

CP violation in the time evolution of the decay $K^0 \rightarrow \pi^0 e^+ e^-$

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We examine the possibility of extracting *CP*-violating terms in the decay $K^0 \rightarrow \pi^0 e^+ e^-$ by studying the time evolution of a K^0 beam. We focus on the interference region and search for clear effects. We find that experiments which average over the electron and positron momenta can detect *CP* violation as an oscillation in the decay rate. The branching ratio is $(3-5) \times 10^{-12}$ and direct *CP* violation dominates over a wide range of the parameters.

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I. INTRODUCTION

One property of the standard model which is still under active consideration is the origin of *CP* violation. Up to now *CP*-odd contributions have been observed only in the decays of K^0 mesons [1]. In the decay one distinguishes two types of *CP* violation effects: direct *CP* violation occurring in the amplitudes (described by ϵ') and *CP*-asymmetric terms in the mass matrix which is called indirect (described by ϵ). The value of ϵ is precisely known ($|\epsilon| = 2.258 \times 10^{-3}$), but there are still uncertainties concerning the value of ϵ' . The CERN experiment NA31 found [2]

$$\text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = (2.3 \pm 0.7) \times 10^{-3} \quad (1)$$

while the measurement of the Fermilab experiment E731 is [3]

$$\text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = (0.74 \pm 0.59) \times 10^{-3}, \quad (2)$$

which is still consistent with the predictions of the superweak theory. Thus it is still interesting to investigate other processes in order to find if direct *CP* violation is different from zero and to provide another crucial test of the standard model. Examples for such processes are *B* meson and rare K^0 meson decays, which are actively investigated both theoretically and experimentally. A promising decay channel is $K^0 \rightarrow \pi^0 e^+ e^-$, where *CP* violation may be relatively large. The specific decay $K_L \rightarrow \pi^0 e^+ e^-$ has been studied extensively and its status was recently reviewed [4,5].

We study the time development of this decay channel starting with a pure K^0 beam and pose the question if one could identify a *CP*-violating signal in the interference region. In particular, we are interested in a signature of direct *CP* violation which we present in this article. We will show that an experiment which studies the time development of K^0 decays and averages over the momenta of electron and positron is sensitive to *CP*-violating terms. The new effect appears in the interfer-

ence region of the K_S and K_L components and manifests itself as a time oscillation. The effect follows from general symmetry considerations as is explained in the next section. In addition, we present an estimate for the magnitude of the effect. This experiment is especially suited for laboratories with intense *K* beams such as Brookhaven [6].

Our paper is organized as follows. In Sec. II we derive the formula describing the time evolution of a pure K^0 state and classify the different contributions. In Sec. III we discuss the calculations available for the amplitudes and their dependence on the parameters. Furthermore, we give the range of parameters, which is used later on in the numerical analysis. In Sec. IV we present the numerical results for the time evolution of the K^0 state with special emphasis on *CP* violation in the interference region. Finally, the reader who is interested in the experimental possibility can study Sec. II and the conclusions in Sec. IV.

II. CLASSIFICATION OF THE VARIOUS AMPLITUDES

The time evolution of a pure K^0 state is given in terms of the time development of the physical states K_L and K_S as

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}} [e^{-iX_L t} (|K_2\rangle + \epsilon |K_1\rangle) + e^{-iX_S t} (|K_1\rangle + \epsilon |K_2\rangle)] \quad (3)$$

The decay proceeds through two intermediate states $K^0 \rightarrow \pi^0 \gamma$ and $K^0 \rightarrow \pi^0 \gamma \gamma$ with the single or two photons converting into electron-positron pairs. The decaying kaon has spin 0, and angular momentum conservation demands the intermediate state $\pi^0 \gamma$ to be in a *p* wave. It follows, then, that the *CP* eigenvalue of $\pi^0 \gamma$ is $(-1)_\pi (+1)_\gamma (-1)^{l=1} = +1$. Thus K_1 can decay through the $\pi^0 \gamma$ channel in terms of *CP*-conserving parts, whereas K_2 decays through the *CP*-violating parts of the Hamiltonian. We denote these amplitudes as

$$A_1 = \langle \pi^0 e^+ e^- | \mathcal{H}_\gamma | K_1 \rangle \quad \textit{CP} \text{ conserving (it gives indirect } \textit{CP} \text{ violation through } \epsilon) , \quad (4)$$

$$B = \langle \pi^0 e^+ e^- | \mathcal{H}_\gamma | K_2 \rangle \quad \textit{CP} \text{ violating (direct)} . \quad (5)$$

These would be the only two amplitudes if there were no higher order terms. In fact the decay of a K^0 can also proceed through the intermediate state $\pi^0\gamma\gamma$, which is higher order in the electromagnetic coupling. This contribution to the decay width is not *a priori* negligible, because, as we will show, it has to be compared with CP -violating terms which are suppressed. To be specific, the intermediate state of a pion and two photons has many partial waves so that both $CP = +1$ and $CP = -1$ states are allowed. Thus the dominant decay in

$$K_2 \rightarrow \pi^0\gamma\gamma \rightarrow \pi^0 e^+ e^- \quad (6)$$

is CP conserving with the final state odd under the CP transformation. In fact, the decay $K_2 \rightarrow \pi^0\gamma\gamma$ has already been observed. We define the relevant amplitude as

$$A_2 = \langle \pi^0 e^+ e^- | \mathcal{H}_{\gamma\gamma} | K_2 \rangle \quad CP \text{ conserving} . \quad (7)$$

The decay of a pure K^0 beam has the general form

$$\begin{aligned} \langle \pi^0 e^+ e^- | \mathcal{H} | K^0 \rangle(t) = & \frac{1}{\sqrt{2}} \{ e^{-iX_L t} \bar{u}(k_-) \not{p}_K [(B^+ + \epsilon A_1^+ + A_2^+) + B^- \gamma_5] v(k_+) \\ & + e^{-iX_S t} \bar{u}(k_-) \not{p}_K [A_1^+ + \epsilon(B^+ + A_2^+) + \epsilon B^- \gamma_5] v(k_+) \} , \end{aligned} \quad (8)$$

where the $+, -$ indicate that the spinors are written out explicitly, with A_1^+, A_2^+ , and B^+ being vector amplitudes and B^- being the axial vector. For explicit definitions see Eqs. (12), (17), and (24).

The term $\epsilon(B + A_2)$ is small in comparison to A_1 for several reasons: (i) B is small, being CP violating; (ii) A_2 is small, being higher order in electromagnetism; (iii) these two small terms are multiplied by the small parameter ϵ . Neglecting $\epsilon(B + A_2)$, the K_S decays are CP conserving. The K_L decays contain A_2 , which is CP conserving, and the CP -violating amplitudes B and ϵA_1 . The amplitude B represents direct CP violation, whereas the violation in ϵA_1 arises through the mass matrix. Since both terms are very likely suppressed, it becomes necessary to consider the A_2 term, as mentioned above.

Next, we compute the time evolution of the decays:

$$\begin{aligned} \frac{d\Gamma}{ds d\Delta}(t) = & \frac{1}{512\pi^3 m_K^3} \{ e^{-\Gamma_L t} [|B^+ + A_2^+ + \epsilon A_1^+|^2 + |B^-|^2] + e^{-\Gamma_S t} |A_1^+|^2 \\ & + \exp\left(-\frac{\Gamma_L + \Gamma_S}{2} t\right) 2 \operatorname{Re}[e^{-i\Delta m_K t} (B^+ + A_2^+ + \epsilon A_1^+) A_1^{+*}] \frac{1}{2} [\lambda(s, m_K^2, m_\pi^2) - \Delta^2] \} , \end{aligned} \quad (9)$$

where

$$\begin{aligned} s &= (p_K - p_\pi)^2 , \\ \lambda(s, m_K^2, m_\pi^2) &= s^2 + m_K^4 + m_\pi^4 - 2sm_K^2 - 2sm_\pi^2 \\ &\quad - 2m_K^2 m_\pi^2 , \end{aligned}$$

and

$$\Delta = (p_K - k_-)^2 - (p_K - k_+)^2 .$$

This expression shows three time intervals: decays for K_S and K_L mesons and an interference region. The first two show the typical exponential behavior for the decays, and the interference has an oscillatory term as well. We point out an important property of the interference term. The A_2 amplitude is odd under the CP transformation and thus antisymmetric under the exchange of the electron and positron energies or momenta. This is borne out by explicit calculation, with Eq. (25) being linear in Δ . The A_1^+ amplitude is even under exchange of the electron and positron momenta. Therefore the term $A_2^+ A_1^{+*}$ drops out in an experiment which symmetrizes over the electrons and positrons. The remaining interference terms in Eq. (9) are CP odd. Thus the presence of an oscillation in the interference region is a clear indication of CP violation. In the remaining article we estimate each of the amplitudes, calculate the magnitude of the effect, and demonstrate it with formulas and several figures.

III. ESTIMATES FOR THE AMPLITUDES

A. The direct CP -violating amplitude B

Among the amplitudes, B is the best known in the standard model. As discussed by several authors, the B amplitude is calculated according to Fig. 1 by means of an effective Hamiltonian for $\Delta S = 1$ semileptonic transitions derived by using the operator-product expansion [7–10]. One starts at a high energy scale, where the interaction is pointlike, and scales down to low energies by means of the renormalization group equation. The procedure also includes electromagnetic and strong effects contained in the Wilson coefficients. Since the B amplitude is CP violating, it involves the imaginary parts of the Wilson coefficients, and the dominant terms are [11]

$$\operatorname{Im}(C_7) O_7 \quad \text{and} \quad \operatorname{Im}(C_8) O_8 \quad (10)$$

with

$$O_7 = (\bar{s}_L \gamma_\mu d_L)(\bar{e} \gamma^\mu e) \quad \text{and} \quad O_8 = (\bar{s}_L \gamma_\mu d_L)(\bar{e} \gamma^\mu \gamma_5 e) . \quad (11)$$

These Wilson coefficients receive their main contribution from energy scales between m_t and m_c where pertur-

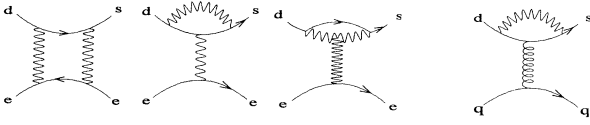


FIG. 1. Short distance contributions to the decay $K \rightarrow \pi^0 e^+ e^-$.

bative QCD is more reliable. The reduced matrix elements of the operators involve quark currents between hadronic states and can be related to K_{l3} decays through an isospin rotation [11]. Neglecting the mass of the electron, the final form of the amplitude is

$$B = \bar{u}(k_-) \not{p}_K [B^+ + B^- \gamma_5] v(k_+) \quad (12)$$

with

$$B^+ = i \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \alpha 2\sqrt{2} f_+(s) \text{Im}(C_7),$$

$$B^- = i \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \alpha 2\sqrt{2} f_+(s) \text{Im}(C_8).$$

Following the Kobayashi-Maskawa parametrization for the quark mixing matrix we use for the coefficients the values [10]

$$\text{Im}(C_7) = -\frac{1}{V_{ud} V_{us}^*} \text{Im}(V_{td} V_{ts}^*) 0.74$$

and

$$\text{Im}(C_8) = -\frac{1}{V_{ud} V_{us}^*} \text{Im}(V_{td} V_{ts}^*) (-0.70) \quad (13)$$

with $m_t = 170$ GeV. The main uncertainty comes from the factor

$$\text{Im}(\lambda_t) = \text{Im}(V_{td} V_{ts}^*) = -s_1 s_2 s_3 c_2 \sin \delta, \quad (14)$$

for which we will allow the range $(1.0-2.0) \times 10^{-4}$. The branching ratio from the B amplitude alone is given by the formula

$$\frac{d\Gamma}{ds d\Delta} = \frac{1}{512\pi^3 m_K^3} [|B^+|^2 + |B^-|^2] \times \frac{1}{2} [\lambda(s, m_K^2, m_\pi^2) - \Delta^2]. \quad (15)$$

Varying the parameters, we obtain the range

$$B(K_L \rightarrow \pi^0 e^+ e^-)_{\text{direct}} = (2.4 - 9.7) \times 10^{-12}. \quad (16)$$

Later on we will use the B amplitude as given above with the corresponding ranges of the parameters in order to study the development of a pure K^0 beam.

B. The CP-conserving amplitude A_1

The amplitude A_1 has the same diagrams (see Fig. 1) as the amplitude B . But the approach of using the same effective Hamiltonian as in Sec. III A involves the real

parts of the Wilson coefficients with large contributions from regions far below m_c , where perturbative QCD is not reliable. For this reason one does not use the QCD effective Hamiltonian, but resorts to other low energy methods like chiral perturbation theory (χ PT). We define A_1 through the equation

$$A_1 = \langle \pi^0 e^+ e^- | \mathcal{H}_\gamma | K_1 \rangle = \bar{u}(k_-) A_1^+ \not{p}_K v(k_+), \quad (17)$$

and A_1^+ is given by

$$A_1^+ = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \frac{\alpha}{\pi} 2 \left[w_+ + \frac{1}{6} \ln \frac{m_\pi^2}{m_K^2} + 2\phi(s) \right]$$

with the loop function $\phi(s)$,

$$\phi(s) = -\frac{4m_K^2}{3s} + \frac{5}{18} + \frac{1}{3} \left(\frac{4m_K^2}{s} - 1 \right)^{3/2} \times \arctan \left(1 / \sqrt{\frac{4m_K^2}{s} - 1} \right), \quad (18)$$

as calculated in [12]. Most of the factors here have standard definitions except for g_8 , which is the coupling constant of the octet of pseudoscalar mesons, and w_+ , which is a dimensionless coupling constant. Both have to be determined experimentally. From the decay $K \rightarrow \pi\pi$ it was found that $g_8 = 5.1$. w_+ was determined from a χ^2 analysis of the spectrum for the decay $K^+ \rightarrow \pi^+ e^+ e^-$, based on a calculation of the spectrum in χ PT [12], including the same set of parameters. From their data the BNL E777 group [13] derived a value of

$$w_+ = 0.89_{-0.14}^{+0.24}. \quad (19)$$

The decay width from the A_1 amplitude reads

$$\frac{d\Gamma}{ds d\Delta} = \frac{1}{512\pi^3 m_K^3} |A_1^+|^2 \frac{1}{2} [\lambda(s, m_K^2, m_\pi^2) - \Delta^2]. \quad (20)$$

This yields a branching ratio for the decay,

$$B(K_L \rightarrow \pi^0 e^+ e^-) = 1.71 \times 10^{-15} - 1.14 \times 10^{-12}, \quad (21)$$

if the decay occurs only through the CP-violating piece of the K^0 - \bar{K}^0 mass matrix.

In the same way as B , the A_1 amplitude with the corresponding range of parameters will be used as an input for the time development of a pure K^0 beam.

C. The CP-conserving amplitude A_2

We have already mentioned that we should include the A_2 amplitude even though it is of $O(\alpha^2)$. Since the decay $K^0 \rightarrow \pi^0 e^+ e^-$ has not yet been observed it was suggested to study the intermediate decay $K_L \rightarrow \pi^0 \gamma \gamma$, which has recently been observed, with the branching ratio [14]

$$B(K_L \rightarrow \pi^0 \gamma \gamma) = (1.7 \pm 0.3) \times 10^{-7}. \quad (22)$$

Starting from these studies, one should couple the two photons to the final electron-positron pair. This is useful but not very direct, because some amplitudes contributing to $K_L \rightarrow \gamma\gamma$ are suppressed in the $K_2 \rightarrow \pi^0 e^+ e^-$ amplitude, being proportional to m_e , and their contribution is negligible. As will become clear later on, one amplitude which gives a significant contribution to the $m_{\gamma\gamma}$ distribution gives a very small contribution to the semileptonic decay.

The amplitude for the decay $K_L \rightarrow \pi^0 \gamma\gamma$ has been estimated by two different methods. One method uses a two-component model developed by Sehgal and collaborators [15,16]. The two contributions included are (a) a diagram with a charged pion loop to which the photons are attached and (b) vector meson intermediate states. The second method applies χ PT [12]. The two approaches differ in several respects, but for the amplitude which is dominant in our investigation they agree. This comes about as follows: the decay $K_L \rightarrow \pi^0 \gamma\gamma$ has several amplitudes, but only one of them is significant for A_2 . It is fortunate that estimates of this amplitude give similar results in the two methods. We describe the results of the two-component model [17,18].

The amplitude for $K_L(p_K) \rightarrow \pi^0(p_\pi)\gamma(k)\gamma(k')$ has the general structure

$$M = \epsilon^\mu \epsilon^\nu [F(k'_\mu k_\nu - g_{\mu\nu} k' \cdot k) + G(g_{\mu\nu} k \cdot p_K k' \cdot p_K + p_{K\mu} p_{K\nu} k' \cdot k - p_{K\mu} k'_\nu k' \cdot p_K - k'_\mu p_{K\nu} k \cdot p_K)] . \quad (23)$$

The pion-loop diagrams contribute only through the F

amplitude, whose contribution to A_2 is proportional to m_e and thus small. This follows by considering the general structure of the loop integral and the tensor structure of the term that multiplies the F amplitude in Eq. (23). The vector meson pole diagram contributes to the amplitude $A_2^+(s, \Delta)$ defined by

$$A_2 = A(K_2 \rightarrow \pi^0 e^+ e^-)_{2\gamma} = \bar{u}(k_-) A_2^+ \not{p}_K v(k_+) . \quad (24)$$

Conservation of CP demands that $A_2^+(s, \Delta)$ is an odd function of Δ . Defining $\beta = \sqrt{1 - 4m_e^2/s}$, the absorptive part of $A_2^+(s, \Delta)$ is

$$\begin{aligned} \text{Im} A_2^+ &= \frac{\alpha}{16} \frac{G_{\text{eff}}}{m_V^2} \frac{\Delta}{\beta} \left[\frac{2}{3} + \frac{2}{\beta^2} - \left(\frac{1}{\beta^2} - \beta^2 \right) \frac{1}{\beta} \ln \frac{1+\beta}{1-\beta} \right] , \\ &= \frac{\alpha}{16} \frac{G_{\text{eff}}}{m_V^2} \frac{8}{3} \Delta \quad \text{when } \beta \rightarrow 1 . \end{aligned} \quad (25)$$

We notice that $\text{Im} A_2^+$ is an odd function of Δ and in addition the limit $\beta \rightarrow 1$ is justified for the decay $K_2 \rightarrow \pi^0 e^+ e^-$. In this limit, Eq. (25) is in agreement with formula (28) of [19]. The formula above has only one coupling G_{eff} which is chosen in such a way that it reproduces the branching ratio in Eq. (22).

The dispersive part is calculated with the help of a dispersion relation [18]

$$\text{Re} A_2^+ = \frac{1}{\pi} \int_{s_{\min}(\Delta)}^{\Lambda^2 = m_\rho^2} \frac{\text{Im} A_2^+(\Delta)}{z - s} dz , \quad (26)$$

where the lower limit is given by

$$s_{\min}(\Delta) = 4m_e^2 \left[1 - \frac{\Delta^2}{[(m_K + m_\pi)^2 - 4m_e^2][(m_K - m_\pi)^2 - 4m_e^2]} \right]^{-1} \quad (27)$$

and the upper limit is determined by the heaviest particle considered, this being the ρ meson. The differential decay width reads

$$\frac{d\Gamma}{ds d\Delta} = \frac{1}{512\pi^3 m_K^3} |A_2^+|^2 \frac{1}{2} [\lambda(s, m_K^2, m_\pi^2) - \Delta^2] . \quad (28)$$

Inserting the numerical values we obtain a branching ratio dominated by the vector meson coupling constant

$$\begin{aligned} B(K_L \rightarrow \pi^0 e^+ e^-)_{2\gamma} &= 4.61 \times 10^{-12} \left(\frac{G_{\text{eff}} m_K^2}{0.25 \times 10^{-7}} \right)^2 (1 + \rho) \\ &= 4.15 \times 10^{-12} \end{aligned} \quad (29)$$

with

$$\rho = \Gamma_{\text{disp}}/\Gamma_{\text{abs}} = 1.5 \quad (30)$$

and

$$G_{\text{eff}} m_K^2 = 0.15 \times 10^{-7} .$$

The second method for calculating the decay $K_L \rightarrow \pi^0 \gamma\gamma$ is χ PT. The authors include effects of order p^4 [20] and p^6 [21] in the momentum expansion of χ PT, as well as vector mesons [22]. This enables them to reproduce the observed decay rate and spectrum. In this approach there are again two effective coupling constants which have to be fixed experimentally. Here the vector meson coupling constant was chosen in such a way that the measured decay rate is reproduced.

The calculation of the two-photon-exchange contribution to the decay $K_L \rightarrow \pi^0 e^+ e^-$ on the basis of the χ PT prediction is analogous. The branching ratio achieved in this manner is dominated by the vector meson intermediate state and reads

$$\begin{aligned} B(K_L \rightarrow \pi^0 e^+ e^-)_{2\gamma} &= 1.8 \times 10^{-12} (1 + \rho) \\ &= 4.5 \times 10^{-12} \end{aligned} \quad (31)$$

with the same ρ . In the numerical analysis of Sec. IV we can choose any of the two calculations for $K_2 \rightarrow \pi^0 e^+ e^-|_{2\gamma}$, because the term relevant for our purpose is practically the same. In addition, we shall demonstrate that in experiments which average over the e^+ and e^-

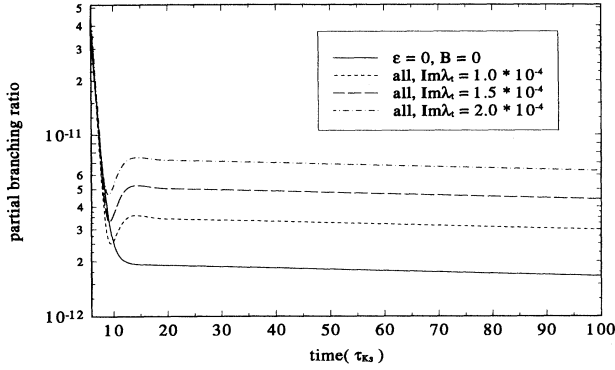


FIG. 2. Time development of the partial branching ratio $\Gamma(K^0 \rightarrow \pi^0 e^+ e^-)(t)/\Gamma(K_L \rightarrow \text{all})$ for the time interval $(6-100) \times \tau_{K_S}$ together with the modified decay rate in which the CP -violating terms are set to zero.

momenta this A_2 amplitude drops out in the interference region.

IV. NUMERICAL RESULTS

With the amplitudes developed and the general equation (9) we calculate the time evolution for the decay $K^0 \rightarrow \pi^0 e^+ e^-$. As already discussed, there are three regions of physical interest.

(1) The K_S region, where the CP -conserving amplitude contributes to the decay width. A measurement in this region will determine parameters of the K_S decay such as ω_+ .

(2) The K_L decay region, which has been studied in several articles interested in CP phenomena (see the reviews [4,5]). The relevant amplitude in this case is $|B + A_2 + \epsilon A_1|^2$, which gives several terms. The interference term $\text{Re}(A_2 B^+ + \epsilon A_1 A_2)$ is odd in Δ and one could define an asymmetry in Δ in order to extract this term. The sum of the absolute values squared is even in Δ , and we will need precise measurements of the amplitudes in order to observe an excess of events.

(3) More interesting is the interference region occurring in the time interval $(8-9) \times \tau_{K_S}$. This term has an oscillatory behavior. The term $A_2 A_1^+$ is a linear function in Δ and drops out when we average over the electron-positron pair. The remaining terms BA_1^+ and $\epsilon|A_1|^2$ are both CP violating. Thus the appearance of an oscillation in the interference region gives evidence for CP violation.

We have studied this phenomenon numerically and show the effect in two figures. In Fig. 2 we show the logarithm of the branching ratio as a function of time. We note that an oscillation is evident at $\tau = 10\tau_{K_S}$. For the curves we use $\omega_+ = 0.89$ and three values for $\text{Im}\lambda_t = 1.0 \times 10^{-4}$, 1.5×10^{-4} , and 2.0×10^{-4} . We plot in the same figure a solid line for which the CP -violating terms BA_1^+ and $\epsilon|A_1|^2$ are set equal to zero. We notice that the curve without CP violation does not oscillate.

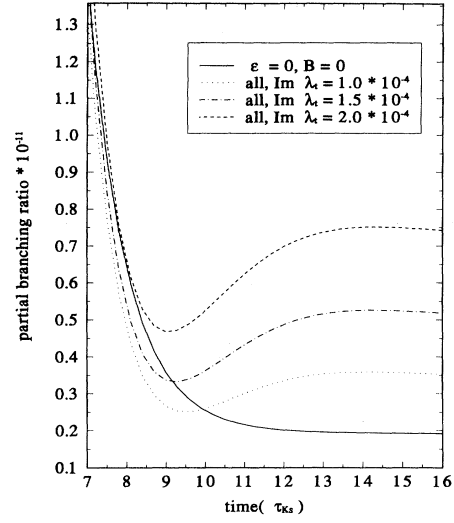


FIG. 3. Same as in Fig. 2 for the time interval $(7-16) \times \tau_{K_S}$.

In the interference region there is a clear oscillation and for very long times the curves which contain the CP amplitudes lie above the curve without the CP -violating terms. We show in Fig. 3 the same result on linear scales, where the minimum is now clearly evident. Experiments which study K_L decays for long times require a precise measurement of magnitudes in order to establish a CP -violating signal. The branching ratio is in the range $(3-5) \times 10^{-12}$. In contrast to this situation, the oscillation in the interference region is unambiguous. We compared the magnitudes of the various terms and found that BA_1^+ dominates over $\epsilon|A_1|^2$. Only if we choose ω_+ at the upper bound ($\omega_+ = 1.13$) are the two contributions of comparable size. But the direct CP -violating term is still larger by $(6-113)\%$ for $\text{Im}(\lambda_t) = (1.0-2.0) \times 10^{-4}$.

We conclude that an experiment searching for a branching ratio down to 10^{-12} and sensitive to the time development of the decay can observe CP violation as an oscillation in the interference region. The experiment does not require a measurement of the $e^+ e^-$ energy asymmetry.

Note added in proof: With the parameters used in this article the branching ratio $B(K_S \rightarrow \pi^0 e^+ e^-)$ can be as large as 3×10^{-10} . When this becomes measurable experiments can start looking for interference effects at times much shorter than $(8-10) \times \tau_{K_S}$.

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