CP violation in $\tau \to 3\pi\nu_\tau$

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(Received 27 January 1995)

We consider CP-violating effects in the decays $\tau \to (3\pi)\nu_\tau$ where both the $J^P = 1^+$ resonance a_1, and $J^P = 0^-$ resonance π' can contribute. The interference between the a_1 and π' resonances can lead to enhanced CP-violating asymmetries whose magnitudes depend crucially on the π' decay constant $f_{\pi'}$. We make an estimate of $f_{\pi'}$ with a simplified chiral Lagrangian coupled to a massive pseudoscalar field, and we compare the estimates from the nonrelativistic quark model and from the QCD sum rule with the estimate from the "mock" meson model. We then estimate quantitatively the size of CP-violating effects in a multi-Higgs-doublet model and scalar-leptoquark models, by assuming that the rest frame of the τ can be reconstructed for three-prong decays. We find that, while CP-violating effects in the scalar-leptoquark models may require more than 10^{10} τ leptons, CP-violating effects from the multi-Higgs-doublet model can be seen at the 2σ level with about 10^7 τ leptons for the maximal value of the CP-violation parameter allowed by present experiments and for the chiral Lagrangian estimate of $f_{\pi'} = (1\text{–}5) \times 10^{-3}$ GeV.

PACS number(s): 13.35.Dx, 11.30.Er, 14.40.Cs

I. INTRODUCTION

The τ has the same interaction structure as the e and the μ in the standard model (SM), apart from their masses. However, for practical purposes [1] the τ lepton, the most massive of the known leptons, behaves quite differently from the e and μ leptons in that (i) the τ has hadronic decay modes [2,3] (e.g., $\tau \to \pi\nu, \rho\nu, a_1\nu, ...$) which allow an efficient measurement of its polarization [2–4] and (ii) the couplings of the τ to neutral and charged Higgs bosons [5–7] are expected to dominate those of the e and μ. These features make the τ a rather special experimental probe of new physics.

One phenomenon where new physics can play a crucial role is CP violation. Currently CP violation has been detected only in the K meson system. The SM explains the effect adequately through a phase in the Kobayashi-Maskawa (KM) matrix [8]. However, it is also possible that other sources of CP violation exist in nature. Recently the τ decays into hadrons have been considered as probes of such a non-KM-type of CP violation in the scalar sector of physics beyond the SM. Nelson *et al.* [9] have considered the so-called stage-2 spin-correlation functions to detect CP violation in the decay $\tau \to (2\pi)\nu_\tau$, while in the same two-pion decay mode Tsai [10] has suggested tests of CP violation with longitudinally polarized electron and positron beams at τ-charm factories. On the other hand, Kilan *et al.* [11] have studied $\tilde{T}$-odd triple momentum correlations in τ decays into $K\pi\pi$ and $K\pi K$.

In the present work we consider the possibility of probing CP violation in the decay of the τ into three charged pions, $\tau \to (3\pi)\nu_\tau$, where both the $J^P = 1^+$ resonance a_1 and the $J^P = 0^-$ resonance π' can contribute. In particular, we investigate whether the large widths of these resonances in the decay of the τ can be used to enhance CP-violation effects in extensions of the SM with scalar-fermion interactions which are consistent with the symmetries of the SM; the importance of broad resonances has been emphasized in the context of the top quark [12] and the B meson [13] in the last few years.

In order to observe CP-violating effects there should exist not only a CP-violating phase but also processes interfering with different CP-conserving phases. In τ decays one can in general have a CP-violating phase between the W-exchange diagram and scalar-exchange diagrams in extensions of the SM such as multi-Higgs-doublet models and scalar-leptoquark models. On the other hand, resonance enhancements of transition amplitudes and their coherent superposition can provide a large CP-conserving phase difference which leads to a significant enhancement of CP-violating observables.

The decay amplitudes of a τ lepton into three pions, $\tau \to (3\pi)\nu_\tau$, have contributions of the two overlapping resonances a_1 and π' with different spins and relatively large width-to-mass ratios [14]. Here we should note that the parameters of the a_1 and π' are not so accurately determined [15]. In the 3π decay mode of the τ lepton various phenomenological parametrizations [17–19,16] of the form factors have been employed to analyze experimental data [20,21]. Keeping in mind the uncertainty of the resonance parameters, we will simply adopt through the paper the parametrization of the τ-decay library TAUOLA [16] for the masses and widths of the a_1 and π' resonances:

$$\begin{aligned} a_1 &: \ J^P = 1^+, \ m_{a_1} = 1.251 \ \text{GeV}, \ \Gamma_{a_1} = 0.599 \ \text{GeV}, \\ \pi' &: \ J^P = 0^-, \ m_{\pi'} = 1.300 \ \text{GeV}, \ \Gamma_{\pi'} = 0.300 \ \text{GeV}. \end{aligned} \tag{1.1}$$

The three-charged-pion decay mode of the τ is promising for the detection of CP violation for the following reasons. First, no tagging of the other τ is necessary. Second, the three-charged-pion mode can be measured not only at the conventional machines [e.g., the Cornell Electron Storage Ring (CESR) and CERN e^+e^- Collider LEP [1]] but also at the planned B factories and at τ-charm factories [22] where many τ leptons (yearly 10^7 to 10^8) are expected to be produced. Third, reconstruction of the τ rest frame is easy for the three-prong decay modes. Since at least two neutrinos escape detection it is in general difficult to reconstruct the τ rest frame in $\tau^+\tau^-$ production. There are a few situations where the rest frame of the τ can actually be reconstructed. One is τ-pair production close to threshold where τ leptons are produced at rest. This possibility can be realized at future τ-charm factories [22]. Another is when both τ leptons decay into hadrons. In this case impact-parameter methods [23] allow us to reconstruct the rest frame of the τ even for τ's in flight. However, in the three-charged-pion decay mode, the direction of the τ can be directly reconstructed through the precise and simultaneous determination of the τ production and decay points.

On the other hand, it is often difficult to make a reliable quantitative prediction for CP violation in hadronic τ decay modes due to the uncertainty in the hadronic matrix elements. This problem can, however, be cured to some extent in the case where the final hadronic states are dominated by at least two neighboring resonances. Then the precise experimental determination of the widths and masses of the resonances will give a rather reliable handle on the calculation of CP asymmetries. In fact, the parametrization of the three-pion decay mode has been investigated quite thoroughly by many authors. Exhaustive discussion of the decay $\tau \to 3\pi\nu_\tau$ is given in Refs. [24,25,18]. Especially, a parametrization of the $a_1/\pi' \to 3\pi$ decay currents can be found in TAUOLA [16]. Since we will use for actual numerical analysis the TAUOLA parametrization, let us mention its limitations. The parametrization is based on the ansatz that (a) the a_1 contribute only to the spin-1 form factors and the π' dominates the spin-0 form factor, and (b) a specific momentum dependence for the π' mode is used. Possible a_1 contribution to the spin-0 form factor is discussed in Ref. [26] and a different form of the scalar form factor is adopted in Ref. [27]. Within the TAUOLA parametrization, the value of the π' decay constant $f_{\pi'} = 0.02$–0.08 GeV estimated in Ref. [18] and $f_{\pi'} = 0.02$ GeV quoted in TAUOLA may not be valid because the mixing between the chiral pion field and a massive pseudoscalar $q\bar{q}$ bound state should be considered. Since any CP-violating effects in the decay $\tau \to 3\pi\nu_\tau$ depend on $f_{\pi'}$ in the TAUOLA parametrization, it is crucial to estimate its magnitude. We devote one section to this issue.

The paper is organized as follows. In Sec. II the decay distribution for $\tau \to (3\pi)\nu_\tau$ is presented. Its general form and our resonance parametrizations are given explicitly. In Sec. III CP-violating asymmetries in the decay $\tau \to 3\pi\nu_\tau$ are introduced. In Sec. IV the multi-Higgs-doublet (MHD) model [29,6] and scalar-leptoquark (SLQ) models [30] are introduced as examples of models which can generate CP-violating effects in the decay $\tau \to 3\pi\nu_\tau$. In Sec. V we estimate the magnitude of $f_{\pi'}$ by employing a simplified chiral Lagrangian coupled to a massive pseudoscalar field. Finally, in Sec. VI the possibility to detect CP violation in the three-charged-pion decay mode of the τ is discussed quantitatively. Section VII summarizes our findings.

II. DISTRIBUTIONS

The matrix element for the decay $\tau^- \to (3\pi)^-\nu_\tau$ is written in the form

$$M = \sqrt{2}G_F\Big[(1+\chi)\bar{u}(k,-)\gamma^\mu P_- u(p,\sigma)J_\mu + \eta\bar{u}(k,-)P_+ u(p,\sigma)J_P\Big], \tag{2.1}$$

with $P_\pm = (1\pm\gamma_5)/2$. Here G_F is the Fermi constant; p and k are the four-momenta of the τ lepton and the τ neutrino, respectively. χ and η are complex numbers parametrizing the contribution from physics beyond the SM. The spin-quantization direction of the τ^- is taken to be the direction opposite to the neutrino momentum [see Fig. 1(a)] and its helicity is denoted by σ ($\sigma = \pm 1$). The τ neutrino is left handed, so its helicity is -1 as indicated by $u(k,-)$. J_μ and J_P are the vector and scalar hadronic matrix elements, respectively, and are given by

$$J_\mu = \cos\theta_C\langle(3\pi)^-|\bar{d}\gamma_\mu(1-\gamma_5)u|0\rangle,$$
$$J_P = \cos\theta_C\langle(3\pi)^-|\bar{d}(1+\gamma_5)u|0\rangle, \tag{2.2}$$

where θ_C is the Cabibbo angle.

Generally the hadronic matrix elements in the decay $\tau \to 3\pi\nu_\tau$ can be parametrized by four form factors [25]. Two of them are dominated by the axial-vector resonance a_1, one vanishes due to G parity, and the other is the so-called scalar form factor. In the chiral limit of vanishing light quark masses, the axial hadronic current should be transverse and the scalar form factor vanishes. However, the chiral symmetry is explicitly broken due to the nonvanishing quark masses and to a small nonconservation of the axial-vector current. Then there can be two types of contributions to the scalar form factor from pseudoscalar resonances such as π' [16,25] and from off-mass-shell contributions from the axial-vector resonances [26]. In the present work we neglect the latter contributions and we assume the dominance of the π' resonance in the scalar mode and take the same form of the scalar form factor

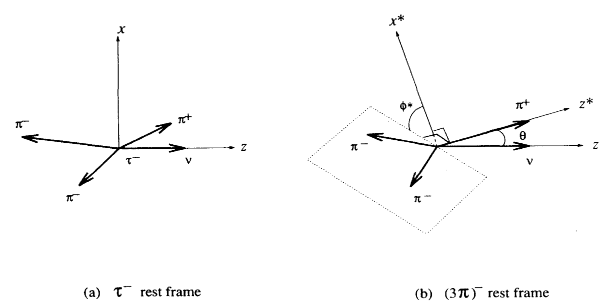

FIG. 1. The decay $\tau^- \to (3\pi)^-\nu_\tau$ viewed from the (a) τ^- and (b) $(3\pi)^-$ rest frames. The two frames are related by a Lorentz boost along the τ neutrino direction and through a rotation by θ with respect to the common y axis.

[27] as in TAUOLA for simplicity. Then the explicit forms of the J_μ and the J_P, which is determined by the Dirac equation from the expression for J_μ, are given by

$$J_\mu = N\left\{ \frac{2\sqrt{2}}{3} T^{\mu\nu}[(q_2-q_3)_\nu F_1(q^2,s_1) + (q_1-q_3)_\nu F_1(q^2,s_2)] + q^\mu C_{\pi'}[s_1(s_2-s_3)F_0(q^2,s_1) + s_2(s_1-s_3)F_0(q^2,s_2)] \right\}, \quad (2.3)$$

$$J_P = \frac{1}{m_u+m_d} q^\mu J_\mu \approx \frac{m_{\pi'}^2}{m_u+m_d} N C_{\pi'}[s_1(s_2-s_3)F_0(q^2,s_1) + s_2(s_1-s_3)F_0(q^2,s_2)], \quad (2.4)$$

with F_1 and F_0 being a_1 and π' contributions, respectively. Here q_1 and q_2 are the four-momenta of two identical π^-'s, q_3 is the four-momentum of the π^+, and q is the four-momentum of the $(3\pi)^-$ system in the decay of the τ^-. m_u and m_d are the current masses of the u and d quarks, respectively. The invariant mass squared of the 3π system, q^2, and the three kinematic invariants s_i $(i=1,2,3)$ are defined in terms of the three-pion momenta, q_i $(i=1,2,3)$, as

$$q^2 = (q_1+q_2+q_3)^2, \quad s_1 = (q_2+q_3)^2, \quad s_2 = (q_3+q_1)^2, \quad s_3 = (q_1+q_2)^2. \quad (2.5)$$

For the tensor $T^{\mu\nu}$, we take the form

$$T^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2, \quad (2.6)$$

which corresponds to the assumption that the a_1 meson contributes only to the spin-1 form factors in accordance with the chiral symmetry. The massless pole $1/q^2$ in (2.6) corresponds to the massless pion in the chiral limit. We ignore the effect of the explicit chiral symmetry breaking in the a_1 contribution [28]. The form factors F_i $(i=0,1)$ are normalized such that $F_i(0,0)=1$, and the coefficient $2\sqrt{2}/3$ in (2.3) is fixed by the soft pion theorem. TAUOLA uses the parameters

$$N = \frac{\cos\theta_C}{f_\pi}, \qquad C_{\pi'} = \frac{g_{\pi'\rho\pi}\, g_{\rho\pi\pi}\, f_{\pi'}\, f_\pi}{m_\rho^4 m_{\pi'}^2}, \quad (2.7)$$

and adopts the parameter values

$$\cos\theta_C = 0.973, \quad m_\rho = 0.773 \ \text{GeV}, \quad f_\pi = 0.0933 \ \text{GeV}, \quad f_{\pi'} = 0.02 \ \text{GeV}, \quad g_{\pi'\rho\pi} = 5.8, \quad g_{\rho\pi\pi} = 6.08. \quad (2.8)$$

In the following numerical analyses, we adopt the above parameters except for $f_{\pi'}$.

It is convenient to cast the decay amplitude (2.1) into the form

$$M = \sqrt{2} G_F (1+\chi)\left[\sum_\lambda L_{\sigma\lambda} H_\lambda + (1+\xi) L_{\sigma s} H_s\right], \quad (2.9)$$

where the parameter ξ is given in terms of the χ and η in Eq. (2.1) by

$$\xi = \frac{m_{\pi'}^2}{(m_u+m_d)m_\tau}\left(\frac{\eta}{1+\chi}\right). \quad (2.10)$$

The $\tau^- \to \nu_\tau$ transition amplitudes $L_{\sigma\lambda}$ $(\lambda = 0,\pm)$ and $L_{\sigma s}$ in Eq. (2.9) are defined as

$$L_{\sigma\lambda} = \bar{u}(k,-)\gamma^\mu P_- u(p,\sigma)\epsilon_\mu^*(q,\lambda), \quad L_{\sigma s} = \bar{u}(k,-)P_+ u(p,\sigma), \quad (2.11)$$

where we have introduced the spin-1 polarization vector $\epsilon(q,\lambda)$ satisfying

$$\sum_{\lambda=0,\pm} \epsilon^\mu(q,\lambda)\epsilon^{*\nu}(q,\lambda) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}. \quad (2.12)$$

The purely leptonic amplitudes are known functions of the kinematic variables and are given in the convention of Ref. [31] by

$$L_{\sigma+} = 0, \quad L_{\sigma 0} = \frac{m_\tau}{\sqrt{q^2}}\sqrt{m_\tau^2 - q^2}\,\delta_{\sigma+}, \quad L_{\sigma -} = \sqrt{2}\sqrt{m_\tau^2-q^2}\,\delta_{\sigma -}, \quad L_{\sigma s} = \sqrt{m_\tau^2-q^2}\,\delta_{\sigma+}. \quad (2.13)$$

The hadronic matrix elements H_λ $(\lambda=0,\pm)$ and H_s are

$$H_\lambda = -\frac{2\sqrt{2}}{3} N \epsilon_\mu(q,\lambda)[(q_2-q_3)^\mu F_1(q^2,s_1) + (q_1-q_3)^\mu F_1(q^2,s_2)], \quad (2.14)$$

$$H_s = N m_\tau C_{\pi'}[s_1(s_2-s_3)F_0(q^2,s_1) + s_2(s_1-s_3)F_0(q^2,s_2)]. \quad (2.15)$$

Assuming single meson dominance and imposing Bose symmetry in each channel we express the form factors F_i $(i=0,1)$ in terms of the meson propagators as

$$F_1(q^2,s_i) = B_{a_1}(q^2)B_\rho(s_i), \quad F_0(q^2,s_i) = B_{\pi'}(q^2)B_\rho(s_i). \quad (2.16)$$

Here, for simplicity, we have neglected the possible ρ' contribution, which is implemented in TAUOLA. On the other hand, following TAUOLA we parametrize the a_1, π', and ρ meson propagators in the Breit-Wigner form with momentum-dependent widths

$$B_X(q^2) = \frac{m_X^2}{m_X^2 - q^2 - i m_X \Gamma_X(q^2)}, \quad \Gamma_X(q^2) = \Gamma_X\left(\frac{f_X(q^2)}{f_X(m_X^2)}\right), \quad (2.17)$$

for $X = a_1$, π', or ρ. The momentum dependence of all widths has been determined from experimental data. The parametrizations of the widths available from the TAUOLA are

$$f_{a_1}(q^2) = \begin{cases} q^2[1.623 + 10.38/q^2 - 9.32/q^4 + 0.65/q^6] & \text{for } q^2 > (m_\rho + m_\pi)^2, \\ 4.1(q^2 - 9m_\pi^2)^3[1 - 3.3(q^2 - 9m_\pi^2) + 5.8(q^2 - 9m_\pi^2)^2] & \text{elsewhere,} \end{cases} \tag{2.18}$$

$$f_{\pi'}(q^2) = \begin{cases} \frac{m_{\pi'}^2}{q^2}\left(\frac{P_\pi(q^2)}{P_\pi(m_{\pi'}^2)}\right)^5 & \text{for } q^2 > (m_\rho + m_\pi)^2 \\ 0 & \text{elsewhere,} \end{cases} \tag{2.19}$$

$$f_\rho(q^2) = \begin{cases} \frac{m_\rho}{\sqrt{q^2}}\left(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} & \text{for } q^2 > (2m_\pi)^2 \\ 0 & \text{elsewhere.} \end{cases} \tag{2.20}$$

P_π in Eq. (2.19) denotes the momentum of the π in the (virtual) π' rest frame. In addition to the parameters (1.1) and (2.8) TAUOLA adopts for the ρ meson width

$$\Gamma_\rho = 0.145 \text{ GeV}. \tag{2.21}$$

The amplitude for the τ^+ decay into $(3\pi)^+$, which is the CP-conjugated process of the τ^- decay into $(3\pi)^-$, can be determined in the same manner as that of the τ^- decay into $(3\pi)^-$. For the sake of discussion we use the same kinematic variables as in the τ^- decay. In the τ^+ decay q_1 and q_2 are the four-momenta of two identical π^+'s, q_3 is the four-momentum of the π^-, and q is the four-momentum of the $(3\pi)^+$ system. The decay amplitude $\bar{M}$ for the process $\tau^+ \to (3\pi)^+\bar{\nu}_\tau$ can be written in the form

$$\bar{M} = \sqrt{2}G_F(1+\bar{\chi})\left[\sum_\lambda \bar{L}_{\sigma\lambda}\bar{H}_\lambda + (1+\bar{\xi})\bar{L}_{\sigma s}\bar{H}_s\right]. \tag{2.22}$$

The $\tau^+ \to \bar{\nu}_\tau$ transition amplitudes $\bar{L}_{\sigma\lambda}$ ($\lambda = 0, \pm$) and $\bar{L}_{\sigma s}$ are given by

$$\begin{aligned} \bar{L}_{\sigma\lambda} &= \bar{v}(p,\sigma)\gamma^\mu P_- v(k,+)\epsilon_\mu^*(q,\lambda), \\ \bar{L}_{\sigma s} &= -\bar{v}(p,\sigma)P_- v(k,+), \end{aligned} \tag{2.23}$$

where $\epsilon(q,\lambda)$ is the same polarization vector as in Eq. (2.11). We note that the hadronic amplitudes $\bar{H}_\lambda$ ($\lambda = 0, \pm$) and $\bar{H}_s$ for the decay $\tau^+ \to (3\pi)^+\bar{\nu}_\tau$ are the same as the hadronic amplitudes H_λ ($\lambda = 0, \pm$) and H_s for the decay $\tau^- \to (3\pi)^-\nu_\tau$:

$$\bar{H}_\lambda = H_\lambda, \qquad \bar{H}_s = H_s. \tag{2.24}$$

Hence it is straightforward to obtain each amplitude of the τ^+ decay into $(3\pi)^+$ from the corresponding τ^- decay amplitude (2.1) through the CP relations among the amplitudes and the couplings

$$\bar{L}_{\sigma\lambda} = -L_{-\sigma,-\lambda}, \qquad \bar{L}_{\sigma s} = -L_{-\sigma s}, \tag{2.25}$$

$$\bar{\chi} = \chi^*, \qquad \bar{\eta} = \eta^*, \tag{2.26}$$

in the approximation where the imaginary parts of the intermediate W and scalar propagators are neglected.

The decay of the $\tau^\pm$ into three charged pions depends on five independent kinematic variables. Because of two intermediate resonances, $a_1^\pm$ and $\pi'^\pm$, decaying into $(3\pi)^\pm$ it is convenient to consider both the $\tau^\pm$ rest frame and the $(3\pi)^\pm$ rest frame. We define two coordinate systems as shown in Fig. 1. The two coordinate systems have a common y axis which is chosen along the $\vec{k} \times \vec{q}_3$ direction. Here $\vec{k}$ is the three-momentum of the τ neutrino. In the (x,y,z) coordinate system for the τ rest frame, the z axis is along the direction of the τ neutrino momentum $\vec{k}$ and the momentum $\vec{q}_3$ is in the positive-x half-plane. The starred coordinate system (x^*, y^*, z^*) is obtained from the (x,y,z) frame by boost along the negative-z axis and the rotation within the x-z plane by θ, the angle between $\vec{k}$ and $\vec{q}_3$, in the $(3\pi)^\pm$ rest frame, to align $\vec{q}_3$ along the positive z^* axis. The azimuthal angle ϕ^* of $\vec{q}_1$ about the z^* axis is then measured from the x^* axis [see Fig. 1(b)]. The decay process is now expressed in terms of the five kinematical variables q^2, s_1, s_2, θ, and ϕ^*.

The range of the variables is given by

$$\begin{aligned} &(3m_\pi)^2 \le q^2 \le m_\tau^2, \quad (2m_\pi)^2 \le s_1 \le (\sqrt{q^2} - m_\pi)^2, \\ &s_{2\min} \le s_2 \le s_{2\max}, \quad 0 \le \theta \le \pi, \quad 0 \le \phi^* \le 2\pi, \end{aligned} \tag{2.27}$$

where for given q^2 and s_1 the minimum and maximum values of s_2 values are

$$\begin{aligned} s_{2\min} &= \frac{1}{2}[q^2 + 3m_\pi^2 - s_1 \\ &\quad -\sqrt{(1 - 4m_\pi^2/s_1)}\lambda^{1/2}(q^2, m_\pi^2, s_1)], \\ s_{2\max} &= \frac{1}{2}[q^2 + 3m_\pi^2 - s_1 \\ &\quad +\sqrt{(1 - 4m_\pi^2/s_1)}\lambda^{1/2}(q^2, m_\pi^2, s_1)], \end{aligned} \tag{2.28}$$

with the well-known triangle function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$. For simplicity we introduce the notation

$$\begin{aligned} \lambda_i &= \lambda(q^2, m_\pi^2, s_i) \quad (i = 1,2,3), \\ G &= \sqrt{-\lambda(\lambda_1, \lambda_2, \lambda_3)}. \end{aligned} \tag{2.29}$$

Using the kinematic variables and notations we obtain analytic expressions of the hadronic matrix elements (2.15):

$$\begin{aligned} H_\pm = &\mp \frac{N}{6\sqrt{q^2\lambda_3}}B_{a_1}(q^2) \\ &\times [A\sin\theta + B(\cos\theta\cos\phi^* \mp i\sin\phi^*)], \end{aligned} \tag{2.30}$$

$$H_0 = -\frac{\sqrt{2}N}{6\sqrt{q^2\lambda_3}} B_{a_1}(q^2)\left[A\cos\theta - B\sin\theta\cos\phi^*\right], \quad (2.31)$$

$$H_s = \frac{Nm_\tau C_{\pi'}}{2} B_{\pi'}(q^2)\left[C\right], \quad (2.32)$$

where

$$\begin{aligned} A &= 3\lambda_3[B_\rho(s_1) + B_\rho(s_2)] \\ &\quad -(\lambda_1 - \lambda_2)[B_\rho(s_1) - B_\rho(s_2)], \\ B &= G[B_\rho(s_1) - B_\rho(s_2)], \\ C &= 2s_1(s_2 - s_3)B_\rho(s_1) + 2s_2(s_1 - s_3)B_\rho(s_2). \end{aligned} \quad (2.33)$$

The hadronic amplitudes H_λ ($\lambda = 0, \pm$) and H_s are symmetric under the interchange of two identical pions. The functions A and C also are symmetric, while the function B is antisymmetric.

We denote the differential decay rates of the $\tau^\mp$ into three pions as

$$G(q^2, s_1, s_2, \cos\theta, \phi^*) = \frac{d^5}{dq^2 ds_1 ds_2 d\cos\theta d\phi^*}\Gamma[\tau^- \to (3\pi)^-\nu_\tau],$$

$$\bar{G}(q^2, s_1, s_2, \cos\theta, \phi^*) = \frac{d^5}{dq^2 ds_1 ds_2 d\cos\theta d\phi^*}\bar{\Gamma}[\tau^+ \to (3\pi)^+\bar{\nu}_\tau], \quad (2.34)$$

and examine the consequences of CP invariance in the $\tau^\mp$ differential decay rates. The CP transformation should relate the decay $\tau^- \to (3\pi)^-\nu_\tau$ to the decay $\tau^+ \to (3\pi)^+\bar{\nu}_\tau$. We find that CP invariance leads to the following relation in the differential decay rates:

$$G(q^2, s_1, s_2, \cos\theta, \phi^*) \overset{CP}{=} \bar{G}(q^2, s_1, s_2, \cos\theta, -\phi^*). \quad (2.35)$$

The relation enables us to construct a CP-conserving sum Σ and a CP-violating difference Δ of the differential $\tau^\pm$ decay rates:

$$\begin{aligned} \Sigma &= G(q^2, s_1, s_2, \cos\theta, \phi^*) + \bar{G}(q^2, s_1, s_2, \cos\theta, -\phi^*), \\ \Delta &= G(q^2, s_1, s_2, \cos\theta, \phi^*) - \bar{G}(q^2, s_1, s_2, \cos\theta, -\phi^*). \end{aligned} \quad (2.36)$$

It is rather straightforward to obtain from the amplitudes (2.9) and (2.22) the analytic forms of Σ and Δ:

$$\begin{aligned} \Sigma &= F(q^2)\Bigg\{ 2|H_-|^2 + \frac{m_\tau^2}{q^2}|H_0|^2 + |1+\xi|^2|H_s|^2 \\ &\quad + 2\frac{m_\tau}{\sqrt{q^2}}\Big[1 + \mathrm{Re}(\xi)\Big]\Big[\mathrm{Re}(H_0 H_s^*)\Big]\Bigg\}, \\ \Delta &= 2F(q^2)\frac{m_\tau}{\sqrt{q^2}}\Big[\mathrm{Im}(\xi)\Big]\Big[\mathrm{Im}(H_0 H_s^*)\Big], \end{aligned} \quad (2.37)$$

where for notational convenience we have introduced the overall function $F(q^2)$,

$$F(q^2) = \frac{G_F^2 m_\tau}{2^7\pi^6}\frac{(1 - q^2/m_\tau^2)^2}{q^2}|1+\chi|^2. \quad (2.38)$$

From the expression (2.37) we note two important features of the CP-violating distribution Δ: (i) Every CP-violating asymmetry requires not only a nonvanishing $\mathrm{Im}(\xi)$ but also requires interference between the spin-1 (helicity-0) and spin-0 amplitudes, and (ii) the difference Δ depends only on $\cos\phi^*$, but does not depend on $\sin\phi^*$ which is odd under the naive time reversal $\tilde{T}$ [see Eqs. (2.31) and (2.32)]. Here the transformation $\tilde{T}$ means $t \to -t$ without the interchange of initial and final states.

III. ASYMMETRIES

As shown in Sec. II every CP asymmetry requires not only a nonvanishing $\mathrm{Im}(\xi)$ but also the interference between the longitudinal a_1 mode and the pseudoscalar π' mode; the interference is proportional to the $f_{\pi'}$. More explicitly Δ is proportional to the product of $f_{\pi'}$ and $\mathrm{Im}(\xi)$:

$$\Delta \propto [\mathrm{Im}(\xi)]\,[f_{\pi'}]. \quad (3.1)$$

In order to observe Δ it is useful to form an observable with an appropriate real weight function $w(q^2, s_1, s_2, \cos\theta, \phi^*)$. A CP-violating scalar quantity is then obtained as

$$\langle w\Delta\rangle = \int \left[w(q^2, s_1, s_2, \cos\theta, \phi^*)\Delta\right] \times dq^2 ds_1 ds_2 d\cos\theta d\phi^*, \quad (3.2)$$

where $\langle X\rangle$ means the integration of the quantity X over the allowed phase space of q^2, s_1, s_2, $\cos\theta$, and ϕ^*. The statistical significance of this observable can be determined by the quantity

$$\varepsilon = \frac{\langle w\Delta\rangle}{\sqrt{\langle\Sigma\rangle\langle w^2\Sigma\rangle}}, \quad (3.3)$$

with Σ given in Eq. (2.37). The number of τ leptons required to observe the effect at the 1σ level is then

$$N_\tau = \frac{1}{\mathcal{B}\varepsilon^2}, \quad (3.4)$$

where $\mathcal{B}$ denotes the branching ratio of the τ decay into three charged pions, which is 6.8% [21]. By appropriately choosing the weight function w, the CP asymmetry $\langle w\Delta\rangle$ can be made large. The optimal weight function maximizing the quantity ε in Eq. (3.3) is known [32] to be

$$w_{\mathrm{opt}}(q^2, s_1, s_2, \cos\theta, \phi^*) = \frac{\Delta(q^2, s_1, s_2, \cos\theta, \phi^*)}{\Sigma(q^2, s_1, s_2, \cos\theta, \phi^*)}. \quad (3.5)$$

We can also consider an observable with $w = \pm 1$ in phase space, which corresponds to the usual definition of an asymmetry. The distribution property of Δ in Eq. (2.37) suggests that we should consider two types of CP-violating forward-backward asymmetries, $A_{1\mathrm{FB}}$ and $A_{2\mathrm{FB}}$, whose weight functions are given by

$$\begin{aligned} w_1(q^2, s_1, s_2, \cos\theta, \phi^*) &= -\mathrm{sgn}[\cos\theta], \\ w_2(q^2, s_1, s_2, \cos\theta, \phi^*) &= \mathrm{sgn}[s_1 - s_2]\mathrm{sgn}[\cos\phi^*], \end{aligned} \quad (3.6)$$

respectively. Note that the factor $\mathrm{sgn}[s_1 - s_2]$ is included in the definition of the $A_{2\mathrm{FB}}$ asymmetry because the CP-violating asymmetry with only $\mathrm{sgn}[\cos\phi^*]$ as a weight function vanishes due to Bose symmetry.

IV. MODELS

In the SM CP violation arises from a nontrivial phase in the KM flavor-mixing matrix [8] in the hadronic charged current, but the KM-type CP violation cannot be detected in τ decays. As possible new sources of CP violation detectable in the τ decay we consider new scalar-fermion interactions which preserve the symmetries of the SM. Then it can be proven that only four types of scalar-exchange models [33] contribute to the decay $\tau \to (3\pi)\nu_\tau$. One of them is the multi-Higgs-doublet (MHD) model [6,29] and the other three models are scalar-leptoquark (SLQ) models [34,30].

A. Multi-Higgs-doublet (MHD) models

In this subsection we consider a MHD model with n Higgs doublets. The Yukawa interaction of the MHD model is

$$\mathcal{L}_{\mathrm{MHD}} = \bar{Q}_{L_i} F^D_{ij}\Phi_d D_{R_j} + \bar{Q}_{L_i} F^U_{ij}\tilde{\Phi}_u U_{R_j} + \bar{L}_{L_i} F^E_{ij}\Phi_e E_{R_j} + \mathrm{H.c.} \tag{4.1}$$

Here Q_{L_i} denotes left-handed quark doublets, and L_{L_i} denotes left-handed lepton doublets. D_{R_i} (U_{R_i}) and E_{R_i} are for right-handed down (up) quark singlets and right-handed charged lepton singlets, respectively. The subindex i is a generation index ($i = 1, 2, 3$). Φ_j ($j = 1$ to n) are n Higgs doublets and $\tilde{\Phi}_j = i\sigma_2\Phi_j^*$. Subindices d, u, and e denote the Higgs doublets that couple to down-type quarks, up-type quarks, and charged leptons, respectively. F^U and F^D are general 3×3 Yukawa matrices of which one matrix can be taken to be real and diagonal. Since neutrinos are massless F^E can be chosen real and diagonal. The MHD model has $2(n-1)$ charged and $2n-1$ neutral physical scalars, and the Yukawa interactions of the $2(n-1)$ physical charged scalars with fermion mass eigenstates read

$$\mathcal{L}_{\mathrm{MHD}} = \sqrt{2\sqrt{2}G_F}\sum_{i=2}^{n}[X_i(\bar{U}VM_D D_R) + Y_i(\bar{U}_R M_U V D_L) + Z_i(\bar{N}_L M_E E_R)]H_i^+ + \mathrm{H.c.} \tag{4.2}$$

Here M_D, M_U, and M_E denote the diagonal mass matrices of down-type quarks, up-type quarks, and charged leptons, respectively. H_i^+ are the positively charged Higgs particles. N_L are for left-handed neutrino fields and V for the KM matrix. X_i, Y_i, and Z_i are complex coupling constants which arise from the mixing matrix for charged scalars.

Within the framework of the MHD model, CP violation in charged scalar exchange can arise for more than two Higgs doublets [35,36]. There are two mechanisms which give rise to CP violation in the scalar sector. In one mechanism [37,35] CP symmetry is maintained at the Lagrangian level but broken through complex vacuum expectation values. However, this possibility has been shown to have some phenomenological difficulties [38,6]. In the other mechanism CP is broken by complex Yukawa couplings and possibly by complex vacuum expectation values so that CP violation can arise both from charged scalar exchange and from $W^\pm$ exchange. CP violation in both mechanisms is commonly manifest in phases that appear in the combinations XY^*, XZ^*, and YZ^*.

One crucial condition for CP violation in the MHD model is that not all the charged scalars should be degenerate. Then, without loss of generality and for simplicity, we can assume that all but the lightest of the charged scalars effectively decouple from fermions. The couplings of the lightest charged scalar to fermions are described by a simple Lagrangian

$$\mathcal{L}_{\mathrm{MHD}} = (2\sqrt{2}G_F)^{1/2}[X(\bar{U}VM_D D_R) + Y(\bar{U}_R M_U V D_L) + Z(\bar{N}_L M_E E_R)]H^+ + \mathrm{H.c.} \tag{4.3}$$

This Lagrangian gives the effective Lagrangian for the decay $\tau\to 3\pi\nu_\tau$,

$$\mathcal{L}^{\mathrm{MHD}}_{\mathrm{eff}} = 2\sqrt{2}G_F\cos\theta_C m_\tau\left[m_d\frac{X^*Z}{M_H^2}(\bar{u}_L d_R)(\bar{\nu}_{\tau_L}\tau_R) + m_u\frac{Y^*Z}{M_H^2}(\bar{u}_R d_L)(\bar{\nu}_{\tau_L}\tau_R)\right], \tag{4.4}$$

at energies which are low compared to the mass of the charged Higgs boson. Then one can show that the contribution from the MHD model in the $\tau\to 3\pi\nu_\tau$ decay distribution of Eq. (2.1) is represented by the parameters

$$\chi_{\mathrm{MHD}} = 0, \quad \eta_{\mathrm{MHD}} = \frac{m_d}{M_H^2}\left[X^*Z - \left(\frac{m_u}{m_d}\right)Y^*Z\right], \tag{4.5}$$

and CP violation in the MHD model is determined by the parameter

$$\mathrm{Im}(\xi_{\mathrm{MHD}}) = -\left(\frac{m_d}{m_u+m_d}\right)\left(\frac{m_{\pi'}^2}{M_H^2}\right) \times\left[\mathrm{Im}(XZ^*) - \left(\frac{m_u}{m_d}\right)\mathrm{Im}(YZ^*)\right]. \tag{4.6}$$

B. Scalar-leptoquark (SLQ) models

In this subsection we discuss CP-violating effects from leptoquark exchange. There are three types of SLQ models [33,34] which can contribute to the decay $\tau\to 3\pi\nu_\tau$ at the tree level. The quantum numbers of the three leptoquarks under the gauge group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ are

$$\Phi_1 = \left(3, 2, \frac{7}{6}\right) \quad \text{(model I)},$$
$$\Phi_2 = \left(3, 1, -\frac{1}{3}\right) \quad \text{(model II)},$$
$$\Phi_3 = \left(3, 3, -\frac{1}{3}\right) \quad \text{(model III)}, \qquad (4.7)$$

respectively. The hypercharge Y is defined to be $Q = I_3 + Y$. The Yukawa couplings of the leptoquarks to fermions are given by

$$\mathcal{L}^{\rm I}_{\rm SLQ} = [-x_{ij}\bar{Q}_{L_i} i\tau_2 E_{R_j} + x'_{ij}\bar{U}_{R_i} L_{L_j}]\Phi_1 + \text{H.c.},$$
$$\mathcal{L}^{\rm II}_{\rm SLQ} = [y_{ij}\bar{Q}_{L_i} i\tau_2 L^c_{L_j} + y'_{ij}\bar{U}_{R_i} E^c_{R_j}]\Phi_2 + \text{H.c.},$$
$$\mathcal{L}^{\rm III}_{\rm SLQ} = z_{ij}[\bar{Q}_{L_i} i\tau_2 \vec{\tau} L^c_{L_j}]\cdot\vec{\Phi}_3 + \text{H.c.} \qquad (4.8)$$

Here the coupling constants $x^{(\prime)}_{ij}$, $y^{(\prime)}_{ij}$, and z_{ij} are complex when CP violation arises from the Yukawa interactions. $\bar{Q}_{L_i} = (\bar{u}_i, \bar{d}_i)_L$ and $L_{L_i} = (\bar{\nu}_i, \bar{e}_i)_L$. The superscript c denotes charge conjugation, i.e., $\psi^c_{R,L} = i\gamma^0\gamma^2\bar{\psi}^T_{R,L}$ for a spinor field ψ. $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ and τ_i $(i = 1, 2, 3)$ are the Pauli matrices. In terms of the charge component of the leptoquarks, the Lagrangian relevant to the $\tau \to 3\pi\nu_\tau$ decay is given by

$$\mathcal{L}^{\rm I}_{\rm SLQ} = [x_{13}\bar{d}_L\tau_R + x'_{13}\bar{u}_R\nu_{\tau_L}]\phi_1^{(2/3)} + \text{H.c.},$$
$$\mathcal{L}^{\rm II}_{\rm SLQ} = [-y_{13}(\bar{u}_L\tau^c_L - \bar{d}_L\nu^c_{\tau_L}) + y'_{13}\bar{u}_R\tau^c_R]\phi_2^{(-1/3)} + \text{H.c.},$$
$$\mathcal{L}^{\rm III}_{\rm SLQ} = -z_{13}[\bar{u}_L\tau^c_L + \bar{d}_L\nu^c_{\tau_L}]\phi_3^{(-1/3)} + \text{H.c.} \qquad (4.9)$$

After Fierz rearrangement we obtain the effective SLQ Lagrangians for the decay $\tau \to 3\pi\nu_\tau$:

$$\mathcal{L}^{\rm I}_{\rm eff} = -\frac{x_{13}x'^*_{13}}{2M^2_{\phi_1}}\left[(\bar{d}_L u_R)(\bar{\nu}_{\tau_L}\tau_R) + \frac{1}{4}(\bar{d}_L\sigma^{\mu\nu}u_R)(\bar{\nu}_{\tau_L}\sigma_{\mu\nu}\tau_R)\right] + \text{H.c.},$$
$$\mathcal{L}^{\rm II}_{\rm eff} = -\frac{y_{13}y'^*_{13}}{2M^2_{\phi_2}}\left[(\bar{d}_L u_R)(\bar{\tau}^c_R\nu^c_{\tau_L}) + \frac{1}{4}(\bar{d}_L\sigma^{\mu\nu}u_R)(\bar{\tau}^c_R\sigma_{\mu\nu}\nu^c_{\tau_L})\right] + \frac{|y_{13}|^2}{2M^2_{\phi_2}}(\bar{d}_L\gamma_\mu u_L)(\bar{\tau}^c_L\gamma^\mu\nu^c_{\tau_L}) + \text{H.c.},$$
$$\mathcal{L}^{\rm III}_{\rm eff} = -\frac{|z_{13}|^2}{2M^2_{\phi_3}}(\bar{d}_L\gamma_\mu u_L)(\bar{\tau}^c_L\gamma^\mu\nu^c_{\tau_L}) + \text{H.c.} \qquad (4.10)$$

The tensor part as well as the scalar part in model I and model II can contribute to the $\tau \to (3\pi)\nu_\tau$ decay. For simplicity, however, we consider only the scalar contribution in the present work. Then the size of new contributions from these three SLQ models is parametrized by the parameters (2.1):

$$\chi^{\rm I}_{\rm SLQ} = 0, \qquad \eta^{\rm I}_{\rm SLQ} = -\frac{x_{13}x'^*_{13}}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_1}}, \qquad (4.11)$$

$$\chi^{\rm II}_{\rm SLQ} = -\frac{|y_{13}|^2}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_2}},$$
$$\eta^{\rm II}_{\rm SLQ} = -\frac{y_{13}y'^*_{13}}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_2}}, \qquad (4.12)$$

$$\chi^{\rm III}_{\rm SLQ} = \frac{|z_{13}|^2}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_2}}, \quad \eta^{\rm III}_{\rm SLQ} = 0. \qquad (4.13)$$

Note that the vector-type interaction terms have only real coupling constants, and thus they cannot generate any CP-violating effects. In particular, model III does not contribute to CP violation in the decay $\tau \to 3\pi\nu_\tau$ so that the model will no longer be considered [39]. In model I and model II, the parameters governing CP violation are

$$\text{Im}(\xi^{\rm I}_{\rm SLQ}) = -\frac{m^2_{\pi'}}{(m_u + m_d)m_\tau}\frac{\text{Im}[x_{13}x'^*_{13}]}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_1}},$$
$$\text{Im}(\xi^{\rm II}_{\rm SLQ}) = -\frac{m^2_{\pi'}}{(m_u + m_d)m_\tau}\frac{\text{Im}[y_{13}y'^*_{13}]}{4\sqrt{2}G_F\cos\theta_C M^2_{\phi_2}}, \qquad (4.14)$$

respectively, where all CP-conserving contributions from new physics are neglected in the normalization. This is justified because the contributions from new physics are expected to be small compared to those from the SM.

C. Phenomenological constraints

The constraints on the CP-violation parameters, (4.6) and (4.14), depend upon the values chosen for the u and d quark masses. In the present work we use, for the light u and d quark masses [40],

$$m_u = 5.0 \text{ MeV}, \qquad m_d = 9.0 \text{ MeV}, \qquad (4.15)$$

and for the W boson mass we use $M_W = 80$ GeV. Inserting these values in (4.6) and (4.14) we obtain

$$\text{Im}(\xi_{\rm MHD}) \simeq -1.7\times 10^{-4}\frac{[\text{Im}(XZ^*) - (5/9)\text{Im}(YZ^*)]}{(M_H/M_W)^2}, \qquad (4.16)$$

$$\begin{bmatrix} \text{Im}(\xi^{\rm I}_{\rm SLQ}) \\ \text{Im}(\xi^{\rm II}_{\rm SLQ}) \end{bmatrix} \simeq -7.7\times 10^2 \begin{bmatrix} \left(\frac{M_W}{M_{\phi_1}}\right)^2 \text{Im}(x_{13}x'^*_{13}) \\ \left(\frac{M_W}{M_{\phi_2}}\right)^2 \text{Im}(y_{13}y'^*_{13}) \end{bmatrix}. \qquad (4.17)$$

Clearly, sizable CP-violating effects require that $\text{Im}(XZ^*)$ and $\text{Im}(YZ^*)$ are large and M_H is small compared to M_W in the MHD model, and similarly that $\text{Im}(x_{13}x'^*_{13})$ and $\text{Im}(y_{13}y'^*_{13})$ are large and M_{ϕ_i} $(i = 1, 2)$ are small compared to M_W in the SLQ models.

In the MHD model the strongest constraint [6] on $\text{Im}(XZ^*)$ comes from the measurement of the branching ratio $\mathcal{B}(B \to X\tau\nu_\tau)$ which actually gives a constraint on $|XZ|$. For $M_H < 440$ GeV, the bound on $\text{Im}(XZ^*)$ is given by

$$\mathrm{Im}(XZ^*) < |XZ| < 0.23 M_H^2 \ \mathrm{GeV}^{-2}. \tag{4.18}$$

On the other hand, the bound [6] on $\mathrm{Im}(YZ^*)$ is mainly given by $K^+ \to \pi^+ \nu\bar{\nu}$. The present bound is

$$\mathrm{Im}(YZ^*) < |YZ| < 110$$

$$\text{for } m_t = 140 \text{ GeV and } M_H = 45 \text{ GeV}. \tag{4.19}$$

Combining the above bounds and assuming $M_H = 45$ GeV, we obtain the bound on $\mathrm{Im}(\xi_{\mathrm{MHD}})$ as

$$|\mathrm{Im}(\xi_{\mathrm{MHD}})| < 0.28. \tag{4.20}$$

The constraints on the leptoquark couplings can be obtained through several rare processes [41–43,30]. As a typical bound we find from the experimental bound $\mathcal{B}(K_L \to \mu e) < 3.3 \times 10^{-11}$ [14] for the branching ratio of the lepton-family-number-violating process $K_L \to \mu e$,

$$\frac{|x_{21}x_{12}^*|}{M_{\phi_1}^2} < 3 \times 10^{-11} \ \mathrm{GeV}^{-2}, \tag{4.21}$$

and from $\Gamma(\mu \mathrm{Ti} \to e\mathrm{Ti})/\Gamma(\mu\mathrm{Ti} \to \mathrm{capture}) < 4.3 \times 10^{-12}$ [44,14],

$$\frac{|y_{11}y_{12}^*|}{M_{\phi_2}^2} < 1.9 \times 10^{-11} \ \mathrm{GeV}^{-2}. \tag{4.22}$$

On the other hand, the helicity-suppressed π_{e_2} decay [45] can set a strong bound on $|x_{11}x_{11}'^*|$ and $|y_{11}y_{11}'^*|$ under the assumption that $\mathrm{Im}(x_{11}x_{11}'^*) \sim \mathrm{Re}(x_{11}x_{11}'^*)$ and $\mathrm{Im}(y_{11}y_{11}'^*) \sim \mathrm{Re}(y_{11}y_{11}'^*)$. Using the experimental value $R_{\mathrm{expt}} = (1.230 \pm 0.004) \times 10^{-4}$ [14] and the SM value $R = 1.235 \times 10^{-4}$ [46] for the ratio of $\mathcal{B}(\pi \to e\nu_e(\gamma))$ to $\mathcal{B}(\pi \to \mu\nu_\mu(\gamma))$ we obtain

$$\frac{|\mathrm{Im}(x_{11}x_{11}'^*)|}{M_{\phi_1}^2} \sim \frac{|\mathrm{Re}(x_{11}x_{11}'^*)|}{M_{\phi_1}^2} < 7 \times 10^{-11} \ \mathrm{GeV}^{-2},$$
$$\frac{|\mathrm{Im}(y_{11}y_{11}'^*)|}{M_{\phi_2}^2} \sim \frac{|\mathrm{Re}(y_{11}y_{11}'^*)|}{M_{\phi_2}^2} < 7 \times 10^{-11} \ \mathrm{GeV}^{-2}. \tag{4.23}$$

If we assume $\mathrm{Im}(x_{13}x_{13}'^*) \sim \mathrm{Im}(x_{21}x_{12}^*) \sim \mathrm{Im}(x_{11}x_{11}'^*)$ and $\mathrm{Im}(y_{13}y_{13}'^*) \sim \mathrm{Im}(y_{11}y_{12}^*) \sim \mathrm{Im}(y_{11}y_{11}'^*)$, then we obtain from (4.21) and (4.22) the constraints

$$|\mathrm{Im}(\xi_{\mathrm{SLQ}}^{\mathrm{I}})| < 1.5 \times 10^{-3}, \quad |\mathrm{Im}(\xi_{\mathrm{SLQ}}^{\mathrm{II}})| < 0.9 \times 10^{-3}. \tag{4.24}$$

Compared to the constraint (4.20) on $\mathrm{Im}(\xi_{\mathrm{MHD}})$, the constraints (4.24) on these SLQ CP-violation parameters are much more severe such that the SLQ models require a larger number of τ leptons than are required by the MHD model to detect any effects of CP violation. However, we should mention here that the strong constraints (4.24) of the leptoquark couplings are based on the assumption that all the leptoquark Yukawa couplings are of the same size. This assumption can fail in a class of leptoquark models where the lepton-family-number symmetry and the electron chirality are softly broken. In such leptoquark models one possible scenario is that the lepton-family-number-conserving couplings are much larger than the lepton-family-number-violating couplings, and also the couplings involving the third lepton generation (τ and ν_τ) are much larger than those involving the first lepton generation (e and ν_e). In such a scenario the constraints (4.24) may be too stringent.

One direct constraint on $|x_{13}x_{13}'^*|$ and $|y_{13}y_{13}'^*|$ can be provided through the measurement of $\mathcal{B}(\tau \to \pi\nu_\tau)$. We assume that $\mathrm{Im}(x_{13}x_{13}'^*) \sim \mathrm{Re}(x_{13}x_{13}'^*)$ and $\mathrm{Im}(y_{13}y_{13}'^*) \sim \mathrm{Re}(y_{13}y_{13}'^*)$. Then, employing the SM value $\Gamma = (2.480 \pm 0.025) \times 10^{-13}$ GeV [46] and the experimental value $\Gamma_{\mathrm{expt}} = (2.605 \pm 0.093) \times 10^{-13}$ GeV [14] for the $\tau \to \pi\nu_\tau$ decay width, we find that

$$\frac{|\mathrm{Im}(x_{13}x_{13}'^*)|}{M_{\phi_1}^2} \sim \frac{|\mathrm{Re}(x_{13}x_{13}'^*)|}{M_{\phi_1}^2} < 3 \times 10^{-6} \ \mathrm{GeV}^{-2},$$
$$\frac{|\mathrm{Im}(y_{13}y_{13}'^*)|}{M_{\phi_2}^2} \sim \frac{|\mathrm{Re}(y_{13}y_{13}'^*)|}{M_{\phi_2}^2} < 3 \times 10^{-6} \ \mathrm{GeV}^{-2}. \tag{4.25}$$

It is clear that the direct constraints (4.25) are by far weaker than the indirect constraints, (4.21) and (4.22); therefore the parameters $\mathrm{Im}(\xi_{\mathrm{SLQ}}^{\mathrm{I}})$ and $\mathrm{Im}(\xi_{\mathrm{SLQ}}^{\mathrm{II}})$ can in principle be significantly larger than the upper bounds (4.24). We expect the precise measurement of $\Gamma(\tau \to \pi\nu_\tau)$ at future τ-charm factories to give a stronger direct constraint on $|x_{13}x_{13}'^*|$ and $|y_{13}y_{13}'^*|$.

V. THE π' DECAY CONSTANT

As clearly shown in Sec. II every CP-violating observable requires interference between the spin-1 and the spin-0 amplitudes, and thus the size of CP violation in the 3π decay mode of the τ crucially depends on $f_{\pi'}$, which determines the coupling of the π' to the weak current. It is, therefore, important to make a reliable estimate of its magnitude.

The most commonly used value of the π' decay constant in the literature is obtained by a quark model calculation (the "mock" meson model) by Isgur *et al.* [18]. The value of the $f_{\pi'}$ in Ref. [18] is in the range

$$f_{\pi'} = 0.02 - 0.08 \ \mathrm{GeV}, \tag{5.1}$$

where $f_{\pi'} = 0.02$ GeV is used in TAUOLA as its default value.

However, the decay constant of π' should be suppressed by the chiral symmetry which is not taken into account in the mock-meson calculation, and thus the value (5.1) might be overestimated [27,47]. In this section we reconsider $f_{\pi'}$ in the chiral Lagrangian framework and show that it actually vanishes in the chiral limit due to the mixing [48] of π' with the chiral pion field. $f_{\pi'}$ is then proportional to the square of the pion mass m_π and may hence be much smaller than (5.1).

In order to establish our notation we start with the usual chiral Lagrangian with the flavor $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry [49]:

$$\mathcal{L} = f^2 \mathrm{tr}(\alpha_{\mu\perp}\alpha_\perp^\mu) + \frac{f^2}{4}\mathrm{tr}(\hat{\chi} + \hat{\chi}^\dagger), \tag{5.2}$$

where the matrix-valued $\hat{\chi}$ and the Maurer-Cartan one-form $\alpha_{\mu\perp}$ are defined as

$$\hat{\chi} = 2b\xi_L(s+ip)\xi_R^\dagger,$$
$$\alpha_{\mu\perp} = -\frac{i}{2}\Big[(\partial_\mu\xi_L)\,\xi_L^\dagger + i\xi_L L_\mu \xi_L^\dagger - (\partial_\mu\xi_R)\,\xi_R^\dagger - i\xi_R R_\mu \xi_R^\dagger\Big], \quad (5.3)$$

with the chiral fields ξ_L and ξ_R given by

$$\xi_R = \xi_L^\dagger = \exp\left[\frac{i\pi^a T^a}{f}\right], \quad (5.4)$$

and π^a ($a = 1,2,3$) and T^a are the Nambu-Goldstone fields and the flavor SU(2) group generators, respectively. The parameter b is given in terms of u quark condensate $\langle\bar{u}u\rangle$ and the π decay constant f by $b = -\langle\bar{u}u\rangle/f^2 \simeq$ 1.3 GeV. The chiral fields ξ_L and ξ_R transform under $SU(2)_L\times SU(2)_R$ as

$$\xi_L \to \xi_L' = h\xi_L g_L^\dagger, \qquad \xi_R \to \xi_R' = h\xi_R g_R^\dagger, \quad (5.5)$$

where the SU(2) group element h is introduced to maintain the relation $\xi_R' = \xi_L'^\dagger$. In the Lagrangian (5.2) we have also introduced external scalar and pseudoscalar fields, s and p, and external left- and right-handed flavor gauge fields, L_μ and R_μ [50]. In the real world the external scalar and pseudoscalar fields are fixed to be $s = \mathrm{diag}(m_u, m_d)$ and $p = 0$.

We next describe how to introduce a massive pseudoscalar π' in the chiral Lagrangian formalism to discuss its physical properties. Since π' is not a Nambu-Goldstone boson it should be treated as a matter field transforming as

$$P \to hPh^\dagger, \quad (5.6)$$

where P is the π' field. The low-energy effective Lagrangian of π and π' with the lowest derivatives can be written as

$$\mathcal{L}_P = \mathcal{L}_{\rm chiral} + \mathrm{tr}(D_\mu P D^\mu P) - M_P^2 \mathrm{tr}(P^2) + iG_P \mathrm{tr}[(\hat{\chi} - \hat{\chi}^\dagger)P] + 2F_P \mathrm{tr}(\alpha_{\mu\perp} D^\mu P), \quad (5.7)$$

where F_P and G_P are parameters describing the low-energy properties of the π'. The covariant derivative of the field P is given by

$$D_\mu P = \partial_\mu P - i[\alpha_{\mu\|}, P], \quad (5.8)$$

with the Maurer-Cartan one-form

$$\alpha_{\mu\|} = -\frac{i}{2}\left[\partial_\mu\xi_L\xi_L^\dagger + i\xi_L L_\mu \xi_L^\dagger + \partial_\mu\xi_R\xi_R^\dagger + i\xi_R R_\mu \xi_R^\dagger\right]. \quad (5.9)$$

First, we discuss the chiral limit where $\hat{\chi} = 0$. In this limit the Lagrangian $\mathcal{L}_P$ is expanded as

$$\mathcal{L}_P = \frac{1}{2}(\partial_\mu\pi^a)^2 + \frac{1}{2}(\partial_\mu P^a)^2 - \frac{F_P}{f}(\partial_\mu\pi^a)(\partial^\mu P^a) - \frac{M_P^2}{2}(P_a)^2 + f\partial_\mu\pi^a A^{a\mu} - F_P\partial_\mu P^a A^{a\mu} + \cdots, \quad (5.10)$$

where A_μ^a is an external axial-vector field, $A_\mu^a = (R_\mu^a - L_\mu^a)/2$. Although the nonvanishing F_P in the last term of (5.10) seems to directly indicate the existence of the π' decay constant the existence of F_P simultaneously causes a kinetic mixing of π and P through the third term of (5.10), and thus we need to diagonalize the kinetic mixing so as to evaluate the actual size of the π' decay constant.

We resolve this kinetic mixing by the redefinition of fields

$$\pi_0^a = \pi^a - \left(\frac{F_P}{f}\right)P^a, \quad P_0^a = \sqrt{1-\left(\frac{F_P}{f}\right)^2}\,P^a. \quad (5.11)$$

We find that the field redefinition pushes the π' mass to a higher value,

$$m_{\pi'}^2 = \frac{M_P^2}{1-(F_P/f)^2}, \quad (5.12)$$

and it forces the π' decay constant to vanish in the chiral limit:

$$f_{\pi'} = 0. \quad (5.13)$$

Although our calculation is based on the effective Lagrangian with the lowest derivatives, the result (5.13) itself is rather general. Actually, we can show that the existence of the higher-derivative terms does not change this result.

In the real world u and d masses are not zero and break the chiral symmetry explicitly. Including the π mass term due to the finite u and d quark masses and using the previous field redefinition (5.11) we obtain the mass term of the π and π' chiral Lagrangian as

$$\mathcal{L}_M = -\frac{\hat{m}_\pi^2}{2}(\pi_0^a)^2 - \hat{m}_\pi^2\frac{F_P + 2G_P}{f\sqrt{1-F_P^2/f^2}}\pi_0^a P_0^a - \frac{\hat{m}_\pi^2}{2(1-F_P^2/f^2)}\left(\frac{M_P^2}{\hat{m}_\pi^2} + 4\frac{G_P F_P}{f^2}\right)(P_0^a)^2, \quad (5.14)$$

with $\hat{m}_\pi^2 = (m_u + m_d)b$. We diagonalize the Lagrangian (5.14) in the perturbation of $\hat{m}_\pi^2$:

$$\pi_0^a = \pi_{\rm phys}^a - \hat{m}_\pi^2\delta P_{\rm phys}^a + \cdots,$$
$$P_0^a = P_{\rm phys}^a + \hat{m}_\pi^2\delta\pi_{\rm phys}^a + \cdots. \quad (5.15)$$

Plugging (5.15) into the Lagrangian (5.14) we find that the parameter δ removing the mixing term is

$$\delta = -\sqrt{1-\frac{F_P^2}{f^2}}\left(\frac{F_P + 2G_P}{M_P^2 f}\right). \quad (5.16)$$

The complete diagonalization leads to

$$m_\pi^2 = \hat{m}_\pi^2,$$
$$m_{\pi'}^2 = \frac{M_P^2 + 4\hat{m}_\pi^2 G_P F_P/f^2}{1 - F_P^2/f^2} \simeq \frac{M_P^2}{1 - F_P^2/f^2}. \quad (5.17)$$

On the other hand, the meson coupling terms to the axial-vector current A_μ^a are written in terms of the physical fields $\pi_{\rm phys}^a$ and $P_{\rm phys}^a$ as

$$f A_\mu^a \partial^\mu \pi_0^a = f A_\mu^a \partial^\mu \pi_{\rm phys}^a + \frac{\hat{m}_\pi^2}{M_P^2}\sqrt{1 - F_P^2/f^2}$$
$$\times (F_P + 2G_P) A_\mu^a \partial^\mu P_{\rm phys}^a, \quad (5.18)$$

and then the π and π' decay constants directly read

$$f_\pi = f, \qquad f_{\pi'} = \frac{m_\pi^2}{M_P^2}\sqrt{1 - F_P^2/f^2}(F_P + 2G_P). \quad (5.19)$$

As a result, we obtain the following expression:

$$r_\pi \equiv \frac{f_{\pi'}^2 m_{\pi'}^4}{f_\pi^2 m_\pi^4} = \frac{F_P^2}{f_\pi^2(1 - F_P^2/f_\pi^2)}\left(1 + 2\frac{G_P}{F_P}\right)^2. \quad (5.20)$$

It is now clear that $f_{\pi'}$ is proportional to the square of the pion mass m_π, and thus it is naturally expected that the value of $f_{\pi'}$ is small.

The parameters F_P and G_P are expected to be of the size of QCD scale. We thus naively expect

$$r_\pi \sim 1. \quad (5.21)$$

On the other hand, the value of $f_{\pi'}$ in Eq. (5.1) leads to a large value of r_π:

$$r_\pi = 300 - 6000, \quad (5.22)$$

which clearly contradicts our naive expectation (5.21).

Although the precise values of those parameters cannot be determined within the chiral Lagrangian framework, we can estimate them from the QCD dynamics. In the present work, we employ the nonrelativistic chiral quark (NRCQ) model of Manohar and Georgi [48] and QCD sum rules [51] to make a rough estimate of the parameters.

In the NRCQ model the lowest S-wave bound states of the chiral quarks are identified as ρ and π'. Neglecting the spin-dependent interaction we can take

$$M_P \simeq m_\rho = 0.773 \text{ GeV}. \quad (5.23)$$

Both G_P and F_P are calculated from the wave function at the origin. In the nonrelativistic approximation we find the relation

$$2bG_P \simeq M_P F_P. \quad (5.24)$$

On the other hand the value of F_P can be estimated from the relation

$$m_{\pi'}^2 \simeq \frac{M_P^2}{1 - F_P^2/f_\pi^2} = (1.3 \text{ GeV})^2, \quad (5.25)$$

with $f_\pi = 0.0933$ GeV. It is now straightforward to show that

$$r_\pi \simeq 4, \qquad f_{\pi'} \simeq 2 \times 10^{-3} \text{ GeV}. \quad (5.26)$$

Let us next discuss the evaluation of r_π with the QCD sum rule technique [51]. By using the operator product expansion of the isotriplet scalar current $j_a^{(a_0)}$ and the pseudoscalar current $j_a^{(\pi)}$, we find a sum rule

$$\int_0^\infty ds e^{-s/M^2} {\rm Im}\Pi(s) \simeq 0, \quad (5.27)$$

where $\Pi(s)$ is defined by

$$\Pi(-q^2)\delta^{ab} \equiv i\int dx e^{iqx}\langle T\{j_a^{(\pi)}(x)j_b^{(\pi)}(0)$$
$$-j_a^{(a_0)}(x)j_b^{(a_0)}(0)\}\rangle \quad (5.28)$$

and M is the scale of the Borel transform. Assuming the a_0 (isotriplet scalar meson), π, and π' meson dominance we can derive an expression [51] for r_π:

$$r_\pi = -\left(\frac{m_{a_0}^2 - m_\pi^2}{m_{a_0}^2 - m_{\pi'}^2}\right)\exp[-(m_\pi^2 - m_{\pi'}^2)/M^2], \quad (5.29)$$

from (5.27) and its first derivative of $1/M^2$. Inserting the π and π' masses and $m_{a_0} = 0.98$ GeV, we find that

$$r_\pi \simeq 7, \qquad f_{\pi'} \simeq 3 \times 10^{-3} \text{ GeV}, \quad (5.30)$$

for the mass scale M around 1 GeV.

Note that despite their very different approaches the NRCQ model [48] and the QCD sum rule [51] give a similar estimate of the r_π. According to those estimates one finds $f_{\pi'} \approx$ (2–3) $\times 10^{-3}$ GeV, which is much smaller than the value (5.1). Considering possible uncertainties in our estimates we use in our numerical analysis a slightly broader range of $f_{\pi'}$:

$$f_{\pi'} = (1 - 5) \times 10^{-3} \text{ GeV}. \quad (5.31)$$

Certainly a more exact value of $f_{\pi'}$ can be determined directly from experiment.

VI. NUMERICAL RESULTS

For the numerical analysis we use for the CP-violation parameter values in the MHD model and the SLQ models

$$\text{Im}(\xi_{\rm MHD}) = 0.28, \quad (6.1)$$
$$\text{Im}(\xi_{\rm SLQ}^{\rm I}) = 1.5 \times 10^{-3}, \quad \text{Im}(\xi_{\rm SLQ}^{\rm II}) = 0.9 \times 10^{-3}, \quad (6.2)$$

which are maximally allowed values according to the constraints (4.20) and (4.24). We mention in passing once more that the constraints (4.24) on the leptoquark couplings may be too stringent in a class of leptoquark models where the leptoquark Yukawa couplings are significantly generation dependent. For the π' decay constant we consider the range: (1–5)$\times 10^{-3}$ GeV as in Eq. (5.31).

Figure 2(a) shows the normalized differential decay width $\Sigma^{-1}[d\Sigma/d\sqrt{q^2}]$ for the decay $\tau \to (3\pi)\nu_\tau$ as a function of the (3π) invariant mass $\sqrt{q^2}$. Note that it

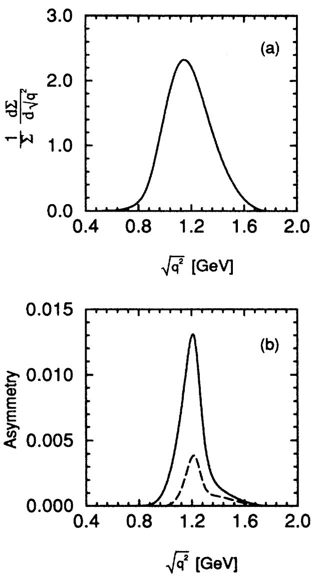

FIG. 2. (a) A plot of the normalized differential decay width $\Sigma^{-1}[d\Sigma/d\sqrt{q^2}]$ as a function of the (3π) invariant mass $\sqrt{q^2}$. (b) CP-violating asymmetries as a function of the (3π) invariant mass $\sqrt{q^2}$, with $f_{\pi'} = 5 \times 10^{-3}$ GeV and $\text{Im}(\xi) = 0.28$ as reference values. The solid line is for the CP-violating forward-backward asymmetry $dA_{1\text{FB}}/d\sqrt{q^2}$, and the long-dashed line is for the CP-violating forward-backward asymmetry $dA_{2\text{FB}}/d\sqrt{q^2}$.

has almost one peak. The reason is not only because $m_{\pi'}$ and m_{a_1} are quite similar in magnitude, but also because the π' contribution is by far smaller than the a_1 contribution. This justifies the approximation where all CP-conserving contributions from new scalar-fermion interactions are neglected in the normalization. In Fig. 2(b) we present the two CP-violating differential forward-backward asymmetries $dA_{1\text{FB}}/d\sqrt{q^2}$ and $dA_{2\text{FB}}/d\sqrt{q^2}$ as a function of the (3π) invariant mass $\sqrt{q^2}$. In this case we take $f_{\pi'} = 5 \times 10^{-3}$ GeV and $\text{Im}(\xi) = 0.28$. The two asymmetries have a similar dependence on $\sqrt{q^2}$, but it is clear that the asymmetry $A_{1\text{FB}}$ is almost four times larger than the asymmetry $A_{2\text{FB}}$.

Table I shows the expected size of the integrated asymmetries $A_{1\text{FB}}$ and $A_{2\text{FB}}$ and the optimal asymmetry ε_{opt} along with the number of τ leptons, N^τ, required to obtain the 2σ signal in the MHD model and the two SLQ models for the CP-violation parameter values (6.2) and for the π' decay constant in the range (5.31). The optimal asymmetry ε_{opt} is optimized with respect to the kinematic variables $(q^2, s_1, s_2, \cos\theta, \phi^*)$ so that its expected size denotes the maximally obtainable asymmetry in the three-pion decay mode. However, we find that the CP-violating forward-backward asymmetry $A_{1\text{FB}}$ is sizable, while the other asymmetry $A_{2\text{FB}}$, is rather small. The $A_{1\text{FB}}$ asymmetry and the corresponding values of N^τ given in Table I show that the CP-violating effects in the MHD model may be seen with less than 10^8 τ's even for a rather small π' decay constant. To see the CP-violating effects with the asymmetries $A_{1\text{FB}}$ and $A_{2\text{FB}}$ in the SLQ models more than 10^{11} τ's may be required. On the other hand, the ε_{opt} and the corresponding N^τ values show that the CP-violating effects from the MHD model can be detected at the 2σ level with about 10^7 τ leptons for the value of the CP-violation parameter $\text{Im}(\xi_{\text{MHD}}) = 0.28$, while still more than 10^{10} τ leptons are required to see CP violation in the SLQ models. However, as previously mentioned, the CP-violating parameters $\text{Im}(\xi^{\text{I}}_{\text{SLQ}})$ and $\text{Im}(\xi^{\text{II}}_{\text{SLQ}})$ may be considerably larger so that the required number of τ leptons could be significantly reduced. In light of the fact that about 10^7 and 10^8 τ leptons are produced yearly at B factories and τ-charm factories, respectively, we conclude that CP-violating effects for the MHD model may be seen at the τ-charm factories through the CP-violating forward-backward asymmetry $A_{1\text{FB}}$, and even at the B factories with the optimal asymmetry ε_{opt} for the π' decay constant in the range (5.31). Finally, we note that, if we take $f_{\pi'} = 0.02$ GeV, the value estimated by Isgur *et al.* and quoted in TAUOLA, the number of τ leptons needed is reduced by a factor of at least 15.

VII. CONCLUSION

In this paper, we have investigated the possibility of probing CP violation through the semileptonic decays $\tau \to (3\pi)\nu_\tau$ where two resonances a_1 and π' contribute. The interference of the a_1 and π' resonances leads to enhanced CP-violating asymmetries whose magnitude crucially depends on the π' decay constant $f_{\pi'}$. We made an estimate of $f_{\pi'}$ with a simplified chiral Lagrangian coupled to a massive pseudoscalar field, and we compared the estimates from the nonrelativistic chiral quark model and from the QCD sum rule with the estimate from the "mock" meson model. We found that the mixing between the pion field and the massive pseudoscalar field renders $f_{\pi'}$ proportional to the square of the pion mass, m_π, and hence it is much smaller than the value quoted in the τ library TAUOLA. We considered two CP-

TABLE I. The CP-violating forward-backward asymmetries A_{iFB} $(i = 1, 2)$ and the optimal asymmetry ε_{opt} are determined for the range $f_{\pi'} = (1\text{–}5) \times 10^{-3}$, and the number of τ leptons, N^τ, needed for detection at the 2σ level for $f_{\pi'} = 5\times10^{-3}$ GeV as a reference value with $\text{Im}(\xi_{\text{MHD}}) = 0.28$ in the MHD model, and $\text{Im}(\xi^{\text{I}}_{\text{SLQ}}) = 1.5 \times 10^{-3}$ and $\text{Im}(\xi^{\text{II}}_{\text{SLQ}}) = 0.9 \times 10^{-3}$ in the two SLQ models.

Asymmetry	Size (%)	N^τ_{MHD} (10^7)	N^τ_{SLQI} (10^{11})	N^τ_{SLQII} (10^{11})
$A_{1\text{FB}}$	0.056–0.28	0.75–19	2.7–68	7.4–185
$A_{2\text{FB}}$	0.014–0.07	12–300	43–1070	120–3000
ε_{opt}	0.13–0.67	0.13–3.3	0.47–11	1.3–33

violating forward-backward asymmetries A_{1FB} and A_{2FB} and the optimal asymmetry $\varepsilon_{\rm opt}$, which is optimized with all the five kinematic variables $(q^2, s_1, s_2, \cos\theta, \phi^*)$. With these asymmetries we quantitatively estimated the size of CP-violating effects from the multi-Higgs-doublet model and the scalar-leptoquark models. We found that, while the scalar-leptoquark models may require more than 10^{10} τ leptons for the detection of any effect of CP violation, CP-violating effects from the multi-Higgs-doublet model may be seen at the 2σ level with about 10^7 τ leptons for the value of the CP-violation parameter $\text{Im}(\xi_{\rm MHD}) = 0.28$ and for the chiral Lagrangian estimate of $f_{\pi'} = (1\text{–}5) \times 10^{-3}$ GeV.

ACKNOWLEDGMENTS

S.Y. Choi would like to thank the Japanese Ministry of Education, Science and Culture (MESC) for financial support. The authors would like to thank R. Szalapski for carefully reading the paper. This work was supported in part by the Grant-in-Aid for Scientific Research from MESC (No. 05228104).

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