Effective action for high-energy scattering in gravity

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The multi-Regge effective action is derived directly from the linearized gravity action. After excluding the redundant field components we separate the fields into momentum modes and integrate over modes which correspond neither to the kinematics of scattering nor to that of exchanged particles. The effective vertices of scattering and of particle production are obtained as sums of the contributions from the triple and quartic interaction terms and the fields in the effective action are defined in terms of the two physical components of the metric fluctuation.

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I. INTRODUCTION

The study of high-energy scattering in gravity is considered as a way to learn about the yet unknown quantum theory of gravitation. At energies of the Planck scale quantum gravity effects become important.

At high energy and small momentum transfer the elastic scattering is described by the eikonal approximation. In this approximation the amplitude can be obtained by summing all graphs with the exchange of an arbitrary number of noninteracting gravitons between the scattering particles [1, 2] as well as from the classical gravitational shock wave solution [3], i.e., the gravitational field of a particle moving with the speed of light [4, 5].

In Yang-Mills theories the contributions of s-channel multiparticle intermediate states dominate the contributions of eikonal-type. Unlike in gravity, simply because of the higher spin, the exchange of one more graviton results in an additional power of s and the contribution of multiparticle intermediate states appears as a correction to the eikonal approximation. Because the eikonal contributions sum up to a phase, these corrections are more important than it seems from the first glance to the perturbative expansion. Quantum effects enter in fact just with these multiparticle contributions.

Corrections to the eikonal have been calculated [6–8]. There is an approach to the improved eikonal [7], where the multi-Regge effective action is used. This action involves the effective vertices of scattering and particle production appearing in the multiparticle amplitudes at high energy with all pairs of particles in s channel having large subenergies, i.e., in the multi-Regge kinematics.

The multiparticle amplitudes in this kinematics can be obtained from the elastic amplitude at high energy by t-channel unitarity. It is enough to know the elastic amplitude to get by unitarity and gauge invariance the effective vertices [9–11] and the multi-Regge effective action [12]. The effective vertices can also be obtained from string amplitudes [13, 14].

In the case of Yang-Mills theory (including fermions, QCD) Lipatov and the authors have found a way to derive the multi-Regge effective action directly from the original action [15]. We write the action in the axial gauge, choosing the momentum of one of the incoming particles as the gauge vector. After eliminating the redundant fields we split the fields into parts, corresponding to momentum ranges determined by the multi-Regge kinematics. The essential step is the (approximate) integration over the "heavy" modes.

In the case of gravity the direct relation in this spirit between the original action and the multi-Regge effective action has not been investigated. The aim of the present paper is to fill this gap.

The multi-Regge effective action is a tool to study the high-energy peripheral scattering both in gravity and in Yang-Mills theory. In the latter case it allows reproducing easily the results of the leading logarithmic approximation (gluon Reggeization, perturbative pomeron) and provides the basis for a systematic improvement (generalized leading logarithmic approximation) including the exchange of an arbitrary number of Reggeized gluons interacting with each other in order to obey the unitarity in all subenergy channels.

There is an effective action for high-energy peripheral scattering both in Yang-Mills theory and in gravity [16] obtained by shrinking the longitudinal dimensions. It reproduces the eikonal approximation and the first correction involving the effective particle production vertex. The contributions with more than one additional particle in the s-channel intermediate state deviate from the ones from the multi-Regge effective action. In particular the leading logarithmic approximation is not reproduced.

In the present paper we extend our procedure of separating modes and of integrating over heavy modes to the case of pure gravity. We choose the axial gauge with the

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momentum of an incoming particle as the gauge vector. The physical degrees of freedom can be represented by two independent matrix elements γ_{11}, γ_{12}, where γ_{ij} is defined by the transverse components of the metric g_{ij}, $i,j = 1,2$:

$$g_{ij} = e^{\psi} \ \gamma_{ij}, \ \ \det(\gamma_{ij}) = 1. \tag{1.1}$$

The elimination of the redundant field components in gravity is much more involved compared to the Yang-Mills theory. The result appropriate for our purposes is obtained [17–19] by specifying the gauge fixing as

$$g_{--} = g_{-i} = 0, \ \ g_{-+} = 2e^{\psi/2}. \tag{1.2}$$

This leads to constraints, which can be solved in closed form to eliminate g_{++}, g_{+i} and ψ. In particular one finds

$$\psi = \frac{1}{4}\partial_-^{-2}[\partial_- \gamma^{ij} \partial_- \gamma_{ij}]. \tag{1.3}$$

The action is determined by the Lagrangian [17],

$$16\pi G \ \mathcal{L} = e^{\psi}(4\partial_+\partial_-\psi - \partial_+\gamma^{ij}\partial_-\gamma_{ij})$$
$$-e^{\psi/2}\gamma^{ij}\left(\frac{1}{2}\partial_i\partial_j\psi - \frac{3}{8}\partial_i\psi\partial_j\psi - \frac{1}{4}\partial_i\gamma^{kl}\partial_j\gamma_{kl} + \frac{1}{2}\partial_i\gamma^{kl}\partial_k\gamma_{jl}\right) - \frac{1}{2}e^{-3/2\psi}\gamma^{ij}\partial_-^{-1}R_i\partial_-^{-1}R_j,$$
$$R_i = \frac{1}{2}e^{\psi}(\partial_-\gamma^{jk}\partial_i\gamma_{jk} + \partial_i\psi\partial_-\psi - 3\partial_-\partial_i\psi) + \partial_k(e^{\psi}\gamma^{jk}\partial_-\gamma_{ij}). \tag{1.4}$$

We parametrize

$$\gamma_{ij} = (e^h)_{ij}, \qquad \mathrm{Sp}\ h = 0, \tag{1.5}$$

and use the complex field defined by the two independent elements of the matrix h by

$$h = \frac{1}{\sqrt{2}}(h_{11} - ih_{12}). \tag{1.6}$$

Complex notation will be used also for two-dimensional transverse momentum and position vectors as in [15]. Our notation is close to that in [20]. Unlike [20] we define $x_\pm = x_0 \pm x_3$ and this leads to the coefficient 2 in g_{+-} in (1.2). The representation of the linearized action given in the latter paper turned out to be a good starting point for our analysis:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \cdots,$$
$$\mathcal{L}^{(2)} = -2h^*(\partial_+\partial_- - \partial\partial^*)h,$$
$$\mathcal{L}^{(3)} = 2\alpha\{(\partial_-h^*\partial_-h)\partial^{*2}\partial_-^{-2}h + \partial_-h^*h\partial^{*2}\partial_-^{-1}h - 2\partial_-h^*\partial^*h\partial^*\partial_-^{-1}h + \text{c.c.}\},$$
$$\mathcal{L}^{(4)} = 2\alpha^2\{-2|\partial_-^{-2}(\partial_-^3h^*\partial^*\partial_-^{-1}h - \partial_-^2\partial^*h^*h)|^2 + |\partial_-^{-2}(\partial_-^2h^*\partial^*h - \partial_-\partial^*h^*\partial_-h)|^2$$
$$+|\partial_-^{-1}(\partial_-h^*\partial^*h - \partial^*h^*\partial_-h)|^2 - 3|\partial_-^{-1}(\partial_-h^*\partial^*h)|^2 + 3\partial_-^{-1}(\partial_-h^*\partial_-h)\partial_-^{-1}(\partial h^*\partial^*h)$$
$$+[\partial_-^{-2}(\partial_-h^*\partial_-h) - h^*h][\partial h^*\partial^*h + \partial^*h^*\partial h - \partial\partial^*\partial_-^{-1}h^*\partial_-h - \partial_-h^*\partial\partial^*\partial_-^{-1}h]\}. \tag{1.7}$$

A factor $(8\pi G)^{1/2}$ has been included into h, $\alpha = (4\pi G)^{1/2}$. For simplicity of notation we understand the differentiations acting on the nearest field only and put brackets otherwise.

We understand the inverse derivatives as operators defined by Fourier transformation from the momentum representation with the zero mode excluded.

Compared to the Yang-Mills case our analysis here is more involved not only because there are more interaction terms but also because the integration over the heavy modes has to be performed with higher accuracy.

After introducing the separation of modes we study first the contributions in (1.7), which are relevant for the elastic and inelastic scattering of a high-energy particle in an external gravitational field. The source of this field is actually the other incoming particle. It is natural to choose the momentum of that particle as the gauge vector. It can be useful to think of such processes, when reading Secs. II, III, and IV. We obtain the effective vertices for scattering (for particles with momenta close to the momentum of the incoming particle which is not the gauge vector) and for particle production. For the latter vertex we have first to derive the induced vertex arising from the integration over the heavy modes.

In Secs. IV and VI we study the contributions in (1.7), which are necessary beyond the ones considered before for elastic and inelastic particle-particle scattering. The resulting effective vertex for scattering with momenta close to the one of the other incoming particle, the momentum of which is the gauge vector, can be written without derivation by parity symmetry. To obtain this vertex as a sum of contributions from the quartic terms $\mathcal{L}^{(4)}$, the original triple vertices $\mathcal{L}^{(3)}$, and the induced vertices is essential for making our argument complete. In this way we show that in the multi-Regge kinematics the leading contributions of all these terms result in the multi-Regge effective action with three relatively simple vertices.

The involved analysis for the other scattering vertex is the price for the simplicity in obtaining the first two vertices by working in the particular axial gauge. For extending our procedure to covariant gauges it would be necessary to understand how to introduce the separation of modes in the presence of redundant fields. The trace of the gauge choice in our final result is erased, when introducing a complex scalar field for the scattering gravitons by the nonlocal relation $\phi = -\partial^{-2}h$. The multi-Regge effective action for pure gravity can be written with the Lagrangian [12]

$$\mathcal{L}_{\rm eff} = -2\phi^*(\partial_+\partial_- - \partial\partial^*)\partial^2\partial^{*2}\phi + 2\mathcal{A}_{++}\partial\partial^*\mathcal{A}_{--} + 2\alpha(\partial_-\partial^{*2}\phi^*\partial_-\partial^2\phi)\mathcal{A}_{++} + 2\alpha(\partial_+\partial^2\phi^*\partial_+\partial^{*2}\phi)\mathcal{A}_{--}$$
$$+2\alpha\{(\partial^{*2}\mathcal{A}_{--}\partial^2\mathcal{A}_{++} - \partial\partial^*\mathcal{A}_{--}\partial\partial^*\mathcal{A}_{++})\phi + \text{c.c.}\}. \tag{1.8}$$

We shall obtain the relation of the fields involved to modes of the field h.

There is no doubt about the gauge independence of the result, because we know an independent derivation operating merely with on-shell amplitudes.

II. SEPARATION OF MODES

The multi-Regge effective action applies to high-energy scattering $p_A p_B \to k_0 k_1 \cdots k_{n+1}$, where the momenta of the produced particles obey the conditions of multi-Regge kinematics. We write these conditions using the notation $s = (p_A + p_B)^2, s_i = (k_i + k_{i-1})^2, k_i = q_i - q_{i-1}$, and referring to the Sudakov decomposition

$$k^\mu = \frac{1}{\sqrt{s}}(k_+ p_B^\mu + k_- p_A^\mu) + \kappa^\mu \tag{2.1}$$

as

$$s \gg s_i \sim s_j \gg |q_i^2| \sim |q_j^2|, \quad i,j = 0,1,...,n+1,$$
$$k_{+i} \ll k_{+i-1}, \quad k_{-i} \gg k_{-i+1}, \quad \prod_{i=1}^{n+1} s_i = s\prod_{i=1}^{n} |\kappa_i^2|. \tag{2.2}$$

It is known that the leading logarithmic contribution to the scattering ampltitudes arises from s-channel intermediate states obeying this condition. Using the effective action we restrict ourselves to these contributions. Intermediate states not obeying (2.2) lead to corrections to the effective vertices and to additional (nonleading) effective vertices [21]. We use the complex notation for transverse vectors, e.g., for the transverse momenta $\kappa = \kappa^1 + i\kappa^2$, and use the light-cone components for the longitudinal part of vectors. The derivatives with respect to coordinates are defined with the normalization $\partial_+ x_- = \partial_- x_+ = \partial x = \partial^* x^* = 1$.

We consider the linearized gravity action (1.7). The gauge vector corresponds to p_B. We separate the field modes according to the kinematics (2.2):

$$h \to h_1 + h + h_t, \tag{2.3}$$

where h_1, h, and h_t contain correspondingly the modes of the momentum ranges

$$h_1 : |k_+k_-| \gg |\kappa|^2 \sim |q|^2,$$
$$h : |k_+k_- - |\kappa|^2| \sim |q|^2,$$
$$h_t : |k_+k_-| \ll |\kappa|^2 \sim |q|^2. \tag{2.4}$$

The momenta of h_t are typical for exchanged particles, and the ones of h are typical for scattering particles. In the generalized leading logarithmic approximation the dominant contributions correspond to the particles in the s-channel intermediate states strongly ordered in longitudinal momenta and close to mass shell. Therefore we replace the second line in (2.4) by

$$h : \quad |k_+k_- - |\kappa|^2| \ll |q|^2. \tag{2.5}$$

This implies in particular that longitudinal derivatives acting on h can be approximately replaced by transverse ones: $\partial_+\partial_- h \simeq \partial\partial^* h$. We keep the notation h for the particular modes of scattering particles and write $\tilde{h}$, whenever the other modes are included.

The modes h_1 corresponding neither to the kinematics of scattering nor to the one of exchanged particles are to be integrated out.

We consider first the part of the action (1.7) with kinetic and the triple interaction terms, $\mathcal{L}^{(2)} + \mathcal{L}^{(3)}$. With the separation (2.3) the kinetic term decomposes:

$$\mathcal{L}^{(2)} = -2h_1^*(\partial_+\partial_- - \partial\partial^*)h_1 - 2h^*(\partial_+\partial_- - \partial\partial^*)h$$
$$+2h_t^*\partial\partial^* h_t. \tag{2.6}$$

In the first term the longitudinal part in the d'Alembert operator clearly dominates. We shall see that (different to the Yang-Mills case) the contributions proportional to the ratio $|\kappa|^2/k_+k_-$ for the modes h_1 are important.

In the triple interaction terms the kinematical configuration, where a field to which the inverse of ∂_- is applied corresponds to a scattering particle with large k_-, is suppressed. We denote by $\tilde{h}$ the field with all modes and by $\tilde{h}_t$ fields with modes h or h_t with momentum components k_- much smaller than the ones of $\tilde{h}$ involved in the considered vertex. We introduce

$$\tilde{\mathcal{A}}_{++} = \partial_-^{-2}(\partial^{*2}\tilde{h}_t + \partial^2\tilde{h}_t^*), \quad \tilde{\mathcal{A}}'_+ = -i\partial_-^{-1}(\partial^{*2}\tilde{h}_t - \partial^2\tilde{h}_t^*), \quad \tilde{\mathcal{A}}_+ = 2\partial_-^{-1}\partial^*\tilde{h}_t, \quad \tilde{\mathcal{A}}_+^* = 2\partial_-^{-1}\partial\tilde{h}_t^*. \tag{2.7}$$

The tilde will be omitted in the case when only the modes h_t are involved. We define currents as the following bilinear expressions in $\tilde{h}$:

$$\tilde{T}_{--} = \partial_-\tilde{h}^*\partial_-\tilde{h}, \quad \tilde{T}_-^* = \frac{1}{2}(\partial_-\tilde{h}^*\partial^*\tilde{h} + \partial^*\tilde{h}^*\partial_-\tilde{h}),$$
$$\tilde{T}_- = (\tilde{T}_-^*)^*, \quad \tilde{T} = \partial\tilde{h}^*\partial\tilde{h}, \quad \tilde{T}^* = (\tilde{T})^*,$$
$$\tilde{J}_- = i(\tilde{h}^* \overleftrightarrow{\partial}_- \tilde{h}), \quad \tilde{J}^* = i(\tilde{h}^* \overleftrightarrow{\partial}^* \tilde{h}), \quad \tilde{J} = (\tilde{J}^*)^*, \quad \tilde{j} = \tilde{h}^*\tilde{h}. \tag{2.8}$$

With this notation the separation of modes $h \to \tilde{h} + \tilde{h}_t$ leads to the following contribution of the triple interaction $\mathcal{L}^{(3)}$:

$$\mathcal{L}^{(3+)} = 2\alpha\{\tilde{T}_{--}\tilde{\mathcal{A}}_{++} - \tilde{J}_{-}\tilde{\mathcal{A}}'_{+} - \tilde{T}^{*}_{-}\tilde{\mathcal{A}}_{+} - \tilde{T}_{-}\tilde{\mathcal{A}}^{*}_{+} + \tilde{T}^{*}\tilde{h}_t + \tilde{T}\tilde{h}^{*}_t - i\tilde{J}^{*}\partial_{-}\tilde{\mathcal{A}}_{+} + i\tilde{J}\partial_{-}\tilde{\mathcal{A}}^{*}_{+} - 2\tilde{j}\partial^2_{-}\tilde{\mathcal{A}}_{++}\}. \tag{2.9}$$

The interaction of a particle with large k_- is dominated by the first term, giving a contribution $O(k_-^2)$. The next three terms give contributions $O(k_-)$ and the remaining ones contribute to $O(k_-^0)$. There are no helicity flip vertices up to this accuracy.

The effective vertices for the scattering of a graviton with large k_- are the contributions of (2.9) when one restricts the currents to the modes h and the fields $\mathcal{A}$ to the modes h_t. Here we restrict ourselves to the effective vertices leading to contributions $O(s^2)$ to the amplitudes. The leading effective vertex for scattering with large k_- is given by the first term in (2.9), and in the following analysis only the first four terms will be relevant.

III. INTEGRATION OVER THE HEAVY MODES

The essential step in deriving the effective action is the integration over the modes h_1. This will be done approximately by evaluating the action just at the saddle point. To obtain the main contribution it is enough to consider the action determined by $\mathcal{L}^{(3+)}+\mathcal{L}^{(2)}$. The value of the action at the saddle point is determined by

$$\mathcal{L}^{(1)} = 2h_1^{(0)*}(\partial_+\partial_- - \partial\partial^*)h_1^{(0)}, \tag{3.1}$$

where $h_1^{(0)}$ is the solution of the equation obtained by variation with respect to h_1^*:

$$(\partial_+\partial_- - \partial\partial^*)h_1^{(0)} = -\alpha\Big\{ \partial_-(\partial_- h\tilde{\mathcal{A}}_{++}) + \frac{1}{2}\partial_- h(\partial^*\tilde{\mathcal{A}}_+) - \frac{1}{2}\partial_- h\partial\tilde{\mathcal{A}}^*_+ - \partial_-(\partial^* h\tilde{\mathcal{A}}_+) - \partial(\partial_- h\tilde{\mathcal{A}}^*_+)\Big\}. \tag{3.2}$$

We write the solution inverting formally the d'Alembert operator. For the modes h_1 the term with the longitudinal derivatives is the leading one, but the next correction has to be kept when applied to the first term on the right-hand side (RHS). We take into account that the momentum component k_- of $\tilde{\mathcal{A}}$ is much smaller than the one of h and that its component k_+ is much larger. Also here we have to keep the first correction proportional to the ratio of the small to the large k_+ components:

$$h_1^{(0)} \simeq -\alpha\Big\{ \partial_- h\partial_+^{-1}\tilde{\mathcal{A}}_{++} - (\partial_+\partial_- h)\partial_+^{-2}\tilde{\mathcal{A}}_{++} + \partial\partial^*(h\partial_+^{-2}\tilde{\mathcal{A}}_{++}) + \frac{1}{2}h\partial_+^{-1}(\partial^*\tilde{\mathcal{A}}_+ - \partial\tilde{\mathcal{A}}^*_+) - \partial^* h\partial_+^{-1}\tilde{\mathcal{A}}_+ - \partial h\partial_+^{-1}\tilde{\mathcal{A}}^*_+ - h\partial_+^{-1}\partial\tilde{\mathcal{A}}^*_+\Big\}. \tag{3.3}$$

We see immediately that in $\mathcal{L}^{(1)}$ the contribution $O(k_-^3)$ in the product of the leading terms cancels giving up to total derivatives a result proportional to

$$(\partial_- h^*\partial_- h)\partial_-(\partial_+^{-1}\tilde{\mathcal{A}}_{++}\tilde{\mathcal{A}}_{++}). \tag{3.4}$$

This is the reflection of the elementary fact that there is no dipole radiation in gravity.

Evaluating $\mathcal{L}^{(1)}$ we keep only terms $O(k_-^2)$, i.e., with two derivatives acting on h. We obtain

$$\mathcal{L}^{(1)} = \alpha^2 T_{--}\mathcal{I}_{++},$$

$$\mathcal{I}_{++} = -(\partial_+^{-1}\tilde{\mathcal{A}}_{++} \overleftrightarrow{\partial}_- \tilde{\mathcal{A}}_{++}) + \{\partial(\partial_+^{-1}\tilde{\mathcal{A}}_{++}\tilde{\mathcal{A}}^*_+ - \tilde{\mathcal{A}}_{++}\partial_+^{-1}\tilde{\mathcal{A}}^*_+) + \partial\tilde{\mathcal{A}}_{++}\partial_+^{-2}\partial^*\tilde{\mathcal{A}}_{++} + \text{c.c.}\}. \tag{3.5}$$

We have expected the result to be proportional to the current T_{--}. In the calculation it emerges from the cancellation of many terms with other structures.

Now we interpret (3.5) as the quartic terms emerging from the integration over the t-channel modes h_t with an action determined by the kinetic term, the leading effective scattering vertex from (2.9), and the induced triple vertex

$$\mathcal{L}^{(1)}_{\text{ind}} = -\alpha\partial\partial^*\mathcal{A}_{--}\mathcal{I}_{++}. \tag{3.6}$$

Here and in the following we use the notation

$$\mathcal{A}_{--} = \frac{1}{2}\partial_-^2(\partial\partial^*)^{-2}(\partial^{*2}h_t + \partial^2 h_t^*) = \frac{1}{2}\partial_-^4(\partial\partial^*)^{-2}\mathcal{A}_{++},$$

$$\mathcal{A}'_{-} = -\frac{i}{2}\partial_-(\partial\partial^*)^{-2}(\partial^{*2}h_t - \partial^2 h_t^*). \tag{3.7}$$

IV. THE EFFECTIVE PRODUCTION VERTEX

The contribution of the triple interaction terms to the configuration where one field carries a component k_- much smaller than the k_- of the other two fields (2.9) involves two cases. One is the case of scattering, where both fields in the currents are of type h. The other is the case of production, where one field in the currents is of type h_t and the other of type h.

We write the contribution of the leading term in (2.9) in the second case using the notation (3.7):

$$\mathcal{L}^{(3-+)} = -2\alpha\{\partial^{*2}(\mathcal{A}_{--} - i\partial_-\mathcal{A}'_-)h + \text{c.c.}\}\mathcal{A}_{++}. \tag{4.1}$$

The contribution with $\mathcal{A}'_-$ is irrelevant for the leading effective vertices.

Also the induced vertex (3.6) contributes to production. In this case one of the fields in $\mathcal{I}_{++}$ carries the modes h ($\mathcal{A}^{(s)}$) and the other the modes h_t ($\mathcal{A}$). The contributions where the modes h_t are in $\partial_-\mathcal{A}_{++}, \partial_+^{-1}\mathcal{A}_{++}$ or in $\mathcal{A}_+, \mathcal{A}_+^*$ are small. Thus we have

$$\mathcal{L}_{\text{ind}}^{(1-+)} = -2\alpha\partial\partial^*\mathcal{A}_{--}[\partial_-\partial_+^{-1}\mathcal{A}_{++}^{(s)}\mathcal{A}_{++} + \{-\partial(\partial_+^{-1}\mathcal{A}_+^{(s)*}\mathcal{A}_{++}) + \partial_+^{-2}\partial^*\mathcal{A}_{++}^{(s)}\partial\mathcal{A}_{++} + \text{c.c.}\}]. \tag{4.2}$$

Using (2.5) and (2.7) we rewrite this as

$$\mathcal{L}_{\text{ind}}^{(1-+)} = -2\alpha\partial\partial^*\mathcal{A}_{--}[\partial^*\partial^{-2}h\partial\mathcal{A}_{++} - \partial^*(\partial^{-1}h\mathcal{A}_{++}) + \text{c.c.}]. \tag{4.3}$$

We obtain the effective production vertex as the sum of (4.1) and (4.3):

$$\mathcal{L}^{(-+)} = -2\alpha(\partial^{*2}\mathcal{A}_{--}\partial^2\mathcal{A}_{++} - \partial\partial^*\mathcal{A}_{--}\partial\partial^*\mathcal{A}_{++})\partial^{-2}h + \text{c.c.} \tag{4.4}$$

The quartic terms do not give a leading contribution to the production. The elastic and also the inelastic scattering of a particle with large k_- in an external gravitational field is determined only by the triple interaction terms.

V. THE QUARTIC INTERACTION TERMS

To lowest order the peripheral scattering of two quanta at high energy is determined by the quartic and the triple interaction terms. The latter contribution is obtained by contracting one vertex in the kinematics (2.2) with another one, where the field of type h_t now has a momentum component k_- of the same order as the largest k_- of the other two fields involved. Let us extract from $\mathcal{L}^{(3)}$ in (1.7) the contribution to this kinematical configuration, $\mathcal{L}^{(3-)} = \mathcal{L}^{(3-1)} + \mathcal{L}^{(3-2)}$. Each of the three fields in the vertex can be in the modes h_t. $\mathcal{L}^{(3-1)}$ corresponds to the case where the field with the inverse derivative in $\mathcal{L}^{(3)}$ in (1.7) is h_t. This contribution may seem unnatural. Indeed we shall see now that it just cancels to a large extent the contribution from the quartic terms $\mathcal{L}^{(4)}$ to particle-particle scattering. The remaining contributions $\mathcal{L}^{(3-2)}$ will be considered in the next section.

From $\mathcal{L}^{(3)}$ in (1.7) we obtain

$$\mathcal{L}^{(3-1)} = 2\alpha\left\{T_{--}\mathcal{A}_{++} - J_-\mathcal{A}'_+ - T_-^*\mathcal{A}_+ - T_-\mathcal{A}_+^* - \frac{i}{4}J^*\partial_-\mathcal{A}_+ + \frac{i}{4}J\partial_-\mathcal{A}_+^* - \frac{1}{2}j\partial_-^2\mathcal{A}_{++}\right\}. \tag{5.1}$$

We consider an action determined by $\mathcal{L}^{(3+)}$ in (2.9), $\mathcal{L}^{(3-1)}$ in (5.1), and the kinetic term and integrate over h_t. We keep only the terms arising from the leading term in $\mathcal{L}^{(3+)}$, because we are interested in the $O(k_-^2)$ contributions to the scattering only. In the kinematics for which $\mathcal{L}^{(3+)}$ is written the momenta k_- of the fields in $\mathcal{L}^{(3-)}$ are much smaller. We obtain

$$\mathcal{L}_{\text{ind}}^{(3-1)} = -4\alpha^2T_{--}\left\{\partial\partial^*\partial_-^{-4}T_{--} - \partial_-^{-3}(\partial T_-^* + \partial^*T_-) - \frac{1}{2}\partial_-^{-2}\partial\partial^*j + \frac{i}{4}\partial_-^{-2}(\partial J^* - \partial^*J)\right\} + O(k_-). \tag{5.2}$$

The first current T_{--} is understood to carry large k_-.

The contribution of the quartic terms to scattering becomes more transparent when we rewrite them using the currents (2.8) as

$$\begin{aligned}\mathcal{L}^{(4)} = 2\alpha^2\Bigg\{&\left|\partial_-^{-2}\left[\partial^*T_{--} - \partial_-T_-^* - \frac{i}{4}\partial_-\partial^*J_- + \frac{i}{4}\partial_-^2J^*\right]\right|^2 - 2\left|\partial_-^{-2}\left[\partial^*T_{--} - \partial_-T_-^* - \frac{3i}{4}\partial_-\partial^*J_- + \frac{3i}{4}\partial_-^2J^* - \partial_-^2\partial^*j + \partial_-^3(h^*\partial^*\partial_-^{-1}h)\right]\right|^2 \\ &+\left|\partial_-^{-1}\left[\frac{i}{2}\partial^*J_- - \frac{i}{2}\partial_-J^*\right]\right|^2 - 3\left|\partial_-^{-1}\left[T_-^* + \frac{i}{4}\partial^*J_- - \frac{i}{4}\partial_-J^*\right]\right|^2 - 3\partial_-^{-2}T_{--}(\partial h^*\partial^*h) \\ &+(\partial_-^{-2}T_{--} - j)[\partial\partial^*j - \partial_-(\partial\partial^*\partial_-^{-1}h^*h + h^*\partial\partial^*\partial_-^{-1}h)]\Bigg\}. \end{aligned} \tag{5.3}$$

We pick up the terms giving a contribution $O(k_-^2)$ to the scattering,

$$\begin{aligned}\mathcal{L}^{(4)} = 2\alpha^2T_{--}\Bigg\{&\partial\partial^*\partial_-^{-4}T_{--} - \partial_-^{-3}(\partial T_-^* + \partial^*T_-) - 3\partial_-^{-2}\partial\partial^*j + \frac{5i}{4}\partial_-^{-2}(\partial J^* - \partial^*J) - 3\partial_-^{-2}(\partial h^*\partial^*h) \\ &+2[\partial^*\partial_-^{-1}(\partial\partial_-^{-1}h^*h) + \partial\partial_-^{-1}(h^*\partial^*\partial_-^{-1}h) - \partial\partial_-^{-1}h^*\partial^*\partial_-^{-1}h] - (\partial\partial^*\partial_-^{-1}h^*h + h^*\partial\partial^*\partial_-^{-1}h)\Bigg\}. \end{aligned} \tag{5.4}$$

In (5.2) we wrote only the contributions, where T_{--} carries the large k_-. Here in (5.3) each of the two currents in every term can carry the large momentum k_-. Therefore the first term in (5.4) contributes twice and its contribution thus cancels the one of the first term in (5.2) in the sum. The contributions from (5.3) where a nonleading current carries the large k_- can be disregarded.

We analyze $\mathcal{L}^{(4)} + \mathcal{L}_{\text{ind}}^{(3-1)}$ using the fact that the fields in the curly brackets in (5.2) and (5.4) carry relatively small k_- and large k_+ and that the modes h obey (2.5). First we observe that the term in square brackets in (5.4) is approximately equal to $\partial_-^{-1}\partial_+ j$ and therefore does not give a contribution $O(s^2)$ to the scattering. Also the last term in (5.4) is approximately proportional to the latter expression. Further we have in the considered kinematics

$$\partial T_-^* + \partial^* T_- = \partial_+ T_{--} + \frac{1}{2}\partial_-(\partial^* h^* \partial h + \partial h^* \partial^* h), \tag{5.5}$$

where again the first term is negligible. This allows us to write the leading contributions as

$$\mathcal{L}^{(4)} + \mathcal{L}_{\text{ind}}^{(3-1)} = -2\alpha^2 T_{--}\partial_-^{-2}(\partial\partial^* h^* h + h^* \partial\partial^* h + 3\partial h^* \partial^* h). \tag{5.6}$$

We interpret the result as arising from intergrating over h_t with an action determined by the leading term of $\mathcal{L}^{(3+)}$ in (2.9), the kinetic term, and the induced triple vertex

$$\mathcal{L}_{\text{ind}}^{(3-4)} = 4\alpha\partial\partial^*\partial_-^{-2}\mathcal{A}_{--}(\partial\partial^* h^* h + h^*\partial\partial^* h + 3\partial h^* \partial^* h). \tag{5.7}$$

Including this induced vertex we have to remove the contribution $\mathcal{L}^{(3-1)}$, i.e., the one where the fields with ∂_-^{-1} play the role of $\mathcal{A}_{--}$, and the quartic terms $\mathcal{L}^{(4)}$ from which it was generated. Therefore we have now two types of exchanged fields; writing $\mathcal{A}_{++}$ and $\mathcal{A}_{--}$ is not any more convenient notation for the same object. In Feynman graphs the exchange lines obtain arrows related to the longitudinal momentum ordering. When $\mathcal{A}_{++}$ and $\mathcal{A}_{--}$ become independent, the normalization of their kinetic term changes by a factor of 2.

VI. THE OTHER EFFECTIVE SCATTERING VERTEX

We write now the second contribution $\mathcal{L}^{(3-2)}$ of the original triple vertex to the configuration involving one field h_t carrying relatively large k_-:

$$\mathcal{L}^{(3-2)} = 2\alpha\{ \partial^{*2}(\mathcal{A}_{--} - i\partial_-\mathcal{A}'_-)[-h\partial^{*2}\partial_-^{-2}h] - 2\partial_-^{-1}\partial^{*2}(\mathcal{A}_{--} - i\partial_-\mathcal{A}'_-)[\partial^* h\partial^*\partial_-^{-1}h] \\ -\partial_-^{-2}\partial^2(\mathcal{A}_{--} + i\partial_-\mathcal{A}'_-)[\partial_-^2 h^*\partial^{*2}\partial_-^{-2}h] - 2\partial_-^{-2}\partial^*\partial^2(\mathcal{A}_{--} + i\partial_-\mathcal{A}'_-)[\partial_- h^*\partial^*\partial_-^{-1}h] + \text{c.c.}\}. \tag{6.1}$$

The contributions with $\mathcal{A}'_-$ can be ignored here.

We write also the contribution $\mathcal{L}_{\text{ind}}^{(-)}$ of the induced vertex $\mathcal{L}_{\text{ind}}^{(1)}$ in (3.6) to the scattering, i.e., to the case where both fields in $\mathcal{I}_{++}$ carry modes h with k_+ relatively large. We apply (2.5) and (2.7) and disregard contributions which do not give the second power of the large k_+:

$$\mathcal{L}_{\text{ind}}^{(-)} = -\alpha\partial\partial^*\mathcal{A}_{--}\{ [\partial_+^2(\partial^{-1}h + \partial\partial^{*-2}h^*)(\partial^*\partial^{-2}h - 3\partial^{*-1}h^*) + \text{c.c.}] \\ -2\partial_+^2(\partial^{-2}h + \partial^{*-2}h^*)(\partial^*\partial^{-1}h + \partial\partial^{*-1}h^*)\}. \tag{6.2}$$

We extract first the terms, which would contribute to helicity-flip scattering and show that they cancel in the sum. We apply (2.5) and obtain, from (6.1),

$$\mathcal{L}^{(3-2)}|_{hh+h^*h^*} = 2\alpha\partial^{*2}\mathcal{A}_{--}[-h\partial^{*2}\partial_-^{-2}h + \partial^*\partial_-^{-1}h\partial^*\partial^{-1}h] + \text{c.c.} \\ = -2\alpha\partial^{*2}\mathcal{A}_{--}\partial[\partial_+^2\partial^{-2}h\partial^{-1}h] + \text{c.c.} \tag{6.3}$$

Disregarding a term proportional to $\partial_+\mathcal{A}_{--}$ we have, from (6.2),

$$\mathcal{L}_{\text{ind}}^{(-)}|_{hh+h^*h^*} = 2\alpha\partial\partial^*\mathcal{A}_{--}[\partial^*\partial_+^2\partial^{-2}h\partial^{-1}h + \partial_+^2\partial^{-2}h\partial^*\partial^{-1}h] + \text{c.c.}, \tag{6.4}$$

which indeed cancels in the sum against (6.3).

Now we look at the helicity-conserving terms in $\mathcal{L}^{(3-2)}$ in (6.1):

$$\mathcal{L}^{(3-2)}|_{h^*h} = 2\alpha\{ -\mathcal{A}_{--}\partial^2(h^*\partial^{*2}\partial_-^{-2}h) - 2\partial_-^{-1}\mathcal{A}_{--}[\partial^*\partial^2(h^*\partial^*\partial_-^{-1}h) + \partial^2(h^*\partial^{*2}\partial_-^{-1}h)] \\ -\partial_-^{-2}\mathcal{A}_{--}[3\partial^2\partial^*(h^*\partial^* h) - \partial^2(\partial^* h^*\partial^* h)] + \text{c.c.}\}. \tag{6.5}$$

Using (2.5) and disregarding total ∂_+ derivatives we transform the term in the second set of brackets as

$$\partial^*(\partial^2 h^*\partial^*\partial_-^{-1}h + \partial h^*\partial\partial^*\partial_-^{-1}h + h^*\partial^*\partial^2\partial_-^{-1}h + \partial^* h\partial^2\partial_-^{-1}h^*) + \text{c.c.} = \partial_-[\partial^*(\partial^2 h^*\partial_-^{-1}h) + \text{c.c.}] + \partial_+(\cdots). \tag{6.6}$$

In the same way we obtain the relation

$$h^*(\partial\partial^*)^2 h + \text{c.c.} = 2(\partial\partial^* h^*\partial\partial^* h) - \partial_-^2(\partial_+ h^*\partial_+ h) + \partial_+(\cdots), \tag{6.7}$$

which can be used to transform the second term in the last set of square brackets of (6.5) as

$$\partial^2(\partial^* h^*\partial^* h) + \text{c.c.} = 2\partial\partial^* h^*\partial\partial^* h + [\partial\partial^*(\partial\partial^* h^* h) - (\partial\partial^*)^2 h^* h + \text{c.c.}] = -\partial_-^2(\partial_+ h^*\partial_+ h) + \partial\partial^*(\partial\partial^* h^* h + \text{c.c.}). \tag{6.8}$$

Therefore (6.5) reduces to

$$\mathcal{L}^{(3-2)}|_{h^*h} = 2\alpha\Big\{ \mathcal{A}_{--} \left[-\partial^2(h^*\partial^{*2}\partial_-^{-2}h) + \frac{1}{2}(\partial_+h^*\partial_+h) + 2\partial^*(\partial^2\partial_-^{-1}h^*\partial^*\partial_-^{-1}h) + \text{c.c.}\right] - \partial_-^{-2}\partial\partial^*\mathcal{A}_{--}[3\partial(h^*\partial^*h) - (\partial\partial^*h^*h) + \text{c.c.}]\Big\}. \quad (6.9)$$

The second term cancels in the sum with the induced vertex $\mathcal{L}_{\text{ind}}^{(3-4)}$ in (5.7) resulting from the cancellation between the quartic terms and the contribution of $\mathcal{L}^{(3-1)}$. The first term in (6.9) has to be added to the helicity conserving contribution of $\mathcal{L}_{\text{ind}}^{(-)}$ in (6.2) from the induced vertex arising from the integration over h_1. The latter can be written as

$$\begin{aligned}\mathcal{L}_{\text{ind}}^{(-)}|_{h^*h} &= -2\alpha\mathcal{A}_{--}\left\{\frac{1}{2}(\partial\partial^*)^2[\partial_+^2\partial^{*-2}h^*\partial^{-2}h] - 2\partial^2\partial^*[\partial_+^2\partial^{*-1}h^*\partial^{-2}h] + \text{c.c.}\right\}\\ &= -2\alpha\mathcal{A}_{--}\Big\{\partial^2\partial_+h^*\partial_+\partial^{-2}h + 4\partial_+\partial h^*\partial_+\partial^{-1}h + 2\partial_+\partial\partial^{*-1}h^*\partial_+\partial^*\partial^{-1}h + \frac{3}{2}\partial_+h^*\partial_+h\\ &\quad -\frac{1}{2}\partial_+\partial^2\partial^{*-2}h^*\partial_+\partial^{*2}\partial^{-2}h + \text{c.c.}\Big\}. \end{aligned} \quad (6.10)$$

We transform the first term in (6.9) using (2.5):

$$\mathcal{L}^{(3-2)}|_{h^*h} + \mathcal{L}_{\text{ind}}^{(3-4)} = 2\alpha\mathcal{A}_{--}\left\{-\partial^2[h^*\partial_+^2\partial^{-2}h] + 2\partial^*(\partial\partial^{*-1}\partial_+h^*\partial_+\partial^{-1}h) + \frac{1}{2}(\partial_+h^*\partial_+h) + \text{c.c.}\right\}. \quad (6.11)$$

This cancels the contribution of $\mathcal{L}_{\text{ind}}^{(-)}|_{h^*h}$ up to the last term in (6.10). This remaining term is the effective vertex for scattering of partices with large k_+ in the gauge (1.2). Transforming to the gauge, where the other incoming particle momentum p_A plays the role of the gauge vector [i.e. exchanging indices + and − in (1.2)], the result looks similar to the first effective scattering vertex with the indices + and − exchanged:

$$\begin{aligned}\mathcal{L}^{(3-2)} + \mathcal{L}_{\text{ind}}^{(3-4)} + \mathcal{L}_{\text{ind}}^{(-)} &= 2\alpha\mathcal{A}_{--}(\partial_+\partial^2\partial^{*-2}h^*\partial_+\partial^{*2}\partial^{-2}h)\\ &= 2\alpha\mathcal{A}_{--}(\partial_+h^{(g)*}\partial_+h^{(g)}).\end{aligned} \quad (6.12)$$

With the results (2.9), (4.4), and (6.12) for the effective vertices we recover the known result (1.8) for the effective action.

VII. CONCLUSIONS

We have established the direct relation of the multi-Regge effctive action in gravity to the Einstein-Hilbert action. The method of separating modes according to the multi-Regge kinematics and of integrating over heavy modes worked out first for the Yang-Mills case has been extended to the case of gravity. In an axial gauge with the redundant metric components excluded we established the relations of the fields describing scattering (ϕ, ϕ^*) and exchanged $(\mathcal{A}_{++}, \mathcal{A}_{--})$ gravitons to momentum modes of the two independent physical fields of the metric fluctuation (h, h^*). The fields describing exchanged gravitons can be considered as pre-Reggeons.

The extension turned out to be more than a straightforward exercise. We had to understand the contributions of involved interaction terms to the peripheral high-energy scattering. Compared to the Yang-Mills case higher accuracy is required in the kinematical approximations referring to the momentum orderings imposed by the multi-Regge kinematics and in particular by the mode separation. The physical reason for this is simply the absence of dipole radiation.

Being an effective action the applicability of (1.8) is restricted clearly to the kinematical region for which it has been derived. For example, the exchanged fields $\mathcal{A}$ have a purely transverse propagator. But this would generate wrong contributions, if one forgets about the condition (2.4) on the momentum range. Actually these conditions should be incorporated into the action by damping factors or by cutoffs.

In the case of QCD we have understood earlier [15] how to deal with fermions in the derivation of the effective action. Combining this with the experience from the present analysis it will be not difficult to generalize it to supergravity and to the coupling of gravity to matter.

We have restricted ourselves to contributions resulting in the leading effective vertices which are related to the exchanges of gravitons contributing each with an addition power of s. Including s-channel intermediate gravitons results in corrections proportional to one power of $\ln s$ for each loop. In view of this it would be desirable to include into the effective action also the nonleading graviton exchanges, which do not change the power of

s of a given contribution to the amplitude. The corresponding effective scattering vertices are the second to fourth terms in $\mathcal{L}^{(3+)}$ in (2.9). However this requires a further improvement of the kinematical approximations.

There are ideas about the extension of the approach to covariant gauges, which should be tried first in the Yang-Mills case. Also the approximation used here in the integration over the heavy modes can be improved with some effort.

It is important to extend the analysis to the full gravity action, including the terms of all orders in the metric fluctuation h. We see now good reasons to hope that this can be done.

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