PHYSICAL REVIEW D VOLUME 52, NUMBER 3 1 AUGUST 1995

Electroweak symmetry breaking by vectorlike fermion condensation with small S and T parameters

Nobuhiro Maekawa
Department of Physics, Tohoku University, Sendai 980, Japan
(Received 21 February 1995)

In order to build realistic models in which electroweak symmetry is dynamically broken we should explain the smallness of the Peskin-Takeuchi S and T parameters. In accordance with the decoupling theorem, these parameters must be suppressed by $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant masses. From this fact we can expect that if fermions with large $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant masses undergo condensation and break electroweak symmetry, the S and T parameters can be small. It is interesting that not only the S but also the T parameter can become small even if there exists a large isospin violation in fermion condensation. In this paper we examine the possibility that, by the strong four-Fermi interaction, massive vectorlike fields undergo condensation and break electroweak symmetry. The model becomes almost the same as the standard model at a low energy scale and predicts a heavy Higgs boson. Moreover, we discuss a model in which the four-Fermi interaction can be induced by massive gauge boson exchange. In this model, the masses of ordinary matter fermions (quark and lepton) are enhanced.

PACS number(s): 12.60.Nz, 12.15.Lk

I. INTRODUCTION

Dynamical electroweak symmetry breaking is one of the most attractive solutions to the naturalness problem in the Higgs sector [1]. Unfortunately, however, it is hard to build realistic models because of some difficulties. One of them is in regards to the Peskin-Takeuchi S and T parameters, which are defined in terms of the new physics' contribution to the vacuum polarization $\Pi_{XY}^{\rm new}(q^2)$ $(X,Y=1,2,3,Q)$:

$$S \equiv 16\pi \frac{d^2}{dq^2}\left[\Pi_{33}^{\rm new}-\Pi_{3Q}^{\rm new}\right]\Big|_{q^2=0},$$
$$T \equiv \frac{4\pi}{\sin^2\theta_W m_W^2}\left[\Pi_{11}^{\rm new}(0)-\Pi_{33}^{\rm new}(0)\right]$$
$$\sim \frac{1}{\alpha}(\rho-\rho_{\rm SM}),$$

where we have adopted the notation of Peskin and Takeuchi [2], and m_W and θ_W are the mass of the W boson and the Weinberg angle, respectively. Recent fits to these parameters indicate that they are small ($S\sim -0.12\pm 0.20$, $T\sim 0.32\pm 0.20$ [3] for top quark mass $m_t=175$ GeV and Higgs boson mass $m_H=1000$ GeV). The smallness of these parameters severely constrains new physics. For example, for N generic left-handed doublets $(U,D)_L$ with right-handed singlets U_R and D_R, these parameters can be calculated as

$$S=\frac{N}{6\pi}\left[1-2Y\ln\left(\frac{m_U^2}{m_D^2}\right)\right], \tag{1}$$

$$T=\frac{N}{16\pi\sin^2\theta_W m_W^2}\left[m_U^2+m_D^2-\frac{2m_U^2m_D^2}{m_U^2-m_D^2}\ln\left(\frac{m_U^2}{m_D^2}\right)\right]$$
$$\geq 0, \tag{2}$$

in which Y is the hypercharge of the doublets. Only when isospin symmetry is not broken ($m_U=m_D$), does $T=0$ in Eq. (2). If $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is mainly broken by these fermions' condensation without the isospin symmetry, the typical value of the T parameter is $\gtrsim 10$ because these fermions' masses are around 1 TeV. On the other hand, the experimental value is almost zero, or at least less than 1. Therefore we should require that these doublet fermion masses must be degenerate. This degeneracy can be naturally understood by introducing the so-called custodial symmetry, which ordinary technicolor (TC) theories have. Even in this case, isospin violation in nature should be explained by introducing other physics with a large isospin violation such as extended TC (ETC), which largely contribute the T parameter [4], especially in walking TC models [5]. Moreover, the smallness of the S parameter severely constrains the dynamical electroweak symmetry-breaking scenario. If custodial symmetry exists in order to realize a small T parameter, there exists a positive contribution to the S parameter. The smallness of the S parameter means that not so many $\mathrm{SU}(2)_L$ doublets are allowed. Since ordinary TC theory usually has a lot of $\mathrm{SU}(2)_L$ doublet technifermions, it is difficult to understand the smallness of the S parameter by TC models.

Top-quark condensation [6–8] is attractive as a point that no new particle exists. Unfortunately, however, it is not a solution for the naturalness problem in the Higgs sector. This is caused by the smallness of the T parameter (strictly speaking, the $\Delta\rho$ parameter). In the top-quark condensation model, the lower the composite scale is, the heavier the top quark is. If we take the composite scale around 1 TeV in order to avoid fine-tuning, the mass of top quark becomes more than 600 GeV, which is inconsistent with the smallness of the T parameter. In order to avoid the naturalness problem, one may introduce fourth generation fermion condensation. In this case, since the masses of fourth up-type quark and down-type quark should be degenerate, the constraint from the S parameter may become severe.

In this paper we analyze a mechanism which was proposed by the author [10] in order to reduce S and T parameters. It is interesting that not only S but also T parameters can be made small even if large isospin violation exists. The point is simple and as follows. The S and T parameters must be suppressed by $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant masses M because of the decoupling theorem [9] when M are larger than the weak scale. From this fact, we can expect that S and T parameters can be made small if massive vectorlike fields can undergo condensation and break $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ [10–14].

The plan of this paper is as follows. After this introduction, first we discuss the S and T parameters. We will see that these parameters are suppressed by $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant masses and try to answer how large invariant masses are needed for small S and T parameters. Second, we discuss the condensation of massive vectorlike fermions by a four-Fermi interaction, and show that these models predict a heavy Higgs particle in the leading N approximation. Third we make some models in which this mechanism can work. Finally we discuss fine-tuning, which appears in order to realize this mechanism.

II. S AND T PARAMETERS

The Peskin-Takeuchi S and T parameters are so-called "nondecoupling" parameters. Actually particles with large $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$-breaking masses (for example, top quark) are not decoupled in a sense. On the other hand, particles with large $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant masses [for example, supersymmetry (SUSY) particles or heavy particles in grand unified theories (GUT's)] must be decoupled because of the decoupling theorem [9]. When the condensation fermions have large breaking masses around 1 TeV, how large an invariant mass M is needed for small S and T parameters? In order to answer this question, we would like to estimate the S and T parameters in a theory with massive vectorlike fields [15, 16]. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &\bar{Q}(iD_\mu\gamma^\mu + M_Q)Q + \bar{U}(iD_\mu\gamma^\mu + M_U)U + \bar{D}(iD_\mu\gamma^\mu + M_D)D\\ &+(y_U\bar{Q}_L U_R\phi + y'_U\bar{Q}_R U_L\phi + y_D\bar{Q}_L D_R\tilde{\phi} + y'_D\bar{Q}_R U_L\tilde{\phi} + \mathrm{H.c.}).\end{aligned}$$

Here $Q=(Q^U,Q^D)$, U and D are Dirac fields which have vectorlike couplings to $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge bosons as $(2,Y)$, $(1,Y+1/2)$ and $(1,Y-1/2)$ representations, respectively, and D_μ are covariant derivatives. The suffix L and R represent the chirality, M_Q, M_U, and M_D are gauge invariant masses, and y_U, y'_U, y_D, and y'_D are Yukawa couplings. After $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is broken by the vacuum expectation value of the Higgs field $\langle\phi\rangle = (v,0)$ ($\tilde{\phi}$ is the charge conjugate field of the Higgs field ϕ, which will be regarded as a composite field later), the fermion mass part of the Lagrangian becomes

$$\begin{aligned}\mathcal{L}_M &= (\bar{Q}^U_L,\bar{U}_L)\begin{pmatrix} M_Q & m_U\\ m'_U & M_U\end{pmatrix}\begin{pmatrix} Q^U_R\\ U_R\end{pmatrix} + (\bar{Q}^D_L,\bar{D}_L)\begin{pmatrix} M_Q & m_D\\ m'_D & M_D\end{pmatrix}\begin{pmatrix} Q^D_R\\ D_R\end{pmatrix} + \mathrm{H.c.}\\ &= (\bar{U}_{L1},\bar{U}_{L2})\begin{pmatrix} m_{U1} & 0\\ 0 & m_{U2}\end{pmatrix}\begin{pmatrix} U_{R1}\\ U_{R2}\end{pmatrix} + (\bar{D}_{L1},\bar{D}_{L2})\begin{pmatrix} m_{D1} & 0\\ 0 & m_{D2}\end{pmatrix}\begin{pmatrix} D_{R1}\\ D_{R2}\end{pmatrix} + \mathrm{H.c.},\end{aligned} \tag{3}$$

in which

$$\begin{pmatrix} U_{(L,R)1}\\ U_{(L,R)2}\end{pmatrix} = V_{U(L,R)}\begin{pmatrix} Q^U_{(L,R)}\\ U_{(L,R)}\end{pmatrix},$$

$$\begin{pmatrix} D_{(L,R)1}\\ D_{(L,R)2}\end{pmatrix} = V_{D(L,R)}\begin{pmatrix} Q^D_{(L,R)}\\ D_{(L,R)}\end{pmatrix},$$

and $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$-breaking masses $m_U = y_U v$, $m'_U = y'_U v$, $m_D = y_D v$, and $m'_D = y_D v$. Here $V_{(U,D)(L,R)}$ are unitary matrices. The S and T parameters can be estimated via the fermion loops (see Fig. 1) [17]. The general expression is presented in Appendix A. Here, only for simplicity, we take $M \equiv M_Q = M_U = M_D \gg m \equiv m_U$ and $m'_U = m_D = m'_D = 0$ ($m_D = m'_D = 0$ means that the isospin in the fermion condensation is almost maximally violated). In this case, we can use a perturbation in terms of the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$-breaking mass m (see Fig. 2). The results are

$$S = \frac{2N}{15\pi}\left(\frac{m}{M}\right)^2 + O\left(\left(\frac{m}{M}\right)^4\right),$$

$$T = \frac{13Nm^2}{480\pi\sin^2\theta_W m_W^2}\left[\left(\frac{m}{M}\right)^2 + O\left(\left(\frac{m}{M}\right)^4\right)\right].$$

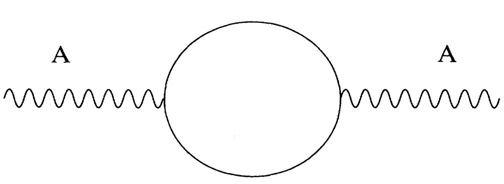

FIG. 1. S and T parameters are estimated by these Feynman diagrams.

We are sure that the decoupling theorem works. For example, if we take m =1 TeV and M = 10 TeV, then $S\sim 0.0004N$ and $T\sim 0.06N$. Notice that the parameter T is fairly small in spite of such a large isospin violation (m_U = 1 TeV and $m_D = m'_D = 0$) [18].

Can this situation be realized in dynamical models?

This situation is realized in models in which the Georgi-Kaplan mechanism [13] works. They discussed the misalignment of vectorlike field condensation in or-

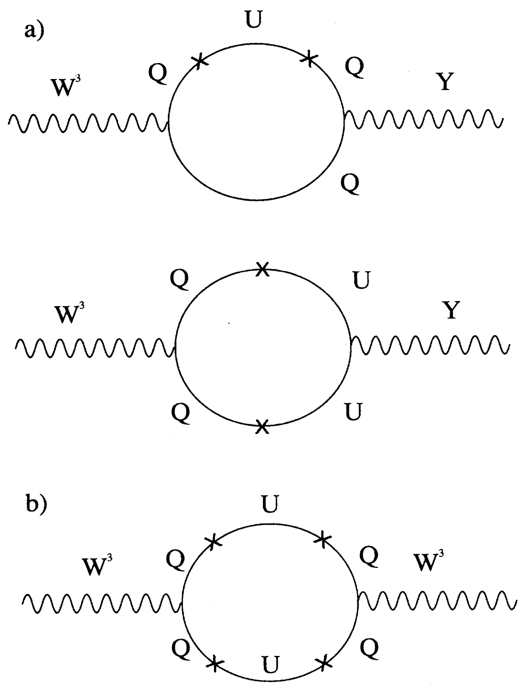

FIG. 2. In the limit that $M \gg m$, we can estimate the S parameter (a) and T parameter (b) by calculating the above Feynman diagrams. You can easily find that the other diagrams with lower order of m are cancelled or zero.

der to avoid the flavor-hanging neutral current (FCNC) problem. This possibility is attractive, but in this paper, we would like to discuss another possibility that heavy particles with $SU(2)_L \times U(1)_Y$ invariant masses undergo condensation and break electroweak symmetry.

III. CONDENSATION OF MASSIVE VECTORLIKE FIELDS BY FOUR-FERMI INTERACTION

Is it possible that such heavy particles ($\sim$ 10 TeV) undergo condensation and the condensation scale becomes the electroweak scale? If it is possible, the situation discussed in the previous section is realized. In this section we discuss the massive vectorlike field condensation by four-Fermi interaction [19]. Here we adopt the following Lagrangian with a four-Fermi coupling G/N:

$$\mathcal{L}_4 = \bar{Q}(iD^\mu\gamma_\mu - M_Q)Q + \bar{U}(iD^\mu\gamma_\mu - M_U)U + \frac{G}{N}(\bar{Q}_L U_R)(\bar{U}_R Q_L), \tag{4}$$

where we neglected the down-type fermion (D) and every four-Fermi interaction except that in Eq. (4) for simplicity. Here we only assume the chiral structure of the four-Fermi interaction, which we will discuss later [21, 22]. The Lagrangian (4) is rewritten by using auxiliary field method as

$$\mathcal{L}_Y = \bar{Q}(iD^\mu\gamma_\mu - M_Q)Q + \bar{U}(iD^\mu\gamma_\mu - M_U) \times U - \frac{N}{G}\phi^\dagger\phi + (\bar{Q}_L U_R \phi + \text{H.c.}).$$

If we integrate out these fermion fields, we get the effective action of the Higgs field ϕ and gauge fields A_μ:

$$S_\phi = N\left(\int d^4x\left[-\frac{1}{G}\phi^\dagger\phi + \mathcal{L}_{\mathbf{gauge}}\right] - i\ln\mathrm{Det}\begin{bmatrix} iD_\mu\gamma^\mu - M_Q & \frac{1+\gamma_5}{2}\phi \\ \frac{1-\gamma_5}{2}\phi^\dagger & iD_\mu\gamma^\mu - M_U\end{bmatrix}\right). \tag{5}$$

The effective potential of the Higgs field in the leading N approximation $V[\langle\phi\rangle = (v,0)^T] = S_\phi[\phi(x) = (v,0)^T, A_\mu(x) = 0]/\int d^4x$ becomes

$$V = \frac{N}{G}v^2 - \frac{N}{8\pi^2}I + \text{const} \sim \left\{ \frac{N}{G}v^2(v\to\infty),\ \frac{Nv^2}{G}\left(1 - \frac{f(x_Q,x_U)G\Lambda^2}{8\pi^2}\right) \quad (v \to 0), \right. \tag{6}$$

$$I = \frac{\Lambda^4}{2}\left[\ln(1+2\alpha+x_Q x_U) + 2\alpha - (\alpha+\beta)^2\ln\frac{1+\alpha+\beta}{\alpha+\beta} - (\alpha-\beta)^2\ln\frac{1+\alpha-\beta}{\alpha-\beta}\right], \tag{7}$$

$$x_{(Q,U)} = \frac{M^2_{(Q,U)}}{\Lambda^2},$$

$$\alpha = \frac{1}{2}(x_Q + x_U + v^2/\Lambda^2), \quad \beta = \sqrt{\alpha^2 - x_Q x_U},$$

$$f(x,y) = \frac{1}{2(1+x)(1+y)} + \frac{1}{2} - \left\{\frac{x^2}{2(x-y)}\left[2\ln\left(\frac{1+x}{x}\right) - \frac{1}{1+x}\right] + (x \leftrightarrow y)\right\},$$

with the cutoff Λ. From Eq. (6) and a fact that $0 < f(x,y) \leq 1$, we can expect that for any fixed values of M_U and M_Q, there exists a critical coupling $G_c(x_Q,x_U) = 8\pi^2/[\Lambda^2 f(x_Q,x_U)]$ (see Fig. 3). If the four-Fermi coupling G is smaller than G_c, the electroweak symmetry is not broken. On the other hand, when $G > G_c$, the electroweak symmetry is spontaneously broken (see Fig. 4). On the contrary, for any fixed value of G greater than $8\pi^2/\Lambda^2$, the critical line exists in the (M_Q, M_U) plane, at which v drops to zero. This is intuitively understandable. If the interaction is so strong that the binding energy becomes larger than sum of the bare masses, the

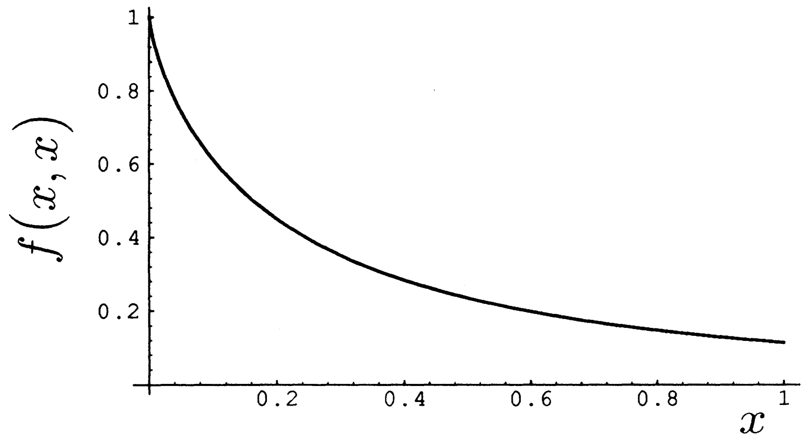

FIG. 3. The inverse of critical coupling $G_c(0,0)/\ G_c(x,x) = f(x,x)$ with $x = M^2/\Lambda^2$. You will easily find that the larger M requires the stronger four-Fermi interaction.

symmetric vacuum becomes unstable. Therefore larger invariant masses need a stronger four-Fermi interaction for dynamical electroweak symmetry breaking.

The breaking mass of the fermion m_U and the Higgs boson mass m_H can be roughly estimated by using the W boson mass m_W as follows. If the weak scale Λ_{weak} is much smaller than the invariant mass scale M and the cutoff Λ, the induced effective Lagrangian in the leading N approximation will be calculated via fermion loops as the form

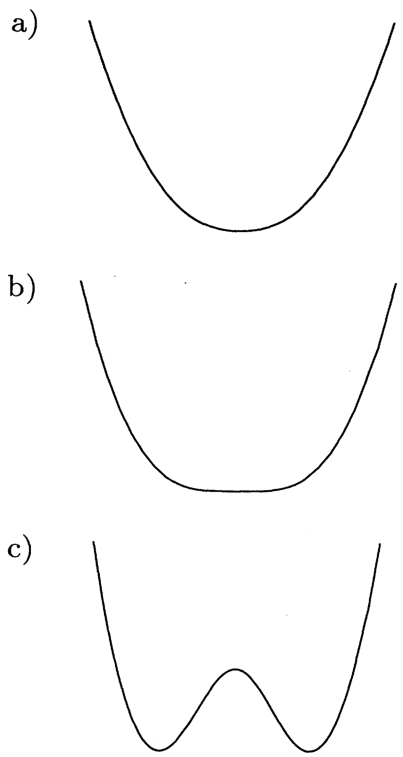

FIG. 4. The potential in the cases (a) $G < G_c$, (b) $G = G_c$, and (c) $G > G_c$. These graphs imply that the transition is second order.

$$\begin{aligned}\mathcal{L}_{\text{eff}} = &\bar{Q}(iD^\mu\gamma_\mu - M_Q)Q + \bar{U}(iD^\mu\gamma_\mu - M_U)U \qquad (8)\\ &+Z_\phi|D_\mu\phi|^2 - m_\phi^2\phi^\dagger\phi - \frac{\lambda}{2}(\phi^\dagger\phi)^2\\ &+(\bar{Q}_L U_R\phi + \text{H.c.}) + \mathcal{L}_{\text{gauge}}, \qquad (9)\end{aligned}$$

where

$$Z_\phi = \frac{N}{16\pi^2}\left(\ln\frac{1+x}{x} - \frac{1}{1+x} - \frac{1}{3(1+x)^2}\right),$$
$$\lambda = \frac{N}{8\pi^2}\left(\ln\frac{1+x}{x} - \frac{1}{1+x} - \frac{1}{2(1+x)^2} - \frac{1}{3(1+x)^3}\right).$$

Here $x = M^2/\Lambda^2$. As implied by the above discussion, this model becomes almost the same as the standard model at the low energy scale. From the above equations, we can get the relations

$$\begin{aligned}m_U &= v,\\ m_W^2 &= \frac{1}{2}g_2^2 Z_\phi v^2, \qquad (10)\\ m_H^2 &= \frac{2}{Z_\phi}\lambda v^2.\end{aligned}$$

Therefore, we can easily find that

$$m_U^2 = \frac{2m_W^2}{g_2^2 Z_\phi} \propto \frac{1}{N} \qquad (11)$$
$$m_H^2 = \frac{4\lambda m_W^2}{g_2^2 Z_\phi^2} \propto \frac{1}{N}. \qquad (12)$$

Typical values of these masses are ~ 1 TeV (see Fig. 5); namely, this model predicts one heavy Higgs boson. For example, if we take $x = 0.1$, $N = 4$, and $v = 1$ TeV, we get $m_H \sim 1.5$ TeV, $S \sim 0.0016$, and $T \sim 0.23$. You should notice that this prediction about the Higgs boson mass is caused by the leading N approximation. Actually, it will be discussed later that the prediction looks inconsistent with the triviality bound because subleading effects play important roles in the estimation of the triviality bound.

In order to suppress S and T parameters, we should take $m_U \ll M$, which requires a kind of fine-tuning (see

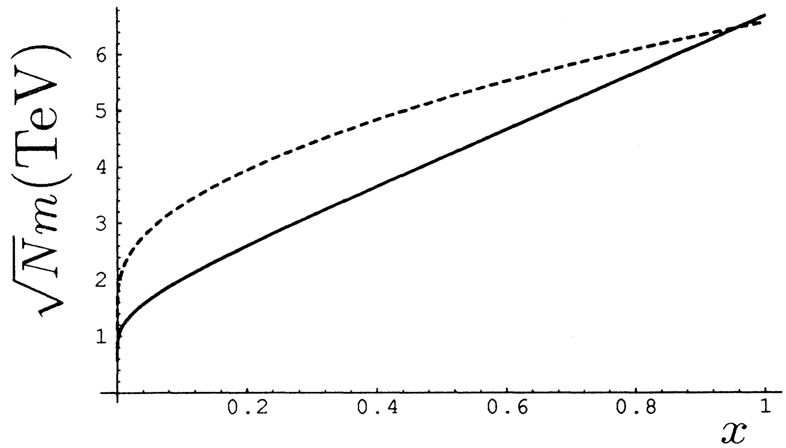

FIG. 5. $SU(2)_Y \times U(1)_Y$-breaking mass of fermion m_U (solid line) and the Higgs boson mass m_H (dashed line) with $x = M^2/\Lambda^2$. In the limit $\Lambda \to \infty$, i.e., $x \to 0$, the ratio m_H/m_U becomes 2, which is the so-called Nambu–Jona-Lasinio relation.

Fig. 6). This is not so strong a fine-tuning as top-quark condensation, and what is important here is that the condensation of the massive vectorlike fields is possible under four-Fermi interaction.

IV. MODELS

What models can realize this mechanism? If I introduce any four-Fermi interaction by hand, the top quark condensation scenario is naturally extended to the fourth family and antifamily scenario. In addition to the strong four-Fermi interaction, we introduce four-Fermi interactions instead of Yukawa terms; then we can make a dynamical model which is almost equivalent to the standard model in the low energy scale. This model is interesting because it does not need so strong a fine-tuning and because large isospin violation can be realized. What models can induce these four-Fermi interactions?

In the following, we try to induce the strong four-Fermi interaction. We would like to discuss an extended model of the one-family extended TC model. This model is interesting because the strong four-Fermi interaction can be induced by massive gauge boson exchange. The model follows. In addition to ordinary one-family technifermions, three families, and ETC gauge group $SU(N_{TC}+3)_G$, we prepare antitechnifermions and antigeneration group $SU(N_{TC})_{AG}$. Namely we introduce the anomaly free set

$$
\begin{aligned}
\mathrm{TQ},\ q &: (N_{\mathrm{TC}}+3,1,3,2,1/6)_L,\ (N_{\mathrm{TC}}+3,1,3,1,2/3)_R,\ (N_{\mathrm{TC}}+3,1,3,1,-1/3)_R,\\
\mathrm{TL},\ l &: (N_{\mathrm{TC}}+3,1,1,2,1/2)_L,\ (N_{\mathrm{TC}}+3,1,1,1,0)_R,\ (N_{\mathrm{TC}}+3,1,1,1,-1)_R,\\
\mathrm{ATQ} &: (1,N_{\mathrm{TC}},3,2,1/6)_R,\ (1,N_{\mathrm{TC}},3,1,2/3)_L,\ (1,N_{\mathrm{TC}},3,1,-1/3)_L,\\
\mathrm{ATL} &: (1,N_{\mathrm{TC}},1,2,1/2)_R,\ (1,N_{\mathrm{TC}},1,1,0)_L,\ (1,N_{\mathrm{TC}},1,1,-1)_L,
\end{aligned}
$$

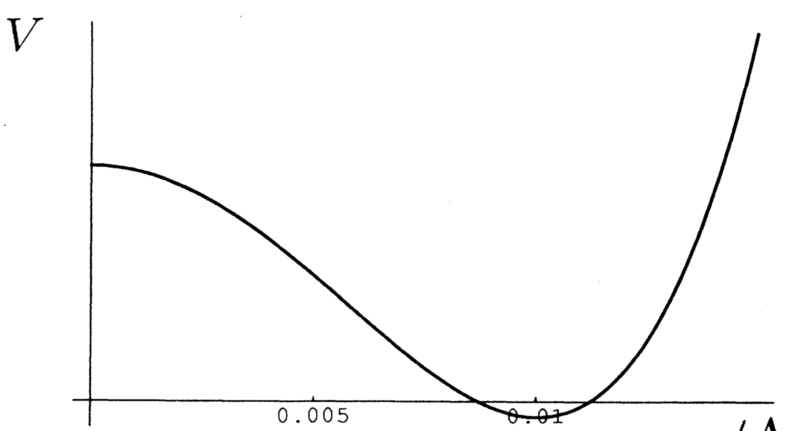

FIG. 6. The potential with fine-tuning $\hat{v} = v/\Lambda \ll 1$. Here we take $M^2 = 0.1\Lambda^2$ and $G = 1.636 G_c(0,0)$.

where these numbers are quantum numbers under gauge groups $SU(N_{TC}+3)_G \times SU(N_{TC})_{AG} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and TQ, TL, ATQ, ATL, q, and l mean techniquark, technilepton, antitechniquark, antitechnilepton, quark, and lepton, respectively. We assume that some other physics (for example, some scalar fields or other strong gauge group) induce the breaking pattern

$$\begin{aligned} & SU(N_{TC}+3)_G \times SU(N_{TC})_{AG} \times G_{SM} \\ \Lambda_1 & \\ & \to SU(N_{TC}+2)_G \times SU(N_{TC})_{AG} \times G_{SM} \\ \Lambda_2 & \\ & \to SU(N_{TC}+1)_G \times SU(N_{TC})_{AG} \times G_{SM} \\ \Lambda_3 & \\ & \to SU(N_{TC})_G \times SU(N_{TC})_{AG} \times G_{SM} \\ \Lambda & \\ & \to SU(N_{TC})_V \times G_{SM}, \end{aligned}$$

where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ and $SU(N_{TC})_V$ is a vectorlike technicolor group. Here the breaking $SU(N_{TC})_G \times SU(N_{TC})_{AG} \to SU(N_{TC})_V$ at the Λ is important for inducing the strong four-Fermi interaction. The breaking can be realized by forming the vacuum expectation value (VEV) $\langle \Phi_b^a \rangle = \mathrm{diag}(V,V,V)$ of a scalar field Φ_b^a, which transforms as $(N_{TC}, \bar{N}_{TC})$ under the groups $SU(N_{TC})_G \times SU(N_{TC})_{AG}$. (This situation is similar to the top color model introduced by Hill [8].) If we introduce Yukawa interactions to technifermions and antitechnifermions,

$$y_Q \bar{Q}_L^a Q_{Rb} \Phi_a^b + y_U \bar{U}_R^a U_{Lb} \Phi_a^b + y_D \bar{D}_R^a D_{Lb} \Phi_a^b + \text{technilepton} + \text{H.c.}, \qquad (13)$$

then technifermions have masses of yV, which are $SU(2)_L \times U(1)_Y$ invariant masses. If we take $SU(N_{TC})_G \times SU(N_{TC})_{AG}$ couplings as g_G and g_{AG}, respectively, massless $SU(N_{TC})_V$ gauge bosons V_μ^a with the gauge coupling g_V and massive vector bosons A_μ^a ("technicolorons") are defined by

$$A_\mu^a = \cos\theta A_{G\mu}^a - \sin\theta A_{AG\mu}^a \qquad (14)$$

$$V_\mu^a = \sin\theta A_{G\mu}^a + \cos\theta A_{AG\mu}^a, \qquad (15)$$

where $A_{G\mu}^a$ and $A_{AG\mu}^a$ are gauge fields of the groups $SU(N_{TC})_G$ and $SU(N_{TC})_{AG}$, respectively, and

$$\tan\theta = g_{AG}/g_G, \quad 1/g_V^2 = 1/g_G^2 + 1/g_{AG}^2. \qquad (16)$$

The mass of the massive vector fields A_μ^a is given by

$$M_A = \sqrt{g_G^2 + g_{AG}^2}\, V. \qquad (17)$$

The currents of $SU(N_{TC})_V$ and $SU(N_{TC})_A$ will be

$$J_{V\mu}^a = g_V[\bar{Q}\gamma_\mu(T^a)Q + \bar{U}\gamma_\mu(T^a)U + \bar{D}\gamma_\mu(T^a)D + \text{technilepton}], \qquad (18)$$

$$\begin{aligned} J_{A\mu}^a = {} & g_V \cot\theta[\bar{Q}_L\gamma_\mu(T^a)Q_L + \bar{U}_R\gamma_\mu(T^a)U_R + \bar{D}_R\gamma_\mu(T^a)D_R + \text{technilepton}] && (19) \\ & - g_V \tan\theta[\bar{Q}_R\gamma_\mu(T^a)Q_R + \bar{U}_L\gamma_\mu(T^a)U_L + \bar{D}_L\gamma_\mu(T^a)D_L + \text{technilepton}]. && (20) \end{aligned}$$

Suppose that $g_G \gg g_{AG}$, namely, $\cot\theta \gg 1$, which can be naturally expected because $N_{TC}+3 > N_{TC}$. Moreover, we assume that the gauge coupling g_G at the scale Λ is large enough to induce the strong four-Fermi interaction as we discussed in the previous section (if the phase transition is second order, it is possible to induce effectively the strong four-Fermi interaction by tuning the parameter g_G [20]). Note that TC interaction under the scale Λ plays little role in breaking the electroweak symmetry (namely, the composite scale of the Higgs field is Λ). Effective four-Fermi interactions $J_A^{\mu a} J_{A\mu}^a / M_A^2$ are induced by technicoloron exchange. Since $\cot\theta \gg 1$, the four-Fermi interactions between TF's becomes stronger than between TF and ATF or between ATF's. Namely, at the scale Λ, the strong four-Fermi interaction between an up-type TQ induced by the ETC interaction is

$$\frac{g_V^2 \cot^2\theta}{M_A^2} \sum_a \bar{Q}_L\gamma_\mu(T^a)Q_L \bar{U}_R\gamma^\mu(T^a)U_R. \qquad (21)$$

The above interaction is rewritten by Fierz transformation as

$$\sim -\frac{N_{TC} g_V^2 \cot^2\theta}{3M_A^2}\left[\bar{Q}_L U_R \bar{U}_R Q_L + \sum_b \bar{Q}_L(\lambda^b)U_R\bar{U}_R(\lambda^b)Q_L\right], \qquad (22)$$

in which T^a and λ^b are generators of $SU(N_{TC})$ and of $SU(3)_C$, respectively. Here we take $\mathrm{tr} T^a T^b = \delta_a^b/2$ and $\mathrm{tr}\lambda^a\lambda^b = \delta_a^b/2$. The first term in Eq. (22) is nothing but the four-Fermi interaction discussed previously. Therefore if $g_G(\Lambda)$ is so strong that the induced ETC interaction can break $SU(2)_L \times U(1)_Y$ the S and T parameters in this model can be made small when the breaking scale m_U is much smaller than the gauge invariant mass scale M. In this model, in addition to the above four-Fermi interactions, there exist strong down-type four-Fermi interactions of down-type TQ's (and of TL's):

$$\frac{g_V^2 \cot^2\theta}{M_A^2} \sum_a \bar{Q}_L\gamma_\mu(T^a)Q_L \bar{D}_R\gamma^\mu(T^a)D_R, \qquad (23)$$

$$\sim -\frac{N_{TC} g_V^2 \cot^2\theta}{3M_A^2}\left[\bar{Q}_L D_R \bar{D}_R Q_L + \sum_b \bar{Q}_L(\lambda^b)D_R\bar{D}_R(\lambda^b)Q_L\right]. \qquad (24)$$

By these four-Fermi interactions, the down-type TQ's (or TL's) may condensate. For models with fine-tuning such as this model, however, the small difference between the up and down sector can induce the large difference between the VEV's of up- and down-type TF's [23, 24], which will be understood by the previous discussions about the critical coupling. In this model, $SU(2)_R$ violation is introduced by the difference between Yukawa couplings y_U and y_D in Eq. (13) and the difference of the hypercharge. By using this criticality, this model can realize large isospin violation in ordinary fermions. Ordinary fermions (for example, top and bottom quarks) get their masses through ETC gauge boson exchange as in Fig. 7:

$$\frac{g_G(\Lambda_3)^2}{\Lambda_3^2}\bar{t}t\bar{U}U \to m_t \sim \frac{g_G(\Lambda_3)^2}{\Lambda_3^2}\langle\bar{U}U\rangle, \tag{25}$$

$$\frac{g_G(\Lambda_3)^2}{\Lambda_3^2}\bar{b}b\bar{D}D \to m_b \sim \frac{g_G(\Lambda_3)^2}{\Lambda_3^2}\langle\bar{D}D\rangle. \tag{26}$$

These vacuum expectation values are

$$\langle\bar{U}U\rangle \sim \frac{1}{(4\pi)^2}\int_0^{\Lambda^2} dk^2 \frac{k^2 m_U}{k^2+M_U^2} \sim \frac{1}{(4\pi)^2} m_U \Lambda^2, \tag{27}$$

$$\langle\bar{D}D\rangle \sim \frac{1}{(4\pi)^2}\int_0^{\Lambda^2} dk^2 \frac{k^2 m_D}{k^2+M_D^2} \sim \frac{1}{(4\pi)^2} m_D \Lambda^2. \tag{28}$$

What is important here is that the condensation of up-type TQ and down-type TQ are proportional to the up-type TQ's breaking mass m_U and the down-type one m_D, respectively, because of the chirality. Therefore, large isospin violation in TF sector causes large isospin violation in ordinary matter sector. Moreover, mass hierarchy of ordinary matters between different generations can be induced by the hierarchy of breaking scales of gauge groups as in ordinary one-family TC theory. Unfortunately, however, this model seems not to induce the Kobayashi-Maskawa matrix [25]. Therefore this model is not a realistic model. Building realistic models will be a future subject.

V. DISCUSSION

In order to realize our mechanism, we need fine-tuning. We should take the weak scale Λ_W around 1 TeV, and in order to make S and T parameters small we should take the invariant masses around 10 TeV. If we take the composite scale $\Lambda = 30$ TeV, $(\Lambda_W/\Lambda)^2 \sim 10^{-3}$ fine-tuning is needed. This fine-tuning is not so strong as in the top quark condensation scenario. Moreover, this fine-tuning enhances the masses of ordinary matter. This is because the vacuum expectation value of $\bar{U}U$ is enhanced by factor $(\Lambda/\Lambda_W)^2$. For ordinary TC models, the vacuum expectation value of technifermions $\langle\bar{U}U\rangle \sim \Lambda_W^3/(4\pi)^2$. On the other hand, for four-Fermi models, $\langle\bar{U}U\rangle \sim m_U\Lambda^2/(4\pi)^2$, as in Eq. (27). Therefore there exists $(\Lambda/\Lambda_W)^2$ enhancement. Of course such an enhancement is also good for solving the FCNC problem.

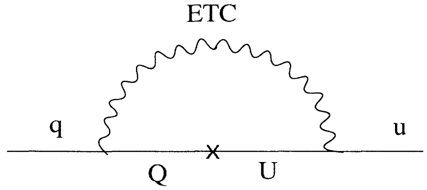

FIG. 7. A Feynman diagram contributing to the fermion masses induced by ETC gauge boson loops.

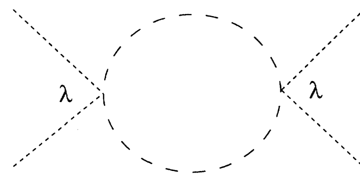

FIG. 8. A Feynman diagram contributing to the renormalization group equation of the Higgs quartic coupling λ.

Since these models are almost the same as the standard model at low energy scale, the above prediction of Higgs boson mass naively looks inconsistent with the triviality bound. The triviality bound of Higgs boson mass are derived as follows. From the renormalization group equation of quartic coupling of the Higgs field,

$$16\pi^2\frac{d\lambda}{dt} = 12\lambda^2, \tag{29}$$

it is seen that if we take a large value of the coupling, it is diverging at some scale. Here we take $t = \ln\mu$ and μ is a renormalization point. If the standard model is correct until a scale Λ, we can get the upper bound of Higgs boson mass,

$$m_H^2 < \frac{8\pi^2 v^2}{3\ln(\Lambda/m_H)}, \tag{30}$$

from the condition that the running coupling is not divergent at the scale Λ. Here we estimated the Higgs boson mass by the relation $m_H^2 = 2\lambda(\mu = m_H)v^2$. If we take the $\Lambda = 10$ TeV, the upper bound of the Higgs boson mass is 526 GeV. The triviality bound of the Higgs boson mass can be derived mainly by the $1/N$ subleading graph (see Fig. 8), in which the Higgs particle propagates. On the other hand, in this paper the estimation of the Higgs boson mass is done by taking account of only the $1/N$ leading graph. Therefore we can expect that the Higgs boson will become lighter than the value estimated in this paper if we take account of the $1/N$ subleading graph. The CERN Large Hadron Collider (LHC) may find such a heavy Higgs boson.

VI. CONCLUSION

We have seen that if massive vectorlike fields undergo condensation and break $SU(2)_L\times U(1)_Y$ S and T parameters can be made small even if there exists large isospin violation. The point is that S and T parameters must be suppressed by $SU(2)_L\times U(1)_Y$ invariant masses which vectorlike fermions can have. When the condensation is induced by strong four-Fermi interactions, we can predict a heavy Higgs boson. Though we need one fine-tuning in order to break the gauge symmetry by massive vectorlike field condensation, it is interesting that such an enhancement may solve some fermion mass problems (heavy top and FCNC).

ACKNOWLEDGMENTS

We thank T. Yanagida, M. Peskin, and M. Luty for stimulating discussions. We also thank S. Matsumoto for the information about recent S and T parameter fittings. This work was supported in part by the Japan Society for the Promotion of Science. A part of this work was done at SLAC.

APPENDIX A: CALCULATION OF S AND T PARAMETERS

In this appendix, we calculate the general expressions of S and T parameters induced by one-loop correction:

$$\begin{aligned}
S = \frac{N}{\pi} \sum_{A=U,D} \Bigg\{ & -\frac{2}{3} Y I_3^A \sum_{i=1}^{2} \sum_{\alpha=L,R} |(V_{A\alpha})_{i1}|^2 \ln m_{Ai}^2 \\
& + \frac{1}{4} \sum_{i,j=1}^{2} \chi_+(m_{Ai}, m_{Aj}) \left(\sum_{\alpha=L,R} (V_{A\alpha})_{1i} (V^\dagger_{A\alpha})_{i2} (V^\dagger_{A\alpha})_{j1} (V_{A\alpha})_{2j} \right) \\
& + \frac{1}{4} \sum_{i,j=1}^{2} \left[\frac{m_{Ai} m_{Aj}}{6} \left(\frac{1}{m_{Ai}^2} + \frac{1}{m_{Aj}^2} \right) + \chi_-(m_{Ai}, m_{Aj}) \right] \left[(V_{AL})_{1i} (V^\dagger_{AR})_{i2} (V^\dagger_{AL})_{j1} (V_{AR})_{2j} + (L \leftrightarrow R) \right] \Bigg\}, \qquad \text{(A1)}
\end{aligned}$$

$$\begin{aligned}
T = \frac{N}{16\pi \sin^2\theta_W m_W^2} \Bigg\{ & \sum_{i,j=1}^{2} \Bigg[\theta_+(m_{Ui}, m_{Dj}) \left(\sum_{\alpha=L,R} |(V_{U\alpha})_{i1}|^2 |(V_{D\alpha})_{j1}|^2 \right) \\
& + 2\theta_-(m_{Ui}, m_{Dj}) \mathrm{Re}\{ (V_{UL})_{1i} (V_{UR})^\dagger_{i1} (V_{DL})^\dagger_{j1} (V_{DR})_{1j} \} \Bigg] \\
& - \sum_{A=U,D} \Bigg[\theta_+(m_{A1}, m_{A2}) \left(\sum_{\alpha=L,R} |(V_{A\alpha})_{11}|^2 |(V_{A\alpha})_{21}|^2 \right) \\
& + 2\theta_-(m_{A1}, m_{A2}) \mathrm{Re}\{ (V_{AL})_{11} (V_{AL})^\dagger_{21} (V_{AR})^\dagger_{11} (V_{AR})_{12} \} \Bigg] \Bigg\}, \qquad \text{(A2)}
\end{aligned}$$

where the functions

$$\begin{aligned}
\theta_+(x,y) &= x^2 + y^2 - \frac{2x^2y^2}{x^2-y^2} \ln \frac{x^2}{y^2}, \\
\theta_-(x,y) &= 2xy \left[\frac{x^2+y^2}{x^2-y^2} \ln \frac{x^2}{y^2} - 2 \right], \\
\chi_+(x,y) &= \frac{5}{9} - \frac{4x^2y^2}{3(x^2-y^2)^2} - \frac{x^6 + y^6 - 3x^2y^2(x^2+y^2)}{3(x^2-y^2)^3} \\
&\quad \times \ln \frac{x^2}{y^2}, \\
\chi_-(x,y) &= xy \left[-\frac{1}{6x^2} - \frac{1}{6y^2} + \frac{x^2+y^2}{(x^2-y^2)^2} - \frac{2x^2y^2}{(x^2-y^2)^3} \right. \\
&\quad \left. \times \ln \frac{x^2}{y^2} \right]
\end{aligned}$$

are introduced by Lavoura and Silva [16]. N means the number of $SU(2)_L$ doublets, $I_3^U = 1/2$ and $I_3^D = -1/2$. Here we used the relation

$$\begin{aligned}
M_Q &= \left[V^\dagger_{UR} \begin{pmatrix} m_{U1} & 0 \\ 0 & m_{U2} \end{pmatrix} V_{UL} \right]_{11} \\
&= \left[V^\dagger_{UL} \begin{pmatrix} m_{U1} & 0 \\ 0 & m_{U2} \end{pmatrix} V_{UR} \right]_{11} \qquad \text{(A3)} \\
&= \left[V^\dagger_{DR} \begin{pmatrix} m_{D1} & 0 \\ 0 & m_{D2} \end{pmatrix} V_{DL} \right]_{11} \\
&= \left[V^\dagger_{DL} \begin{pmatrix} m_{D1} & 0 \\ 0 & m_{D2} \end{pmatrix} V_{DR} \right]_{11}. \qquad \text{(A4)}
\end{aligned}$$

If the mass matrices in Eq. (3) are symmetric and real, we can take the unitary matrices

$$V_{UL} = V_{UR} \equiv \begin{pmatrix} c & -s \\ s & c \end{pmatrix}, \qquad \text{(A5)}$$

$$V_{DL} = V_{DR} \equiv \begin{pmatrix} \bar{c} & -\bar{s} \\ \bar{s} & \bar{c} \end{pmatrix}. \qquad \text{(A6)}$$

In this case, the S and T parameters become

$$\begin{aligned}
S = \frac{N}{6\pi} \Bigg[& -4Y(c^2 \ln m_{U1}^2 + s^2 \ln m_{U2}^2 - \bar{c}^2 \ln m_{D1}^2 \\
& - \bar{s}^2 \ln m_{D2}^2) \\
& - c^2 s^2 \left(6\chi(m_{U1}, m_{U2}) + \frac{m_{U1}^2 + m_{U2}^2}{m_{U1} m_{U2}} - 2 \right) \\
& - \bar{c}^2 \bar{s}^2 \left(6\chi(m_{D1}, m_{D2}) + \frac{m_{D1}^2 + m_{D2}^2}{m_{D1} m_{D2}} - 2 \right) \Bigg],
\end{aligned}$$

$$\begin{aligned}
T = \frac{N}{8\pi \sin^2\theta_W m_W^2} [& c^2 \bar{c}^2 \theta(m_{U1}, m_{D1}) + c^2 \bar{s}^2 \theta(m_{U1}, m_{D2}) \\
& + s^2 \bar{c}^2 \theta(m_{U2}, m_{D1}) + s^2 \bar{s}^2 \theta(m_{U2}, m_{D2}) \\
& - c^2 s^2 \theta(m_{U1}, m_{U2}) - \bar{c}^2 \bar{s}^2 \theta(m_{D1}, m_{D2})],
\end{aligned}$$

with the functions

$$\chi(x,y) = \chi_+(x,y) + \chi_-(x,y),$$
$$\theta(x,y) = \theta_+(x,y) + \theta_-(x,y).$$

If we take $m_D = 0$, i.e., the isospin is maximally violated [26], and $M = M_Q = M_U = M_D \gg m = m_U$, then the S and T parameters can be expanded by m/M:

$$S = \frac{2N}{3\pi}\left(Y + \frac{11}{20}\right)\left(\frac{m}{M}\right)^2 + O\left(\left(\frac{m}{M}\right)^4\right),$$
$$T = \frac{Nm^2}{10\pi\sin^2\theta_W m_W^2}\left[\left(\frac{m}{M}\right)^2 + O\left(\left(\frac{m}{M}\right)^4\right)\right].$$

These results are also obtained by calculating Feynman diagrams in Fig. 2.

[1] E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981), and references therein.

[2] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992); B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990); T. Appelquist and G. Triantaphyllou, *ibid.* **278**, 345 (1992); T. Appelquist and J. Terning, *ibid.* **315**, 139 (1993).

[3] S. Matsumoto, KEK Report No. 94-147, 1994 (unpublished); K. Hagiwara, D. Haidt, C.S. Kim, and S. Matsumoto, Z. Phys. C **64**, 559 (1994).

[4] R.S. Chivukula, Phys. Rev. Lett. **61**, 2657 (1988); T. Appelquist, T. Takeuchi, M.B. Einhorn, and L.C.R. Wijewardhana, Phys. Lett. B **232**, 211 (1989).

[5] B. Holdom, Phys. Lett. **150B**, 301 (1985); K. Yamawaki, M. Bando, and K. Matsumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Akiba and T. Yanagida, Phys. Lett. **169B**, 432 (1986); T. Appelquist, D. Karabali, and L.C.R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[6] V.A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B **221**, 177 (1989); Mod. Phys. Lett. A **4**, 1043 (1989); Y. Nambu, University of Chicago Report No. EFI 89-08, 1989 (unpublished); W.J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989); Phys. Rev. D **41**, 219 (1990).

[7] W.A. Bardeen, C.T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[8] C.T. Hill, Phys. Lett. B **266**, 419 (1991).

[9] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975); Y. Kazama and Y.P. Yao, Phys. Rev. Lett. **43**, 1562 (1979); Phys. Rev. D **21**, 1116 (1980); **21**, 1138 (1980); **25**, 1605 (1982).

[10] N. Maekawa, Prog. Theor. Phys. **93**, 919 (1995).

[11] J. Preskill and S. Weinberg, Phys. Rev. D **24**, 1059 (1981).

[12] S. Dimopoulos and J. Preskill, Nucl. Phys. **B199**, 206 (1982).

[13] H. Georgi and D.B. Kaplan, Phys. Lett. **145B**, 216 (1984); M.J. Dugan, H. Georgi, and D.B. Kaplan, Nucl. Phys. **B254**, 299 (1985); D. Suematsu, Z. Phys. C **46**, 151 (1990).

[14] T. Banks, Nucl. Phys. **B243**, 125 (1984).

[15] G. Bhattacharyya and A. Raychaudhuri, Phys. Lett. B **296**, 448 (1992); F. Caravaglios, *ibid.* **324**, 105 (1994).

[16] L. Lavoura and J.P. Silva, Phys. Rev. D **47**, 2046 (1993).

[17] In this paper the S and T parameters are estimated only via the fermion loops, though we should estimate contributions from the bound states. If four-Fermi interactions are induced by ETC interactions, there must exist vector resonances in general. If the masses of the vector bound states are light, the S and T parameters will be seriously changed. Our estimation in this paper seems to suggest that the masses of the vector states remain heavy, $O(M)$ in spite of the strong four-Fermi interaction. This may be because in the vector channel quadratic divergence does not exist. More study is needed at this point.

[18] If a theory remains isospin symmetry (for example, $m \equiv m_U = m_D$, $m'_U = m'_D = 0$, and $M \equiv M_Q = M_U = M_D$), the T parameter is zero, while the S parameter is suppressed by m/M. In the above example,

$$S = \frac{4N}{15\pi}\left(\frac{m}{M}\right)^2 + O\left(\left(\frac{m}{M}\right)^4\right).$$

In this case, even if we take N large, we can make the S parameter small by taking $m < M$.

[19] The four-Fermi interaction is only an effective description of a more fundamental theory [8, 20].

[20] M.A. Luty, Phys. Rev. D **48**, 1295 (1993).

[21] In this model, the Vafa-Witten theorem [22] cannot be applied because chiral structure plays an important role in breaking electroweak symmetry.

[22] C. Vafa and E. Witten, Nucl. Phys. **B234**, 173 (1984).

[23] V.A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 129 (1989); T. Appelquist, M.B. Einhorn, T. Takeuchi, and L.C.R. Wijewardhana, Phys. Lett. B **220**, 223 (1989); B. Holdom, *ibid.* **226**, 137 (1989); T. Takeuchi, Phys. Rev. D **40**, 2697 (1989).

[24] Y. Nagoshi, K. Nakanishi, and S. Tanaka, Prog. Theor. Phys. **85**, 131 (1991); M. Bando, T. Kugo, and K. Suehiro, *ibid.* **85**, 1299 (1991); M. Bando, T. Kugo, N. Maekawa, and H. Nakano, Phys. Rev. D **44**, 2957 (1991).

[25] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[26] It is interesting that there exists a region in which the S parameter can be negative when $M \sim m_U$, $m_D = 0$, and $Y = -1/2$ (leptonlike), though the mass scale should be rather small ($\sim$ 100 GeV) in order to make the T parameter small.