PHYSICAL REVIEW D VOLUME 52, NUMBER 3 1 AUGUST 1995

Predicting the masses of baryons containing one or two heavy quarks

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(Received 31 January 1995)

The Feynman-Hellmann theorem and semiempirical mass formulas are used to predict the masses of baryons containing one or two heavy quarks. In particular, the mass of the Λ_b is predicted to be 5620 ± 40 MeV, a value consistent with measurements.

PACS number(s): 12.40.Yx, 12.15.Ff, 14.20.−c, 14.40.−n

In a recent paper [1], the Feynman-Hellmann theorem [2,3] and semiempirical mass formulas [4,5,1] were used as tools enabling the prediction of the masses of a number of as yet undiscovered mesons and baryons. The major point of the paper [1] is that one can exploit regularities in the pattern of known hadron masses to obtain estimates of the masses of unknown hadrons. Although results are obtained within the framework of the constituent quark model, no explicit Hamiltonian is used. Therefore, the results given in Ref. [1] are complementary to calculations which use specific potentials between quarks to calculate hadron masses.

In this Brief Report, we extend the considerations of Ref. [1] in order to (a) predict the mass of the Λ_b, (b) make slightly revised predictions of masses of other baryons containing one heavy quark, and (c) predict masses of baryons containing two heavy quarks. A number of authors have recently considered baryons [6–12] containing two heavy quarks in anticipation of future experiments which may discover these particles. Again, our work is complementary to that which uses a specific model, and provides predictions which do not depend upon the exact functional form of any explicit Hamiltonian. However, our results do depend on certain general properties of the Hamiltonian: namely, that the constituent quark picture is valid and that for ground-state hadrons the interaction is flavor independent except for a color magnetic interaction whose properties are motivated from QCD.

In Ref. [1], it was shown that, in the constituent quark model, the Feynman-Hellmann theorem can be applied to give useful information about the masses of mesons and baryons even in systems with relativistic kinematics. Under plausible assumptions, application of the Feynman-Hellmann theorem leads to the conclusion that the energy eigenvalues (excluding quark masses) of certain mesons and baryons are smooth, monotonically decreasing functions of the parameter μ, given by

$$\mu^{-1} = \sum_i m_i^{-1}, \tag{1}$$

where the m_i are the masses of the constituent quarks. These results were found to hold for ground-state spin-1 mesons and spin-$\frac{3}{2}$ baryons even in the presence of the color magnetic (spin-spin) interaction between pairs of quarks. On the other hand, the eigenenergies of spin-0 mesons and spin-$\frac{1}{2}$ baryons do not necessarily decrease monotonically with increasing μ.

The relation between the eigenenergy $E(12...)$ of a hadron containing quarks $q_1, q_2, ...$ and its mass $M(12...)$ is

$$M(12...) = E(12...) + \sum_i m_i. \tag{2}$$

The Feynman-Hellmann theorem is useful in allowing us to obtain estimates of $E(12...)$, not $M(12...)$. Therefore we can obtain predictions about hadron masses $M(12...)$ from Eq. (2) only if we assume values for the quark masses.

However, we are not allowed to assume arbitrary values for the quark masses, as we can obtain constraints on the quark mass differences from the Feynman-Hellmann theorem. It was shown in Ref. [1] that the quark mass differences must satisfy the inequalities

$$m_s - m_q > M(K^*) - M(\rho) = 126 \pm 4 \text{ MeV}, \tag{3}$$

$$m_c - m_s > M(D^*) - M(K^*) = 1115 \pm 4 \text{ MeV}, \tag{4}$$

$$m_b - m_c > M(B^*) - M(D^*) = 3316 \pm 6 \text{ MeV}, \tag{5}$$

where the numbers in (3)–(5) come from the experimental values of the vector meson masses as given by the Particle Data Group [13].

The derivation of (3)–(5) was given in Ref. [1], but we briefly repeat it here for (3). From Eq. (2), we have

$$M(K^*) = m_q + m_s + E(qs) \tag{6}$$

and

$$M(\rho) = 2m_q + E(qq). \tag{7}$$

Subtracting (7) from (6) and noting from the Feynman-Hellmann theorem that $E(qs) < E(qq)$, we get the inequality in (3).

We can obtain stronger inequalities from spin-$\frac{3}{2}$ baryons in an analogous manner to the derivation of the inequality (3). In Ref. [1], the following inequality was derived:

$$m_s - m_q > M(\Sigma^*) - M(\Delta) = 153 \pm 4 \text{ MeV}, \quad (8)$$

where the numbers are from baryon masses given in Ref. [13]. This inequality for $m_s - m_q$ is stronger than the inequality given in (3). However, the analogous inequalities

$$m_c - m_s > M(\Sigma_c^*) - M(\Sigma^*) \quad (9)$$

and

$$m_b - m_c > M(\Sigma_b^*) - M(\Sigma_c^*) \quad (10)$$

do not immediately help us because the masses of the baryons Σ_c^* and Σ_b^* are not known experimentally.

However, we can obtain inequalities for $m_c - m_s$ and $m_b - m_c$ in MeV from (9) and (10) if we obtain theoretical estimates of the masses of Σ_c^* and Σ_b^*. We do this using a semiempirical mass formula for the spin splittings among baryons containing a given quark content [1]. Using the known masses [13] of the Λ_c, Σ_c, and Λ_b as input, we get from the semiempirical formula $M(\Sigma_c^*) = 2525$ MeV and $M(\Sigma_b^*) = 5855$ MeV. Then we obtain the inequalities

$$m_c - m_s > M(\Sigma_c^*) - M(\Sigma^*) = 1140 \pm 20 \text{ MeV}, \quad (11)$$

$$m_b - m_c > M(\Sigma_b^*) - M(\Sigma_c^*) = 3340 \pm 60 \text{ MeV}, \quad (12)$$

where the errors are larger than for the previous inequalities because of uncertainties in the semiempirical mass formula [1] and in the experimental value of the mass of the Λ_b [13]. Of course, we should use the experimental values of the Σ_c^* and Σ_b^* in (11) and (12) when these data become available.

In Ref. [1], the following quark masses were used for the discussion of baryons: $m_q = 300$ MeV, $m_s = 475$ MeV, $m_c = 1640$ MeV, and $m_b = 4990$ MeV. We see that these masses satisfy the inequalities (8), (11), and (12), and therefore are potentially good candidates for use with the Feynman-Hellmann theorem to enable us to obtain estimates of the masses of baryons with two heavy quarks. For a detailed discussion of how these quark masses were arrived at, see Ref. [1]. We use the same masses in this paper except that we take the mass of the b quark to be 4985 MeV, 5 MeV lower than the value used in Ref. [1]. Thus we use the quark mass values (in MeV)

$$m_q = 300, \quad m_s = 475, \quad m_c = 1640, \quad m_b = 4985. \quad (13)$$

The small (5 MeV) difference in the value of m_b comes about as follows. In Ref. [1], m_b was obtained with the mass of the Λ_b as input. However, the value of the Λ_b mass (5641 ± 50 MeV), as given in the baryon table of the Particle Data Group [13], has a rather large error. Therefore, in this paper we do not use the Λ_b mass as input but obtain the value of m_b from the vector mesons. This revised procedure allows us to predict the value of the Λ_b mass. We use as input the quark masses $m_q = 300$ MeV, $m_s = 475$ MeV, and $m_c = 1640$ MeV as found in Ref. [1] for the baryons. We also use as input the observed masses [13] of the ground-state vector mesons ρ, K^*, ϕ, D^*, D_s^*, B^*, B_s^*, J/ψ, and Υ. In addition, we use as input the predicted mass of the B_c^*, which in Ref. [1] was found to be 6320 MeV. Then, applying the Feynman-Hellmann theorem as in Ref. [1] we fit the masses of the vector mesons with a monotonically decreasing function of the reduced mass μ. The specific functional form has no theoretical significance. In practice, we use a three-parameter exponential curve similar to that used in Ref. [1], but with different parameters, as the parameters depend on the input quark masses. We vary the three parameters of the curve and the b quark mass so as to get a best fit to the data. This procedure yields $m_b = 4985$ MeV (rounded to the nearest 5 MeV). We show the fit to the meson eigenenergies in Fig. 1.

We obtain predictions for unobserved spin-$\frac{3}{2}$ baryon masses as follows. We first use as input the observed masses Δ, Σ^*, Ξ^*, and Ω. Then we use the semiempirical mass formula of Ref. [1] (with slightly changed parameters because the parameters depend on the input quark masses) to obtain the masses of the Σ_c^* and Ξ_c^*, using the observed masses of the Λ_c, Σ_c, and Ξ_c as input. We next subtract the quark masses of Eq. (13) from the masses of the spin-$\frac{3}{2}$ baryons Δ, Σ, Ξ^*, Ω, Σ_c^*, Ξ_c^* [see Eq. (2)] to obtain the corresponding energy eigenvalues. We then fit a monotonically decreasing three-parameter exponential curve through these energies. The parameters of this curve are slightly different from the parameters used to fit the baryon eigenenergies in Ref. [1] because we take the b quark mass to be 5 MeV less than in Ref.

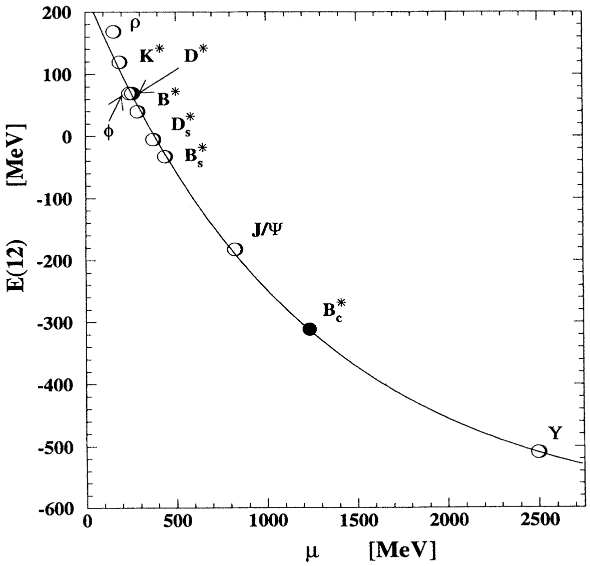

FIG. 1. Fit to the energy eigenvalues of vector mesons with an exponential curve. Open circles denote mesons whose eigenenergies are obtained from experiment by subtracting the quark masses of Eq. (13) and the solid circle denotes an eigenenergy obtained from the mass of the B_c^* predicted in Ref. [1].

[1]. We then extrapolate the curve to give us the energy eigenvalues of unknown spin-$\frac{3}{2}$ baryons including the Σ_b^*, Ω_c^*, and baryons containing two heavy quarks. We show this extrapolated curve in Fig. 2. Because we have to extrapolate rather a long way in μ, we assign errors of as much as 100 MeV to the eigenenergies.

The next step is to add back the quark masses to obtain the masses of these spin-$\frac{3}{2}$ baryons. Lastly, we use the semiempirical mass formula to obtain the masses of spin-$\frac{1}{2}$ baryons, including the Λ_b, Σ_b, Ω_c, and baryons with two heavy quarks. In Table I we give the masses of baryons containing one or two heavy quarks. Masses of known baryons used as input are marked with a superscript a, and masses already predicted in Ref. [1] are marked with a superscript b. In some of the cases marked with a superscript b, our present predictions differ by 10 MeV from those given in Ref. [1]. These differences are well within our estimated errors.

From Table I we see that we predict the mass difference $\Xi_c' - \Xi_c$ to be 110 ± 20 MeV. A preliminary experimental value has been reported [14]: $\Xi_c' - \Xi_c \approx 95$ MeV, in agreement with our value within the errors.

Note that in Table I we predict that the mass of the Λ_b is 5620±40 MeV. (In Ref. [1] the observed mass of the Λ_b was used as input.) We also predict in Table I that the mass of the Ω_c is 2710 ± 30 MeV. (This is the same mass found in Ref. [1], but here the error is smaller.) Both these masses are consistent with the values 5641±50 MeV

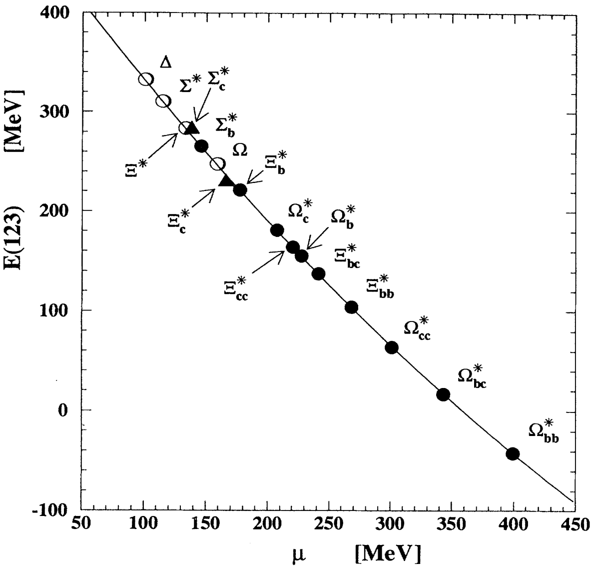

FIG. 2. Energy eigenvalues of baryons with spin $\frac{3}{2}$. Open circles denote baryons whose eigenenergies are obtained from experiment [13] by subtracting the quark masses of Eq. (13), triangles denote eigenenergies obtained with the aid of the baryon semiempirical mass formula of Ref. [1], and solid circles denote predicted eigenenergies obtained from an exponential curve by extrapolation.

TABLE I. Masses of baryons containing one or two heavy quarks. In column 3 we show predictions for ground-state spin-1/2 baryons (denoted by M_A in the first row of this column) whose first two quarks have an antisymmetric spin wave function. In column 4 we show predictions for ground-state spin-1/2 baryons (denoted by M_S) with symmetric spin wave functions in the first two quarks. In column 5 we show predictions for the ground-state spin-3/2 baryons (denoted by M^*).

Name	Quark content	M_A (MeV)	M_S (MeV)	M^* (MeV)
$\Lambda_c, \Sigma_c, \Sigma_c^*$	*qqc*	2285 ± 1[a]	2453 ± 3[a]	2520 ± 20
Ξ_c, Ξ_c', Ξ_c^*	*qsc*	2468 ± 3[a]	2580 ± 20	2650 ± 20
Ω_c, Ω_c^*	*ssc*		2710 ± 30[b,c]	2770 ± 30[b]
$\Lambda_b, \Sigma_b, \Sigma_b^*$	*qqb*	5620 ± 40	5820 ± 40[b]	5850 ± 40[b]
Ξ_b, Ξ_b', Ξ_b^*	*qsb*	5810 ± 40[b]	5950 ± 40[b]	5980 ± 40[b]
Ω_b, Ω_b^*	*ssb*		6060 ± 50[b]	6090 ± 50[b]
Ξ_{cc}, Ξ_{cc}^*	*ccq*		3660 ± 70	3740 ± 70
$\Omega_{cc}, \Omega_{cc}^*$	*ccs*		3740 ± 80	3820 ± 80
$\Xi_{cb}, \Xi_{cb}', \Xi_{cb}^*$	*qcb*	6990 ± 90	7040 ± 90	7060 ± 90
$\Omega_{cb}, \Omega_{cb}', \Omega_{cb}^*$	*scb*	7060 ± 90	7090 ± 90	7120 ± 90
$\Xi_{bb}, , \Xi_{bb}^*$	*bbq*		10340 ± 100	10370 ± 100
$\Omega_{bb}, \Omega_{bb}^*$	*bbs*		10370 ± 100	10400 ± 100

[a] Masses of known baryons used as input.

[b] Masses also predicted in Ref. [1]. The masses in this table sometimes differ from the masses of Ref. [1] by up to 10 MeV because of slightly different input parameters. These differences are well within our estimated errors.

[c] This prediction agrees with the value 2710 ± 5 MeV given in the full listings of the Particle Data Group [13]. We did not use the Particle Data Group value as input because the Ω_c was omitted from their summary baryon table.

and 2710 ± 5 MeV, respectively, given by the Particle Data Group [13].

Various authors have estimated the masses of baryons containing at least one heavy quark [15–21]. For comparison, we give here predictions of a few of those authors for the mass of Λ_b. Some predictions, obtained with a variety of methods including bag models [15], potential models [16], Skyrme models [17], and soliton models [18], differ from our estimate by 20 MeV or less. However, other models [19–21] give masses which are lower by about 35 to 70 MeV. For example, using a bag model approach, Ponce [19] estimates the Λ_b mass at 5555 MeV. Basdevant and Boukraa [20] and Capstick and Isgur [21], using potential models, get 5547 and 5585 MeV, respectively.

The predicted baryon masses given in Table I satisfy an inequality derived by Bagan *et al.* [22], which says that, if quarks 1, 2, and 3 are ordered according to increasing mass, then

$$M(123) \leq M(113) + M(112) - M(111). \qquad (14)$$

If we compare the masses of baryons with two heavy quarks given in Table I with masses calculated with a potential model [12], we find that our predicted masses are from 20 to 130 MeV larger.

Before ending this paper, we say a few words about meson masses. In Ref. [1], in order to obtain best fits to the data on mesons and baryons separately, different input quark masses were used for mesons and baryons. The quark masses used for mesons were (in MeV)

$$m_q = 300, \quad m_s = 440, \quad m_c = 1590, \quad m_b = 4920. \qquad (15)$$

Comparing these masses with the ones in Eq. (13), we see that the differences range from 0 (for m_q) to 65 MeV (for m_b). Although there is no good theoretical reason why effective constituent light quark masses need to be exactly the same in mesons and baryons, the heavy quark masses ought to be pretty much independent of their environment. Furthermore, it is economical in parameters to use the same light quark masses in all hadrons. Therefore, in this paper, we depart from Ref. [1] and attempt to use one set of quark masses in mesons and baryons.

We immediately see that we cannot use the quark masses of Eq. (15) for the baryons, as these mass differences do not satisfy the baryon inequality (8). In confirmation, we have verified numerically that we obtain poor agreement with known baryon masses when using the quark mass set of Eq. (15). We also find that using an average of the quark masses of Eqs. (13) and (15) is unsatisfactory, although these average masses do satisfy all the baryon inequalities (8), (11), and (12). However, as we see from Fig. 1, if we use the quark masses of Eq. (13) as input, we get reasonable agreement with the experimental vector meson masses [although the fit is not statistically as good as with the masses of Eq. (15)]. As we have stated, we used the predicted mass of the B_c^* from Ref. [1] as input. However, all other predictions of the masses of as yet undiscovered mesons are the same as those given in Ref. [1] within the errors given in that paper. We conclude that the quark masses of Eq. (13) are suitable to use for both mesons and baryons in applications involving the Feynman-Hellmann theorem.

This work was begun while one of us (D.B.L.) was visiting the Department of Theoretical Physics of the University of Torino, and he is grateful to the members of that department for their kind hospitality. The work was supported in part by the U.S. Department of Energy, the U.S. National Science Foundation, and the Italian National Institute for Nuclear Physics (INFN).

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