PHYSICAL REVIEW D VOLUME 52, NUMBER 4 15 AUGUST 1995

ARTICLES

Parallel beam interferometric detectors of gravitational waves

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(Received 29 August 1994)

We discuss interferometric detection of gravitational waves using multiple bounce parallel beam systems. We consider as an example the simplest design that allows us to remove the laser frequency fluctuations, and yet gives a remaining nonzero gravitational wave signal, viz., an antiparallel pair of folded beams. The resultant sensitivity, however, is about B times smaller than the sensitivity of a two-arm Michelson interferometer optimally operating with B reflections. We have calculated other, less symmetrical, designs with similar results. Parallel beam interferometric detectors could possibly be preferred for engineering reasons, site availability, and simplicity of response.

PACS number(s): 04.80.Nn, 07.60.Ly, 95.55.Ym

I. INTRODUCTION

Nonresonant detectors of gravitational radiation (with frequency content $0 < f < f_0$) are essentially interferometers with one or more arms, in which a coherent train of electromagnetic waves (of nominal frequency $\nu_0 \gg f_0$) is folded into several beams, and at points where these intersect relative fluctuations of frequency or phase are monitored (homodyne detection). Frequency fluctuations in a narrow band can alternatively be described as fluctuating sideband amplitudes and interference of two or more beams, produced and monitored by a (nonlinear) device such as a photo detector, exhibits these sidebands as a low frequency signal again with frequency content $0 < f < f_0$. The observed low frequency signal is due to frequency variations of the source of the beams about ν_0, to relative motions of the source and the mirrors (or amplifying transponders) that do the folding, to temporal variations of the index of refraction along the beams, and, according to general relativity, to any time-variable gravitational fields present, such as the transverse traceless metric curvature of a passing plane gravitational wave train. To observe these gravitational fields in this way, it is thus necessary to control, or monitor, the other sources of relative frequency fluctuations, and, in the data analysis, to use optimally algorithms based on the different characteristic interferometer responses to gravitational waves (the signal) and to the other sources (the noise). Several feasibility studies [1–4] have shown that this can presently be done to astrophysically interesting thresholds for both ground and space-based instruments.

The frequency band in which a ground-based interferometer can be made most sensitive to gravitational waves [2] ranges from about 10 Hz to about a few kHz, with arm lengths ranging from a few tens of meters to a few km. Space-based interferometers, such as the coherent microwave tracking of interplanetary spacecraft [3] and proposed Michelson optical interferometers in planetary orbits [4], are most sensitive to mHz gravitational waves and have arm lengths ranging from 10^6 to 10^8 km.

In present single-spacecraft Doppler tracking observations many of the noise sources can be either reduced or calibrated by implementing appropriate frequency microwave links and by using specialized electronics, so the fundamental limitation is imposed by the frequency (time-keeping) fluctuations inherent to the reference clocks that control the microwave system. Hydrogen maser clocks, currently used in Doppler tracking experiments, achieve their best performance at about 1000 sec integration time, with a fractional frequency stability of a few parts in 10^{16}. This is the reason why these one-arm interferometers in space are most sensitive to mHz gravitational waves. This integration time is also comparable to the microwave propagation (or "storage") time $2L/c$ to spacecraft en route to the outer solar system ($L \simeq 3$ AU), so these one-arm, one-bounce, interferometers have near-optimum response to gravitational radiation, and a simple antenna pattern.

By comparing phases of split beams propagated along nonparallel arms [2,4,7,13], source frequency fluctuations can be removed and gravitational wave signals at levels many orders of magnitude lower can be detected. Especially for interferometers that use light generated by presently available lasers, which have frequency stability roughly a few parts in 10^{13}, it is essential to be able to remove these fluctuations when searching for gravitational waves of dimensionless amplitude less than 10^{-19} in the mHz band [4], or down to 10^{-21}–10^{-23} desired in the kHz frequency band [2]. Combined with the fact that plane gravitational waves have a spin-two polarization symmetry, this implies that the customary right-angled Michelson configuration is optimal. The response to gravitational waves is then maximized in Earth-based systems by having many bounces in each arm.

Practical considerations may, however, intervene. The requirements of the extended vacuum system strongly dominate design of ground-based installations, and a straight (but optically folded) configuration would be simpler. Site constraints may allow construction of only one vacuum pipe [5]. Alternatively, the possibility of implementing independent interferometer detectors with each arm would imply that existing orthogonal-arm vacuum installations would actually be capable of generating two streams of data with simpler antenna patterns. This would provide both redundancy in the data analysis and useful extra directional and polarization information about the signal. In this paper we derive the response of folded, parallel-beam configurations to incident gravitational waves and, as a demonstration of principle, calculate a particular design in which opposed multiple beams are driven by the same laser light, so source fluctuations are directly removed. The response found is not optimal: it is comparable to that of a one-bounce (but two arms) Michelson interferometer of the same scale size.

We cannot exclude *a priori* the existence of better designs that could make parallel-beam interferometry more attractive for multibounce Earth-based detectors. Many configurations suggest themselves when the path lengths of two split beams are allowed to differ, or when independent offset readouts are recorded at different locations. In practice, modern techniques of heterodyne interferometry with unequal arm lengths and/or independent readout stations can still yield data from which source frequency fluctuations can be removed by many orders of magnitude [7–11]. It must be said, however, that we have calculated the responses of several such offset or unequal arm configurations, and have not found them to offer any further improvement. Source frequency fluctuations have instrumental responses, due to delay time effects, very similar to those characteristic of passing gravitational waves.

In Sec. II we deduce from first principles the response function of a single-arm folded beam to a plane gravitational wave train. In the long wavelength limit (arm length $\ll$ gravitational wavelength) the usual expression for the phase shift of a many-bounce system [6] is recovered. In Sec. III we deduce the response function of an opposed arm many-bounce configuration to a plane gravitational wave train. The data from this interferometer will indeed include a nonzero gravitational wave signal, and a remaining laser phase noise of magnitude smaller than the signal, although the usual advantage of having many bounces is lost. Finally in Sec. IV we present our comments and conclusions.

II. THE RESPONSE FUNCTION FOR A FOLDED BEAM

The net effect of a weak gravitational wave train on the frequency of a coherent light beam reflected once in a stationary, freely falling, configuration of source and mirror is the so-called *three-pulse response function* [1,12,13]. A gravitational wave pulse contributes to the interferometrically measured phase shift at three times, namely at the time it is incident on the source, at a time delayed by L/c after it is incident on the end mirror, and at the round-trip light time (delayed by $2L/c$).

In this section we will deduce the general expression for the phase shift due to a gravitational wave when the laser light is made to bounce B times between two freely falling (geodesic) mirrors of very high reflectivity. The source of the light is at the first mirror, and the net frequency change, or equivalent phase fluctuation, is interferometrically measured there.

Let us consider the space-time metric

$$ds^2 = -dt^2 + (1+h)dx^2 + (1-h)dy^2 + dz^2 \ , \qquad (1)$$

where $h = h(t-z) \ll 1$. To first order, this is the general relativistic solution for the strain field of a linearly polarized gravitational wave train propagating in vacuum along the positive z direction. The metric could be generalized by adding in an amplitude for the other possible polarization, but to first order it is just as easy to do this at the conclusion, as needed. Let us also assume that our two mirrors are stationary in the (x, z) plane. The relative geometry is described in Fig. 1; we have denoted by α the cosine of the angle between the direction of propagation of the gravitational wave and the line joining mirror a to mirror b.

In this space-time the mirrors follow a geodesic motion, represented by world lines parallel to the t axis. With the geometry described in Fig. 1, we can visualize our physical system within the space-time diagram shown in Fig. 2. The vertical axis is the time t, while the horizontal axis is the line $\alpha z + \beta x$, where $\beta^2 = 1 - \alpha^2$. The t axis coincides with the world line $x = y = z = 0$ of mirror a, while the world line for mirror b is (to first order in h): $x = \beta L$, $y = 0$, and $z = \alpha L$. The characteristic wave fronts of the gravitational wave are given by $t-z$ =const.

Consider, at an arbitrary time t, a perfectly monochro-

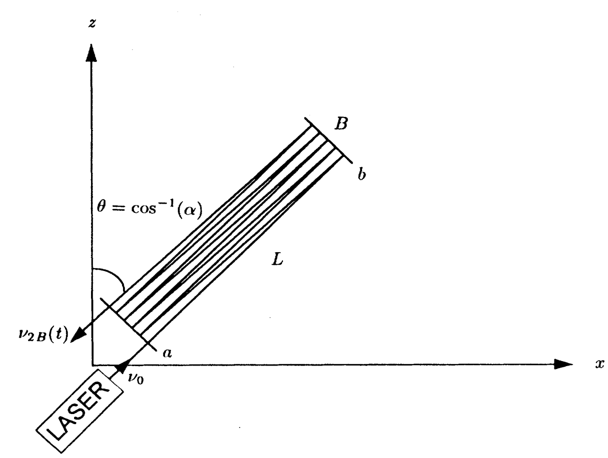

FIG. 1. Laser light of nominal frequency ν_0 is injected inside two highly reflecting mirrors, a and b. It bounces B times against mirror b, and then is made to interfere with the incoming light from the laser. The gravitational wave train propagates along the z direction, and the cosine of the angle between its direction of propagation and the laser light is denoted by α.

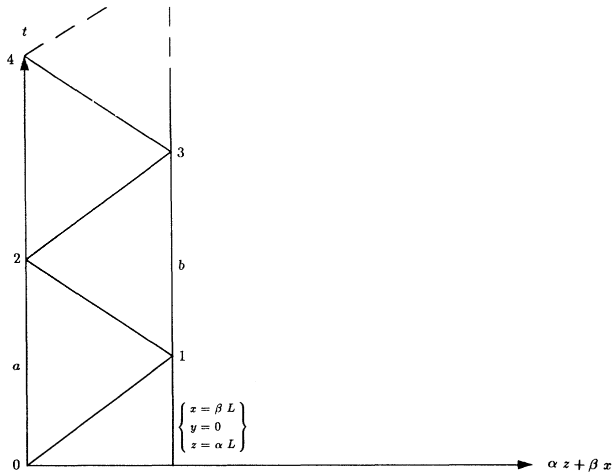

FIG. 2. Space-time diagram describing the optical configuration discussed in Fig. 1. The vertical axis is the time axis t, while the horizontal axis is the line $\alpha z + \beta x$. α is the cosine of the angle between the direction of propagation of the gravitational wave and the direction of the light; β is determined by the relation $\beta^2 = 1 - \alpha^2$. The geodesic world line of mirror a coincides with the time axis t, while the world line of mirror b is given by $x = \beta L$, $y = 0$, $z = \alpha L$.

matic photon of frequency ν_0 (as measured in the rest frame of a) emitted from a laser at a, which bounces off the end mirror b at time $t + L$, and then returns to mirror a at time $t + 2L$. In Fig. 2 this trajectory is represented by two null geodesics, one originating at the event labeled 0 and ending at the event 1; the other connects the event 1 to the event 2. Parallel transport of a null vector along these null geodesics is used to calculate ν_1, the frequency measured at event 1 in the rest frame of b, and ν_2 at event 2 again in the rest frame of a.

The frequency shifts $\nu_1 - \nu_0$, and $\nu_2 - \nu_1$ are related to the gravitational wave amplitude according to the simple "two-pulse" relationships [12] [also see Eqs. (13) and (19) of Ref. [1]]

$$\frac{\nu_1(t+L)}{\nu_0} = 1 + \frac{(1+\alpha)}{2}\left[h(t) - h(t+(1-\alpha)L)\right], \quad (2)$$

$$\frac{\nu_2(t+2L)}{\nu_1(t+L)} = 1 + \frac{(1-\alpha)}{2}\left[h(t+(1-\alpha)L) - h(t+2L)\right], \quad (3)$$

where ν_0 is independent of time, since for the moment we are considering a monochromatic light source (or "atomic" frequency standard).

If we multiply together Eq. (2) and Eq. (3), and disregard second order terms in the wave amplitude h, we deduce the three-pulse response function in its original form [12]

$$\frac{\nu_2(t+2L)}{\nu_0} = 1 + \frac{(1+\alpha)}{2}\, h(t) \;-\; \alpha\, h(t+(1-\alpha)L) - \frac{(1-\alpha)}{2}\, h(t+2L). \quad (4)$$

Equation (4) is then best rewritten to display the fractional frequency change at a as a function of time t:

$$y(t) \equiv \frac{\nu_2(t) - \nu_0}{\nu_0} = -\frac{(1-\alpha)}{2}\, h(t) \;-\; \alpha\, h(t-(1+\alpha)L) + \frac{(1+\alpha)}{2}\, h(t-2L). \quad (5)$$

The phase difference $\Delta\phi^{(1)}(t)$ measured, say, by a photodetector is related to the corresponding frequency change, given by Eq. (5), as

$$y(t) = \frac{1}{2\pi\nu_0}\frac{d\Delta\phi^{(1)}(t)}{dt}. \quad (6)$$

If we define the Fourier transform of the time series $\Delta\phi^{(1)}(t)$ to be given by

$$\widetilde{\Delta\phi^{(1)}}(f) \equiv \int_{-\infty}^{+\infty} \Delta\phi^{(1)}(t)\, e^{2\pi i f t}\, dt\,, \quad (7)$$

we can rewrite Eq. (5) in the Fourier domain as

$$\frac{\widetilde{\Delta\phi^{(1)}(f)}}{2\,\pi\,\nu_0} = -\frac{R(\alpha,f)}{2\pi i f}\,\tilde{h}(f). \quad (8)$$

In Eq. (8) $R(\alpha, f)$ is the three-pulse transfer function

$$R(\alpha,f) = -\frac{(1-\alpha)}{2} \;-\; \alpha\, e^{2\pi i(1+\alpha)fL} + \frac{(1+\alpha)}{2}\, e^{4\pi i f L}. \quad (9)$$

For those who prefer to think in terms of heterodyne detection, of signals on a carrier of amplitude A_0 and frequency ν_0, this phase modulation engenders side bands of amplitude A given by

$$\frac{A(\nu_0+f)}{A_0} = \frac{\nu_0}{f}\left[R(\alpha,f)\; R(\alpha,f)^*\right]^{1/2}\,\tilde{h}(f). \quad (10)$$

If we expand Eq. (9) in the long wavelength limit ($fL \ll 1$), to first order in fL Eq. (8) becomes [13]

$$\frac{\widetilde{\Delta\phi^{(1)}(f)}}{2\,\pi\,\nu_0} \simeq -\,(1-\alpha^2)\; L\; [1 + \pi i(\alpha+2)fL]\;\tilde{h}(f). \quad (11)$$

The factor $(1 - \alpha^2)$ is the "beam pattern" of a single-bounce linear gravitational wave antenna. In the long wavelength limit, its "antenna gain" is $\approx L$.

Let us now assume that the light inside the arm makes B bounces before it is made to interfere with the light of the laser. We want to determine what the corresponding phase change will be in this case. From Fig. 2 we note that the frequencies $\nu_2(t + 2L)$, $\nu_3(t + 3L)$, and $\nu_4(t + 4L)$, for instance, are related among themselves as ν_0, $\nu_1(t + L)$, and $\nu_2(t + 2L)$ assuming proper care of the time argument is taken. We can, for example, easily find that the following expression for $\nu_4(t + 4L)/\nu_2(t + 2L)$ holds:

$$\frac{\nu_4(t+4L)}{\nu_2(t+2L)} = 1 + \frac{(1+\alpha)}{2}\, h(t+2L) \; - \; \alpha\, h(t+2L+(1-\alpha)L) - \frac{(1-\alpha)}{2}\, h(t+4L). \tag{12}$$

If we multiply Eq. (4) by Eq. (12) we get, to first order in h,

$$\begin{aligned}\frac{\nu_4(t+4L)}{\nu_0} = {} & 1 + \frac{(1+\alpha)}{2}\, h(t+2L) \; - \; \alpha\, h(t+2L+(1-\alpha)L) \; - \; \frac{(1-\alpha)}{2}\, h(t+4L) \\ & + \frac{(1+\alpha)}{2}\, h(t) \; - \; \alpha\, h(t+(1-\alpha)L) \; - \; \frac{(1-\alpha)}{2}\, h(t+2L).\end{aligned} \tag{13}$$

If we use the definition of $y(t)$ given in Eq. (5), Eq. (13) can be rewritten as

$$\frac{\nu_4(t)-\nu_0}{\nu_0} = y(t) + y(t-2L). \tag{14}$$

After some simple algebra we can easily deduce the following expression for the frequency change after B bounces:

$$\frac{\nu_{2B}(t)-\nu_0}{\nu_0} = \sum_{k=0}^{B-1} y(t-2kL). \tag{15}$$

Let us now denote by $\Delta\phi^{(B)}(t)$ the phase shift measured at the photodetector for the B bounce configuration. Taking into account Eq. (15), we can write the equation

$$\frac{1}{2\pi\nu_0}\,\frac{d\Delta\phi^{(B)}(t)}{dt} = \sum_{k=0}^{B-1} y(t-2kL), \tag{16}$$

which in the Fourier domain becomes

$$\frac{\widetilde{\Delta\phi^{(B)}}(f)}{2\,\pi\,\nu_0} = -\,\frac{\widetilde{y}(f)}{2\pi i f}\,\sum_{k=0}^{B-1} e^{4\pi i k f L}. \tag{17}$$

From the definition of $y(t)$ [Eq. (5)], and after adding the geometric progression, we can rewrite Eq. (17) as

$$\frac{\widetilde{\Delta\phi^{(B)}}(f)}{2\,\pi\,\nu_0} = -\,\frac{R(\alpha,f)\,\widetilde{h}(f)}{2\pi i f}\left[\frac{1-e^{4\pi i B f L}}{1-e^{4\pi i f L}}\right]. \tag{18}$$

If we expand Eq. (18) in the long wavelength limit, that is to say when $fL \ll 1$ but allow B to be large enough that $4BfL \simeq 1$, for the dominant frequency band of the gravitational wave signal, we get

$$\begin{aligned}\frac{\widetilde{\Delta\phi^{(B)}}(f)}{2\,\pi\,\nu_0} \simeq {} & \frac{(1-\alpha^2)}{2}\;\frac{(1-e^{4\pi i B f L})}{2\pi i f} \\ & \times [1+\pi i(\alpha+2)fL]\,\widetilde{h}(f).\end{aligned} \tag{19}$$

Note that the transfer function given in Eq. (19) does not increase linearly with the arm length, as it did for the one-bounce configuration, $B = 1$. For a given arm length L and for a gravitational wave signal of dominant frequency f, one can *choose* the number of reflections B in such a way that $4BfL \simeq 1$, and the response is optimal, depending only on f and the geometrical factor $(1-\alpha^2)$.

Note that this condition also holds for a Michelson interferometer, since its transfer function is essentially equal to the one given in Eq. (19), apart from a different antenna pattern [14,15]. At 1 kHz an orthogonal-arm interferometer, of 40 m arm length and $B \simeq 2000$ bounces, would experience the same phase shift due to a passing gravitational wave as would an interferometer of 4 km arm length and $B \simeq 20$ bounces.

III. ANTIPARALLEL ARMS

Let us consider the optical configuration described in Fig. 3. We have two opposed but parallel folded beams, each of total length $2BL$, with the laser located between them. This setup will be referred to as antiparallel arms. At an arbitrary time t a laser light of frequency ν_0 is injected simultaneously into the two arms, through a beamsplitter and a highly reflecting mirror. It bounces inside the arms B times, and then the two outcoming beams are made to interfere at a photodetector where phase differences are measured. This physical configuration is represented by the space-time diagram given in Fig. 4.

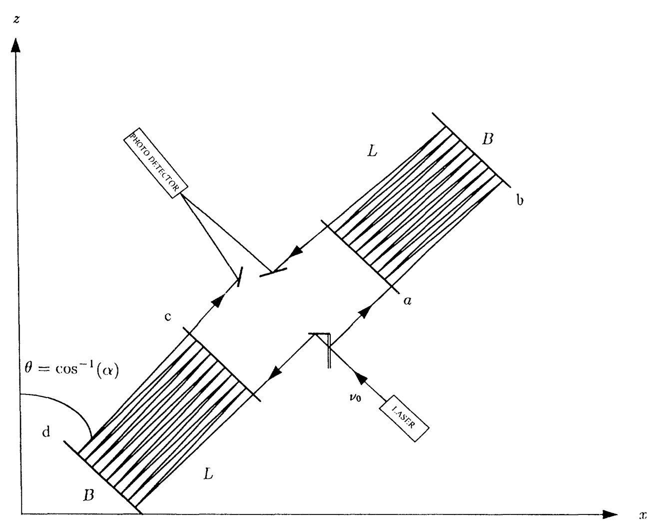

FIG. 3. Two parallel folded beams disposed sequentially, each of total length $2BL$. Laser light of frequency ν_0 is injected into the two arms through a beam splitter and a highly reflecting mirror. After making B bounces, the light is recombined at a photo detector where an interference pattern is monitored.

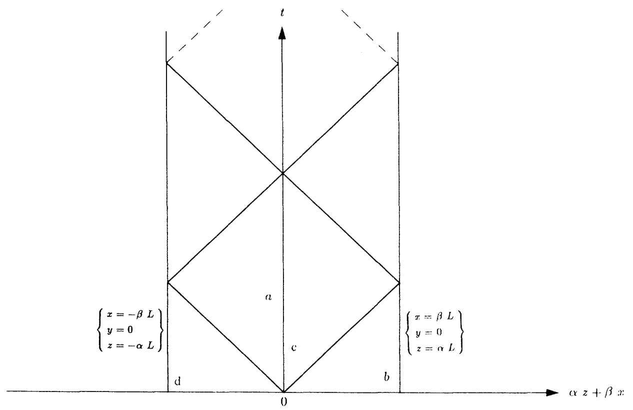

FIG. 4. Space-time diagram describing the optical configuration discussed in Fig. 3. The vertical axis is the time axis t, while the horizontal axis is the line $\alpha z + \beta x$. α is the cosine of the angle between the direction of propagation of the gravitational wave and the direction from mirror a to mirror b; β is determined by the relation $\beta^2 = 1 - \alpha^2$. The world lines of mirrors a and c coincide with the time axis t, while the world line of mirror b is given by $x = \beta L$, $y = 0$, $z = \alpha L$. Mirror d is represented by the world line $x = -\beta L$, $y = 0$, $z = -\alpha L$.

Here we have three world lines for the four mirrors. The mirrors at the input ports of the two arms are at the same space location and their world lines therefore are the same. The time axis t coincides with the world line of mirrors a and c, $x = y = z = 0$. The world lines of mirrors b and d are given (to first order in h) by the equation $x = \beta L$, $y = 0$, $z = \alpha L$ and $x = -\beta L$, $y = 0$, $z = -\alpha L$, respectively. The trajectory of the light is represented by $4B$ null geodesics: $2B$ null geodesics connecting sequentially events on the timelike geodesics of mirrors a and b, and the corresponding $2B$ null geodesics between mirrors c and d.

The overall phase difference measured at the photo detector is equal to the difference between the phase change $\Delta\phi_{ab}(f)$ of the light bouncing B times between mirrors a and b, and the phase change $\Delta\phi_{cd}(f)$ experienced by the light after B bounces between mirrors c and d. From the space-time diagram given in Fig. 4, and using Eq. (18), we see that the two phase differences can be written in the Fourier domain as follows:

$$\frac{\widetilde{\Delta\phi_{ab}}(f)}{2\,\pi\,\nu_0} = -\,\frac{R(\alpha, f)\,\tilde{h}(f)}{2\pi i f}\,\left[\frac{1 - e^{4\pi i B f L}}{1 - e^{4\pi i f L}}\right] + \tilde{C}(f)\,\left[\frac{1 - e^{4\pi i B f L}}{2\pi i f}\right], \tag{20}$$

$$\frac{\widetilde{\Delta\phi_{cd}}(f)}{2\,\pi\,\nu_0} = -\,\frac{R(-\alpha, f)\,\tilde{h}(f)}{2\pi i f}\,\left[\frac{1 - e^{4\pi i B f L}}{1 - e^{4\pi i f L}}\right] + \tilde{C}(f)\,\left[\frac{1 - e^{4\pi i B f L}}{2\pi i f}\right], \tag{21}$$

where $\tilde{C}(f)$ are the Fourier components of the laser frequency fluctuations. Note that they appear in the two phase differences with the same transfer function. Of all the noise sources, the laser frequency noise is the largest, being eight to ten orders of magnitude larger than the amplitude of any other noise source [2]. As in a regular Michelson interferometer, also in our scheme the laser phase fluctuations propagate along the two almost equal length arms, and when the returning beams are recombined after each makes B bounces, the fluctuations are delayed by equal times and so cancel. If we subtract Eq. (21) from Eq. (20) we deduce the overall phase difference $\Delta\phi_p(f)$ measured at the photodetector:

$$\frac{\widetilde{\Delta\phi_p}(f)}{2\,\pi\,\nu_0} = -\,\frac{\alpha\,\left[1 - 2\cos(2\pi\alpha f L) \,+\, e^{4\pi i f L}\right]\,\tilde{h}(f)}{2\pi i f} \times \left[\frac{1 - e^{4\pi i B f L}}{1 - e^{4\pi i f L}}\right] + \frac{1}{2\,\pi\,\nu_0}\,\tilde{n}(f)\,, \tag{22}$$

where we have denoted by $\tilde{n}(f)$ the Fourier components of the random process associated with the remaining phase noise sources affecting the output of the one-arm response.

If we expand Eq. (22) in the long wavelength limit ($fL \ll 1$ but $4BfL \simeq 1$), for the dominant frequency band of the gravitational wave signal, we get

$$\frac{1}{2\pi\nu_0}\widetilde{\Delta\phi_p}(f) \simeq \frac{\alpha(1-\alpha^2)}{2}\,L\,(1 - e^{-4\pi i B f L})\,\tilde{h}(f) + \frac{1}{2\,\pi\,\nu_0}\,\tilde{n}(f)\,. \tag{23}$$

Equation (23) shows some interesting, and somewhat peculiar properties of the remaining gravitational wave signal. In fact the transverse gravitational wave signal goes to zero not only when the wave propagates along the direction of the arms ($\alpha = \pm 1$), but also when it propagates orthogonally to the arms themselves ($\alpha = 0$). For $\alpha = 0$ the "three-pulse" response of any one-arm interferometer, Eq. (19), becomes a "two-pulse" response identical to that for a laser fluctuation in Eq. (20), and therefore the two gravitational wave signals that combine at the photodetector will cancel out. We finally note that the maximum value of the antenna pattern given in Eq. (23) is equal to $\sqrt{3}/9$, while for a regular Michelson interferometer the maximum is equal to 1. This allows us to compare, for each Fourier component of the same wave amplitude h, the maximum value of the phase shift $\widetilde{\Delta\phi^h_p}(f)$ induced by a wave in a parallel-arm interferometer against the corresponding one, $\widetilde{\Delta\phi^h_m}(f)$, experienced by a Michelson interferometer. We find the following ratio of the two phase shifts at an arbitrary Fourier frequency f:

$$\frac{\widetilde{\Delta\phi^h_p}(f)}{\widetilde{\Delta\phi^h_m}(f)} \simeq 1.3 \times i f L. \tag{24}$$

For a gravitational wave signal of dominant frequency 1 kHz, and assuming L to be about 2 km, an antiparallel arm interferometer would observe a gravitational wave effect 100 times smaller than what would be observed by a regular Michelson interferometer. If the number of bounces B are chosen to maximize the signal at this frequency, then Eq. (24) can be rewritten in the form

$$\frac{\widetilde{\Delta\phi_p^h}(f)}{\widetilde{\Delta\phi_m^h}(f)} \simeq 1.3 \times \frac{i}{4B}. \tag{25}$$

IV. CONCLUSIONS

We have discussed a method of interferometric detection of gravitational waves using multiple bounce parallel beam systems. The main result of our analysis, deduced in Eq. (23), shows that it is in principle possible to remove laser frequency fluctuations from a parallel beam interferometer without removing the gravitational wave signal. The magnitude of the remaining gravitational wave's phase shift appears to be, however, about B times smaller than that which an orthogonal two-arm Michelson interferometer with B bounces would measure.

Parallel beam interferometry would be applicable to situations in which engineering and/or site constraints do not allow the construction of two long vacuum pipes along orthogonal directions. It would also imply that orthogonal-arm vacuum installations could be used to generate two streams of data, from independent one-arm systems, providing both redundancy in the data analysis and useful directional information about the gravitational wave signal.

ACKNOWLEDGMENTS

It is a pleasure to thank Ulvi Yurtsever for several useful discussions, and Ron Drever and Kip Thorne for their attention and encouragement during this work. M.T. acknowledges the financial support provided by the Celestial Mechanics Experiment, letter of agreement ARC/PP017, Pioneer Project, NASA/AMES Research Center. This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA.

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