Strong-CP question in $SU(3)_c \times SU(3)_L \times U(1)_N$ models

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We analyze two recent models based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)_N$ where each generation is not anomaly-free, but the anomaly cancels when three generations are taken into account. We show that the most general Yukawa couplings of these models admit a Peccei-Quinn symmetry. This symmetry can be extended to the entire Lagrangian by using extra fields in a very elegant way so that the resulting axion can be made invisible.

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INTRODUCTION

In the standard model, each generation of fermions is anomaly-free. This is true for many extensions of the standard model as well, including popular grand unified models. In these models, therefore, the number of generations is completely unrestricted on theoretical grounds. There is an interesting class of alternative models which do not have this property. Each generation is anomalous, but different generations are not exact replicas of one another, and the anomalies cancel when a number of generations are taken into account. The most economical gauge group which admits such fermion representations is $SU(3)_c \times SU(3)_L \times U(1)_N$. To the best of our knowledge, models based on this gauge group were first proposed by Singer, Valle, and Schechter [1], where they had to introduce exotic quark and lepton fields. Recently, a different model based on this gauge group has been proposed by Pisano and Pleitez [2] and by Frampton [3] (PPF) where exotic leptons are not necessary, although exotic quarks have to be there. The PPF model did not have right-handed neutrinos, but recently Foot, Long, and Tran (FLT) [4] have included them in a nontrivial way in an interesting variation of the model, thereby revitalizing an older model by Valle and Singer [5].

Here we show that the Yukawa couplings of these models automatically contain a Peccei-Quinn (PQ) symmetry. The symmetry can also be extended to the Higgs potential, thereby making it a symmetry of the entire Lagrangian. This solves the strong CP problem in an elegant way in these models. However, the resulting axion can be made consistent with known bounds only by introducing new fields.

PPF MODEL

The fermion representations of the Pisano-Pleitez-Frampton model are [2,3]

$$f_{aL} = \begin{pmatrix} e_a^+ \\ \nu_a \\ e_a \end{pmatrix}_L \sim (1,3,0), \quad (1)$$

$$Q_{1L} = \begin{pmatrix} T_1 \\ u_1 \\ d_1 \end{pmatrix}_L \sim (3,3,\tfrac{2}{3}), \quad (2)$$

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \\ B_i \end{pmatrix}_L \sim (3,\bar{3},-\tfrac{1}{3}), \quad (3)$$

$$u_{aR} \sim (3,1,\tfrac{2}{3}), \quad (4)$$

$$d_{aR} \sim (3,1,-\tfrac{1}{3}), \quad (5)$$

$$T_{1R} \sim (3,1,\tfrac{5}{3}), \quad (6)$$

$$B_{iR} \sim (3,1,-\tfrac{4}{3}). \quad (7)$$

The generation indices are of two types: a goes from 1 to 3, whereas i takes only the values 2 and 3. Notice that in addition to the ordinary quarks, there are exotic ones, with charges $\frac{5}{3}$ and $-\frac{4}{3}$. It is straightforward to check that gauge anomalies cancel in this model.

In order to break the symmetry as well as to give masses to the fermions, the following Higgs boson representations are needed:

$$\chi = \begin{pmatrix} \chi_0 \\ \chi_- \\ \chi_{--} \end{pmatrix} \sim (1,3,-1), \quad (8)$$

$$\rho = \begin{pmatrix} \rho_{++} \\ \rho_+ \\ \rho_0 \end{pmatrix} \sim (1,3,1), \quad (9)$$

$$\eta = \begin{pmatrix} \eta_+ \\ \eta_0 \\ \eta_- \end{pmatrix} \sim (1,3,0), \quad (10)$$

$$S \sim (1,6,0). \quad (11)$$

The most general Yukawa couplings consistent with gauge symmetry can now be written as

$$\begin{aligned}\mathcal{L}_Y = {} & h_1\overline{Q}_{1L}T_{1R}\chi + h_{2ij}\overline{Q}_{iL}B_{jR}\chi^* + h_{3a}\overline{Q}_{1L}d_{aR}\rho \\ & + h_{4ia}\overline{Q}_{iL}u_{aR}\rho^* + h_{5a}\overline{Q}_{1L}u_{aR}\eta + h_{6ia}\overline{Q}_{iL}d_{aR}\eta^* \\ & + \mathcal{G}_{ab}f_{aL}f_{bL}\eta + \mathcal{G}'_{ab}f_{aL}f_{bL}S^* \,. \end{aligned} \tag{12}$$

Notice that the introduction of the sextet S is not essential for the symmetry breaking. In fact, it is easy to see that the gauge symmetry breaks to $SU(3)_c\times U(1)_Q$ if the triplet Higgs multiplets obtain the vaccum expectation values (VEV's)

$$\langle\chi_0\rangle = v_\chi\,, \quad \langle\rho_0\rangle = v_\rho\,, \quad \langle\eta_0\rangle = v_\eta\,. \tag{13}$$

However, in this case, the mass matrix of the charged leptons would be antisymmetric, and for three generations one eigenvalue will be zero and the other two equal in magnitude. This is not realistic, and therefore a VEV of the sextet is needed to produce a realistic mass matrix [3,6].

We now show that the Yukawa couplings of Eq. (12) respect an extra global U(1) symmetry:

$$\begin{array}{lcccccccc} \text{Multiplet} & \chi & \eta & \rho & S & Q_{1L} & Q_{iL} & f_{aL} \\ \text{U(1) charge} & 1 & 1 & 1 & 2 & 1 & -1 & 1 & . \end{array} \tag{14}$$

Since the charges of left and right chiral fermions are unequal, this is a chiral symmetry, of the type envisaged by Peccei and Quinn [7] to solve the strong CP problem. If this symmetry can be extended to the entire Lagrangian, the model can be made free from the strong CP problem.

The most general Higgs potential with these Higgs multiplets is given by [6]

$$\begin{aligned} V(\chi,\rho,\eta,S) = {} & \lambda_1(\chi^\dagger\chi - v_\chi^2)^2 + \lambda_2(\rho^\dagger\rho - v_\rho^2)^2 + \lambda_3(\eta^\dagger\eta - v_\eta^2)^2 + \lambda_4[\text{tr}\,(S^\dagger S) - v_S^2]^2 + \lambda_5[\text{tr}\,(S^\dagger S S^\dagger S) - v_S^4] \\ & + \lambda_6(\chi^\dagger\chi\rho^\dagger\rho - v_\chi^2 v_\rho^2) + \lambda_7(\rho^\dagger\rho\eta^\dagger\eta - v_\rho^2 v_\eta^2) + \lambda_8(\eta^\dagger\eta\chi^\dagger\chi - v_\eta^2 v_\chi^2) \\ & + \lambda_9[\chi^\dagger\chi\,\text{tr}\,(S^\dagger S) - v_\chi^2 v_S^2] + \lambda_{10}[\rho^\dagger\rho\,\text{tr}\,(S^\dagger S) - v_\rho^2 v_S^2] + \lambda_{11}[\eta^\dagger\eta\,\text{tr}\,(S^\dagger S) - v_\eta^2 v_S^2] \\ & + \lambda_{12}\chi^\dagger\rho\rho^\dagger\chi + \lambda_{13}\rho^\dagger\eta\eta^\dagger\rho + \lambda_{14}\eta^\dagger\chi\chi^\dagger\eta + \mu_1\eta\rho\chi + \mu_2\rho^T S^\dagger\chi + \text{H.c.} \end{aligned} \tag{15}$$

If the PQ symmetry is imposed on the Higgs potential as well, the trilinear interaction term $\eta\rho\chi$ drops out. However, from the analysis of the Higgs potential [6], it is easy to see that one still obtains the vacuum given in Eq. (13).

In fact, it is not even necessary to impose the PQ symmetry to get rid of the trilinear term mentioned above. One can, for example, use a discrete symmetry

$$\chi \to -\chi\,, \quad T_{1R} \to -T_{1R}\,, \quad B_{iR} \to -B_{iR}\,. \tag{16}$$

Then the aforementioned term is automatically eliminated, and PQ symmetry emerges as an automatic symmetry of the classical Lagrangian.

This approach has an advantage. Usually the imposition of the PQ symmetry is somewhat awkward. One has to impose it at the classical level only, but the quantum corrections break it through instanton effects. There have been a number of attempts to find models where the PQ symmetry follows as a natural consequence of other symmetries at the classical level [8]. This model seems to be one of that sort if the discrete symmetry of Eq. (16) is imposed.

However, this also shows that the model, as described above, cannot be realistic. This is because the process of symmetry breaking breaks the PQ symmetry spontaneously, and an axion results. Naively, it seems from Eq. (14) that the spontaneous breaking of the PQ symmetry takes place at the scale v_χ, which can be much higher than the weak scale, and therefore the axion can be invisible. This is not true. At the scale v_χ, although the $U(1)_{PQ}$ breaks, the combination $U(1)_{PQ}+U(1)_N$ remains unbroken, and this acts as the PQ symmetry. This symmetry is broken at the weak scale by the VEV's of η and ρ, so that the axion is of Weinberg-Wilczek-type [9], which has been ruled out by experiments. One therefore must extend this model in order to make the axion invisible.

The standard way of achieving this goal [10] is to introduce a gauge singlet field Φ which has a nonzero charge under the $U(1)_{PQ}$ symmetry. This field can then develop a large VEV to break the PQ symmetry at a very high scale, thus making the axion invisible. The problem is that, from gauge symmetry requirements alone, in addition to terms such as $|\Phi|^2$, $|\Phi|^4$ which are invariant under a U(1) phase of the Φ field, the Higgs potential can also have terms such as Φ^2, Φ^3, Φ^4, etc. In this case, one has to impose the PQ symmetry on the classical Lagrangian. As we argued above, this is a somewhat awkward procedure.

In this case, however, other possibilities remain. For example, instead of introducing an extra gauge singlet, we can introduce a Higgs boson multiplet

$$\Delta \sim (1, 10, -3)\,. \tag{17}$$

This multiplet does not have any Yukawa coupling. In the Higgs potential, apart from the pure-Δ terms and the terms which involve the combination $\Delta^\dagger\Delta$, only the following Δ-dependent term can be present:

$$\lambda_{15}\chi^3\Delta^* + \text{H.c.}\,, \tag{18}$$

provided Δ is odd under the discrete symmetry of Eq. (16). This term governs that the PQ charge of the multiplet Δ should be 3. Thus the PQ symmetry need not be imposed on the classical Lagrangian in this case; it follows automatically from gauge symmetry. This Δ has one component which is a singlet of $SU(2)_L\times U(1)_Y$. If this component develops a VEV, the PQ symmetry will be broken. If this symmetry is broken at a high scale, above 10^9 GeV, the resulting axion will be invisible.

VSFLT MODEL

The model discussed above does not have right-handed neutrinos, but there is a nontrivial way to modify the above model to include them. This was shown in an early paper by Valle and Singer [5], and has recently been rediscovered by Foot, Long, and Tran [4]. In this model [5,4], the fermions appear in the gauge multiplets

$$f_{aL} = \begin{pmatrix} \nu_a \\ e_a \\ \widehat{\nu}_a \end{pmatrix}_L \sim (1,3,-\tfrac{1}{3}), \tag{19}$$

$$e_{aR} \sim (1,1,-1), \tag{20}$$

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u'_1 \end{pmatrix}_L \sim (3,3,\tfrac{1}{3}), \tag{21}$$

$$Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ d'_i \end{pmatrix}_L \sim (3,\bar{3},0), \tag{22}$$

$$u_{aR}, u'_{1R} \sim (3,1,\tfrac{2}{3}), \tag{23}$$

$$d_{aR}, d'_{iR} \sim (3,1,-\tfrac{1}{3})\,. \tag{24}$$

Here, $\widehat{\nu}$ denotes antineutrinos, and u' and d' are new quark fields introduced in the model. Thus, like the previous model, one generation of left-handed quarks transforms differently from the other ones. The representations of the leptons are different from the earlier model, which includes the right-handed neutrinos in the triplet. Choice of this representation also allows one [4,5] to eliminate quark fields of exotic charges, and to use a more economical Higgs boson sector which performs the necessary gauge symmetry breaking:

$$\chi = \begin{pmatrix} \chi_0 \\ \chi_- \\ \chi'_0 \end{pmatrix} \sim (1,3,-\tfrac{1}{3}), \tag{25}$$

$$\rho = \begin{pmatrix} \rho_+ \\ \rho_0 \\ \rho'_+ \end{pmatrix} \sim (1,3,\tfrac{2}{3}), \tag{26}$$

$$\eta = \begin{pmatrix} \eta_0 \\ \eta_- \\ \eta'_0 \end{pmatrix} \sim (1,3,-\tfrac{1}{3})\,. \tag{27}$$

It should be noted that the gauge group and the representations mentioned here are identical to an earlier model by Singer, Valle, and Schechter (SVS) [1]. The only difference is that the third member of the lepton triplet was interpreted as a new neutrino field by SVS, and they had to introduce its right-handed partner as well, which is a gauge singlet. Here, since the third member is interpreted as the antineutrino, no extra leptonic fields are necessary.

The most general Yukawa couplings of the model can be written as

$$\begin{aligned}\mathcal{L}_Y = {} & h_1\overline{Q}_{1L}u'_{1R}\chi + h_{2ij}\overline{Q}_{iL}d'_{jR}\chi^* \\ & + h_{3a}\overline{Q}_{1L}u_{aR}\eta + h_{4ia}\overline{Q}_{iL}d_{aR}\eta^* \\ & + h_{5a}\overline{Q}_{1L}d_{aR}\rho + h_{6ia}\overline{Q}_{iL}u_{aR}\rho^* + \mathcal{G}_{ab}f_{aL}f_{bL}\rho \\ & + \mathcal{G}'_{ab}\overline{f}_{aL}e_{bR}\rho + \text{H.c.}\end{aligned} \tag{28}$$

FLT [4] showed that if the vacuum expectation values (VEV's) of the neutral scalar fields are given by

$$\langle\chi'_0\rangle = v_\chi\,, \quad \langle\rho_0\rangle = v_\rho\,, \quad \langle\eta_0\rangle = v_\eta\,, \tag{29}$$

where all others have zero VEV's, the symmetry breaks to $SU(3)_c \times U(1)_Q$, and the fermions obtain masses.

We now show that the Yukawa couplings of this model respect an extra global U(1) symmetry. The charges of various multiplets under this symmetry are

Multiplet	χ	η	ρ	Q_{1L}	Q_{iL}	f_{aL}	e_{aR}
U(1) charge	1	1	1	1	-1	$-\frac{1}{2}$	$-\frac{3}{2}$

(30)

all other multiplets being neutral. Once again, it is obvious that this is a chiral Peccei-Quinn [7] symmetry. The present model, then, has the property that it does not suffer from the strong *CP* problem if this symmetry can be extended to the entire Lagrangian.

The Higgs potential advocated by FLT [4] is not the most general one subject to gauge invariance. In fact, they imposed a discrete symmetry on the Lagrangian:

$$\chi \to -\chi\,, \qquad u'_{1R} \to -u'_{1R}\,, \qquad d'_{jR} \to -d'_{jR}\,. \tag{31}$$

All the terms in the Yukawa couplings given above are allowed under this symmetry. The Higgs potential allowed under this symmetry is [4]

$$\begin{aligned}V(\chi,\eta,\rho) = {} & \lambda_1(\chi^\dagger\chi - v_\chi^2)^2 + \lambda_2(\eta^\dagger\eta - v_\eta^2)^2 + \lambda_3(\rho^\dagger\rho - v_\rho^2)^2 \\ & + \lambda_4(\chi^\dagger\chi - v_\chi^2)(\eta^\dagger\eta - v_\eta^2) + \lambda_5(\eta^\dagger\eta - v_\eta^2)(\rho^\dagger\rho - v_\rho^2) + \lambda_6(\rho^\dagger\rho - v_\rho^2)(\chi^\dagger\chi - v_\chi^2) \\ & + \lambda_7(\chi^\dagger\eta + \eta^\dagger\chi)^2 + \lambda_8(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) + \lambda_{10}(\rho^\dagger\chi)(\chi^\dagger\rho)\,.\end{aligned} \tag{32}$$

Now, if $\lambda_1, \ldots, \lambda_{10} \geq 0$, the vacuum structure mentioned in Eq. (29) can be obtained from the minimization of this potential [4]. The first six terms imply that the magnitude of the VEV's are v_χ, v_η, and v_ρ, whereas the last four terms imply that they must be in orthogonal directions.

It is now trivial to see that the $U(1)_{PQ}$ symmetry given in Eq. (30) is a symmetry of the Higgs potential as well. Thus it is a symmetry of the entire Lagrangian. This is an automatic symmetry, in the sense that it does not have to be imposed separately on the Lagrangian. Rather, it comes as a consequence of the gauge symmetry and the discrete symmetry of Eq. (31).

Unfortunately, like the previous model, here also the symmetry is broken by the instanton effects as well as spontaneously, and therefore an axion results. This spontaneous breaking of the $U(1)_{PQ}$ occurs at the weak scale by the VEV's of η and ρ. Therefore, the axion is of the Weinberg-Wilczek-type [9], and therefore again the model needs to be extended in order to make it realistic.

Here also, one can get rid of the problem by introducing a gauge singlet Higgs boson [10] which transforms nontrivially under the PQ symmetry. But in this case, the PQ symmetry does not arise automatically, and we argued the awkwardness of this procedure.

Fortunately, in this case as well, one can do otherwise. One can introduce a Higgs boson multiplet

$$\Delta \sim (1, 10, -1) . \tag{33}$$

The nontrivial couplings of this field are

$$\lambda_{11}\eta^3\Delta^* + \lambda_{12}\eta\chi^2\Delta^* , \tag{34}$$

provided Δ is even under the discrete symmetry of Eq. (31), and

$$\lambda'_{11}\chi^3\Delta^* + \lambda'_{12}\eta^2\chi\Delta^* \tag{35}$$

provided Δ is odd. In either case, these couplings dictate that the PQ charge of the multiplet Δ should be 3. When the standard model singlet component of Δ develops a VEV along with the VEV of χ, the PQ symmetry is broken. If this symmetry breaking takes place above 10^9 GeV, the axion is invisible.

CONCLUSIONS

We have thus shown that in recently proposed models based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)_N$, the strong CP problem can be solved in a very elegant way. The elegance comes from the fact that the PQ symmetry does not have to be *imposed* in these models. Rather, with a judicious choice of the Higgs boson multiplets, the PQ symmetry can follow automatically from the gauge invariant Lagrangian. We explained that this is a very satisfying feature, since the imposition of the PQ symmetry is awkward at the classical level, knowing that it is anyway broken by instanton effects when quantum effects are considered.

However, in order to achieve this, we needed to extend the Higgs boson sector of the model, which can be considered as an ugly feature. More importantly, our implementation of the PQ symmetry with the decouplet fields imply that the $SU(3)_L$ symmetry has to be broken at a rather high scale, higher than 10^9 GeV. Thus the massive gauge bosons which occur in this model, apart from the $W^\pm$ and the Z, are all very heavy, and the model loses a lot of its rich phenomenological consequences which would have occurred if these gauge bosons were much lighter. If the Peccei-Quinn symmetry is broken by a gauge singlet Higgs boson field, then of course the gauge bosons can be much lighter, because only the global PQ symmetry is broken at high scale and not gauge symmetry, but in this case one has to live with the awkward implementation of the PQ symmetry.

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