Search for η_c' and $h_c(^1P_1)$ states in e^+e^- annihilation

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The production and decay of spin-singlet S-, P-wave charmonium states η_c' and $h_c(^1P_1)$ in e^+e^- annihilation are considered in the QCD multipole expansion neglecting the nonlocality in time coming from the color-octet intermediate states. Our approximation is opposite to the Kuang-Yan model. The results are $B(\psi' \to h_c + \pi^0) \approx 0.3\%$, $B(\psi' \to \eta' + \gamma) \approx 0.34\%$, $\Gamma(\eta_c' \to J/\psi + \gamma) \approx 0.26$ keV, and $\Gamma(h_c \to J/\psi + \pi^0) \approx 2.5$ keV.

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Among the heavy quarkonium states, the spin-singlet P-wave state $[h_{c,b}(^1P_1)]$ is extremely difficult to produce in e^+e^- annihilation, because it has $J^{PC} = 1^{+-}$. Such a state in the charmonium family was first discovered in $p\bar{p}$ annihilation by the E760 Collaboration at Fermilab in 1992 (almost 20 years after the discovery of charm quarks) with the subsequent decay $h_c \to J/\psi\pi^0$ [1]:

$$\begin{aligned} \Gamma(h_c) &\sim 800 \ \text{keV}, \\ \Gamma(h_c \to J/\psi + \pi^0) &\sim 1 \ \text{keV}, \\ \Gamma(h_c \to J/\psi + \pi\pi) &< 0.18 \ \Gamma(h_c \to J/\psi + \pi^0). \end{aligned} \tag{1}$$

The data support Voloshin's approach [2] instead of the Kuang-Yan model [3], as discussed in Refs. [1] and [4]. Another spin-singlet S-wave state η_c' can be reached in e^+e^- annhilation via a spin-flip radiative decay $\psi' \to \eta_c' + \gamma$. However, this state has not been confirmed yet. Various potential models predict $m(\eta_c')$ to be around $\approx$ 3.6 GeV.

Since BES (Beijing Spectrometer) is planning to search for h_c and η_c' in the decay channels $\psi' \to \eta_c'\gamma$, $h_c\pi^0$, and $h_c\pi\pi$, it is worthwhile to estimate the branching ratios of these decays in the framework of QED and QCD multipole expansions, and compare the results with predictions of the Kuang-Yan model as well as with the data from BES and other experiments. This is precisely the purpose of this Brief Report.

First, we discuss $E1$ and $M1$ radiative transitions of charmonium states, $\psi' \to \chi_{cJ} + \gamma$, $\chi_{cJ} \to J/\psi + \gamma$, and $\psi' \to \eta_c + \gamma$, in order to extract various matrix elements such as $\langle 1P|r|nS\rangle$ with $n = 1, 2$ and $\langle 1S|r^2|2S\rangle$ from the experimental data. These matrix elements enter various hadronic transitions between charmonium states, which will be considered later in this work. We also estimate the branching ratio for $\psi' \to \eta_c' + \gamma$. After discussing radiative transitions of charmonium states, we briefly recapitulate how to describe hadronic transitions between heavy quarkonia in QCD multipole expansions and make a simple approximation that ignores the nonlocality in time because of the color-octet intermediate state between subsequent emissions of gluons. One can estimate the size of the color-octet Green's function $[G_8(m_{\psi'})]$ from the measured decay rate for $\psi' \to J/\psi + \pi\pi$ (or η). For this purpose, one has to use the matrix element $\langle 1S|r^2|2S\rangle$ determined from $\psi' \to \eta_c + \gamma$. One can use this G_8 to compute other decays such as $h_c \to J/\psi + \pi^0$(or $\pi\pi$) and $\psi' \to h_c + \pi^0$ (or $\pi\pi$). Our results are summarized at the end.

It should be emphasized that one cannot make precise predictions for decay rates of hadronic transitions between heavy quarkonia because of our ignorance of color confinement in QCD. Any attempts to estimate such quantities are admittedly model dependent. The Kuang-Yan model takes into account the nonlocality of G_8 in time, using the string excitations near the flavor threshold. Our approximation will be opposite to the Kuang-Yan model in the sense that we ignore the nonlocality of G_8 in time, and regard G_8 as a constant. Predictions of both apporaches are a kind of rough estimate, and the actuality may lie somewhere between the two approaches. In this sense, our predictions can be considered as complementary to the Kuang-Yan model.

In order to describe hadronic transitions between heavy quarkonia, we have to know such matrix elements as $\langle nS|r|1P\rangle$ and $\langle 1S|r^2|2S\rangle$. In previous estimations, these quantities were replaced by the radius of the initial or the final quarkonium. However, such estimates can be improved, since these matrix elements are also relevant to electric dipole and spin-flip radiative transitions bewteen heavy quarkonia. In this work, we work in the nonrelativistic quantum mechanics, neglecting the relativistic corrections, LS coupling, and hyperfine splittings. Therefore, the results may be affected by a factor of ~ 2 because of such neglected effects.

Let us first consider electric dipole transitions, whose decay rates are described by

$$\Gamma(E1) = \frac{4}{27}\alpha \ e_q^2 \ \omega^3 \ S_{if} \ (2J_f + 1) \ |\langle nS|r|mP\rangle|^2. \tag{2}$$

Here, J_f is the spin of the final quarkonium, and ω is the energy of the emitted photon. $S_{if} = 3$ for $h_c(^1P_1) \to \eta_c + \gamma$ and $\eta' \to h_c(^1P_1) + \gamma$, whereas $S_{if} = 1$ for other transitions.

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Comparing (2) and the measured decay rates of $\chi_{cJ}(1P) \to J/\psi + \gamma$ and $\psi' \to \chi_{cJ}(1P) + \gamma$, we get

$$|\langle 1S|r|1P\rangle_\psi| \approx 1.58 \text{ GeV}^{-1} = 0.3 \text{ fm}, \tag{3}$$

$$|\langle 2S|r|1P\rangle_\psi| \approx 1.97 \text{ GeV}^{-1} = 0.4 \text{ fm}, \tag{4}$$

where we have taken the weighted average over $J = 0, 1, 2$ states. Using $m(h_c) = 3526$ MeV and taking $m(\eta_c') = 3600$ MeV as a tentative value, we get

$$\Gamma(h_c \to \eta_c + \gamma) \approx 460 \text{ keV}, \tag{5}$$

$$\Gamma(\eta_c' \to h_c + \gamma) \approx 6.5 \text{ keV}. \tag{6}$$

Since $|\langle nS|r|1P\rangle_\psi|$'s with $n = 1, 2$ are determined from the experimental data on $\Gamma(\chi_{cJ} \to J/\psi + \gamma)$ and $\Gamma(\psi' \to \chi_{cJ}(1P) + \gamma)$, our predictions (5) and (6) are independent of specific potential models.

Next, we consider a magnetic dipole radiative transition $n^3S_1 \to m^1S_0 + \gamma$. The decay rate for such a spin-flip radiative decay is given by

$$\Gamma(\psi' \to \eta_c + \gamma) = \frac{4}{3}\, \alpha Q_c^2\, \frac{\omega^3}{m_c^2}\, |\langle 1S|j_0(\omega r/2)|2S\rangle|^2 \tag{7}$$

$$\approx \frac{4}{3}\, \alpha Q_c^2\, \frac{\omega^3}{m_c^2} \left|\left\langle 1S \left| \frac{\omega^2 r^2}{24} \right| 2S \right\rangle\right|^2. \tag{8}$$

In (7), $j_0(x)$ is the zeroth-order spherical Bessel function, $j_0(x) = \sin x/x$. We have used the long-wavelength approximation ($\omega r \ll 1$) in (8). From the measured decay rate $\Gamma(\psi' \to \eta_c + \gamma) = 0.78 \pm 0.19$ keV, one can extract

$$|\langle 1S|r^2|2S\rangle| \approx 2.55 \text{ GeV}^{-2}, \tag{9}$$

which satisfies the bound (20) derived from (19). The main point of this work is the following: one can get the matrix element $|\langle 1S|r^2|2S\rangle|$ from $\psi' \to \eta_c + \gamma$. This matrix element will be used later in order to extract G_8 from $\psi' \to J/\psi + \eta$, as described in the following.

Now, let us consider the possibilty of finding the η_c' state via a radiative transition $\psi' \to \eta_c' + \gamma$. Assuming $m(\eta_c') = 3.60$ GeV, the energy of the emitted photon is $\omega = 84$ MeV, and the matrix element in (7) can be approximated as 1. We have $\Gamma(\psi' \to \eta_c' + \gamma) \approx 0.94$ keV or 3.4×10^{-3} in the branching ratio. Thus, one would get $\sim 340\eta_c'$'s among $10^5\psi'$'s. The subsequent decay of η_c' is mainly hadronic through annihilation into two gluons: $\Gamma(\eta_c' \to 2g) \approx 1-4$ MeV. The decay rate of the spin-flip radiative transition $\eta_c' \to J/\psi + \gamma$ is about 0.26 keV, using the matrix element $|\langle 1S|r^2|2S\rangle|$, which was determined above in Eq. (9).

Let us now discuss our main subject, the hadronic transitions between heavy quarkonium states in QCD multipole expansion [2, 3, 5]. The basic formulas can be found in previous works, and we write down the relevant equations only here. The decay rates for spin-flip transitions such as $1^1P_1 \to 1^3S_1 + \pi^0$ and $n^3S_1 \to 1^1P_1 + \pi^0$ are given by [2]

$$\Gamma(1^1P_1 \to 1^3S_1 + \pi^0) = \Gamma(n^3S_1 \to 1^1P_1 + \pi^0) = \frac{1}{2\pi}\, (A_0\, I_{SP})^2\, |\vec{p}_\pi|. \tag{10}$$

Here, $A_0 = 1.7 \times 10^{-3}$ GeV^3 is one-third of the matrix element $\langle \pi^0|\pi\alpha_s \mathbf{E}^a(0) \cdot \mathbf{B}^a(0)|0\rangle$, and the matrix element I_{SP} is defined as

$$I_{SP} = -\frac{2\sqrt{3}}{9m_Q}\, \langle\, nS \mid G_8(E)\, r + r\, G_8(E) \mid 1P\, \rangle, \tag{11}$$

where $G_8(E)$ is the Green's functions for the color-octet $Q\bar{Q}$ intermediate states:

$$G_8(E) = \sum_k \frac{|\, k\, \rangle\, \langle\, k\, |}{E_k - E}. \tag{12}$$

To get the absolute decay rate for hadronic transitions between quarkonia, it is imperative to know more about "I_{SP}" defined in (11). However, it lies beyond our ability to calculate I_{SP} from the first principle in QCD because of our ignorance of the confinement in QCD. Therefore, we have to make some reasonable approximations. We take the opposite limit to the Kuang-Yan approach; namely, we ignore the nonlocality in time coming from the color-octet Green's function $G_8(E)$. We simply assume that $G_8(E)$ is a constant with the dimension of inverse mass as in Ref. [4], since this assumption does not lead to any contradictions to exsiting data:

$$G_8(E) = G_8 = \text{const}. \tag{13}$$

Under this assumption, we have

$$I_{SP} = -\frac{4\sqrt{3}}{9m_Q}\, G_8\, \langle nS|r|1P\rangle, \tag{14}$$

and one can use the information on the matrix elements on r, Eqs. (3) and (4).

Taking $m_c = 1.65$ GeV and using (3) and (4), we get

$$\Gamma(h_c(1P) \to J/\psi + \pi^0) = 0.095 \left(\frac{1.65}{m_c(\text{GeV})}\right)^2 |G_8(\text{GeV}^{-1})|^2 \text{ keV}, \tag{15}$$

$$\Gamma(\psi' \to h_c(1P) + \pi^0) = 0.032 \left(\frac{1.65}{m_c(\text{GeV})}\right)^2 |G_8(\text{GeV}^{-1})|^2 \text{ keV}. \tag{16}$$

Note that (15) is smaller than the measured value (1) by an order of magnitude if we assume $G_8 \sim 1$ GeV^{-1}, as in Ref. [6].

In order to get informations on $G_8(E)$, let us consider another hadronic transition, $\psi' \to J/\psi + \eta$, which occur through $E1-E1$ and $E1-M2$ transitions in QCD multipole expansion [3, 5] Its amplitude is proportional to the matrix element

$$I_{SS} = \frac{2}{9}\, \langle\, mS \mid r_i\, G(E)\, r_i \mid nS\, \rangle, \tag{17}$$

which is simplified to the following under the assumption (13) on $G_8(E)$:

$$I_{SS} = \frac{2}{9}\, G_8\, \langle mS|r^2|nS\rangle. \tag{18}$$

Therefore, once $\langle 2S|r^2|1S\rangle$ is known, the absolute decay rates for these decays can be readily obtained. However, a potential model calculation of this matrix element is not available in contrast with dipole matrix elements $\langle f|\,r|i\rangle$ and the mean square radius of a quarkonium $\langle i|r^2|i\rangle$. In Ref. [4], I have used a quantum-mechanical sum rule to derive

$$|\langle 1S \mid r^2 \mid nS\rangle|^2 < \frac{4}{m_Q}\,\frac{|\langle 1S \mid r^2 \mid 1S\rangle|}{(E_{nS}-E_{1S})}. \tag{19}$$

From $|\langle 1S|r^2|1S\rangle_\psi| = 4\ \ \mathrm{GeV}^{-2}$, the bound (19) was found to be

$$|\langle 1S|r^2|2S\rangle_\psi|^2 < 16.5\ \ \mathrm{GeV}^{-4}. \tag{20}$$

By considering $\psi' \to J/\psi + \eta$ and using (20), one obtains a lower bound on G_8 [4]:

$$|G_8|^2 > 10.4\ \mathrm{GeV}^{-2}. \tag{21}$$

Assuming $G_8(m_{\psi'}) \approx G_8(m_{h_c})$, we get the following lower bound on $h_c \to J/\psi + \pi^0$ from (15):

$$\Gamma(h_c \to J/\psi + \pi^0) > 1.0\ \mathrm{keV}. \tag{22}$$

This is consistent with the E760 data (1), and moreover, very close to it. Of course, this would be uncertain by a factor of ~ 2, depending on the choice of m_c. (In Ref. [4], the mass of h_c was assumed to be 3.51 GeV, and thus numerical results obtained there are slightly different from the results in the present work.)

One of the main points of this work is that one can actually *improve* the lower bound (22) obtained in Ref. [4], using the estimate on $\langle 1S|r^2|2S\rangle$ obtained from $\psi' \to \eta_c' + \gamma$. From (9) and $\Gamma(\psi' \to J/\psi + \eta) = (7.51 \pm 1.41)$ keV, one obtains

$$|G_8|^2 \approx 26.4\ \ \mathrm{GeV}^{-2}. \tag{23}$$

Using this $|G_8|^2$ in (17) and (18), we finally get

$$\Gamma(h_c \to J/\psi + \pi^0) \approx 2.5\ \ \mathrm{keV}, \tag{24}$$

$$\Gamma(\psi' \to h_c + \pi^0) \approx 0.84\ \ \mathrm{keV}. \tag{25}$$

Note that our prediction for $B(\psi' \to h_c + \pi^0) = 0.3\%$ is very close to the current upper limit, and should be checked in the near future at the Beijing Electron-Positron Collider (BEPC). Also, these two predictions in our approach are remarkably in accord with predictions by the Kuang-Yan model with $\alpha_M \simeq 10\alpha_E$. However, we have only one gauge coupling (namely, $\alpha_M = \alpha_E$) in our model, and this agreement of two approaches must be regarded as accidental.

Another hadronic decay mode $1\,{}^1P_1 \to 1\,{}^3S_1 + \pi\pi$ does not receive any contribution from the trace of the energy-momentum tensor in QCD, and is not enhanced over $1\,{}^1P_1 \to 1\,{}^3S_1 + \pi^0$ [2]. (There is no analogous decay $\psi' \to h_c + \pi\pi$ because the phase space is not available.) Using the result of Ref. [2], we predict

$$\frac{\Gamma(h_c \to J/\psi + \pi\pi)}{\Gamma(h_c \to J/\psi + \pi^0)} \approx \frac{\lambda^2}{30} \approx 0.16. \tag{26}$$

Here, λ measures the gluonic contributions to the energy-momentum of a pion. This is consistent with the E760 data, and very close to the current upper limit for $\lambda \sim 2$. Therefore, $h_c \to J/\psi\pi\pi$ may be observed in the near future.

A remark is in order. As mentioned before, we have made an approximation treating the color-octet Green's function $G_8(E)$ as a constant and determined it from the measured decay rate for $\psi' \to J/\psi + \eta$. One can actually check the consistency of this approximation, making the following observation. The ratio

$$\frac{\Gamma(h_c \to J/\psi + \pi^0)}{\Gamma(h_c \to \eta_c + \gamma)} = 2.1 \times 10^{-4}\ |G_8(\mathrm{GeV}^{-1})|^2 \tag{27}$$

depends on $|G_8^2|$, which can be used as an independent determination of $|G_8^2|$ from the measurements. Another ratio between $\Gamma(\psi' \to h_c + \pi^0)$ and $\Gamma(\psi' \to \chi_{cJ}(1P) + \gamma)$ can be useful as well. Thus, our assumption on G_8 is simple, does lead to predictions on various hadronic transitions between heavy quarkonia, and can be checked through the measurement of the ratio (27).

In conclusion, productions and decays of η_c' and h_c in the e^+e^- annihilations through ψ' decays are considered in the framework of QCD multipole expansion, assuming the Green's function for the color-octet states is a constant. Using the matrix element $|\langle 1S|r^2|2S\rangle|$ extracted from the spin-flip radiative transition $\psi' \to \eta_c + \gamma$, we could estimate the size of $|G_8|^2$ by fitting $\psi' \to J/\psi + \eta$. This leads to predictions of absolute decay rates, $B(\psi' \to \eta_c' + \gamma) \approx 0.34\%$, $\Gamma(\eta_c' \to J/\psi + \gamma) \approx 0.26$ keV, and (24) and (25), which improves the lower bounds on the decay rates for the same processes obtained in Ref. [4]. We also have suggested that our assumption (13) could be checked by measuring the ratio (27). We hope high statistics experiments at the BEPC and other places searching for η_c' and h_c can test predictions made in this work. Having $\sim 10^5\psi'$ decays would be enough to get the spin-singlet η_c' and h_c states and their subsequent decay properties.

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[1] T.A. Armstrong, Phys. Rev. Lett. **69**, 2337 (1992).
[2] M.B. Voloshin, Sov. J. Nucl. Phys. **43**, 1011 (1986).
[3] T.M. Yan, Phys. Rev. D **22**, 1652 (1980); Yu-Ping Kuang and T.M. Yan, *ibid.* **24**, 2874 (1981); Yu-Ping Kuang, San Fu Tuan, and T.M. Yan, *ibid.* **37**, 1210 (1988).
[4] P. Ko, Phys. Rev. D **47**, 2837 (1993).
[5] M.B. Voloshin and V.I. Zakharov, Phys. Rev. Lett. **45**, 688 (1980).
[6] M.B. Voloshin and V.I. Zakharov, DESY Report No. F15-80/03, Hamburg, 1980 (unpublished).