangle\langle|\bar{K}^0|L^{\text{eff}}(\Delta S=-1)|\eta\rangle$$
$$= (\sqrt{2}G_F\sin\theta_C\cos\theta_C)^2\,\frac{m_\eta^2-m_\pi^2}{m_\eta^2-m_K^2}\,\frac{f_\pi^3 m_\eta^2}{2\sqrt{2}}\tilde{\beta}\text{Re}c_5\frac{\text{Im}c_5}{\text{Re}c_5}$$
$$\times\left[-\sqrt{3}\sin\theta_P(c_1+c_2)+\frac{10\cos\theta_P+2\sqrt{2}\sin\theta_P}{\sqrt{6}}c_3+\left(\frac{2\sin\theta_P}{\sqrt{3}}+\frac{\cos\theta_P}{\sqrt{6}}\right)c_4e^{i(\delta_2-\delta_0)}\right], \quad (17)$$

where we have used the fact that c_j are real, for $j \neq 5$ and that only c_5 has an imaginary part arising from the quark mixing matrix. Furthermore, the contribution of the intermediate states K^+ and K^- is suppressed by the factor m_π^2/m_η^2 as compared to the quantity in Eq. (17) because the matrix element

$$\langle\pi^\pm(p_\pi)|L(\Delta S=\pm1)|K^\pm(p_K=p_\pi)\rangle$$

is proportional to p_π^2 and thus, *a priori*, of the same order of magnitude as the higher order SU(3) breaking corrections to Eq. (17).

The above results may also be applied to the decay $\eta' \to \pi^+\pi^-$ where we get

$$\langle\pi^+\pi^-|\eta'\rangle \cong (\sqrt{2}G_F\sin\theta_C\cos\theta_C)^2\,\frac{m_{\eta'}^2-m_\pi^2}{m_{\eta'}^2-m_K^2}\,\frac{f_\pi^3 m_{\eta'}^2}{2\sqrt{2}}\tilde{\beta}\text{Re}c_5\frac{\text{Im}c_5}{\text{Re}c_5}$$
$$\times\left[\sqrt{3}\cos\theta_P(c_1+c_2)+\frac{10\sin\theta_P-2\sqrt{2}\cos\theta_P}{\sqrt{6}}c_3-\left(\frac{2\cos\theta_P}{\sqrt{3}}-\frac{\sin\theta_P}{\sqrt{6}}\right)c_4e^{i(\delta_2-\delta_0)}\right]. \quad (18)$$

Now we are in position to give an estimate of the value of the amplitudes of $\eta \to \pi\pi$ and $\eta' \to \pi\pi$.

III. THE RATES OF $\eta \to \pi^+\pi^-$ AND $\eta \to \pi^0\pi^0$ DECAYS

According to Eq. (17), we obtain, up to a common phase factor,

$$\langle \pi^+\pi^-|\eta\rangle = 6.02\times 10^{-15}\tilde{\beta}\mathrm{Re}c_5\frac{\mathrm{Im}c_5}{\mathrm{Re}c_5} \times\left[-\sqrt{3}\sin\theta_P(c_1+c_2)+\frac{10\cos\theta_P+2\sqrt{2}\sin\theta_P}{\sqrt{6}}c_3+\left(\frac{2\sin\theta_P}{\sqrt{3}}+\frac{\cos\theta_P}{\sqrt{6}}\right)c_4e^{i(\delta_2-\delta_0)}\right]\mathrm{GeV}, \tag{19}$$

where the relative phase between the $I = 0$ final state (described by the terms proportional to c_j, $j = 1-3$) and the $I = 2$ final state (represented by c_4) has been taken into account. The right-hand side (RHS) of Eq. (19) may be considered as a coupling constant in the effective Lagrangian

$$L(\eta\to\pi^+\pi^-) = g_{\eta\ \pi^+\pi^-}\eta\pi^+\pi^-. \tag{20}$$

For the width of $\eta\to\pi\pi$ decays we find

$$\Gamma(\eta\to\pi^+\pi^-) = \frac{|g_{\eta\pi^+\pi^-}|^2}{16\pi m_\eta}\sqrt{1-4m_\pi^2/m_\eta^2}, \tag{21}$$

$$\Gamma(\eta\to\pi^0\pi^0) = \frac{|g_{\eta\pi^0\pi^0}|^2}{32\pi m_\eta}\sqrt{1-4m_\pi^2/m_\eta^2}. \tag{22}$$

From Eqs. (13) and (15) it follows that the coupling constants $g_{\eta\pi^+\pi^-}$ and $g_{\eta\pi^0\pi^0}$ differ only due to terms proportional to c_4. This difference vanishes at $\sin\theta_P = -1/3$, i.e., when $\theta_P = -19.47°$. But this value of θ_P is very close to the experimental value [18]. For simplicity, we shall use $\theta_P = -19.47°$ in our numerical estimates. Then

$$g_{\eta\pi^+\pi^-} = g_{\eta\pi^0\pi^0} = 3.48\times 10^{-15}\tilde{\beta}\mathrm{Re}c_5\frac{\mathrm{Im}c_5}{\mathrm{Re}c_5}(c_1+c_2 +6c_3)\ \mathrm{GeV}. \tag{23}$$

The c's may be determined from an analysis of the decays $K_S\to\pi\pi$ and $K^+\to\pi^+\pi^0$. One finds [15]

$$c_4 = 0.328, \quad c_1-c_2-c_3+\tilde{\beta}\mathrm{Re}c_5 = -10.13. \tag{24}$$

Using the theoretical values of $c_1 - c_3$ from Ref. [19],

$$c_1 = -2.75, \quad c_2 = 0.06, \quad c_3 = 0.08, \tag{25}$$

and the estimate[1] [20]

$$\left|\frac{\mathrm{Im}c_5}{\mathrm{Re}c_5}\right| = (1\pm 0.4)|s_2s_3\sin\delta|, \tag{26}$$

where s_2, s_3, and δ are parameters of the quark mixing matrix in the manner of Kobayashi and Maskawa [21], we find that

$$g_{\eta\pi^+\pi^-} = g_{\eta\pi^0\pi^0} \le (1.45\pm 0.58)10^{-16}\sin\delta\ \mathrm{GeV}, \tag{27}$$

where we have used that $s_2s_3 \le 2.6\times 10^{-3}$ as given by Ref. [22].

Thus

$$R_{+-} = \frac{\Gamma(\eta\to\pi^+\pi^-)}{\Gamma(\eta\to\ \mathrm{all})} \le (0.66^{+0.63}_{-0.32})10^{-27}\sin^2\delta, \tag{28}$$

and the rate of $\eta\to\pi^0\pi^0$ is half as big.

Thus the above-detailed calculation leads to a result which is two orders of magnitude smaller than the crude estimate made in Ref. [9].

We wish to point out that many authors give for c_5 a value considerably smaller than the one used in this paper. The point made is that the usual technics of computing c_5, including the gluonic corrections, work only when the virtual momenta are larger than $\Lambda\sim 1$ GeV. Since c_5 is proportional to $\ln(m_c/m_u)$ the authors in question replace m_u by $\Lambda\sim 1$ GeV and thus obtain a small value for c_5. Such an approach is logically incorrect. Inability of computing in a self-consistent manner the contribution of low virtual momenta to c_5 by no means implies that such a contribution is absent or negligible. In particular, such a contribution could be sufficient to explain the $\Delta I = 1/2$ rule. We assume that indeed this is the case and extract a value for the coupling constants in Eq. (24) directly from the experimental data on $K\to\pi\pi$ decays.

Our result for $g_{\eta\pi\pi}$, given above, could be compared with that obtained [8] from possible CP violation in strong interactions due to the θ term:

$$|g_{\eta\pi\pi}| \approx \theta\times(0.085)\ \mathrm{GeV}. \tag{29}$$

At present [23] the limit on θ is 6×10^{-10} and consequently

$$R_{+-} \le 0.78\times 10^{-16}. \tag{30}$$

Thus CP violation due to the θ term could exhibit itself in a very broad range of R_{+-} values, depending on the value of the θ parameter.

IV. THE RATES OF $\eta'\to\pi^+\pi^-$ AND $\eta'\to\pi^0\pi^0$ DECAYS

From Eqs. (18), (13), and (15), for $\sin\theta_P = -1/3$, we have

$$g_{\eta'\pi^+\pi^-} = 2.52\times 10^{-15}\tilde{\beta}\mathrm{Re}c_5\frac{\mathrm{Im}c_5}{\mathrm{Re}c_5}2\sqrt{2}\left[c_1+c_2-\frac{3}{2}c_3-\frac{3}{4}c_4e^{i(\delta_2-\delta_0)}\right]\ \mathrm{GeV}, \tag{31}$$

[1] Although the calculation was done for the case of $m_t < M_W$ the result may be used here because of the weak dependence of $\mathrm{Im}c_5$ on m_t in the region $50\le m_t\le 250$ GeV (see Refs. [11–14]).

$$g_{\eta'\pi^0\pi^0} = 2.52 \times 10^{-15}\tilde{\beta}\text{Re}c_5 \frac{\text{Im}c_5}{\text{Re}c_5} 2\sqrt{2}\left[c_1 + c_2 - \frac{3}{2}c_3 + \frac{3}{2}c_4 e^{i(\delta_2-\delta_0)}\right] \text{ GeV}. \quad (32)$$

Using the numerical values of the c's given in Eqs. (24) and (25) we get

$$g_{\eta'\pi^+\pi^-} \approx (4 \pm 1.6) \times 10^{-16} \sin\delta,$$
$$g_{\eta'\pi^0\pi^0} \approx (3.3 \pm 1.3) \times 10^{-16} \sin\delta, \quad (33)$$

where we have neglected the small imaginary parts of $g_{\eta'\pi\pi}$.

For the rates of $\eta' \to \pi\pi$ decays we have

$$R'_{+-} = \frac{\Gamma(\eta' \to \pi^+\pi^-)}{\Gamma(\eta' \to \text{ all})} \leq (1.66^{+1.5}_{-1.6}) \times 10^{-29} \sin^2\delta, \quad (34)$$

$$R'_{00} = \frac{\Gamma(\eta' \to \pi^0\pi^0)}{\Gamma(\eta' \to \text{ all})} \leq (0.55^{+0.5}_{-0.35}) \times 10^{-29} \sin^2\delta. \quad (35)$$

V. CONCLUSIONS

The calculated rates of $\eta \to \pi^+\pi^-$ and $\eta \to \pi^0\pi^0$ decays are too small to allow the observation of these decays at the existing facilities (SATURN) or the η factories under construction (CELSIUS, DAPHNE) where the expected number of produced η mesons per year does not exceed $10^{11}, 10^{10}$, and 10^9, respectively.

An observation of such decays with branching ratios considerably larger than 10^{-27} would be a signal of mechanisms for CP violation lying beyond the standard model.

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APPENDIX

Here we show why the operators corresponding to so-called electromagnetic penguins and box diagrams [11–14] do not give any significant contribution to our result, contrary to the case of ϵ/ϵ' ratio in the neutral kaon system.

According to Eqs. (13) and (15), in the tree approximation, we have

$$\left\{\begin{array}{c} \langle\pi^+\pi^-|K^0\rangle \\ \langle\pi^+\pi^-|\bar{K}^0\rangle \\ \langle\pi^0\pi^0|K^0\rangle \\ \langle\pi^0\pi^0|\bar{K}^0\rangle \end{array}\right\} = \frac{-iG_F \sin\theta_C \cos\theta_C f_\pi}{2}(p_K^2 - p_\pi^2)\left\{\begin{array}{c} be^{i\Delta} - c_4 \\ -be^{-i\Delta} + c_4 \\ be^{i\Delta} + 2c_4 \\ -be^{-i\Delta} - 2c_4 \end{array}\right\},$$

where

$$b = c_1 - c_2 - c_3 + \tilde{\beta}\text{Re}c_5,$$

$$\Delta = \tilde{\beta}\text{Im}c_5/b.$$

Here we have used the fact that $\text{Im}c_5$ is small.

As the phase of K^0 is arbitrary, one redefine it by letting $K^0 \to K^0e^{-i\Delta}$. Then in terms of the new neutral kaon fields we have

$$\langle\pi^+\pi^-|K_2^0\rangle = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_C (m_K^2 - m_\pi^2) c_4 f_\pi \Delta.$$

This formula demonstrates that in order for the *direct CP-violation* to manifest itself experimentally the existence of the $\Delta I = 3/2$ amplitude is mandatory. The latter amplitude is changed considerably when the electromagnetic penguins and other diagrams contributing to $\Delta I = 3/2$ amplitude are taken into account. In spite of the suppression factor $\alpha_{\text{em}}/\alpha_S$ this contribution is not negligible and modifies c_4 to $c_4(1 - \frac{\alpha_{\text{em}}}{\alpha_S}\frac{A_{1/2}}{A_{3/2}}\gamma)$. This leads to a considerable decrease of the value of the ϵ' parameter [11–14].

In our case we have the transitions

$$\langle\pi^+\pi^-|K^0\rangle\langle K^0|\eta\rangle \quad \text{and} \quad \langle\pi^+\pi^-|\bar{K}^0\rangle\langle\bar{K}^0|\eta\rangle,$$

where $\pi^+\pi^-$ and η states are self-conjugate states. It is evident that passing to the new fields $K^0e^{-i\Delta}$ and $\bar{K}^0e^{+i\Delta}$ the result (17) does not change at all. Note that in the formula (19) a contribution coming from the $\Delta I = 3/2$ amplitude (proportional to c_4) does not play any significant role due to the fact that c_4 is small. We have neglected it in our estimate keeping in mind that for $\sin\theta_P = -1/3$ this contribution exactly vanishes. Therefore, the diagrams of higher order in electroweak coupling constants can give only small corrections of order $\alpha_{\text{em}}/\alpha_S$ to $\Delta I = 1/2$ part and thus leave our results practically unchanged. This is why we may restrict ourselves to the effective nonleptonic Lagrangian given in Eq. (3).

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