More on direct CP violation in $b \to dJ/\psi$ decays

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Direct CP violation can occur in B-meson decays of the type $b \to dc\bar{c}$, where the charm-anticharm pair forms a J/ψ. The CP asymmetry requires the contribution to the amplitude from decays into other states, which rescatter into dJ/ψ *via* final state interactions. In particular, states with the same quark content as dJ/ψ can contribute. A perturbative calculation, based on a quark level description of the rescattering process, gives an asymmetry of about 1%, due to this effect. This makes it the dominant contribution to the asymmetry, as suggested by an earlier estimate, based on a hadronic picture.

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I. INTRODUCTION

Direct CP violation in B-meson decays may generate an asymmetry

$$a_{CP} = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}, \tag{1}$$

between the rates for the CP-conjugated processes $B \to f$ and $\bar{B} \to \bar{f}$, even in the absence of B-$\bar{B}$ mixing. In the standard model, it is predicted that the most significant asymmetries of this type, which tend to be fairly small [1–3], occur for Cabibbo suppressed decays. Hence, a large number of B mesons are necessary, and the observation of the asymmetries in Eq. (1) may be best achieved at hadronic accelerators. The decays $b \to qc\bar{c}$ ($q = s, d$), where the charm-anticharm pair forms a J/ψ, are particularly suitable for a hadronic machine, given the clean signature from $J/\psi \to l^+ l^-$. It was pointed out recently by Dunietz [4], and later investigated in more detail in Ref. [5], that a CP asymmetry of type 1 appears in these decays. A rough estimate suggested that it could be of order 1%, in decays of the type $b \to dJ/\psi$, which is the expected reach of an experiment with a sample of 10^{10} B mesons (the analogous asymmetry in $b \to sJ/\psi$ is suppressed by a factor of $\sin^2\theta_C$).

The CP asymmetry in $b \to dJ/\psi$ stems from the interference between the dominant tree amplitude and a small absorptive amplitude that contributes coherently. That is due to the process $b \to i \to dJ/\psi$, where i denotes on-mass-shell intermediate states with the quark content $du\bar{u}$ or $dc\bar{c}$. For the former, the process is Okubo-Zweig-Iizuka (OZI) suppressed, and the rescattering to dJ/ψ occurs at the order α_s^3 or *via* the electromagnetic interaction. It generates an asymmetry of the order of *a few* $\times 10^{-3}$ [5]. As for the intermediate states with the same quark content $dc\bar{c}$ as the final state, it was first pointed out by Wolfenstein [6] that they can contribute to the CPasymmetry in exclusive decays, or in semi-inclusive decays such as $b \to dJ/\psi$ (although their contribution is absent in the case of the inclusive decay [2]). In Ref. [5], the nature of this effect was discussed in terms of the on-mass-shell hadronic states that form the intermediate state. This approach is reviewed in Sec. II; it only allows for a rough estimate which suggests that a contribution to the CP asymmetry of about 1% is possible. In Sec. III, I look at this effect from a different perspective, and try to obtain a better estimate of its contribution to the asymmetry. In the spirit of quark-hadron duality, the collection of hadronic intermediate states is replaced by the corresponding $dc\bar{c}$ quark configuration. The final state scattering is then treated perturbatively in α_s; the absorptive part of the amplitude and the ensuing CP asymmetry are evaluated at the lowest order. The results obtained are discussed in Sec. IV.

II. THE HADRONIC DESCRIPTION

The effect of the final state interactions in a given decay amplitude $A_i \equiv A(B \to i)$ can be described by the S matrix, $S = 1 + iT$ for the scattering among different final states. When the amplitudes in T can be treated perturbatively [6, 7]:

$$A_i \simeq A_i^{(0)} + i\frac{1}{2}\sum_j T_{ij} A_j^{(0)}, \tag{2}$$

where $A_j^{(0)}$ are the weak decay amplitudes in the absence of the final state interactions. It is the interference between the different terms on the right-hand side (RHS) that generates the CP-violating quantity

$$\Delta_i \equiv |A_i|^2 - |\bar{A}_i|^2 = \sum_j \Delta_i^j \ , \tag{3}$$

with

$$\Delta_i^j = 2T_{ij}\mathrm{Im}\{A_j^{(0)*} A_i^{(0)}\}. \tag{4}$$

Clearly, $\Delta_i^i = 0$: the rescattering of the final state does not contribute to the asymmetry, since it does not generate a term in Eq. (2) with a different CP-odd phase than that of $A_i^{(0)}$. For the case of the inclusive decay $b \to dc\bar{c}$, this means that there is no contribution to the asymmetry from the intermediate state $dc\bar{c}$. However, the situation is different for the exclusive or semi-inclusive cases

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[6]. In Ref. [5] the decay $B^- \to J/\psi\pi^-$ was examined, as an example. Including the absorptive part that is due to the intermediate states $X = D^0 D^-, D^{*-}D^0, J/\psi\rho^-, \ldots$, that have the same quark content as the final state $J/\psi\pi^-$, the decay amplitude is

$$\begin{aligned} A(B^- \to J/\psi\pi^-) &= V_{cb}V_{cd}^* T_{\psi\pi^-} + V_{tb}V_{td}^* P_{\psi\pi^-} \\ &+ i\frac{1}{2}\sum_X (V_{cb}V_{cd}^* T_X + V_{tb}V_{td}^* P_X) \\ &\times A(X \to J/\psi\pi^-). \end{aligned} \tag{5}$$

The weak amplitudes include both tree and penguin contributions, proportional to $V_{cb}V_{cd}^*$ and $V_{tb}V_{td}^*$, respectively. The penguin/tree ratios $P_{\psi\pi^-}/T_{\psi\pi^-}$ and P_X/T_X will in general be different, and so the dispersive and absorptive parts of the amplitude in Eq. (5) will have different CP-odd phases. Then, the states X will contribute to the CP asymmetry with

$$\begin{aligned} a_{\rm CP} &\simeq \mathrm{Im}\left\{\frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*}\right\}\sum_X \frac{T_{\psi\pi^-}^* T_X A(X \to J/\psi\pi^-)}{|T_{\psi\pi^-}|^2} \\ &\times \left(\frac{P_X}{T_X} - \frac{P_{\psi\pi^-}}{T_{\psi\pi^-}}\right). \end{aligned} \tag{6}$$

There is no reliable way of calculating the scattering amplitudes $A(X \to J/\psi\pi^-)$ at the hadronic level (moreover, a large number of hadronic states, including those with larger multiplicity, should be included). In Ref. [5], the asymmetry due to some of the intermediate states X was estimated, leaving the ratio

$$\xi_X \equiv \frac{T_{\psi\pi^-}^* T_X A(X \to J/\psi\pi^-)}{|T_{\psi\pi^-}|^2} \tag{7}$$

as an undetermined parameter. Contributions to the asymmetry of about

$$a_{\rm CP} = \xi_X \times 1\% \times \frac{\eta}{0.4} \tag{8}$$

were found [$\eta = -\mathrm{Im}\{(V_{tb}V_{td}^*)/(V_{cb}V_{cd}^*)\}$, in the Wolfenstein parametrization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix].

III. QUARK LEVEL DESCRIPTION

In view of the difficulties of the approach outlined in the previous section, an approximate prescription may provide a more complete calculation of the absorptive amplitude. It amounts to replacing the collection of the hadronic intermediate states by the corresponding quark configuration $dc\bar{c}$ (in the spirit of the quark-hadron duality). Then, the scattering to dJ/ψ is treated perturbatively in the strong coupling constant, which is chosen at the m_b scale (for $m_b \simeq 5.0$ GeV and $\Lambda_{\overline{\rm MS}}^{(4)} \simeq 200$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme, $\alpha_s(m_b) \simeq 0.23$). Notice that at zeroth order in α_s, there are no intermediate states (other than dJ/ψ) that contribute to the absorptive amplitude, as the J/ψ resonance is below the D-$\bar{D}$ threshold, and its overlap with ψ' can be neglected. At the order α_s, the presence of the gluon removes the kinematical constraint, and the intermediate states where $c\bar{c}$ appears in a color octet (i.e., states in the $c-\bar{c}$ continuum) will contribute. The intermediate states with $c\bar{c}$ in a color singlet will be neglected: they can only contribute at higher orders in α_s, due to the color selection rule, and the weak decay amplitude into that configuration is color suppressed.

The amplitude for the decay $b \to dJ/\psi$, in the absence of final state scattering, is

$$A_{d\psi}^{(0)} = V_{cb}V_{cd}^* T_{d\psi} + V_{tb}V_{td}^* P_{d\psi}. \tag{9}$$

The tree and penguin terms are calculated from the effective Hamiltonian

$$\begin{aligned} H_{\rm eff} = -\frac{G_F}{\sqrt{2}}\Bigg[& V_{ub}V_{ud}^*(C_1 Q_1^u + C_2 Q_2^u) \\ & + V_{cb}V_{cd}^*(C_1 Q_1^c + C_2 Q_2^c) \\ & + V_{tb}V_{td}^* \sum_{k=3}^{6} C_k Q_k + \text{H.c.}\Bigg], \end{aligned} \tag{10}$$

where

$$\begin{aligned} Q_1^l &= \bar{d}\gamma^\mu(1-\gamma_5)b\ \bar{l}\gamma_\mu(1-\gamma_5)l\ , \\ Q_2^l &= \bar{l}\gamma^\mu(1-\gamma_5)b\ \bar{d}\gamma_\mu(1-\gamma_5)l\ , \\ Q_3 &= \sum_{l=u,d,s,c,b} \bar{d}\gamma^\mu(1-\gamma_5)b\ \bar{l}\gamma_\mu(1-\gamma_5)l\ , \\ Q_4 &= \sum_{l=u,d,s,c,b} \bar{l}\gamma^\mu(1-\gamma_5)b\ \bar{d}\gamma_\mu(1-\gamma_5)l\ , \\ Q_5 &= \sum_{l=u,d,s,c,b} \bar{d}\gamma^\mu(1-\gamma_5)b\ \bar{l}\gamma_\mu(1+\gamma_5)l\ , \\ Q_6 &= -2\sum_{l=u,d,s,c,b} \bar{l}(1-\gamma_5)b\ \bar{d}(1+\gamma_5)l. \end{aligned} \tag{11}$$

In the leading-logarithm approximation, the Wilson coefficients at the scale m_b (and for $\Lambda_{\overline{\rm MS}}^{(4)}$ as above) are [8]

$$\begin{aligned} C_1 &= 0.25\ , \\ C_2 &= -1.11\ , \\ C_3 &= 0.011\ , \\ C_4 &= -0.026\ , \\ C_5 &= 0.008\ , \\ C_6 &= -0.032. \end{aligned} \tag{12}$$

Then,

$$T_{d\psi} = \frac{G_F}{\sqrt{2}}\left(C_1 + \frac{1}{N_c}C_2\right) m_\psi f_\psi \epsilon_\mu^* \bar{u}_d\gamma^\mu(1-\gamma_5)u_b \tag{13}$$

and

$$\begin{aligned} P_{d\psi} = \frac{G_F}{\sqrt{2}}&\left[C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6)\right] \\ &\times m_\psi f_\psi \epsilon_\mu^* \bar{u}_d\gamma^\mu(1-\gamma_5)u_b. \end{aligned} \tag{14}$$

The internal momentum of the $c\bar{c}$ pair that forms the J/ψ is neglected, and the decay constant f_ψ is defined by

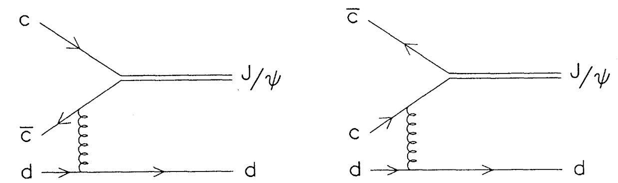

FIG. 1. Diagrams that contribute to the scattering amplitude $A(d(c\bar{c})_8 \to dJ/\psi)$, at the order α_s.

$$\langle J/\psi|\bar{c}\gamma_\mu c|0\rangle = m_\psi f_\psi \epsilon^*_\mu. \tag{15}$$

The same Hamiltonian gives the amplitude for the decay $b \to d(c\bar{c})_8$, where the charm-anticharm pair forms a color octet:

$$A_8^{(0)} = V_{cb}V^*_{cd}(T_8^V + T_8^A) + V_{tb}V^*_{td}(P_8^V + P_8^A). \tag{16}$$

The superscripts V and A designate the terms that correspond to the $c\bar{c}$ pair in a vector and in an axial-vector state, respectively. The latter are given by

$$T_8^A = -\frac{G_F}{\sqrt{2}} C_2 \tfrac{1}{2}\bar{u}_d\gamma^\mu(1-\gamma_5)\lambda^a u_b \bar{u}_c\gamma_\mu\gamma_5\lambda^a v_{\bar{c}} \tag{17}$$

and

$$P_8^A = -\frac{G_F}{\sqrt{2}}(C_4 - C_6)\tfrac{1}{2}\bar{u}_d\gamma^\mu(1-\gamma_5)\lambda^a u_b \bar{u}_c\gamma_\mu\gamma_5\lambda^a v_{\bar{c}}. \tag{18}$$

As for the former, it is shown below that they do not contribute to the absorptive part of the $b \to dJ/\psi$ amplitude, if the final state scattering is treated at the order α_s.

The $c\bar{c}$ color octet can scatter to the J/ψ by exchanging a gluon with the d quark (see Fig. 1). As pointed out earlier, this gluon exchange is required not only by the color constraint, but also for kinematical reasons, since the J/ψ is below the $c\bar{c}$ continuum (a similar effect was discussed in Ref. [3] in relation to the CP asymmetry in the radiative b decays). The convolution of the scattering amplitude $A(d(c\bar{c})_8 \to dJ/\psi)$ with $A_8^{(0)}$ gives the contribution to the absorptive part of the $b \to dJ/\psi$ amplitude:

$$A_{d\psi}^{\text{absorptive}} = i\frac{1}{2}\sum \int d\Phi A(d(c\bar{c})_8 \to dJ/\psi) A_8^{(0)}. \tag{19}$$

The summation is over the spin and color, and the integral is over the phase space of the intermediate state quarks. The integration was done analytically, and the rather cumbersome result is given in the Appendix.

The scattering amplitude $A(d(c\bar{c})_8 \to dJ/\psi)$ is the sum of two terms that correspond to the diagrams in Fig. 1, with a gluon exchanged between the d quark and either the c or the $\bar{c}$ quarks. As long as the internal momentum of the J/ψ is neglected, the corresponding terms in the expression for $A_{d\psi}^{\text{absorptive}}$ are related by charge conjugation, and they are equal in magnitude. Because J/ψ is a vector state ($C=-1$), the two terms have the same sign when the $A_8^{(0)}$ amplitude produces $c\bar{c}$ as an axial vector, whereas they have the opposite sign and cancel each other when the $c\bar{c}$ octet forms a vector (this is nothing else than a manifestation of Furry's theorem). The amplitude for the decay $b \to dJ/\psi$, with the final state scattering included to order α_s, is then

$$\begin{aligned} A_{d\psi} &= V_{cb}V^*_{cd}T_{d\psi} + V_{tb}V^*_{td}P_{d\psi} \\ &\quad + i\frac{1}{2}\sum \int d\Phi A(d(c\bar{c})_8 \to dJ/\psi) \\ &\quad \times (V_{cb}V^*_{cd}T_8^A + V_{tb}V^*_{td}P_8^A), \end{aligned} \tag{20}$$

where the dispersive terms are given in Eqs. (13) and (14), and the absorptive part is given in the Appendix. The interference between the terms with relative CP-odd and CP-even phases in the RHS gives

$$\begin{aligned} \Delta &\equiv |A_{d\psi}|^2 - |\bar{A}_{d\psi}|^2 \\ &= 2\text{Im}\{V_{cb}V^*_{cd}V^*_{tb}V_{td}\}\sum \int d\Phi A(d(c\bar{c})_8 \to dJ/\psi) \\ &\quad \times (P_8^A T_{d\psi}^\dagger - T_8^A P_{d\psi}^\dagger), \end{aligned} \tag{21}$$

where the summation includes the spin and color of the b and d quarks, and the J/ψ polarization. It follows that

$$\begin{aligned} \Delta = -\text{Im}\{V_{cb}V^*_{cd}V^*_{tb}V_{td}\} &\Bigg[(C_4 - C_6)(C_1 + \frac{1}{N_c}C_2) \\ &\quad -C_2\left(C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6)\right)\Bigg] \\ &\quad \times G_F^2 \alpha_s m_b^3 m_\psi f_\psi^2 \frac{32}{3}\frac{(1-z)^2(z^2-3)}{\sqrt{z}(2-z)^2}, \end{aligned} \tag{22}$$

with $z = (m_\psi/m_b)^2$. This gives the CP asymmetry, in the semi-inclusive decay $b \to dJ/\psi$:

$$\begin{aligned} a_{\rm CP} &\simeq \frac{|A_{d\psi}|^2 - |\bar{A}_{d\psi}|^2}{2|V_{cb}V^*_{cd}T_{d\psi}|^2} \\ &= \text{Im}\left\{\frac{V_{tb}V^*_{td}}{V_{cb}V^*_{cd}}\right\} \frac{(C_4 - C_6)(C_1 + \frac{1}{N_c}C_2) - C_2[C_3 + C_5 + \frac{1}{N_c}(C_4 + C_6)]}{(C_1 + \frac{1}{N_c}C_2)^2} \alpha_s \frac{8}{9}\frac{(1-z)(z^2-3)}{(1+2z)(2-z)^2}. \end{aligned} \tag{23}$$

Following the usual prescription of setting $N_c = \infty$ [so that the strength of the color suppression in $\Gamma(b \to dJ/\psi)$ is in good agreement with what is measured in the analogous decays of the type $b \to sJ/\psi$] [9],

$$a_{\rm CP} = 1.1\% \times \frac{\eta}{0.4}. \tag{24}$$

IV. CONCLUSION

The value of the CP asymmetry in Eq. (24) confirms the earlier suspicion [5] that the contribution from the $dc\bar{c}$ intermediate states may be important. Indeed, if the assumptions on which this calculation is based are correct (namely, the use of a quark configuration for the intermediate state, and an expansion in α_s for the final state scattering), the absorptive part of the $b \to dJ/\psi$ amplitude is dominated by the rescattering from states that contain a $c\bar{c}$ pair in a color octet, i.e., states in the continuum above the $D - \bar{D}$ threshold. Their contribution to the CP asymmetry is somewhat lowered by the fact that it must be proportional to a ratio of penguin to tree amplitudes (both are necessary in order to generate a relative CP-odd phase). Still, it dominates over the contribution from the OZI suppressed process $b \to du\bar{u} \to dJ/\psi$ [5, 10].

An important source of uncertainty is the very bothersome fact that, at present, the strength of the color suppression in decays such as $b \to qJ/\psi$ ($q = s$ or d) is not well understood. The prescription of dropping all nonleading terms in $1/N_c$, that I adopted in here, allows us to reproduce the branching ratios that have been measured, but it is neither well founded theoretically, nor confirmed by data from other types of B decays [9]. For the moment, it provides a systematic framework to derive quantitative predictions. However, it is quite possible that some new mechanism is at work that would dominate the decay rate, and most likely affect the value of the CP asymmetry, hence, the interest in pursuing an experimental search, given the potential of present and future facilities for probing the asymmetry close to the level predicted in here.

It should be pointed out, however, that the comparison of the theoretical predictions with experimental results is not trivial. The CP asymmetry in the inclusive decay $B \to J/\psi X_d$ that was calculated in here would be difficult to measure, as it is necessary to exclude from X_d the strange hadrons. Moreover, there would be contributions from the decay $B \to \psi' + X_d \to J/\psi + X_d$, that were not considered. Most likely, the experimental data will correspond to exclusive decays, such as $B \to J/\psi\pi$. Although unusually strong enhancements or cancellations in some channels cannot be excluded, it is likely that the result in Eq. (24) provides an order of magnitude estimate for the CP asymmetry in the exclusive channels. A more thorough calculation could be attempted; it would require a specific model for the mesons, and ambiguities regarding IR divergences and mass singularities would arise requiring an arbitrary cutoff [12]. It is unclear whether this would yield a more reliable estimate.

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APPENDIX

The scattering amplitude $A(d(c\bar{c})_8 \to dJ/\psi)$ is the sum of a term

$$\bar{t} = \alpha_s \pi \bar{u}_d \gamma^\mu \lambda^a u'_d \bar{v}'_{\bar{c}} \gamma^\mu \lambda^a v_{\bar{c}} \frac{1}{(p'_d - p_d)^2} \tag{A1}$$

(the quantities u'_d, $\bar{v}'_{\bar{c}}$, p'_d, and later $p'_{\bar{c}}$, correspond to the intermediate state quarks) that corresponds to the gluon exchange between the d and the $\bar{c}$ quarks, and an analogous term t due to the gluon exchange between the d and the c quarks. In the expression for the absorptive amplitude in Eq. (19), the contributions from t and $\bar{t}$ are related by charge conjugation, and it follows that

$$A_{d\psi}^{\rm absorptive} = i \sum \int d\Phi \bar{t} (V_{cb} V_{cd}^* T_8^A + V_{tb} V_{td}^* P_8^A). \tag{A2}$$

The tree and penguin terms T_8^A and P_8^A are given in Eqs. (17) and (18), and

$$d\Phi \equiv (2\pi)^4 \delta^4 (p_{\bar{c}} + p_d - p'_{\bar{c}} - p'_d) \frac{d^3 p'_{\bar{c}}}{(2\pi)^3 2 p'^0_{\bar{c}}} \frac{d^3 p'_d}{(2\pi)^3 2 p'^0_d}. \tag{A3}$$

The hadronization of the $c\bar{c}$ pair, which forms the J/ψ in the final state, is described by a single parameter: the magnitude of the J/ψ wave function at the origin or, equivalently, the decay constant defined in Eq. (15). The relations

$$\langle J/\psi | \bar{c} \sigma_{\mu\nu} c | 0 \rangle = i f_\psi (p_{\psi\mu} \epsilon_\nu^* - p_{\psi\nu} \epsilon_\mu^*) \ ,$$
$$\langle J/\psi | \bar{c} \sigma_{\mu\nu} \gamma_5 c | 0 \rangle = \frac{1}{2} i \epsilon_{\mu\nu\alpha\beta} \langle J/\psi | \bar{c} \sigma^{\alpha\beta} c | 0 \rangle \tag{A4}$$

are also useful. Summing over the spin and color of the intermediate state quarks, and integrating over their phase space, I obtain the result

$$A_{d\psi}^{\rm absorptive} = -i \frac{G_F}{2\sqrt{2}} \alpha_s f_\psi [V_{cb} V_{cd}^* C_2 + V_{tb} V_{td}^* (C_4 - C_6)] \frac{8}{9} \Omega \ , \tag{A5}$$

where

$$\Omega = m_\psi \epsilon^{\sigma *}\Bigg\{ \bar{u}_d\gamma_\sigma(1-\gamma_5)u_b[(\mathcal{C}_1+\mathcal{C}_2)\frac{p_b\cdot p_d}{m_\psi^2}+\mathcal{D}+3\mathcal{G}]$$
$$-\bar{u}_d(1+\gamma_5)u_b(\mathcal{C}_1+\mathcal{C}_2)\frac{m_b p_{b\sigma}}{2m_\psi^2} - \bar{u}_d\gamma_\nu\gamma_\sigma\gamma_\mu(1-\gamma_5)u_b(\mathcal{C}_1-\mathcal{C}_2)i\epsilon^{\alpha\mu\beta\nu}\frac{p_{b\alpha}p_{d\beta}}{4m_\psi^2}\Bigg\}$$
$$+\frac{1}{4}(p_\psi^\sigma\epsilon^{\delta *}-p_\psi^\delta\epsilon^{\sigma *})\Bigg\{-\bar{u}_d\gamma_\sigma(1+\gamma_5)u_b\frac{m_b}{m_\psi^2}[(\mathcal{C}_1+\mathcal{C}_2)p_{d\delta}+2\mathcal{D}p_{\psi\delta}]$$
$$-\bar{u}_d(1-\gamma_5)u_b 2(\mathcal{C}_1+\mathcal{C}_2-2\mathcal{D})\frac{p_{b\delta}p_{d\sigma}}{m_\psi^2}+\bar{u}_d\gamma_\sigma(1-\gamma_5)u_b\frac{2}{m_\psi}[(\mathcal{B}-\mathcal{A})p_{d\delta}+\mathcal{A}p_{b\delta}]$$
$$-\bar{u}_d\gamma_\sigma\gamma_\delta(1-\gamma_5)u_b\left[(\mathcal{C}_1+\mathcal{C}_2)\frac{p_b\cdot p_d}{2m_\psi^2}+\frac{1}{2}\mathcal{D}+4\mathcal{G}\right]+\bar{u}_d\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\mu(1-\gamma_5)u_b(\mathcal{C}_1-\mathcal{C}_2)i\epsilon^{\alpha\mu\beta\nu}\frac{p_{b\alpha}p_{d\beta}}{4m_\psi^2}\Bigg\}$$
$$+\frac{1}{4}i\epsilon^{\sigma\delta\lambda\rho}p_{\psi\lambda}\epsilon_\rho^*\Bigg\{\bar{u}_d\gamma_\sigma(1+\gamma_5)u_b\frac{m_b}{m_\psi^2}[(\mathcal{C}_1+\mathcal{C}_2)p_{d\delta}+2\mathcal{D}p_{\psi\delta}]$$
$$+\bar{u}_d(1-\gamma_5)u_b 2(\mathcal{C}_1+\mathcal{C}_2-2\mathcal{D})\frac{p_{b\delta}p_{d\sigma}}{m_\psi^2}+\bar{u}_d\gamma_\sigma(1-\gamma_5)u_b\frac{2}{m_\psi}[(\mathcal{B}-\mathcal{A})p_{d\delta}-\mathcal{A}p_{b\delta}]$$
$$-\bar{u}_d\gamma_\sigma\gamma_\delta(1-\gamma_5)u_b\left[(\mathcal{C}_1+\mathcal{C}_2)\frac{3p_b\cdot p_d}{2m_\psi^2}+\frac{3}{2}\mathcal{D}+4\mathcal{G}\right]$$
$$+\bar{u}_d\gamma_\sigma\gamma_\delta(1+\gamma_5)u_b\mathcal{A}\frac{m_b}{m_\psi}-\bar{u}_d\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\mu(1-\gamma_5)u_b(\mathcal{C}_1-\mathcal{C}_2)i\epsilon^{\alpha\mu\beta\nu}\frac{p_{b\alpha}p_{d\beta}}{4m_\psi^2}\Bigg\}. \qquad \text{(A6)}$$

The quantities $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{D}$, $\mathcal{F}$, and $\mathcal{G}$, are defined by

$$4\pi m_\psi^2\int d\Phi\frac{1}{(p_d'-p_d)^2}p_{d\alpha}'=\mathcal{A}p_{\psi\alpha}+\mathcal{B}p_{d\alpha}\ ,$$
$$8\pi m_\psi^2\int d\Phi\frac{1}{(p_d'-p_d)^2}p_{d\alpha}'p_{\bar{c}\beta}'=\mathcal{C}_1p_{\psi\alpha}p_{d\beta}+\mathcal{C}_2p_{\psi\beta}p_{d\alpha}+\mathcal{D}p_{\psi\alpha}p_{\psi\beta}+\mathcal{F}p_{d\alpha}p_{d\beta}+\mathcal{G}m_\psi^2g_{\alpha\beta}. \qquad \text{(A7)}$$

Performing the integrations, it follows that

$$\mathcal{A}=-\frac{z}{2-z}\ ,$$
$$\mathcal{B}=I+\frac{2z}{(1-z)(2-z)}\ ,$$
$$\mathcal{C}_1=-\left(\frac{z}{2-z}\right)^2\ ,$$
$$\mathcal{C}_2=I+\frac{2z}{1-z}-\left(\frac{z}{2-z}\right)^2\ ,$$
$$\mathcal{D}=-\frac{z}{(2-z)^2}\ ,$$
$$\mathcal{G}=-\frac{1}{2}\left(\frac{1-z}{2-z}\right) \qquad \text{(A8)}$$

$[z=(m_\psi/m_b)^2]$. The divergent integral

$$I=-\frac{z}{1-z}\int_{-1}^{+1}dx\frac{1}{1-x} \qquad \text{(A9)}$$

corresponds to a mass singularity (due to taking $m_s=0$ or $m_c=m_\psi/2$). However, the dependence on I cancels in the expression for Δ, and so the CP asymmetry is free of mass singularities (and of IR divergences [11]).

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[9] See the discussion in Ref. [5], and references therein.

[10] The contribution from the $du\bar{u}$ intermediate state, with $u\bar{u}$ in a color octet, was not included in Ref. [5]. Unlike the color singlet case, there is no one-photon final state scattering and only the contribution of order α_s^3 remains.

[11] That IR divergences do not appear at this order in the CP asymmetry can be understood from a similar argument to that used in Ref. [3], for the case of the radiative decays. The cancellation of the IR divergences requires that all the cuts in the Kinoshita diagram that corresponds to $\Gamma(B\to f)-\Gamma(\bar{B}\to\bar{f})$ be included. At order α_s, this yields the results presented here. At higher orders, it means that degenerate intermediate and final states, where soft gluons accompany $dc\bar{c}$ and dJ/ψ, re-

spectively, must be taken into account. Similarly, in order to cancel mass singularities in the CP asymmetry, at higher orders, degenerate states with massless and collinear quark-antiquark pairs must be considered.

[12] See, for example, the calculation of the CP asymmetry in the analogous process $B \rightarrow K^{*}\gamma$ in C. Greub, H. Simma, and D. Wyler, Nucl. Phys. **B434**, 39 (1995).